## EXERCISE 5.6

BASIC

Sketch the following curves

1. $x = 0, y = \cos t, z = \sin t$
2. $x = \cos t, y = -1, z = \sin t$
3. $x = \sin t, y = \cos t, z = 1$

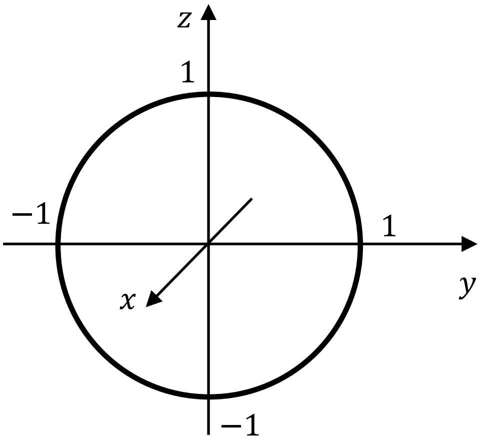
MEDIUM

4. $x = 2\cos^2 t, y = \sin t$
5. $x = 2\cos t, y = 1 + \sin^2 t$
6. $x = \sin t, y = \cos t, z = t$ for $t \geq 0$.
7. $x = \cos t, y = t, z = \sin t$ for $t \leq 0$.
8. $x = \sin t, y = \cos t, z = \sin t$
9. $x = \cos t, y = \sin t, z = \sin 3t$

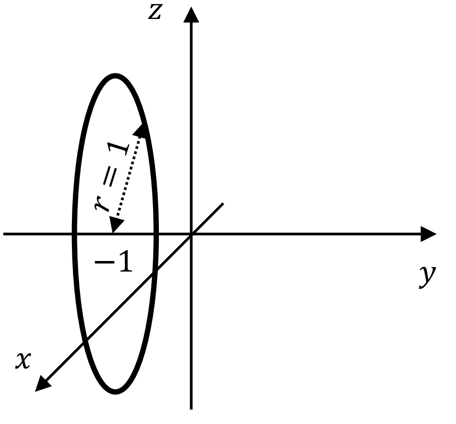
CHALLENGING

10. $x = \cos t, y = \sin 2t$
11. $x = t, y = t^2, z = t^2$

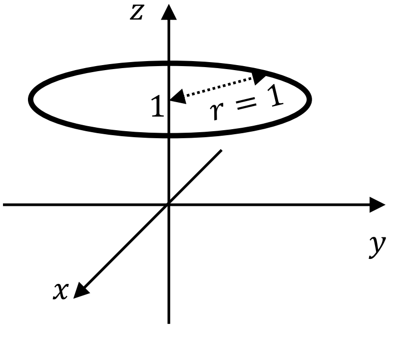
**1** We see that in the $yz$ plane this is the unit circle, with an $x$-value of zero, so it is a vertical unit circle.



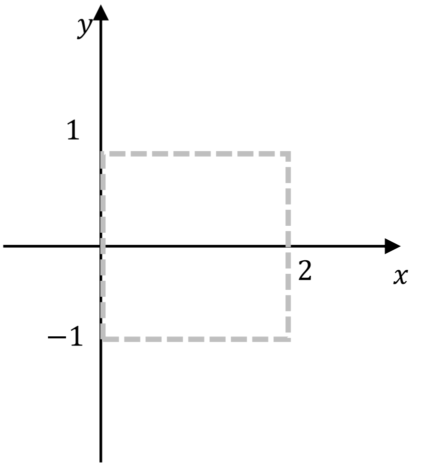

**2** We see that in the $xz$ plane this is the unit circle, but with $y = -1$, so it is a vertical unit circle but moved 1 unit to the left.




**3** Swapping $\cos t$ and $\sin t$ makes no difference to the final curve, so here we have a unit circle in the $xy$ plane, but moved 1 unit up.



**4** With this unknown curve we will start by considering the domain and range then use a table to plot some points. There no restrictions for $t$, though we note in the domain that $t^2 \geq 0$.

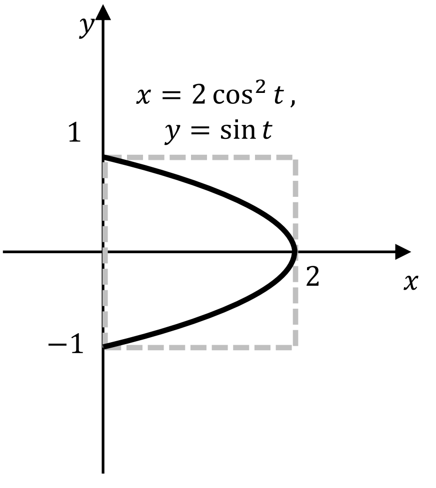


domain: $[0,2]$ since $0 \leq \cos^2 t \leq 1$
range: $[-1,1]$ since $-1 \leq \sin t \leq 1$

The sketch must fit within the square of side length 2 centred at $(1,0)$ as shown.

