# I. COIN CLAIRVOYANCE GAME

Setup: two players, $A$ and $B$. The game is *not* zero-sum: at the beginning there's $P$ in the pot. Player $A$ flips a coin and looks at it. Player $B$ cannot see the coin. Player $A$ wins when the coin is head and loses otherwise. Player $A$ can make a bet of $b$ after he sees the card. Player $B$ can call or fold if $A$ bets. If $A$ just checks then $B$ also checks. The expected winnings for $A$ strategy in which it bluffs $q$ of the tails and bets every head, and $B$ calls $r$ of the times, is

$$E[W_A] = \frac{1}{2}\left(r\left(P+b\right)+(1-r)\,P\right)+\frac{1}{2}q\left(r\left(-b\right)+(1-r)\,P\right) \tag{1}$$

$$= \frac{1}{2}\,rb+\frac{1}{2}P+\frac{1}{2}q\left(P-(P+b)r\right), \tag{2}$$

$$E[W_B] = \frac{1}{2}\,r\left(-b\right)+\frac{1}{2}\left(q\,r\left(P+b\right)+(1-q)\,P\right) \tag{3}$$

$$= \frac{1}{2}\,P(1-q)+\frac{1}{2}\,r\left(q(P+b)-b\right). \tag{4}$$

Therefore $A$ in order to minimize earnings of $B$, and at the same time make $B$ indifferent to $B$'s call rate, will bluff $q = b/(P+b)$ times. Similarly $B$ in order to make $A$ indifferent to its bluff rate, and to minimize $A$'s earnings, will call $r = P/(P+b)$ times.

