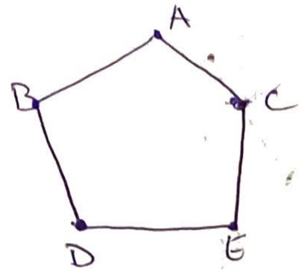
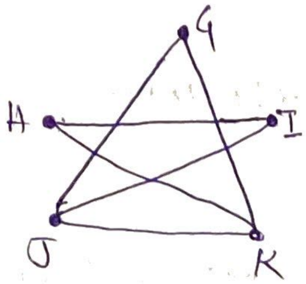
AMAT 583 Test-1 Daneetu Reddy Tadiparthi

UAlbany ID: 001491692

1) Let us consider a C_5 graph below.



Now, we will consider the compliment of C_5 graph below is $\overline{C_5}$



Now consider

$F: C_5 \to \overline{C_5}$

$F(A) = G$

$F(B) = H$

$F(C) = I$

$F(D) = J$

$F(E) = K$

Now Total number of vertices for $C_5 = 5$ & Total number of edges for $C_5 = 5$

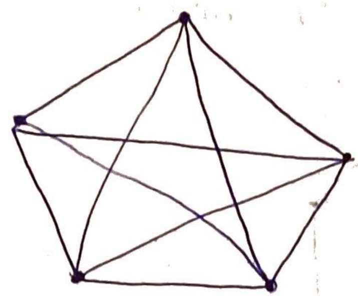
Now total number of vertices for $\overline{C_5} = 5$ & Total number of Edges for $\overline{C_5} = 5$

where F is bijection b/w vertex set and Edge set, which preserves adjacency.

$\therefore$ F is isomorphic

From above statments, we can say that C_5 graph and it's complement $\overline{C_5}$ is isomorphic. ($\because$ Hence proved).

2) Let us consider the graph K_5



Degree of each vertex $= 4$

Calculating the all edges with 5 vertexs in the graph K_5

$$\text{Total Edges} = 4+4+4+4+4$$

$$= 20$$

e = number of edges $= 10$ [$\because$ In K_5 graph]

v = number of vertexs $= 5$

Let us assume K_5 is Planar

$$v - e + r = 2$$

$$5 - 10 + r = 2$$

$$r = 2 + 5$$

$$\boxed{r = 7}$$

If a graph is a Planar, then

$$3r \leq 2e$$

$$3(7) \leq 2(10)$$

$$21 \leq 20 \quad \text{--- (1)}$$

consider (1) which is not true, we can say that our assumption is wrong, which clearly indicates graph K_5 is not Planar.

$\therefore K_5$ is not Planar.

3) Let us explain this by induction. The only tree on 2 vertices is K_2, which is can be clearly a bipartite.

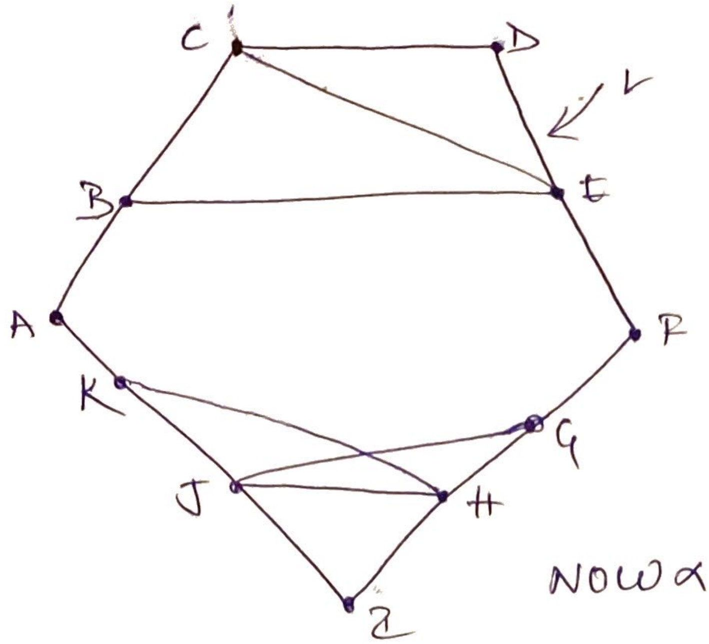
Now, Let us assume that every tree on 'n' vertices is a bipartite graph, where its vertexes set can be de-composed into two sets. Let T be a tree on 'n+1' vertices, we know that the vertex set of 'T' contains a leaf 'v'. Further more $T' = T - v$ is a tree on 'n' vertices by the induction hypothesis the vertex set of T' can be decomposed into disjoints sets X & Y.

