Mobile Device Localization in 5G Wireless Networks

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Abstract—As wireless networks are evolving into 5G, tremendous amount of data will be shared on the newly developed open source platforms. These data can be used in developing new services. Among which, location information of mobile devices are extremely useful. For example, the location information can be used to assist wireless operators to trouble shoot the network performance. It can also be used to provide some location assisted service. However, some of these devices may be designed for limited budget that do not have the capability of GPS. Furthermore, operators may not have access to the GPS information on the mobile devices. In this paper, we propose a novel machine learning based approach to estimate the location of the mobile devices based on the measurement data that mobiles reported during every call and session. Our proposed algorithm utilizes the advanced features of 5G wireless network, such as the beam information. Simulation shows that the proposed solution can achieve 4m accuracy for LoS enviorment and 12m accuracy for mixed LoS and NLoS environment. And the proposed algorithm can also work even with only the information from one base station.

I. INTRODUCTION

Fueled with the emerging cloud technologies (cloud computing, cloud storage, etc), software defined network (SDN) together with network functions virtualization (NFV) is transforming the technology, and business models in telecommunication industry. Several open source platforms (ONAP, XRAN, etc) have emerged to provide a platform to collect data and share technologies among different vendors and operators. As the wireless network is now evolving into 5G, large amounts of data will be shared in these open source platform and thus create a lot of opportunity to provide better service using these data. The shared data can include a wide variety of measurements, such as the service throughput of the mobile device, the serving cell, the signal strength, etc. In [10], a novel localization algorithm has been proposed to estimate the location when measurement reports are made in LTE systems. The paper shows that the medium accuracy of 20m can be achieved for outdoor mobiles. When cellular network evolves from 4G LTE to 5G, a lot of changes have been made to support lower latency and higher throughput. While multiple input multiple output (MIMO) has been extensively used in LTE, the most profound advancement of technology adopted in 5G network is the utilization of a high number of (massive) MIMO antennas especially in the high band. With the use of massive MIMO, transmit power can be effectively beamformed and the mobile will report beam related metric

to the base station. Thus, the same signal characteristics (such as received signal strength, signal to noise ration, etc) may be the same even if the user is served by different beams (thus different locations). Therefore, the existing algorithms without considering the beam information can not be applied directly on 5G and new positioning algorithms need to be developed. In [7] [8], the authors propose a two-stage extended Kalman filter (EKF) that is based on reference signal received power (RSRP) measurements. However, the proposed algorithm only works when there are at least two base stations. In 5G network, especially for high band, the cell coverage is limited and 5G cell sites are used to increase the capacity in the hot spot. Thus, there may only be one base station observed by the mobile device. [9] presents a message passing-based estimator which jointly estimates the position and orientation of the mobile terminal. However, the scheme in [9] needs extra message passing between nodes. There are other schemes focusing on indoor positioning, which also need the presence of multiple base stations, such as [?]. In this paper, we propose a new localization algorithm to estimate the location of mobile devices utilizing the new reported metric introduced in 5G network.

In this paper, we design the localization algorithm combining the unique RF fingerprinting in 5G network and probablitistic path-tracking used for robot localization. The unique property of 5G network, such as the beam related information is used in this paper. The focus of this paper is on estimating location for outdoor mobiles. At a high level, our approach has two steps:

- Instead of viewing each NR UE measurement data (NUMD) record in isolation, for each mobile, we *stitch* together NUMD records from that mobile over a "session duration" and model it as a suitable Markovian time series. The problem now reduces to identifying locations (states) of the entire path of the mobile.
- 2) The above solution method assumes that the probabilities characterizing the underlying Markovian structure can be learned. This is done by performing supervised learning using the beam information of 5G networks. The training data for supervised learning may come from drive test carried out by network providers once 5G is deployed commonly in the field. However, given that 5G network deployment is not available in the field during

this study, we generate the drive training data and testing data from a 5G simulator.

The details of the above two steps are provided in Section IV.

Our main contribution in this paper is that we have proposed a new localization algorithm appliable to 5G network. Based on our best knowledge, this is the first localization algorithm designed for 5G network.

