

## I. CLAIRVOYANCE COIN GAME

Setup: two players,  $A$  and  $B$ . The game is *not* zero-sum: at the beginning there's  $P$  in the pot. Player  $A$  flips a coin and looks at it. Player  $B$  cannot see the coin. Player  $A$  wins when the coin is head and loses otherwise. Player  $A$  can make a bet of  $b$  after he sees the card. Player  $B$  can call or fold if  $A$  bets. If  $A$  just checks then  $B$  also checks. The expected winnings for  $A$  strategy in which it bluffs  $q$  of the tails and bets every head, and  $B$  calls  $r$  of the times, is

$$E[W_A] = \frac{1}{2} (r (P + b) + (1 - r) P) + \frac{1}{2} q (r (-b) + (1 - r) P) \quad (1)$$

$$= \frac{1}{2} r b + \frac{1}{2} P + \frac{1}{2} q (P - (P + b) r) , \quad (2)$$

$$E[W_B] = \frac{1}{2} r (-b) + \frac{1}{2} (q r (P + b) + (1 - q) P) \quad (3)$$

$$= \frac{1}{2} P (1 - q) + \frac{1}{2} r (q (P + b) - b) . \quad (4)$$

Therefore  $A$  in order to minimize earnings of  $B$ , and at the same time make  $B$  indifferent to  $B$ 's call rate, will bluff  $q = P/(P + b)$  times. Similarly  $B$  in order to make  $A$  indifferent to its bluff rate, and to minimize  $A$ 's earnings, will call  $r = b/(P + b)$  times.