EXERCISE 4.6

BASIC

Find the following indefinite integrals:

1 $\int \cos^3 x \ dx$

2 $\int \sec^4 x \, dx$

3 Prove $\int \operatorname{cosec} x \, dx = -\ln|\cot x + \operatorname{cosec} x| + c$

4 $\int \tan x \sec^3 x \, dx$

5 $\int (\cos 2x + \cos 4x)^2 \, dx$

6 $\int \cos^4 x \, dx$

7 $\int \cot^3 x \operatorname{cosec}^2 x \, dx$

MEDIUM

8 $\int \operatorname{cosec}^4 x \, dx$

9 $\int \sin 3x \sin 2x \, dx$

10 $\int \frac{\cos 2x}{\cos x} dx$

11 $\int \frac{\tan x}{\cos^2 x} dx$

12 $\int (\cos^4 x - \sin^4 x) \, dx$

13 $\int \sin mx \sin nx \, dx$ for positive integral m, n and $m \neq n$

CHALLENGING

14 $\int \frac{(\cos x + \sin x)^3}{\sin 2x + 1} dx$

15 Prove $\int \sin x \sin 2x \, dx = \frac{2\sin^3 x}{3} + c$

i Using the double angle results **ii** Using the Product to Sum results.

16 $\int \frac{\sqrt{1 + \sin x}}{\sec x} dx$

17 $\int \sqrt{1 + \sin x} \, dx$

1

$$\int \cos^3 x \ dx$$

$$= \int \cos^2 x \cos x \ dx$$

$$= \int (1 - \sin^2 x) \cos x \, dx$$

$$= \int \cos x \, dx - \int \cos x \sin^2 x \, dx$$

$$= \sin x - \frac{\sin^3 x}{3} + c$$

2

$$\int \sec^4 x \, dx$$

$$= \int \sec^2 x \sec^2 x \, dx$$

$$= \int \sec^2 x \, (\tan^2 x + 1) \, dx$$

$$= \int \sec^2 x \, (\tan x)^2 dx + \int \sec^2 x \, dx$$

$$= \frac{\tan^3 x}{3} + \tan x + c$$

3

$$\int \operatorname{cosec} x \, dx$$

$$= \int \operatorname{cosec} x \times \frac{\cot x + \operatorname{cosec} x}{\operatorname{cosec} x + \cot x} dx$$

$$= \int \frac{\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x}{\operatorname{cosec} x + \cot x} dx$$

$$= -\int \frac{-\operatorname{cosec} x \cot x - \operatorname{cosec}^2 x}{\operatorname{cosec} x + \cot x} dx$$

$$= -\ln|\cot x + \operatorname{cosec} x| + c$$

4

$$\int \tan x \sec^3 x \, dx$$

$$= \int \tan x \sec x \, (\sec x)^2 \, dx$$

$$= \frac{\sec^3 x}{3} + c$$

5

$$\int (\cos 2x + \cos 4x)^2 \, dx$$

$$= \int (\cos^2 2x + 2\cos 2x \cos 4x + \sin^2 4x) \, dx$$

$$= \int \left(\frac{1}{2}(1 + \cos 4x) + 2\left(\frac{1}{2}[\cos(4x + 2x) + \cos(4x - 2x)]\right) + \frac{1}{2}(1 + \cos 8x)\right) dx$$

$$= \int \left(1 + \frac{1}{2}\cos 4x + \cos 6x + \cos 2x + \frac{1}{2}\cos 8x\right) dx$$

$$= x + \frac{\sin 8x}{16} + \frac{\sin 6x}{6} + \frac{\sin 4x}{8} + \frac{\sin 2x}{2} + c$$

6

$$\int \cos^4 x\, dx$$

$$= \int (\cos^2 x)^2$$

$$= \int \left(\frac{1}{2}(1+\cos 2x)\right)^2 dx$$

$$= \frac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)\, dx$$

$$= \frac{1}{4}\int \left(1 + 2\cos 2x + \frac{1}{2}(1+\cos 4x)\right) dx$$

$$= \frac{1}{4}\left(x + \sin 2x + \frac{1}{2}\left(\frac{1}{4}\sin 4x + x\right)\right) + c$$

$$= \frac{x}{4} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + \frac{x}{2} + c$$