The rest of the paper is organized as follows. Section II provides some background and introduces relevant terminologies. Section III presents the problem setting and states the precise localization problem. Section IV presents the main localization algorithm and Section V describes how to get the likelihood of the observation using machine learning approach. We present experimental validation in Section VI and finally we conclude in Section VII.

II. RELEVANT 5G TERMINOLOGIES

Though our techniques could apply to any future cellular system, we use 5G New Radio (NR) terminologies for convenience. The terminologies [3] relevant for our purpose are described below.

UE (user equipment): UE refers to the mobile end-device. Cell: In 5G networks, a cell refers to coverage footprint of a gNodeB transmitter. Since 5G networks are expected operate in diverse spectrum bands, the cell coverage can vary from as low 100m to 5km. Typically, each cell is expected to cover an azimuth range of 120° using sectorized antennas.

gNodeB (gNB): The gNB is the network element that interfaces with the UE and hosts critical protocol layers like PHY, MAC, and Radio Link Control (RLC) etc. Each gNB could have multiple transmission and reception points (TRP). Typical number number of TRPs is 3 for full coverage around the gNB.

Secondary Synchronization Reference Signal Received Power (SS-RSRP): In 5G networks, UEs make certain measurements of received signal strength on the secondary synchronization (SS) signal for each nearby cell transmitter. SS-RSRP is the total measured time-average received power at UE from the SS signals from a given cell transmitter.

Beam Indices: In 5G-NR, there is the concept of a SS/PBCH block on the downlink which comprises of a set of symbols (4) during which beamformed transmissions in a particular direction from gNB could take place. Each SS/PBCH block is identified by an index which the UE uses to report back measurements during a given SS/block. The beam indices we refer to in this paper correspond to the SS/PBCH block index and can have a range from 0 to 63 in standards. We use a grid of beams with upto 56 possible directions in the performance evaluation section.

ONAP This stands for Open Network Automation Platform and provides comprehensive platform for real time policy driven orchestration and automation of physical and virtual network functions that will enable software, network, IT and cloud providers and developers to rapidly create new services. Figure 1 shows the high level architecture. It is expected

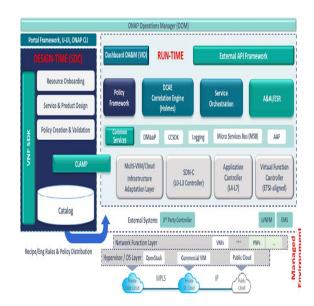


Fig. 1. ONAP architecture

that in the context of 5G networks and localization topic of this paper, ONAP would orchestrate the localization policy and associated cloud storage within the "Policy framework", "Data Collection, Analytics and Events (DCAE)" and "Service orchestration" components of the run-time module.

Measurement data collection architecture: The NR UE measurement data (NUMD) collection process occurs primarily in the 5G network via the gNB and Mobility Management Entity (MME). The MME serves as the coordinator of the NUMD data. After NUMD collection is turned on at the gNodeB, it collects the records and sends the data to the MME. MME aggregates and temporarily saves NUMD from multiple gNodeBs and sends it periodically (typically in minutes timescale) to the data center where NUMD is saved and analyzed (for example, ONAP). Scalable storage of NUMD, which can easily run into TB in a week per metropolitan, in the data center is an important design problem and beyond the scope of this paper.

Contents of NUMD: NUMD record contains data related to signaling performance on per UE, per bearer level for different procedures, user experience such as data throughput and procedure duration, gNodeB internal UE related data such as MIMO decision, SINR, buffer size, and normalized power headroom etc. The information present depends on procedure/event that led to the measurement record. For our purpose, we are interested in RF information, and more specifically the SS-RSRP information contained in measurement records.