# II. EVOLUTIONARY SELECTION OF STRATEGIES

In the previous section we solved the coin clairvoyance game. In this section we apply genetic optimization approach to reproduce the optimal strategies. The results of various simulations are provided below. We initialize the population of $M$ pairs of players, by setting up two groups of players. The players of the first group are characterized by the probability of value bet and the probability of bluff, and the players of the second group are characterized by the probability of call. Each strategy (determined by the specified probabilities) has the corresponding fit function, which we calculate by allowing the players to play for $K$ rounds, and ranking up their final bankrolls. At each round we randomly match up the players from the first group with the players from the second group. Initially we endow each player with 'bank' amount of money. The money in the pot is $P$ and the money bet by player 1 is $b$. We consider various values of $P$ and $b$ below and compare the evolutionary shaped optimal probabilities with those predicted analytically.
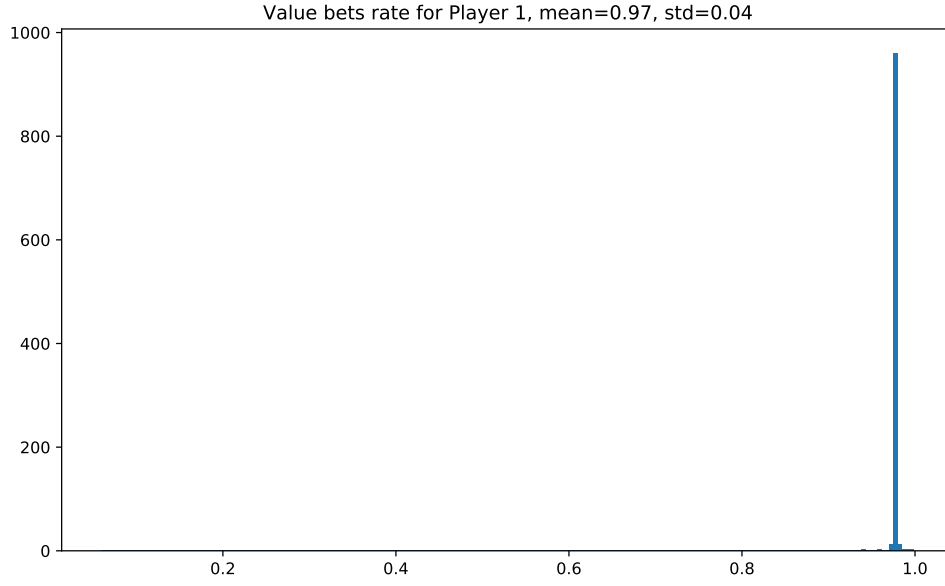
FIG. 1: Player 1 value betting, $P = 2$, $b = 3$, bank $= 10000$, $M = 1000$, $K = 100$, propagate $= 0.98$, selection rate $= 0.5$. Evolve for 2000 rounds. Predicted value is 1.
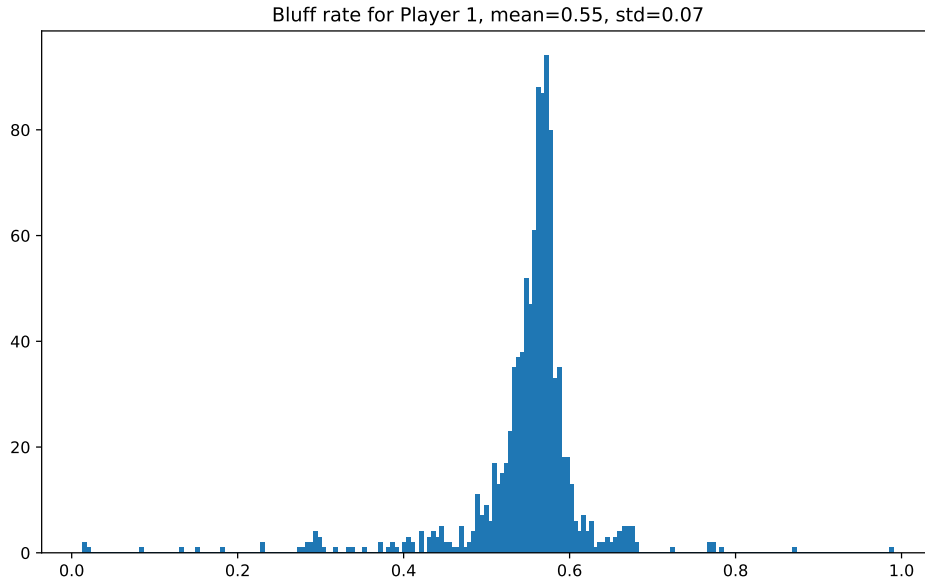