$$= \frac{\sin 4x}{32} + \frac{\sin 2x}{4} + \frac{3x}{8} + c$$

7

$$\int \cot^3 x \operatorname{cosec}^2 x\, dx$$

$$= -\int (-\operatorname{cosec}^2 x)(\cot x)^3\, dx$$

$$= \frac{\cot^4 x}{4} + c$$

8

$$\int \operatorname{cosec}^4 x\, dx$$

$$= \int \operatorname{cosec}^2 x \times \operatorname{cosec}^2 x\, dx$$

$$= \int \operatorname{cosec}^2 x\,(1 + \cot^2 x)\, dx$$

$$= \int (\operatorname{cosec}^2 x + \operatorname{cosec}^2 x \cot^2 x)\, dx$$

$$= -\cot x - \frac{\cot^3 x}{3} + c$$

9

$$\int \sin 3x \sin 2x\, dx$$

$$= \int \frac{1}{2}\Big[\cos(3x-2x) - \cos(3x+2x)\Big]\, dx$$

$$= \frac{1}{2}\int \Big[\cos x - \cos 5x\Big]\, dx$$

$$= \frac{\sin x}{2} - \frac{\sin 5x}{10} + c$$

10

$$\int \frac{\cos 2x}{\cos x}\, dx$$

$$= \int \frac{2\cos^2 x - 1}{\cos x}\, dx$$

$$= \int (2\cos x - \sec x)\, dx$$

$$= 2\sin x - \ln|\tan x + \sec x| + c$$

11

$$\int \frac{\tan x}{\cos^2 x}\, dx$$

$$= \int \tan x \sec^2 x\, dx$$

$$= \int \sec^2 x\,(\tan x)^1\, dx$$

$$= \frac{\tan^2 x}{2} + c$$

12

$$\int (\cos^4 x - \sin^4 x)\, dx$$

$$= \int (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)\, dx$$

$$= \int (\cos^2 x - \sin^2 x)\, dx$$

$$= \int \cos 2x\, dx$$

$$= \frac{1}{2}\sin 2x + c$$

13 $\int \sin mx \sin nx \, dx$

$= \int \frac{1}{2}\left[\cos(mx - nx) - \cos(mx + nx)\right] dx$

$= \int \frac{1}{2}\left[\cos(m - n)x - \cos(m + n)x\right] dx$

$= \frac{\sin(m - n)x}{2(m - n)} - \frac{\sin(m + n)x}{2(m + n)} + c$

14 $\int \frac{(\cos x + \sin x)^3}{\sin 2x + 1} dx$

$= \int \frac{(\cos x + \sin x)^3}{2 \sin x \cos x + \cos^2 x + \sin^2 x} dx$

$= \int \frac{(\cos x + \sin x)^3}{(\cos x + \sin x)^2} dx$

$= \int (\cos x + \sin x)\, dx$

$= \sin x - \cos x + c$

15 **i** $\int \sin x \sin 2x \, dx$

$= \int 2 \sin^2 x \cos x \, dx$

$= 2 \int (\sin x)^2 \times \frac{d}{dx}(\sin x)\, dx$

$= \frac{2 \sin^3 x}{3} + c$

ii $\int \sin x \sin 2x \, dx$

$= \int \frac{1}{2}\left[\cos(2x - x) - \cos(2x + x)\right] dx$

$= \frac{1}{2}\int \left[\cos x - \cos 3x\right] dx$

$= \frac{1}{2}\sin x - \frac{1}{6}\sin 3x + c$

$= \frac{1}{2}\sin x - \frac{1}{6}(\sin 2x \cos x + \cos 2x \sin x) + c$

$= \frac{1}{2}\sin x - \frac{1}{6}(2 \sin x \cos^2 x + \sin x - 2\sin^3 x) + c$

$= \frac{1}{2}\sin x - \frac{1}{3}\sin x\,(1 - \sin^2 x) - \frac{1}{6}\sin x + \frac{1}{3}\sin^3 x + c$

$= \frac{1}{2}\sin x - \frac{1}{3}\sin x + \frac{1}{3}\sin^3 x - \frac{1}{6}\sin x + \frac{1}{3}\sin^3 x + c$

$= \frac{2 \sin^3 x}{3} + c$

16 $\int \frac{\sqrt{1 + \sin x}}{\sec x} dx$

$= \int \cos x\,(1 + \sin x)^{\frac{1}{2}}\, dx$

$= \frac{2}{3}(1 + \sin x)^{\frac{3}{2}} + c$

$= \frac{2}{3}\sqrt{(1 + \sin x)^3} + c$

17 $\int \sqrt{1 + \sin x}\, dx$

$= \int \sqrt{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} + \sin 2\left(\frac{x}{2}\right)}\, dx$

$= \int \sqrt{\cos^2 \frac{x}{2} + 2 \cos \frac{x}{2} \sin \frac{x}{2} + \sin^2 \frac{x}{2}}\, dx$

$= \int \sqrt{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2}\, dx$

$= \int \left(\cos \frac{x}{2} + \sin \frac{x}{2}\right) dx$

$= 2 \sin \frac{x}{2} - 2 \cos \frac{x}{2} + c$