Let 'w' be the neighbour of v in T. Then w is also a vertex in the graph T' without loss of generality (wlog), we can assume that $w \in X \subset V(T')$.

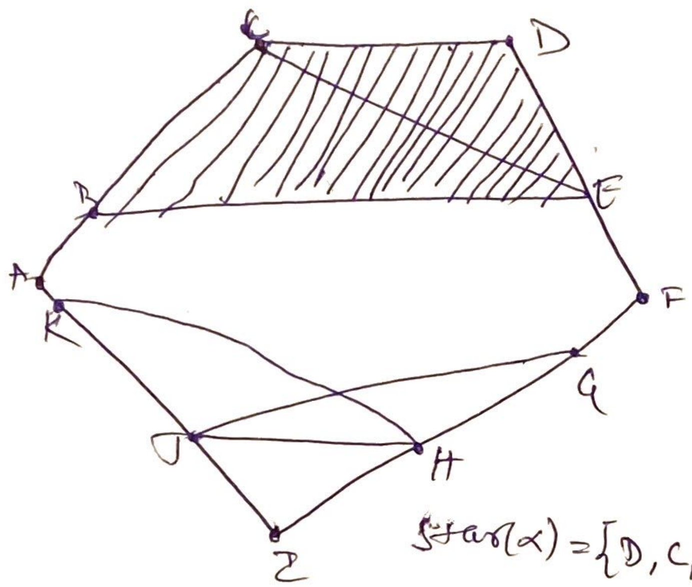
Then $X' = X$ & $Y' = Y \cup \{v\}$ is a bipartiton of $V(2)$ which it makes T as a bipartite graph.

i

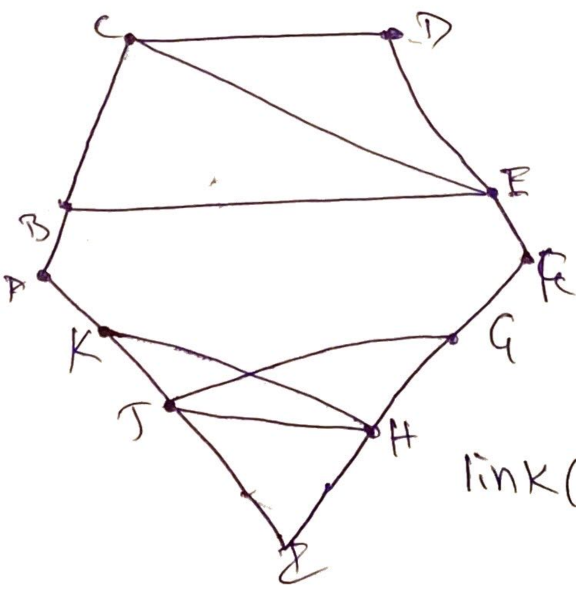
4) consider a Graph bleow



NOW $\alpha = \{D, E, \langle D,E\rangle\}$



$Star(\alpha) = \{D, C, E, B, \langle D,C\rangle, \langle D,E\rangle \langle E,B\rangle\}$



$link(\alpha) = \{B, C, \langle B,C\rangle\}$