Pertemuan ke - 7

DIFERENSIAL FUNGSI TRIGONOMETRI

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i. Turunan $f(x) = \sin x$

$$
\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} \\
&= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} \\
&= \lim_{h \to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h} \\
&= \lim_{h \to 0} \frac{\sin x \cos h - \sin x + \sin h \cos x}{h} \\
&= \lim_{h \to 0} \frac{\sin x (\cos h - 1) + \sin h \cos x}{h} \\
&= \lim_{h \to 0} \frac{\sin x (\cos h - 1)}{h} + \lim_{h \to 0} \frac{\sin h \cos x}{h} \\
&= \lim_{h \to 0} \frac{\sin x (\cos h - 1)}{h} \times \frac{h(\cos h + 1)}{h(\cos h + 1)} + \lim_{h \to 0} \frac{\sin h \cos x}{h} \\
&= \lim_{h \to 0} \frac{\sin x (\cos^2 h - 1) h}{h^2 (\cos h + 1)} + \lim_{h \to 0} \frac{\sin h \cos x}{h} \\
&= \lim_{h \to 0} \frac{\sin x (\cos^2 h - [\sin^h h + \cos^2 h]) h}{h^2 (\cos h + 1)} + \lim_{h \to 0} \frac{\sin h \cos x}{h} \\
&= \lim_{h \to 0} \frac{\sin x (-\sin^2 h) h}{h^2 (\cos h + 1)} + \lim_{h \to 0} \frac{\sin h \cos x}{h} \\
&= \lim_{h \to 0} -\sin x \times \frac{\sin^2 h}{h^2} \times \frac{h}{\cos h + 1} + \lim_{h \to 0} \frac{\sin h}{h} \cos x \\
&= \lim_{h \to 0} -\sin x \times 1 \times \frac{0}{\cos 0 + 1} + (1) \cos x \\
&= 0 + \cos x \\
f'(x) &= \cos x
\end{aligned}
$$

ii. Turunan $f(x) = \cos x$

$$
\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} \\
&= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \\
&= \lim_{h \to 0} \frac{\cos x \cos h - \cos x - \sin x \sin h}{h} \\
&= \lim_{h \to 0} \frac{\cos x (\cos h - 1) - \sin x \sin h}{h} \\
&= \lim_{h \to 0} \frac{\cos x (\cos h - 1)}{h} - \lim_{h \to 0} \frac{\sin x \sin h}{h} \\
&= \lim_{h \to 0} \frac{\cos x (\cos h - 1)}{h} \times \frac{h(\cos h + 1)}{h(\cos h + 1)} - \lim_{h \to 0} \frac{\sin x \sin h}{h} \\
&= \lim_{h \to 0} \frac{\cos x (\cos^2 h - 1)}{h^2} \times \frac{h}{(\cos h + 1)} - \lim_{h \to 0} \frac{\sin x \sin h}{h} \\
&= \lim_{h \to 0} \frac{\cos x (\cos^2 h - [\sin^2 h + \cos^2 h])}{h^2} \times \frac{h}{(\cos h + 1)} - \lim_{h \to 0} \frac{\sin x \sin h}{h} \\
&= \lim_{h \to 0} \frac{\cos x (-\sin^2 h)}{h^2} \times \frac{h}{(\cos h + 1)} - \lim_{h \to 0} \frac{\sin x \sin h}{h} \\
&= \lim_{h \to 0} -\cos x \times \frac{\sin^2 h}{h^2} \times \frac{h}{(\cos h + 1)} - \lim_{h \to 0} \frac{\sin h}{h} \times \sin x \\
&= \lim_{h \to 0} -\cos x \times 1 \times \frac{0}{(\cos 0 + 1)} - \lim_{h \to 0} 1 \times \sin x \\
&= 0 - \sin x \\
f'(x) &= -\sin x
\end{aligned}
$$

DIFERENSIAL FUNGSI TRIGONOMETRI

iii. Turunan $f(x) = \tan x$

Misalkan

$$u(x) = \sin x \qquad\qquad v(x) = \cos x$$
$$u'(x) = \cos x \qquad\qquad v'(x) = -\sin x$$

$$f(x) = \tan x$$

$$f(x) = \frac{\sin x}{\cos x}$$

$$f(x) = \frac{u(x)}{v(x)}$$

$$f'(x) = \frac{u'(x).v(x) - u(x).v'(x)}{[v(x)]^2}$$

$$= \frac{\cos x.\cos x - \sin x.(-\sin x)}{[\cos x]^2}$$

$$= \frac{\cos x.\cos x + \sin x.\sin x}{[\cos x]^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{[\cos x]^2}$$

$$f'(x) = \frac{1}{\cos^2 x}$$

DIFERENSIAL FUNGSI TRIGONOMETRI

▶ Beberapa diferensial fungsi trigonometri adalah sebagai berikut:

1. $y = a \sin x \quad \rightarrow \quad y' = a \cos x$

2. $y = a \cos x \quad \rightarrow \quad y' = -a \sin x$

3. $y = a \tan x \quad \rightarrow \quad y' = a \, sec^2 \, x$

4. $y = a \, cotan \, x \quad \rightarrow \quad y' = - \, cosec^2 \, x$

5. $y = a \sec x \quad \rightarrow \quad y' = \sec x \tan x$

6. $y = a \, cosec \, x \quad \rightarrow \quad y' = - \, cosec \, x \, \, cotan \, x$

