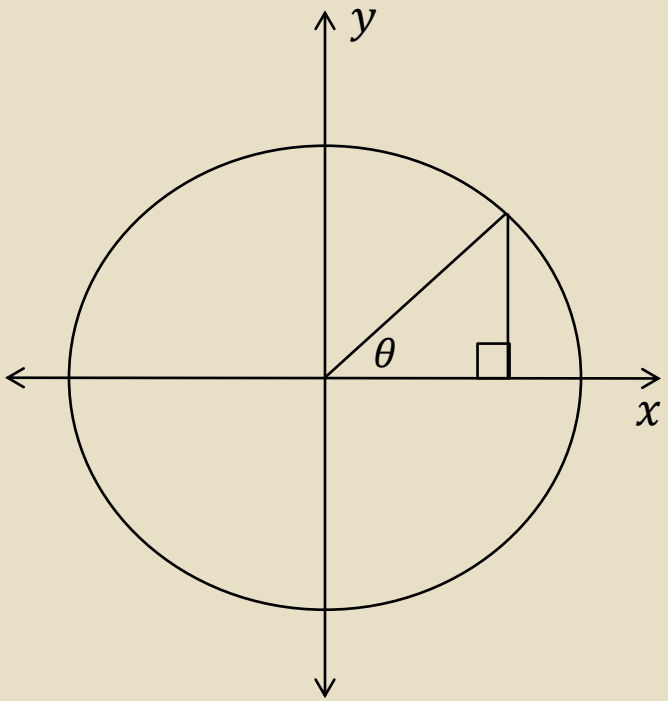
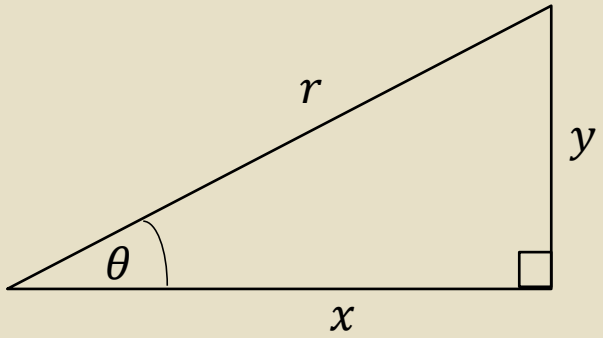
Pertemuan ke - 5

LIMIT FUNGSI TRIGONOMETRI

Oleh:
Santi Arum Puspita Lestari, M.Pd
Teknik Informatika
Universitas Buana Perjuangan Karawang

LIMIT FUNGSI TRIGONOMETRI DI SUATU TITIK





- $\sin\theta = \frac{y}{r}$
- $\cos\theta = \frac{x}{r}$
- $\tan\theta = \frac{y}{x}$

IDENTITAS TRIGONOMETRI

- $sin^2 x + cos^2 x = 1$
- $sin^2 x = 1 - cos^2 x$
- $cos^2 x = 1 - sin^2 x$
- $\tan x = \frac{\sin x}{\cos x}$
- $\cot x = \frac{\cos x}{\sin x}$
- $\sec x = \frac{1}{\cos x}$
- $\csc x = \frac{1}{\sin x}$
- $\cos 2\alpha = 1 - 2sin^2 \alpha$
- $cos^2 \alpha = 1 - sin^2 \alpha$

LIMIT FUNGSI TRIGONOMETRI

Rumus untuk menentukan nilai limit fungsi trigonometri:

1. $\lim_{x \to 0} \frac{ax}{\sin bx} = \frac{a}{b}$
2. $\lim_{x \to 0} \frac{\sin bx}{ax} = \frac{b}{a}$
3. $\lim_{x \to 0} \frac{ax}{\tan bx} = \frac{a}{b}$
4. $\lim_{x \to 0} \frac{\tan bx}{ax} = \frac{b}{a}$
5. $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$
6. $\lim_{x \to 0} \frac{\tan ax}{\tan bx} = \frac{a}{b}$
7. $\lim_{x \to 0} \frac{\tan ax}{\sin bx} = \frac{a}{b}$
8. $\lim_{x \to 0} \frac{\sin bx}{\tan ax} = \frac{b}{a}$

LIMIT FUNGSI TRIGONOMETRI

- Jika terdapat fungsi cos maka ubahlah ke dalam bentuk sebagai berikut:

1. cos x di ubah menjadi $1 - 2sin^2 \frac{1}{2}x$
2. $cos^2 x$ diubah menjadi $1 - sin^2 x$

Contoh 1:

Tentukan nilai limit dari fungsi trigonometri berikut:

1. $\lim\limits_{x\to 0} \dfrac{\sin 6x}{2x}$ 2. $\lim\limits_{x\to 0} \dfrac{\frac{1}{2}x}{\tan \frac{2}{3}x}$ 3. $\lim\limits_{x\to 0} \dfrac{\tan 2x}{\sin 8x}$

Penyelesaian:

1. $\lim\limits_{x\to 0} \dfrac{\sin 6x}{2x} = \dfrac{6}{2} = 3$

2. $\lim\limits_{x\to 0} \dfrac{\frac{1}{2}x}{\tan \frac{2}{3}x} = \dfrac{\frac{1}{2}}{\frac{2}{3}} = \dfrac{1}{2} \times \dfrac{3}{2} = \dfrac{3}{4}$

3. $\lim\limits_{x\to 0} \dfrac{\tan 2x}{\sin 8x} = \dfrac{2}{8} = \dfrac{1}{4}$

Contoh 2:

Tentukan nilai limit dari fungsi trigonometri berikut:

1. $\lim\limits_{x\to 0} \frac{sin^3 2x}{5x^3}$ 2. $\lim\limits_{x\to 0} \frac{5x^2}{tan^2 3x}$ 3. $\lim\limits_{x\to 0} \frac{sin^2 3x}{tan^2 5x}$

Penyelesaian:

1. $$\lim_{x\to 0} \frac{sin^3 2x}{5x^3} = \lim_{x\to 0} \frac{\sin 2x \cdot \sin 2x \cdot \sin 2x}{5x \cdot x \cdot x}$$

$$= \lim_{x\to 0} \frac{\sin 2x}{5x} \cdot \lim_{x\to 0} \frac{\sin 2x}{x} \cdot \lim_{x\to 0} \frac{\sin 2x}{x}$$

$$= \frac{2}{5} \cdot \frac{2}{1} \cdot \frac{2}{1}$$

$$= \frac{8}{5}$$

Lanjutan Contoh 2 :

2. $$\lim_{x\to 0} \frac{5x^2}{tan^2 3x} = \lim_{x\to 0} \frac{5x \cdot x}{\tan 3x \cdot \tan 3x}$$

$$= \lim_{x\to 0} \frac{5x}{\tan 3x} \cdot \lim_{x\to 0} \frac{x}{\tan 3x}$$

$$= \frac{5}{3} \cdot \frac{1}{3}$$

$$= \frac{5}{9}$$

3. $$\lim_{x\to 0} \frac{sin^2 3x}{tan^2 5x} = \lim_{x\to 0} \frac{\sin 3x \cdot \sin 3x}{\tan 5x \cdot \tan 5x}$$

$$= \lim_{x\to 0} \frac{\sin 3x}{\tan 5x} \cdot \lim_{x\to 0} \frac{\sin 3x}{\tan 5x}$$

$$= \frac{3}{5} \cdot \frac{3}{5}$$

$$= \frac{9}{25}$$

Contoh 3:

Tentukan nilai limit dari fungsi trigonometri berikut:

1. $\lim\limits_{x\to 0} \frac{1-\cos x}{3x^2}$ 2. $\lim\limits_{x\to 0} \frac{5x \tan 3x}{1-\cos 6x}$

Penyelesaian:

1. $\lim\limits_{x\to 0} \frac{1-\cos x}{3x^2} = \lim\limits_{x\to 0} \frac{1-\left(1-2sin^2\frac{1}{2}x\right)}{3x^2} = \lim\limits_{x\to 0} \frac{1-1+2sin^2\frac{1}{2}x}{3x^2}$

$= 2 \lim\limits_{x\to 0} \frac{sin^2\frac{1}{2}x}{3x^2} = 2 \cdot \lim\limits_{x\to 0} \frac{\sin\frac{1}{2}x}{3x} \cdot \lim\limits_{x\to 0} \frac{\sin\frac{1}{2}x}{x}$

$= 2 \cdot \frac{\frac{1}{2}}{3} \cdot \frac{\frac{1}{2}}{1} = 2 \cdot \frac{1}{6} \cdot \frac{1}{2}$

$= \frac{2}{12} = \frac{1}{6}$

Lanjutan Contoh 3:

2. $$\lim_{x\to 0}\frac{5x\tan 3x}{1-\cos 6x}=\lim_{x\to 0}\frac{5x\tan 3x}{1-(1-2sin^2 3x)}$$

$$=\lim_{x\to 0}\frac{5x\tan 3x}{1-1+2sin^2 3x}$$

$$=\lim_{x\to 0}\frac{5x\tan 3x}{2sin^2 3x}$$

$$=\lim_{x\to 0}\frac{5x\tan 3x}{2\sin 3x\cdot\sin 3x}$$

$$=\frac{5}{2}\cdot\lim_{x\to 0}\frac{x}{\sin 3x}\cdot\lim_{x\to 0}\frac{\tan 3x}{\sin 3x}$$

$$=\frac{5}{2}\cdot\frac{1}{3}\cdot\frac{3}{3}$$

$$=\frac{5}{6}$$

Contoh 4:

Tentukan nilai limit $\lim\limits_{x\to\frac{\pi}{3}} \dfrac{\tan(3x-\pi)\cos 2x}{\sin(3x-\pi)}$

Penyelesaian:

$$\lim_{x\to\frac{\pi}{3}} \frac{\tan(3x-\pi)\cos 2x}{\sin(3x-\pi)} = \lim_{x\to\frac{\pi}{3}} \cos 2x$$

$$= \cos\frac{2\pi}{3} \qquad \to (\pi = 180°)$$

$$= \cos\frac{360°}{3}$$

$$= \cos 120°$$

$$= -\frac{1}{2}$$

TEOREMA SUBSTITUSI

- Jika *f* suatu fungsi polinomial atau fungsi rasional, maka

$$\lim_{x \to c} f(x) = f(c)$$

asalkan dalam kasus fungsi rasional nilai penyebut di *c* tidak nol.

Contoh:

- Carilah $\lim_{x \to 2} \frac{7x^5 - 10x^4 - 13x + 6}{3x^2 - 6x - 8}$

Penyelesaian:

$$\lim_{x \to 2} \frac{7x^5 - 10x^4 - 13x + 6}{3x^2 - 6x - 8} = \frac{7(2)^5 - 10(2)^4 - 13(2) + 6}{3(2)^2 - 6(2) - 8} = \frac{44}{-8} = -5{,}5$$

SEKIAN
DAN
TERIMA KASIH