Pertemuan ke-12

INTEGRAL TRIGONOMETRI

Oleh:

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INTEGRAL FUNGSI TRIGONOMETRI

- Dengan menggunakan aturan integral tak tentu maka diperoleh rumus-rumus dasar integral tak tentu untuk fungsi trigonometri sebagai berikut:

1. $\int \cos x \, dx = \sin x + C$
2. $\int \sin x \, dx = -\cos x + C$
3. $\int sec^2 \, x \, dx = \tan x + C$
4. $\int cosec^2 \, x \, dx = -\cot x + C$
5. $\int \tan x \, . \sec x \, dx = \sec x + C$
6. $\int \cot x \, . cosec \, x \, dx = -cosec \, x + C$

INTEGRAL FUNGSI TRIGONOMETRI

7. $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$

8. $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$

9. $\int sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$

10. $\int cosec^2(ax+b)\,dx = -\frac{1}{a}\tan(ax+b) + C$

11. $\int \tan(ax+b).\sec(ax+b)\,dx = \frac{1}{a}\sec(ax+b) + C$

12. $\int \cot(ax+b).cosec\,(ax+b)\,dx = -\frac{1}{a}cosec(ax+b) + C$

Contoh 1:

- Tentukan integral-integral di bawah ini:

1) $\int (5x + \cos x)\, dx$

2) $\int (2\cos x - 3\sin x)\, dx$

3) $\int (\sin x - \cos x)^2\, dx$

4) $\int \sin 7x \cos 3x\, dx$

Penyelesaian:

1) $\int (5x + \cos x)\, dx = \int 5x\, dx + \int \cos x\, dx$

$$= \frac{5}{2}x^2 + \sin x + C$$

Lanjutan Contoh 1:

$$2) \int (2\cos x - 3\sin x)\, dx = \int 2\cos x\, dx - \int 3\sin x\, dx$$

$$= 2\sin x + 3\cos x + C$$

$$3) \int (\sin x - \cos x)^2\, dx = \int (1 - \sin 2x)\, dx$$

$$= \int dx - \int \sin 2x\, dx$$

$$= x - \left(-\frac{1}{2}\cos 2x\right) + C$$

$$= x + \frac{1}{2}\cos 2x + C$$

Lanjutan Contoh 1:

$$4) \int \sin 7x \cos 3x \, dx = \int \frac{1}{2}(\sin(7+3)x + \sin(7-3)x) \, dx$$

$$= \int \frac{1}{2}(\sin 10x + \sin 4x) \, dx$$

$$= \frac{1}{2}\left(-\frac{1}{10}\cos 10x - \frac{1}{4}\cos 4x\right) + C$$

$$= -\left(\frac{1}{20}\cos 10x + \frac{1}{8}\cos 4x\right) + C$$

Contoh 2:

Hitunglah nilai dari integral berikut:

$$\int_0^{\pi} 3 \sin x \, dx$$

Penyelesaian:

$$\int_0^{\pi} 3 \sin x \, dx = [-3 \cos x]_0^{\pi}$$

$$= (-3 \times (-1)) - (-3 \times 1)$$

$$= 3 + 3$$

$$= 6$$

Contoh 3

- Hitunglah nilai $\int_0^{\frac{\pi}{6}} (\sin 3x + \cos 3x)\, dx$

Penyelesaian:

$$\int_0^{\frac{\pi}{6}} (\sin 3x + \cos 3x)\, dx$$

$$= \left[-\frac{1}{3}\cos 3x + \frac{1}{3}\sin 3x \right]_0^{\frac{\pi}{6}}$$

$$= \left[-\frac{1}{3}\cos 3 \cdot \frac{\pi}{6} + \frac{1}{3}\sin 3 \cdot \frac{\pi}{6} \right] - \left[-\frac{1}{3}\cos 3 \cdot 0 + \frac{1}{3}\sin 3 \cdot 0 \right]$$

$$= \left(0 + \frac{1}{3} \right) - \left(\frac{1}{3} + 0 \right)$$

$$= \frac{2}{3}$$

SEKIAN
DAN
TERIMAKASIH