Assignment

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Q:2.1 Compute the simplical Betti numbers of the simplicial complex $K = \{1,2,3,4,5,6,7,(1,2),(2,3),(1,3),(1,4),(3,4),(2,4),(2,5),(5,6),(6,7),(5,7),(1,2,3),(1,2,4),(2,3,4),(1,3,4)\}$ a "a zipped up tent with a torn flag". Here you can think of the lists in parentheses as my choices for the orientations of corresponding simplices.

Sol:- $0 \to C_2 \to C_1 \to C_0 \to 0$

$\langle 1,2,3\rangle$	$\langle 1,2\rangle$	$\langle 1\rangle$
$\langle 1,2,4\rangle$	$\langle 2,3\rangle$	$\langle 2\rangle$
$\langle 2,3,4\rangle$	$\langle 1,3\rangle$	$\langle 3\rangle$
$\langle 1,3,4\rangle$	$\langle 1,4\rangle$	$\langle 4\rangle$
	$\langle 3,4\rangle$	$\langle 5\rangle$
	$\langle 2,4\rangle$	$\langle 6\rangle$
	$\langle 2,5\rangle$	$\langle 7\rangle$
	$\langle 5,6\rangle$	
	$\langle 6,7\rangle$	
	$\langle 5,7\rangle$	

$$Z_1 = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_0 = \left[\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right]$$

$$Z_1 = \left[\begin{bmatrix} 1\\1\\1\\1\\0\\0\\0\\0\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\0\\1\\1\\1\\0\\0\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\1\\0\\1\\0\\1\\0\\0\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\1\\1\\1 \end{bmatrix}, \right)$$

$$Z_0 = \left[\begin{bmatrix} 1\\0\\0\\0\\0\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\1\\0\\0\\0\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\1\\0\\0\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\0\\1\\0\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\0\\0\\1\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\0\\0\\0\\1\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\0\\0\\0\\0\\1 \end{bmatrix} \right)$$

$$B_0 = Z_0 \mid B_0 = \left[\begin{array}{ccccccc|ccccccc} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{cccccc|ccccccc} 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \right]$$

$$H_0 = \left[\begin{bmatrix} 1\\0\\0\\0\\0\\0\\0 \end{bmatrix} + B_0 \right]$$

$\beta_0 = 1$

$$Z_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \left[\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right], \quad Z = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$H_1 = \left[\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right] + \beta_1$$

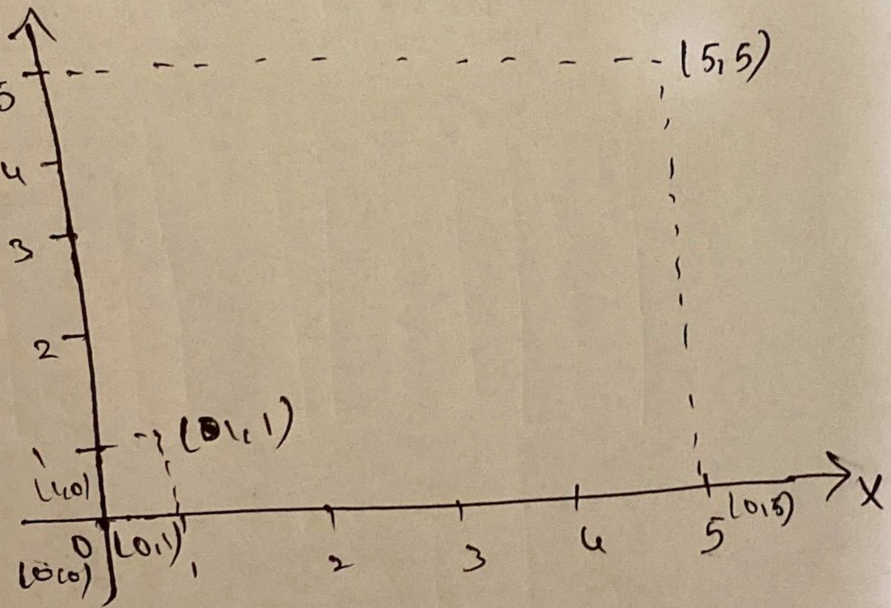
Betti number

$\therefore \boxed{\beta_0 = 1} \quad \boxed{\beta = 1} \quad \boxed{\beta_2 = 1}$

4.5.1 You will construct the persistent homology barcodes in dimensions 0 & 1 only (!) for the Rips complex filtration built from a finite set of points in $\mathbb{R}^2$. The metric we choose in $\mathbb{R}^2$ is the taxi-cab metric. For this problem doing the careful matching analysis of the births & deaths is not necessary since they can be observed from a sketch. You will need to compute homology of each stage in the filtration, then relate the homology computations via maps induced by inclusions.

Here is the sets of points: (0,0), (1,0), (0,1), (1,1), (0,5), (5,5) (5,0)

Sol:- Given points (0,0), (1,0), (0,1), (1,1), (0,5), (5,5), (5,0)



Barcode