AMAT-584
T-2

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1) p^{th} wasserstien distance is

$$d_p(B_1, B_2) = \left(\min_{\theta} P_p(\theta)\right)^{1/p}$$

where $P_p(\theta)$ is the p^{th} wasserstien penalty of θ $\therefore$

$$P_p(\theta) = \sum_{a \in A'} d(I_a, J_{\theta(a)})^p + \sum_{a \in A/A'} d(I_a, \Delta)^p + \sum_{b \in B/B'} d(J_b, \Delta)^p$$

$B_1 = \{I_a\}$, $B_2 = \{J_b\}$ be barcodes.

for a metric we have a property called 0-value property

$$d_p(B_1, B_2) = 0$$

then $\left(\min_{\theta} P_p(\theta)\right)^{1/p} = 0 \Rightarrow$ such that $P_p(\theta) = 0$

θ is bijection on $A' = A$ & $B' = B$ & the Bars matched are exactly same

p^{th} wasserstien penalty of θ ($P_p(\theta)$) can only be zero only when individual terms.

$d(I_a, J_{\theta(a)})$, $d(I_a, \Delta)$, $d(J_b, \Delta)$ are zero. which means Points a,b lie on diagonal.

which shows that bars are exactly same

$B_1 = B_2$ / $d_p(B_1, B_2) \geq 0$

Symmetry :-

we know $\theta : A' \to B'$ is a bijection

$$\theta' : B' \to A'$$

$$P_p(\theta) = P_p(\theta^{-1}) \Rightarrow \min_{\theta} P_p(\theta) = \min_{\theta} P_p(\theta^{-1}) \quad -(1)$$

Hence Wasserstein distance $d_P(\beta_1, \beta_2) = \min_\theta P_P(\theta)$

$$d_P(\beta_2, \beta_1) = \min_\theta P_P(\theta^{-1}) \rightarrow \text{from } 1$$

from this we can state that

$$d_P(\beta_1, \beta_2) = d_P(\beta_2, \beta_1)$$

$\therefore$ d_P is symmetry.

2) Gromov - Hausdorff distance b/w X & Y of metric space Z is

$$d_Z^H(X, Y) = \max(d_Z(X, Y), d_Z(Y, X))$$

$$d_Z(X, Y) = \max_{x \in X} d_Z(x, Y)$$

where $d_Z(x, Y) = \min_{y \in Y} d_Z(x, y)$

$$d_Z(Y, X) = \max_{y \in Y} d_Z(y, X)$$

where $d_Z(y, X) = \min_{x \in X} d_Z(x, y)$

when $d_Z^H(X, Y) = 0 \Rightarrow d_Z(X, Y) = d_Z(Y, X) = 0$

$d_Z(X, Y) = 0 \Rightarrow \max_{x \in X} d_Z(x, Y) = 0$ which implies for each $x \in X$

&

$\min_{y \in X} d_Z(x, y) = 0$ for any $x \in X$, Because Y is finite set, $y \in X$

such that $x = y$

from this we can tell that $x \subset y$ & $y \subset x$

Hence both are possible when $x = y$

$\therefore d_Z^H(X, Y) = 0$ iff $X = Y$

3) If F_1 & F_2 are topologies on X. $F_1 \subseteq F_2$ then we say that f_1 is weaker than F_2. The weaker topoly is course topology.

Given E be a straight line in $\mathbb{R}^2$ horizontal (or) vertical.

It can be defined in two ways

$x = a$ for $a \in \mathbb{R}$ for vertical line

$y = b$ for $b \in \mathbb{R}$ for horizontal line

The possible open sets can be

$\{(x,y) \mid x < a\} \to (-\infty, a) \times \mathbb{R}$, on topology $(\mathbb{R}^2, \tau)$

$\{(x,y) \mid x > a\} \to (a, \infty) \times \mathbb{R}$

$\{(x,y) \mid y > b\} \to \mathbb{R} \times (b, \infty)$

$\{(x,y) \mid y < b\} \to \mathbb{R} (-\infty, b)$

I be the corset topology on $\mathbb{R}^2$. which contains all open half spaces with vertical (or) horizontal lines.

Let $B = \{ (a, \infty) \times \mathbb{R}, (-\infty, a) \times \mathbb{R}, \mathbb{R} \times (b, \infty), \mathbb{R}(-\infty, b) \mid a, b \in \mathbb{R}\}$

The topology by B is coarset topology.

4) Suppose $\{x_i\}$ is a sequence in A that converges to point $P \in X$. By local finitness, we may choose an open neighbourhood U of P. which interacts only finitely many closed cells $e^1, \dots e^k$.

So, we may consider that if $i \geq K$ then $x_i \subset U$ & therefo $x_i \in e^1 \cup \dots \cup e^k$ since there are infinielty many $i > K$ but only finitely many $K = 1, \dots K$ there exists infinite sequ $i_1 < i_2 < i_3 \dots < i_k$ & $K \in \{1, \dots K\}$

such that $x_{i_j} \in e^{-K}$ $\forall i_j$

since the limit of the sequence equals limit of any sub sequence it follows that $P \in e^{-K}$

But $x_{i_j} \in A \cap e^{-k}$

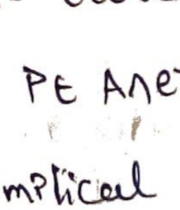
which is closed subset of e^{-K}

So $P \in A \cap e^{-k} \subset A$.

So, a simplical structure on a simplical complex is actually a complex decomposition.

Simlicial map implies that the image of all n-simplexes is contained in all n-simplexes. Then it is cellular

5) X



1-0 cell

2-1 cell

Y

2-0 cells

2-1 cells

The one point cell with endattalhed of the two differ-
-ent cells are possible cellular maps.

$x \oplus y$ is the product of x & y in category of simplical complexes maps to simplical complex.

$$x \oplus y \to x$$
$$x \oplus y \to y$$

Bijection complex with pair maps to simplicial complex

$$Z \to x,\ Z \to y$$

The cartesian product of spaces has 1 cell to same circle & 2-0 cell & 2-1 cells attalhed with end attached to 2 different 0 cell possible cellular.

A homotopy universal property in category of space

$$x \oplus y \to x[x \oplus y] \to y$$ induce the natural map.

$$S : [x \oplus y] \to |x| \times |y|$$

Unless x (or) y is discrete has only 0 simplix this map is far from nomorphism.

$$\Delta[i] + \Delta[j] \longrightarrow \Delta((i+1)(j+1)-1)$$

Projection of 1 all of same circle $|x| \times |y| \to [x \oplus y]$

On the product the topology $|x| \times |y|$ is corsetuan the union topology is not continuous maps x & y is locally finite.

The product $|x| \times |y|$ consides with the union topology

Analysis.