III. PROBLEM STATEMENT

Consider in a 5G network, mobiles travel along a road represented by a graph $G_r = (V, E)$ where V denotes graph nodes represented by a latitude-longitude tuple and E denotes valid direct path between two nodes. To estimate the location

 ${\cal V}$ of this mobile, two types of data are used in our proposed algorithm:

- 1) Training data: This is essentially geo-tagged data sent from a set of locations in the road graph nodes V. The size of the training set is n, which include n locations $\{x_i\}_{i=1}^n$ and the corresponding SS-RSRP and beam indices at each location for each serving cell. Based on [1], the mobile at each location can report up to four best beam indices and the corresponding SS-RSRPs. We denote by $\{B_{i,k}^j\}_{i=1}^n$, the jth best beam indices sent from training location x_i in cell k and $\{R_{i,k}^j\}_{i=1}^n$ are the corresponding signal strength associated with those reported beam indices. Note that, for a location x_i , the data $R_{i,k}^j$ and $R_{i,k}^j$ are only available for a small subset of cells near location x_i . We will also denote the set of training data by \mathcal{D}_{tr} .
- 2) **NUMD data or observed data:** This data is not geotagged but comes with time stamp. Precisely, for every mobile, we are given time instants $t_i, i = 1, 2, ..., T$ for each t_i we are also given SS-RSRP $\{\tilde{R}_k^j(t_i)\}_{j=1}^4$ and beam indices $\{\tilde{B}_k^j(t_i)\}_{j=1}^4$ where $k \in K(t_i)$; $K(t_i)$ denotes the set of cells reported by the mobile at time t. Typically $|K(t_i)|$ takes value one or two. Though we have NUMD for each mobile-m, we drop the dependence of m on $R_k(t)$ and K(t) as we are essentially performing the same algorithm for each mobile separately. The locations of mobiles $\tilde{x}(t_i)$ at different times t_i are unknown.

Thus the problem can be stated as follows:

Problem of localization in 5G network: We are given training data consisting of locations $\{x_i\}_{i=1}^n$ and the associated SS-RSRPs $\{R_{i,k}^j\}_{i=1}^n$ and beam indices $\{B_{i,k}^j\}_{i=1}^n$ of cell-k at location x_i . Assume that the locations are drawn from locations in a road network given by $G_r = (V, E)$. Thus, the problem is to estimate the unknown location of the mobiles when mobiles report a sequence of measurements $\tilde{R}_k^j(t_i)$ and $\tilde{B}_k^j(t_i)$ where $i=1,2,\ldots,L,\ k\in K(t_i),j=1,2,3,4.\ L$ is the length of the sequence of the measurement reports. Note that, in the algorithm illustration in this paper, we only focus on SS-RSRP and beam indices. However, in the field, there may be other measurement reports that can be used to help to improve the localization accuracy. For example, timing alignment, etc. These additional measurement reports can be easily incorporated into our proposed algorithm.

IV. LOCALIZATION ALGORITHMS

The framework we use for tackling the localization problem is hidden markov model (HMM) as illustrated in Figure 2. The hidden states in HMM in our case are the locations and the velocity. The observations corresponding to each hidden state are the reported measurement reports, such as SS-RSRP and the beam indices. The system moves from one hidden state to another hidden state with some underlying mobility model. The goal is to infer the hidden state from the observations based on prior knowledge about the transition probabilities

between hidden states and observations in the states. We assume that the mobile updates its speed according to Gauss-Markov Mobility Model [5].

$$S_t = \alpha S_{t-1} + (1 - \alpha)\bar{S} + \sqrt{(1 - \alpha^2)}S_{x_{t-1}}$$
 (1)

$$d_t = \alpha d_{t-1} + (1 - \alpha)\bar{d} + \sqrt{(1 - \alpha^2)} d_{x_{t-1}}$$
 (2)

where S_t and d_t are the new speed and direction of the mobile at time interval t. $S_{x_{t-1}}$ and $d_{x_{t-1}}$ are random variables from a Gaussian distribution with mean \bar{S} and \bar{d} . Please the mean speed \bar{S} is set to be the average speed of the user, which can be roughly estimated based on the terrian, for example, urban, suburban, etc. At each time interval the next location is calculated based on the current location, speed, and direction of movement. Specifically, at time interval t, a mobile's position is given by the equations.

$$l_t = l_{t-1} + S_{t-1}cos(d_{t-1}) * \Delta t$$
(3)

$$h_t = h_{t-1} + S_{t-1} sin(d_{t-1}) * \Delta t \tag{4}$$

where (l_t, h_t) and (l_{t-1}, h_{t-1}) are the latitude and longitude coordinates of the mobile's position at time intervals t and t-1, respectively. Δt is the duration of the time interval.