We then plot some positive values of $t$, making them multiples of $\pi$.

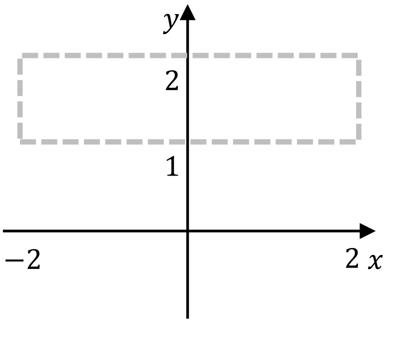
| $t$ | $x$ | $y$ |
|---|---|---|
| 0 | 2 | 0 |
| $\frac{\pi}{8}$ | 1.7 | 0.4 |
| $\frac{\pi}{4}$ | 1 | 0.7 |
| $\frac{3\pi}{8}$ | 0.3 | 0.9 |
| $\frac{\pi}{2}$ | 0 | 1 |



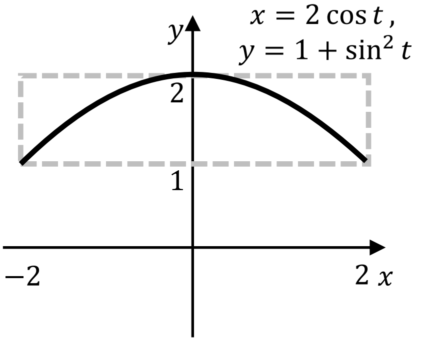

Plot these points, then also reflect them over the $x$-axis as $\sin t$ can be positive or negative, then draw a smooth curve through the points, filling the box.

**5** domain: $[-2,2]$ since $-1 \leq \cos t \leq 1$
range: $[1,2]$ since $0 \leq \sin^2 t \leq 1$

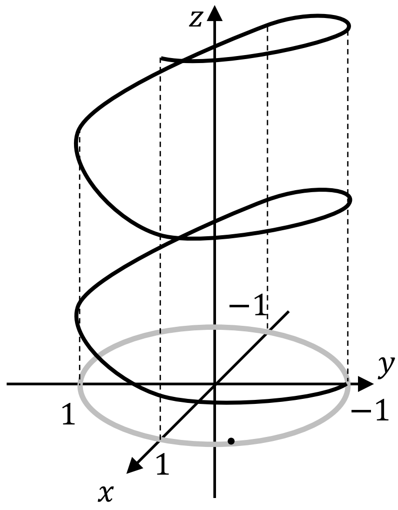
The sketch must fit within the rectangle as shown.



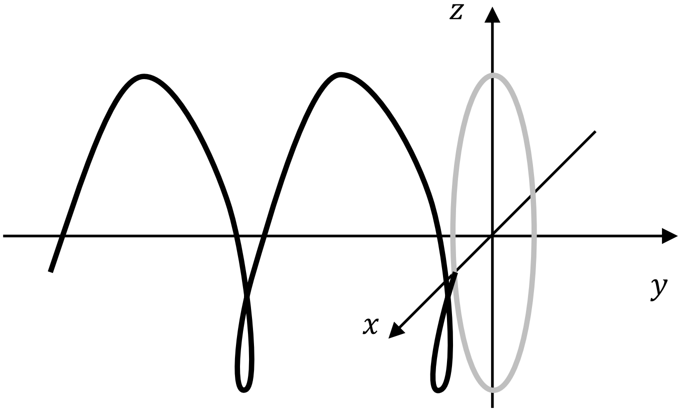
| $t$ | $x$ | $y$ |
|---|---|---|
| 0 | 2 | 1 |
| $\frac{\pi}{8}$ | 1.8 | 1.1 |
| $\frac{\pi}{4}$ | 1.4 | 1.5 |
| $\frac{3\pi}{8}$ | 0.8 | 1.9 |
| $\frac{\pi}{2}$ | 0 | 2 |