FIG. 2: Player 1 bluff, $P = 2$, $b = 3$, bank $= 10000$, $M = 1000$, $K = 100$, propagate $= 0.98$, selection rate $= 0.5$. Evolve for 2000 rounds. Predicted value is 0.6.
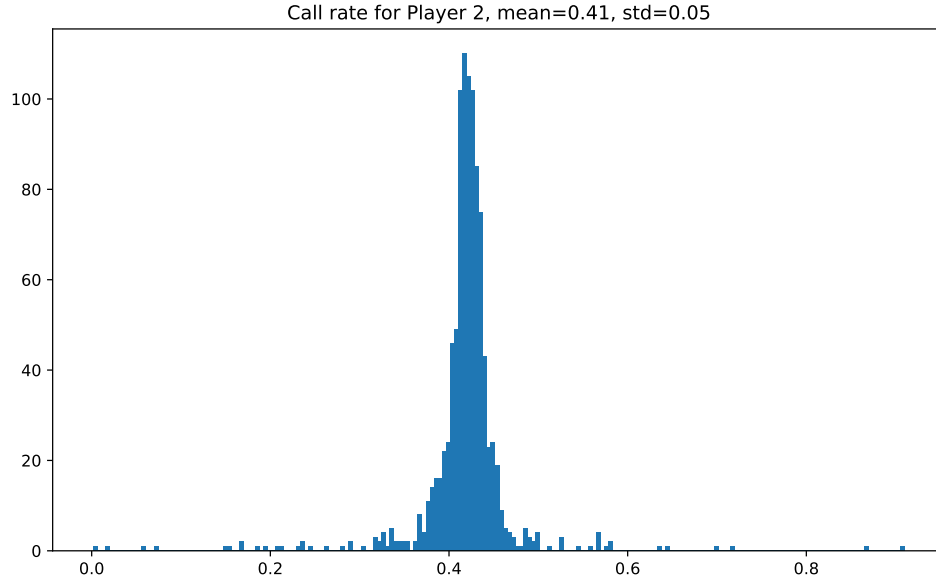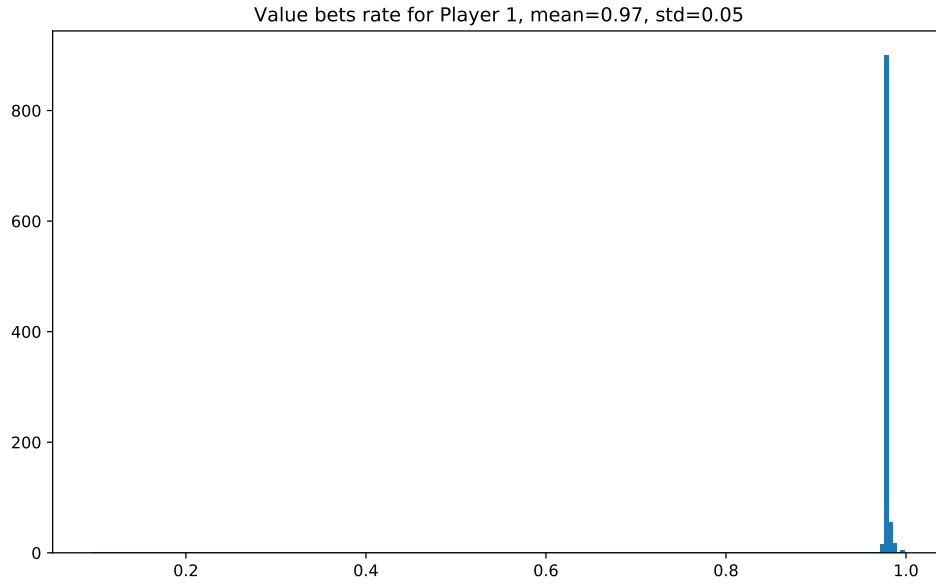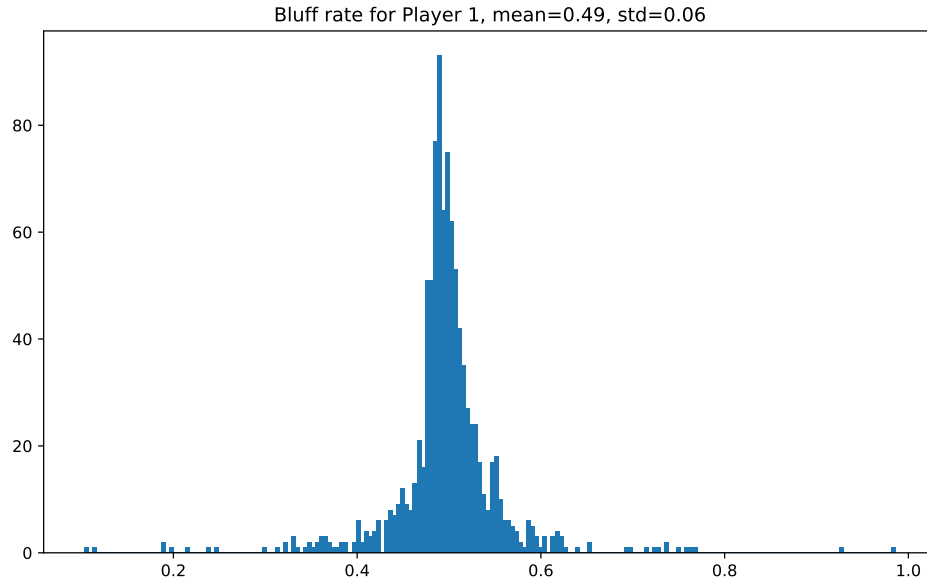
FIG. 3: Player 2 call, $P = 2$, $b = 3$, bank $= 10000$, $M = 1000$, $K = 100$, propagate $= 0.98$, selection rate $= 0.5$. Evolve for 2000 rounds. Predicted value is 0.4.