However, it is difficult to obtain analytic solutions for HMM and particle filter has been widely used to provide approximate solutions to these intractable inference problems [6] [11]. We also use particle filter approach in this paper.

At high level, the proposed localization algorithm, namely 5GLocalizeAlgo, is stated as follows:

- We first initialize a set of N particles based on some priori distribution, for example, the coverage map of a base station. Here, particles are the samples of the possibles locations.
- Each particle has its corresponding weights or likelihoods, which represents the likelihood of achieving the observations at each given particle (i.e., location in this case). This weight/likelihood is obtained using machine learning algorithm as illustrated in Section V.
- Particles move from one state to another state based on the state transition model given in equations (3) and (4).
- In the end, the sequence of the particles which have the largest likelihood are the estimated locations.
- The pseudocode is presented in Algorithm 1. The inputs are training data D_{tr} , the graph G_r and the threshold N_{th} used in resampling particle filter [11].

V. OBSERVATION MODELING USING MACHINE LEARNING

As mentioned in the above section, one of the important inference of HMM is the likelihood of achieving the observation at the given particle. In this section, we discuss how to infer the likelihood using machine learning approach.

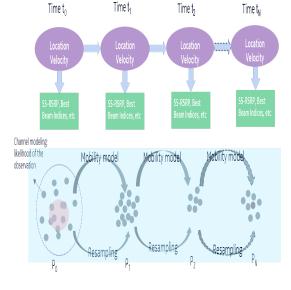


Fig. 2. Hidden Markov Model and Particle filter

A. Inference on likelihood of beam indices

As part of the observations of HMM model, UE reports up to four observed best beam indices. Note that there may be some additional overhead in 5G-NR to request UE to send measurements for more than one beam. The probability distribution (also called the likelihood function) of a particular observed beam index such as its j-th (j = 1, 2, 3, 4) best reported beam indices at a given location is denoted by $p(\hat{B}_i^j|\hat{x}_i)$. In our approach, these probabilities can be learnt from the drive test data using machine learning classification algorithms.

Following are the steps to obtain the inference:

- 1) For each location in the training set, we take the corresponding reported beam indices. Then, pre-processing is done to separate the data based on the cell ID and whether it is best beam index, or second/third/fourth best beam index.
- 2) For each cell, we obtain the likelihood of a certain class (i.e., beam index) using classifier where the latitude and the longitude are taken as features of the model and the likelihood of the beam indices is the output. Different cells are trained using their own data sets. Even for the same cell, each best beam index will be trained separately, meaning, the best beam index, the second best beam index, the third best beam index and the fourth best beam index each has their own trained model. Each such classifier is trained using the data aggregated in the previous step.
- There are different machine learning classifiers in the literature. Considering the strong spatial correlation of the beam index, and also we will need to get the probability of class, random forest classifier is considered as a good choice. In this paper, we use the random

Algorithm 1 $5GLocalizeAlgo(\mathcal{D}_{tr}, G, N_{th})$

1: Offline inference based on training data \mathcal{D}_{tr} to get the importance weight associated with each particle. This does not need real time processing.

2: Sample N particles $\mathcal{P}_j=\{\hat{x}_1^{(j)},\hat{v}_1^{(j)}\},\,j=1,\ldots,N$ from prior distribution $p(\tilde{x}_1,\tilde{v}_1|G)$

 $\begin{array}{lll} \text{3: Initialize} & \text{importance} & \text{weights} & \hat{w}_{1}^{(n)} \\ & p(\{\tilde{R}_{1}^{j}\}_{j=1}^{4}, \{\tilde{B}_{1}^{j}\}_{j=1}^{4}|\hat{x}_{1}^{(n)}), \ n=1,\dots,N \\ \text{4: Normalize} & w_{1}^{(n)} \leftarrow \hat{w}_{1}^{(n)}/\sum_{l=1}^{N}\hat{w}_{1}^{(l)}, \ n=1,\dots,N \end{array}$

5: for i=2 to L do

for n=1 to N do

Sample $\hat{x}_i^{(n)}$ from $\hat{x}_{i-1}^{(n)}$ based on state transition model in equations (3) and (4). 7:

Update weight $\hat{w}_i^{(n)} \leftarrow \hat{w}_{i-1}^{(n)} \times p(\{\tilde{R}_i^j\}_{j=1}^4, \{\tilde{B}_i^j\}_{j=1}^4 | \hat{x}_i^{(n)})$ end for
Normalize $w_i^{(n)} \leftarrow \frac{\hat{w}_i^{(n)}}{\sum_{l=1}^N \hat{w}_i^{(l)}}$ $\hat{N}_{eff} \leftarrow \frac{1}{\sum_{l=1}^N (w_i^{(l)})^2}$ if $\hat{N}_{eff} < N_{th}$, i < L (excluding the last point in the 8:

9:

6:

10:

11:

sequence), resampling condition satisfied then

Sample N particles with replacement from current particle set $\{\mathcal{P}_j\}_{j=1}^N$ with probabilities $\{\hat{w}_i^{(j)}\}_{j=1}^N$. Update particle set with the new sampled set $w_i^{(n)} \leftarrow \frac{1}{N}$ for $n=1,\ldots,N$ 13:

14:
$$w_i^{(n)} \leftarrow \frac{1}{N} \text{ for } n = 1, \dots, N$$

15:

16: end for

17: $n^* = \arg\max_{n=1,...,N} w_L^{(n)}$

18: Output location estimate $\{\hat{x}_i^{(n^*)}\}_{i=1}^L$

19: Output distribution

20: $p(\{\hat{x}^{(n)}\}_{i=1}^{L}|\{\tilde{R}_{i}^{1,2,3,4}\}_{i=1}^{L},\{\tilde{B}_{i}^{1,2,3,4}\}_{i=1}^{L},G)=w_{L}^{(j)}$ for n=1 to N

forest classifier in the machine learning library (sklearn) to predict the probability of reporting a specific beam indices. The function $predict_proba(X)$ can be used to get the probability of class X. Note that different classifiers were tested in the study and we found that neural network and random forest provide similar performance. However, given the more intuitive illustration of random forest, we decided to use random forest classifier.

B. Inference on likelihood of SS-RSRP

Similar to beam indices, the SS-RSRP reported at different states is also part of the observations of HMM model. The probability distribution (also called the likelihood function) of an observed SS-RSRP on a location is denoted by $p(R_i|\hat{x}_i)$. In our approach, these probabilities can be learnt from the training data using machine learning regression algorithms.

Following is the steps to obtain the inference for SS-RSRP:

1) For each location, each base station and each reported set of beam indices, we take the empirical mean and variance of all corresponding drive test data SS-RSRP. Note that for each location, there may be multiple reported SS-RSRP as SS-RSRP varies when measured at different time.

- 2) Similar with the inference on beam index, random forest regressor is a good choice here. For each cell and each beam indices, model the spatial variation of SS-RSRP-statistics (i.e., mean and standard deviation) using *Random Forest* where the latitude and the longitude are taken as features of the model and the SS-RSRP-statistic of the cell is the output. Each such *Random Forest* is trained using data aggregated in previous step. Also, computed is the mean square error (or *cross validation error*) for each random forest.
- 3) Denote by $RndFrst_m(x,b,c)$ ($RndFrst_s(x,b,c)$) the random forest predictor of mean (standard deviation) of SS-RSRP for cell-c, beam-b at location x. Let $(\sigma_{RF}(b,c))^2$ be the corresponding mean square error of the predictor. Then we model

$$p(\tilde{R}_i^j|\hat{x}_i, \tilde{B}_i^j) = \mathcal{N}(RndFrst_m(\hat{x}_i, b, c), \sigma_c^2(\hat{x}_i)) \ , \ (5)$$
 where

$$\sigma_c^2(x) = RndFrst_s(x, b, c) + \sigma_{RF}^2(b, c)$$

and the serving cell-c can be obtained from the NUMD record. $\mathcal N$ represents the Gaussian distribution. In general, we can choose any spatial regressor instead of random forest. However, choosing random forest makes the model robust to cell propagation properties and to the fact that the coverage area of the cell could be disjoint.

Note that in a 5G network, with the existence of beams, the regression algorithm is now implemented per cell and per beam index.

