Pertemuan ke-6

DIFERENSIAL

OLEH:

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DEFINISI DIFERENSIAL

Diferensial fungsi adalah fungsi lain f' yang nilainya pada sembarang bilangan x adalah

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

asalkan limit ini ada.

- Pencarian diferensial disebut pendiferensialan.
- Diferensial pertama fungsi y terhadap x pada fungsi $y = f(x)$ dinotasikan sebagai:

$$y', f'(x), \frac{dy}{dx}, \frac{d\{f(x)\}}{dx}$$

CONTOH 1:

- Andaikan $f(x) = 13x - 6$. carilah $f'(4)$.

<u>penyelesaian:</u>

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$$

$$f'(4) = \lim_{h \to 0} \frac{(13(4+h) - 6) - (13(4) - 6)}{h}$$

$$f'(4) = \lim_{h \to 0} \frac{52 + 13h - 6 - (52 - 6)}{h}$$

$$f'(4) = \lim_{h \to 0} \frac{13h}{h}$$

$$f'(4) = \lim_{h \to 0} 13$$

$$f'(4) = 13$$

CONTOH 2:

Andaikan $f(x) = x^2$. carilah $f'(1)$.

penyelesaian:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{(1+h)^2 - (1^2)}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{1 + 2h + h^2 - 1}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{h(2+h)}{h}$$

$$f'(1) = \lim_{h \to 0} 2 + h$$

$$f'(1) = 2 + 0$$

$$f'(1) = 2$$

CONTOH 3:

- Diketahui $f(x) = x^2 + 5x + 6$. Tentukan diferensial dari $f'(x)$.

  penyelesaian:

$$f'(x) = \lim_{h \to 0} \frac{\left((x+h)^2+5(x+h)+6\right)-\left(x^2+5x+6\right)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2+2xh+h^2+5x+5h+6-x^2-5x-6}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{h^2+2xh+5h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{h(h+2x+5)}{h}$$

$$f'(x) = \lim_{h \to 0} h + 2x + 5$$

$$f'(x) = 0 + 2x + 5$$

$$f'(x) = 2x + 5$$

RUMUS-RUMUS DIFERENSIAL FUNGSI

1. $f(x) = c \Rightarrow f'(x) = 0$ aturan f. konstanta
2. $f(x) = x \Rightarrow f'(x) = 1$ aturan f. identitas
3. $f(x) = x^n \Rightarrow f'(x) = n \cdot x^{n-1}$ aturan pangkat

Contoh 4:

- Tentukan turunan fungsi berikut;

1. $f(x) = 3 \Longrightarrow f'(x) = 0$
2. $f(x) = 12x \Longrightarrow f'(x) = 12$
3. $f(x) = x^4 \Longrightarrow f'(x) = 4 \cdot x^{4-1} = 4x^3$
4. $f(x) = 6x^3 \Longrightarrow f'(x) = 6 \cdot 3 \cdot x^{3-1} = 18x^2$
5. $f(x) = 2x^{-5} - 5 \Longrightarrow f'(x) = 2.(-5).x^{-5-1} - 0 = f'(x) = -10x^{-6}$

SIFAT-SIFAT DIFERENSIAL FUNGSI

- Misalkan u dan v adalah fungsi dalam x, serta maka berlaku sifat-sifat berikut:

1. $f(x) = U \pm V \implies f'(x) = U' \pm V'$
2. $f(x) = U \cdot V \implies f'(x) = U' \cdot V + U \cdot V'$
3. $f(x) = \frac{U}{V} \implies f'(x) = \frac{U' \cdot V - U \cdot V'}{V^2}$
4. $f(x) = U^n \implies f'(x) = n \cdot U^{n-1} \cdot U'$
5. $f(x) = e^U \implies f'(x) = e^U \cdot U'$

CONTOH 5:

- Tentukan turunan pertama dari fungsi-fungsi berikut.

1. $y = x^3 + 5x^2 - 4x - 9$
2. $f(x) = (2x + 3)(x^2 - 1)$
3. $g(x) = \frac{x^2-4}{3x+7}$
4. $f(x) = (2x - 3)^3$
5. $y = e^{x^2+5}$

Penyelesaian:

1. $y = x^3 + 5x^2 - 4x - 9$

$y' = 3 \cdot x^{3-1} + 5 \cdot 2 \cdot x^{2-1} - 4 - 0$

$y' = 3x^2 + 10x - 4$

LANJUTAN CONTOH 5:

2. $f(x) = (2x + 3)(x^2 - 1)$

misalkan : $U = 2x + 3$ maka $U' = 2 + 0 = 2$

$V = x^2 - 1$ maka $V' = 2.x^{2-1} - 0 = 2x$

sehingga;

$f(x) = U \cdot V$

$f'(x) = U' \cdot V + U \cdot V'$

$f'(x) = 2 \cdot (x^2 - 1) + (2x + 3) \cdot 2x$

$f'(x) = 2x^2 - 2 + 4x^2 + 6x$

$f'(x) = 6x^2 + 6x - 2$

LANJUTAN CONTOH 5:

3. $g(x) = \frac{x^2-4}{3x+7}$

misalkan : $U = x^2 - 4$ maka $U' = 2x$

$V = 3x + 7$ maka $V' = 3$

$g(x) = \frac{U}{V}$

$g'(x) = \frac{U' \cdot V - U \cdot V'}{V^2}$

$g'(x) = \frac{2x \cdot (3x+7) - (x^2-4) \cdot 3}{(3x+7)^2}$

$g'(x) = \frac{6x^2+14x-3x^2+12}{9x^2+42x+49}$

$g'(x) = \frac{3x^2+14x+12}{9x^2+42x+49}$

LANJUTAN CONTOH 5:

4. $f(x) = (2x - 3)^3$

misalkan : $U = 2x - 3$ maka $U' = 2$

sehingga,

$f(x) = U^n$

$f'(x) = n \cdot U^{n-1} \cdot U'$

$f'(x) = 3 \cdot (2x - 3)^{3-1} \cdot 2$

$f'(x) = 6 \cdot (2x - 3)^2$

$f'(x) = 6(4x^2 - 12x + 9)$

$f'(x) = 24x^2 - 72x + 54$

LANJUTAN CONTOH 5:

5. $y = e^{x^2+5}$

misalkan: $U = x^2 + 5$ maka $U' = 2x$

sehingga;

$f(x) = e^U$

$f'(x) = e^U \cdot U'$

$f'(x) = e^{x^2+5} \cdot 2x$

$f'(x) = 2xe^{x^2+5}$

SOAL LATIHAN

Tentukan fungsi turunan pertama dari:

1. $f(x) = x^{\frac{1}{2}} + \sqrt[3]{x^2} + 1$
2. $f(x) = (x+1)(x^3 + 2x + 2)$
3. $f(x) = \frac{x+1}{x-1}$
4. $f(x) = \frac{x}{x^2-1}$
5. $f(x) = \frac{x^2-1}{x^2+1}$

SEKIAN
DAN
TERIMA KASIH