FIG. 4: Player 1 value betting, $P = 1$, $b = 1$, bank $= 10000$, $M = 1000$, $K = 100$, propagate $= 0.98$, selection rate $= 0.5$. Evolve for 2000 rounds. Predicted value is 1.

FIG. 5: Player 1 bluff, $P = 1$, $b = 1$, bank = 10000, $M = 1000$, $K = 100$, propagate = 0.98, selection rate = 0.5. Evolve for 2000 rounds. Predicted value is 0.5.
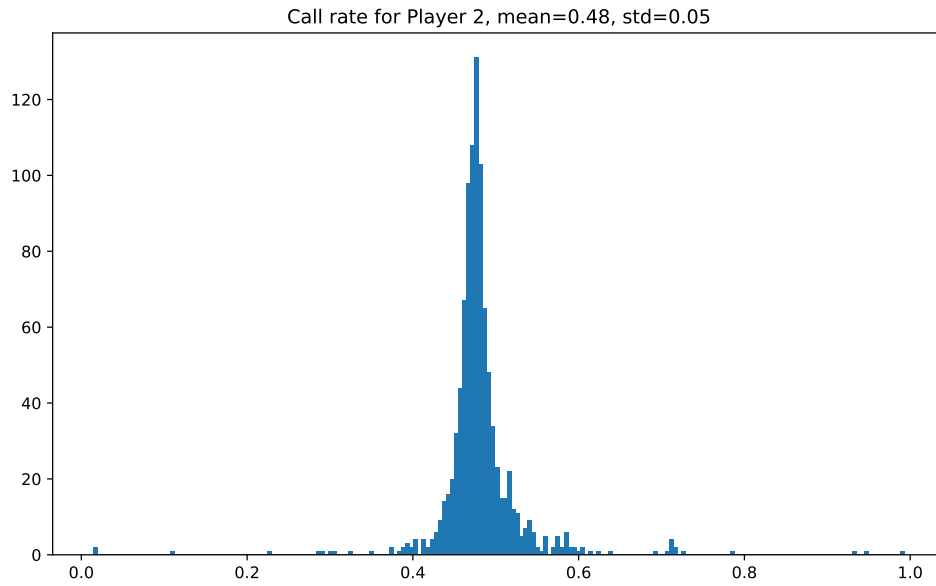