**6** Swapping sine and cosine causes the helix to start at $(0,1,0)$ an spiral in the opposite direction.

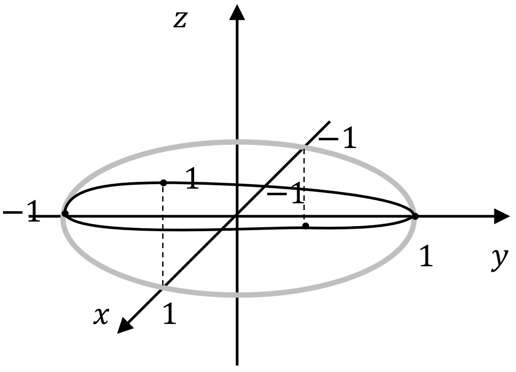



**7** The helix spirals to the left around the $y$-axis.



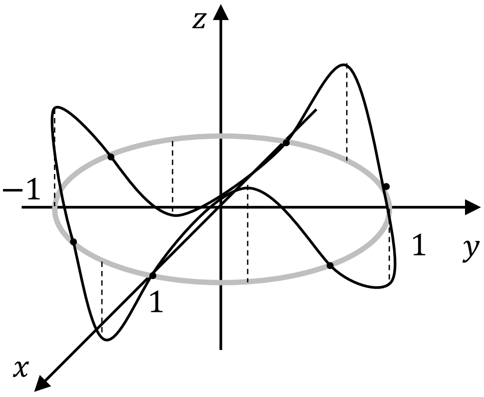

**8** As we move around the unit circle the elevation is 0 where it crosses the $y$-axis, and 1 or -1 where it crosses the $x$-axis.

The shape is an ellipse tilted $45°$ about the $y$-axis. By Pythagoras we could see that $a = \sqrt{2}$ and $b = 1$. Very difficult to see from this perspective!




**9** The elevation completes 3 cycles of the sine curve, with peaks at a height of 1 at $\frac{\pi}{6}, \frac{5\pi}{6}$ and $\frac{3\pi}{2}$, and troughs at $\frac{\pi}{2}, \frac{7\pi}{6}$ and $\frac{11\pi}{6}$. The two peaks will occur at $t = \frac{\pi}{4}, \frac{5\pi}{4}$ and the two troughs at $t = \frac{3\pi}{4}, \frac{7\pi}{4}$, and the elevation will be zero at $t = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ and $\frac{5\pi}{3}$.

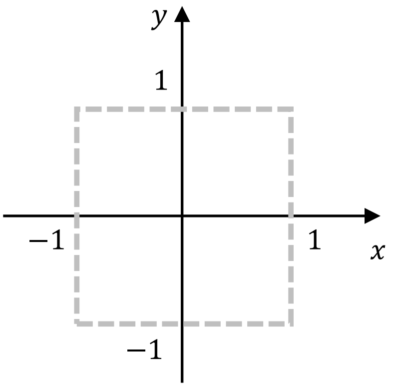
Viewed from the side we see the ABC logo.



**10** domain: $[-1,1]$ since $-1 \le \cos t \le 1$

range: $[-1,1]$ since $0 \le \sin 2t \le 1$

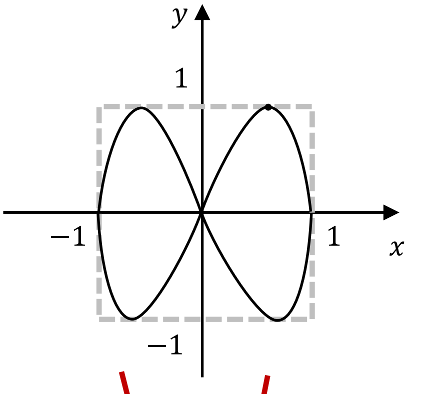
The sketch must fit within the rectangle as shown.



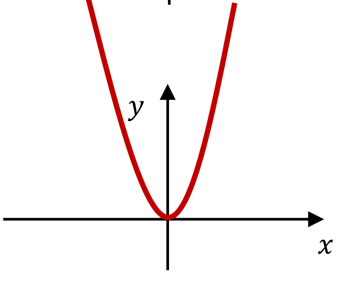

The $2$ in $2t$ indicates that there will be two humps in the Lissajous curve. Let $\mathrm{y} = 1 \;\rightarrow$

$\sin 2t = 1 \;\rightarrow\; t = \frac{\pi}{4} \;\rightarrow x = \cos\frac{\pi}{4} = 0.7.$

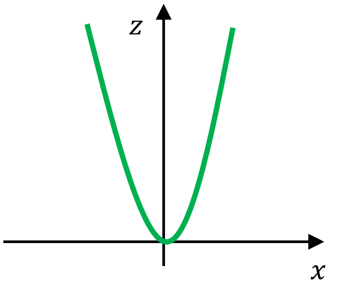
Arrange other humps symmetrically.




**11** Using $x = \mathrm{t}, y = t^2$ we can rearrange to get $y = x^2$. This means that in the $xy$ plane (so viewed from above) the curve is a parabola. Think of this like its shadow.




Using $x = t, z = t^2$ we can rearrange to get $\mathrm{z} = x^2$. This means that in the $xz$ plane (so viewed from the right) the curve is a concave up parabola with vertex at the origin.




So now we convert our two lines into three dimensional space. Remember that the first quadrant is now bottom right, so the parabola looks concave right.

So the only curve that would like a parabola from above and from the right is a parabola with its axis along the line $y = z$, shown in black.