Contoh 1:

- Tentukan nilai diferensial dari :

1) $y = 3 \sin x - 2 \cos x$

2) $y = 3 \sin 2x$

Penyelesaian:

1) $y = 3 \sin x - 2 \cos x$

$y' = 3 \cos x - (-2 \sin x)$

$y' = 3 \cos x + 2 \sin x$

Lanjutan Contoh 1:

2) $y = 3 \sin 2x$

$\quad y = 3 \cdot 2 \sin x \cos x$

$\quad y = 6 \sin x \cos x$

Misal : $U = \sin x$ maka $\quad U' = \cos x$

$\quad\quad V = \cos x \quad$ maka $\quad V' = -\sin x$

$y' = 6 \cdot U' \cdot V + U \cdot V'$

$y' = 6 \cdot (\cos x \cos x + \sin x (-\sin x))$

$y' = 6 \cdot (cos^2 x - sin^2 x)$

$y' = 6 \cos 2x$

Contoh 2:

▶ Tentukan diferensial dari fungsi
$$y = 2\sin(5x^2 - 7x + 8)$$

Penyelesaian:

Misalkan : $U = 5x^2 - 7x + 8$ maka $U' = 10x - 7$

Sehingga,
$y = 2\sin(5x^2 - 7x + 8)$
$y' = 2U' \cdot \cos U$
$y' = 2(10x - 7)\cos(5x^2 - 7x + 8)$
$y' = (20x - 14)\cos(5x^2 - 7x + 8)$

Contoh 3:

▶ Hitunglah nilai diferensial dari fungsi
$$y = \cos 3x \, (1 - 2sin^2 2x)$$

Penyelesaian:

Sifat trigonometri $1 - 2sin^2 2x = \cos 2x$, maka;
$y = \cos 3x \, (1 - 2sin^2 2x)$
$y = \cos 3x \cos 2x$

Misal : $U = \cos 3x \quad \rightarrow \quad U' = -3\sin 3x$

$\quad\quad V = \cos 2x \quad \rightarrow \quad V' = -2\sin 2x$
$y' = -3\sin 3x \cos 2x + \cos 3x \, (-2\sin 2x)$
$y' = -3\sin 3x \cos 2x - 2\cos 3x \sin 2x$

Contoh 4:

- Jika $f(x) = \sin x \cos 3x$ maka tentukan diferensial $f'\left(\frac{\pi}{6}\right)$.

Penyelesaian:

Misal : $U = \sin x$ maka $U' = \cos x$

$\quad\quad V = \cos 3x$ maka $V' = -3 \sin 3x$

Sehingga

$f'(x) = U' \cdot V + U \cdot V'$

$f'(x) = \cos x \cos 3x + \sin x\, (-3 \sin 3x)$

$f'(x) = \cos x \cos 3x - 3 \sin x \sin 3x$

$f'\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) \cos 3\left(\frac{\pi}{6}\right) - 3 \sin\left(\frac{\pi}{6}\right) \sin 3\left(\frac{\pi}{6}\right)$

$f'\left(\frac{\pi}{6}\right) = \left(\frac{1}{2}\sqrt{3} \cdot 0\right) - \left(3 \cdot \frac{1}{2} \cdot 1\right)$

$f'\left(\frac{\pi}{6}\right) = 0 - \frac{3}{2}$

$f'\left(\frac{\pi}{6}\right) = -\frac{3}{2}$

Contoh 3:

- Tentukan diferensial dari fungsi $y = \frac{1+\cos x}{\sin x}$

<u>Penyelesaian:</u>

Misal : $U = 1 + \cos x$ maka $\quad U' = -\sin x$

$\qquad V = \sin x \qquad\qquad$ maka $\quad V' = \cos x$

$y' = \dfrac{-\sin x \sin x - (1 + \cos x)\cos x}{sin^2 x}$

$y' = \dfrac{-sin^2 x - \cos x - cos^2 x}{sin^2 x}$

$y' = \dfrac{-(sin^2 x + cos^2 x) - \cos x}{sin^2 x}$

$y' = \dfrac{-1 - \cos x}{1 - cos^2 x}$

$y' = \dfrac{-(1 + \cos x)}{(1 - \cos x)(1 + \cos x)}$

$y' = \dfrac{-1}{1 - \cos x}$

Contoh 4:

▶ Tentukan diferensial dari fungsi
$$y = (2x - 1)\tan(x^3 + x)$$

<u>Peyelesaian:</u>

Misalkan :

$U = 2x - 1 \qquad \rightarrow \ U' = 2$

$V = \tan(x^3 + x) \rightarrow \ V' = (3x^2 + 1)sec^2(x^3 + x)$

Sehingga;
$y' = 2\tan(x^3 + x) + (2x - 1)(3x^2 + 1)sec^2(x^3 + x)$
$y' = 2\tan(x^3 + x) + (6x^3 - 3x^2 + 2x - 1)sec^2(x^3 + x)$

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