FIG. 6: Player 2 call, $P = 1$, $b = 1$, bank = 10000, $M = 1000$, $K = 100$, propagate = 0.98, selection rate = 0.5. Evolve for 2000 rounds. Predicted value is 0.5.
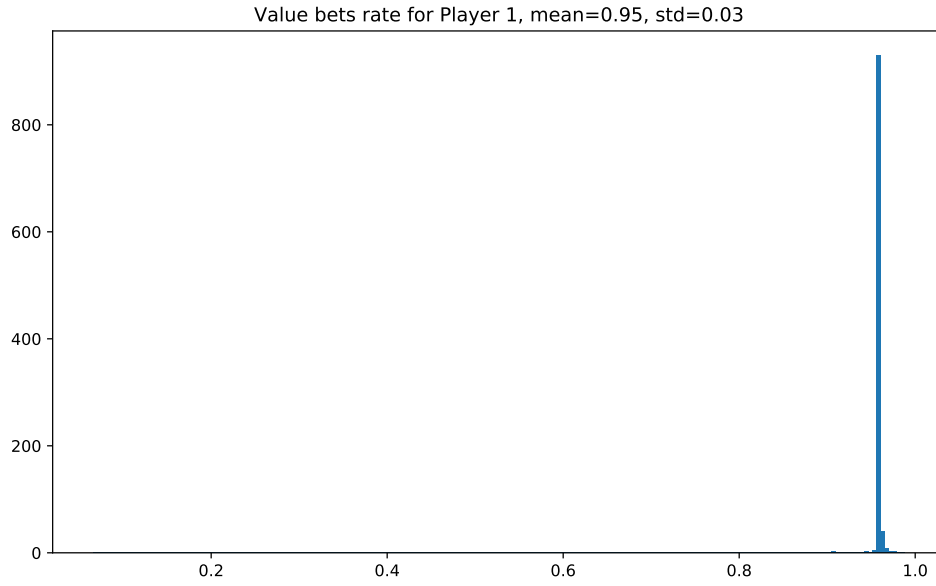
FIG. 7: Player 1 value betting, $P = 1$, $b = 3$, bank $= 10000$, $M = 1000$, $K = 100$, propagate $= 0.995$, selection rate $= 0.5$. Evolve for 2000 rounds. Predicted value is 1.
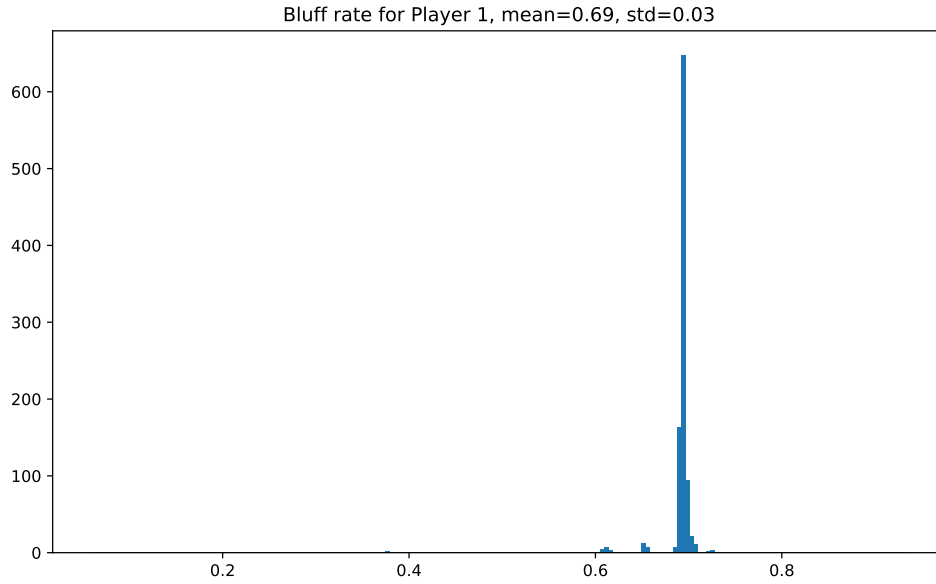