C. Inference on the likelihood of combined observations

The likelihood of seeing all the observations, $\{\tilde{R}_i^j\}_{j=1}^4$ and $\{\tilde{B}_i^j\}_{j=1}^4$ at a given location \hat{x}_i is as follows:

$$p(\tilde{R}_{i}^{1}, \tilde{R}_{i}^{2}, \tilde{R}_{i}^{3}, \tilde{R}_{i}^{4}, \tilde{B}_{i}^{1}, \tilde{B}_{i}^{2}, \tilde{B}_{i}^{3}, \tilde{B}_{i}^{4} | \hat{x}_{i})$$

$$= \prod_{j=1}^{4} p(\tilde{R}_{i}^{j} | \hat{x}_{i}, \tilde{B}_{i}^{j}) p(\tilde{B}_{i}^{j} | \hat{x}_{i}),$$
(6)

where $p(\tilde{R}_i^j|\hat{x}_i, \tilde{B}_i^j)$ and $p(\tilde{B}_i^j|\hat{x}_i)$ are obtained in the section V-A and V-B.

VI. EVALUATION

In this section, we present evaluation results of our proposed algorithm.

A. Methodology

In this paper, we focus on one isolated cell in 5G network, where the measurement report only has the information of the serving cell. In the field, when we have multiple cells, the additional information on the measurement report from neighboring cells can be easily incorporated into the proposed algorithm to improve the localization accuracy. This could include information such as RSRQ and RSSI. As there is no

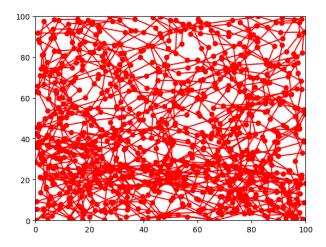


Fig. 3. Training data points

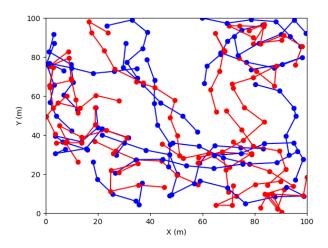


Fig. 4. Comparison of actual (red) and predicted (blue) locations.

commercial deployed 5G network yet, we do not have the drive test data from the field. Instead, we use our 5G system simulator to generate both training data and test data.

The 5G system that we use in this evaluation is operating at mmwave band, and the cell ISD is 200m. The testing data points are illustrated in Figure 3, which has 1000 data points. Instead of diving the training data into training and validation, we use the gridsearch and cross validation functionality in sklearn [2] to avoid the overfitting since we have limited number of training data.

B. Localization Results

In Figure 4, we show the predicted and actual locations of all mobiles. As it can be seen, the actual locations and the estimated locations are quite close. As we only used the information from one base station in our study, some locations may not be tracked closely. This also depends on the shape of the coverage area of each beam. Since beam indices are used in the algorithm, the narrower the coverage area of a particular beam, the better the estimation accuracy for the

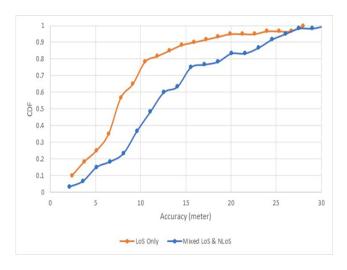


Fig. 5. CDF of accuracy, with different RF channel condition (LoS vs. MixedLoS&NLoS)

points reporting this beam indices. In the following, we present more detailed analysis of the results.

In Figure 5, we show the accuracy distribution under two different type of RF channel condition, pure line of sight (LoS) and mixed LoS & non-LoS(NLoS). More details of these scenarios can be found in [4]. With LoS channel, the median accuracy is around 4m, while with mixed LoS and NLoS channel, the median accuracy is around 12m. These results have not included additional constraints such as users moving on prescribed paths such as walkways etc. as well as additional columns in the data matrix such as TA, etc. With this additional information, the localization accuracy is expected to improve further.

The next step of our work is to evaluate our proposed algorithm in field data once 5G is deployed in the field.

VII. CONCLUDING REMARKS

In this paper, we have developed localization algorithms of mobile devices in 5G networks based on the measurement records We have shown median accuracy of 4m in LoS enviorment and 12m in mixed LoS and NLoS enviorments. The above results have not considered the constraint of the road, terrace etc, which limit the potential location of the mobile devices. With this consideration, the accuracy is expected to be better.

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