FIG. 8: Player 1 bluff, $P = 1$, $b = 3$, bank $= 10000$, $M = 1000$, $K = 100$, propagate $= 0.995$, selection rate $= 0.5$. Evolve for 2000 rounds. Predicted value is 0.75.
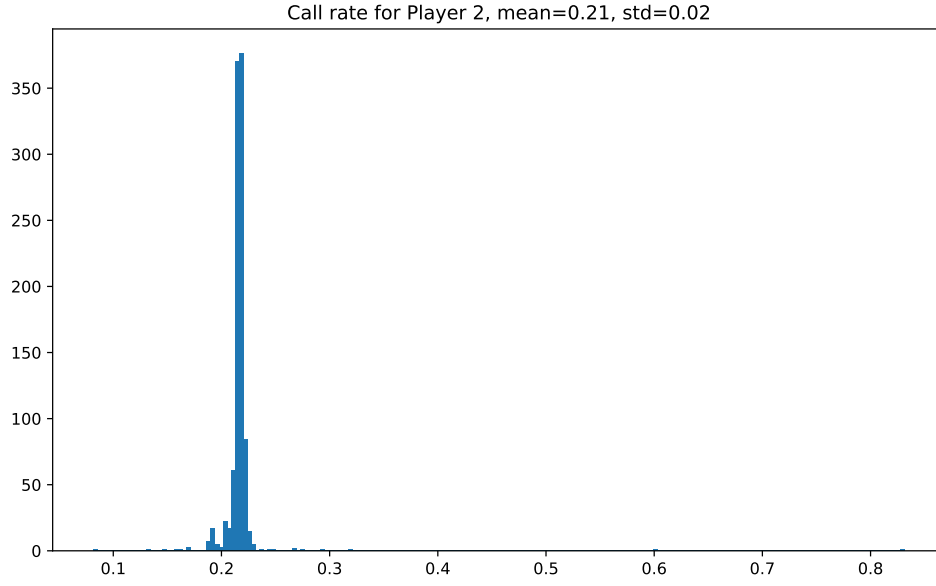
FIG. 9: Player 2 call, $P = 1$, $b = 3$, bank $= 10000$, $M = 1000$, $K = 100$, propagate $= 0.995$, selection rate $= 0.5$. Evolve for 2000 rounds. Predicted value is 0.25.
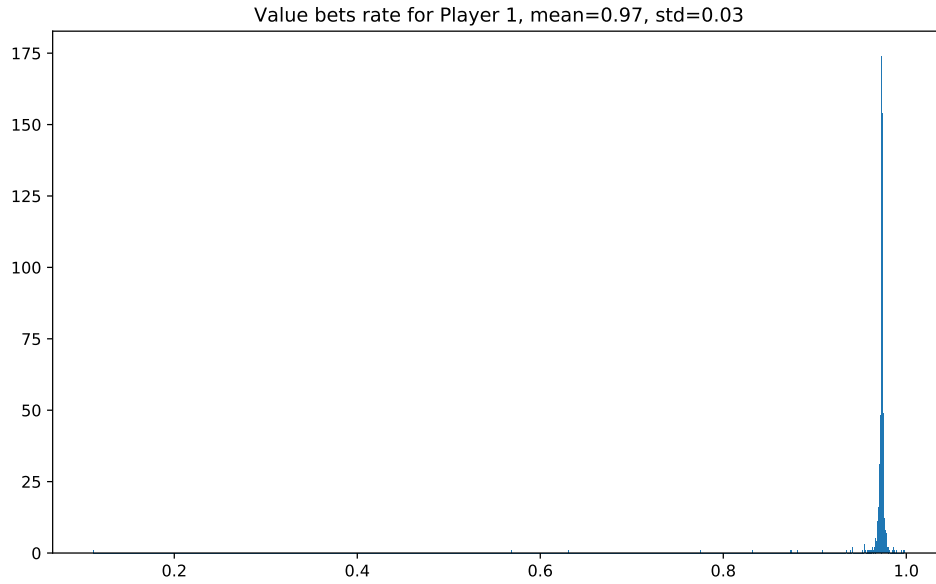


FIG. 10: Player 1 value betting, $P = 2$, $b = 1$, bank $= 10000$, $M = 1000$, $K = 100$, propagate $= 0.98$, selection rate $= 0.5$. Evolve for 2000 rounds. Predicted value is 1.
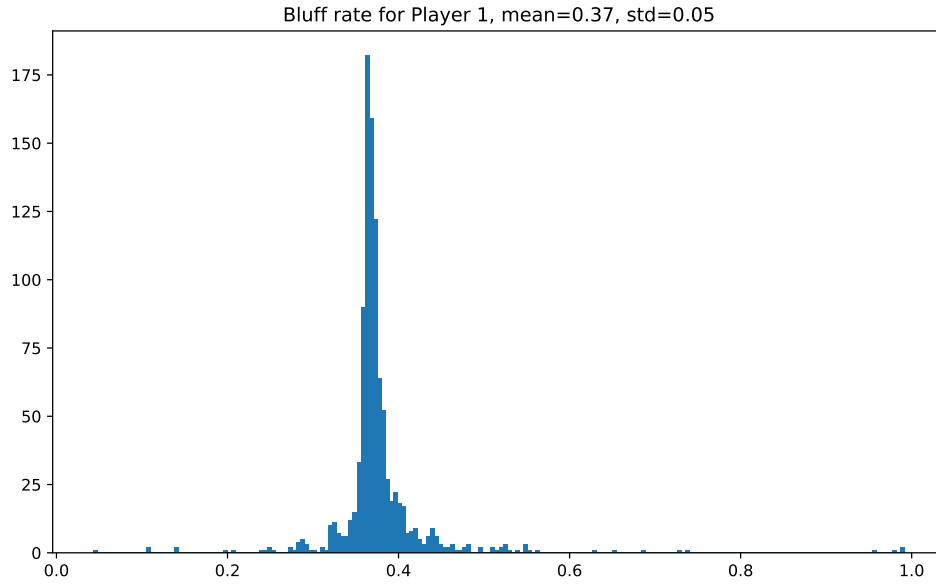
FIG. 11: Player 1 bluff, $P = 2$, $b = 1$, bank $= 10000$, $M = 1000$, $K = 100$, propagate $= 0.98$, selection rate $= 0.5$. Evolve for 2000 rounds. Predicted value is 0.33.
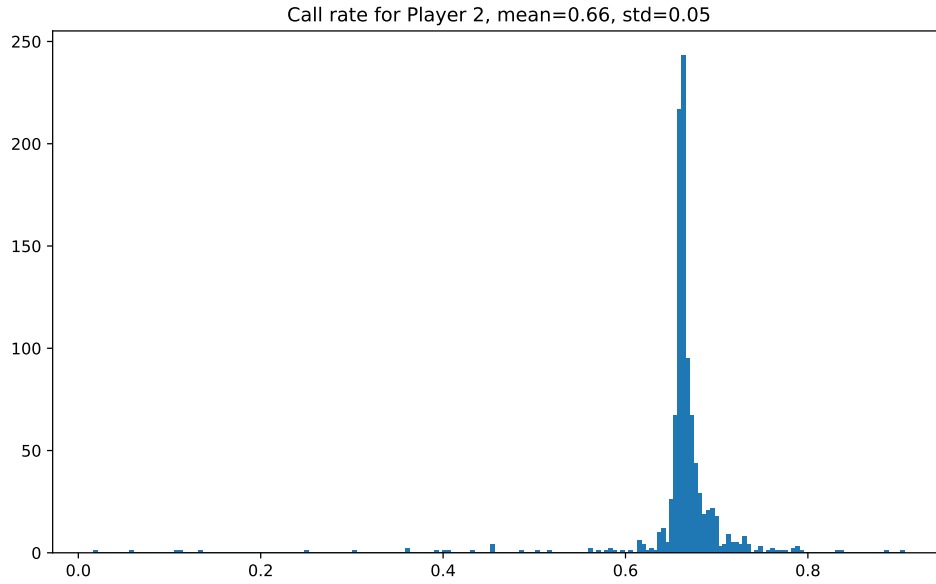


FIG. 12: Player 2 call, $P = 2$, $b = 1$, bank $= 10000$, $M = 1000$, $K = 100$, propagate $= 0.98$, selection rate $= 0.5$. Evolve for 2000 rounds. Predicted value is 0.66.