FIRST EDUCATION

2024 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time - 11 minutes
- Working time - 184 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number on the Question 13 Writing Booklet attached

Total marks: 102

Question 1: ME1 HSC 2013 1 **1**

The polynomial $P(x) = x^3 - 4x^2 - 6x + k$ has a factor $x - 2$.

What is the value of k?

(A) 2
(B) 12
(C) 20
(D) 36

Question 2: ME1 HSC 2015 1 **1**

What is the remainder when $x^3 - 6x$ is divisble by $x + 3$?

(A) -9
(B) 9
(C) $x^2 - 2x$
(D) $x^2 - 3x + 3$

Question 3: ME1 HSC 2017 1 **1**

Which polynomial is a factor of $x^3 - 5x^2 + 11x - 10$?

(A) $x - 2$
(B) $x + 2$
(C) $11x - 10$
(D) $x^2 - 5x + 11$

Question 4: ME2 HSC 2016 2 **1**

Which polynomial has a multiple root at $x = 1$?

(A) $x^5 - x^4 - x^2 + 1$
(B) $x^5 - x^4 - x - 1$
(C) $x^5 - x^3 - x^2 + 1$
(D) $x^5 - x^3 - x + 1$

Question 5: ME1 HSC 2016 2 **1**

What is the remainder when $2x^3 - 10x^2 + 6x + 2$ is divided by $x - 2$?

(A) -66
(B) -10
(C) $-x^3 + 5x^2 - 3x - 1$
(D) $x^3 - 5x^2 + 3x + 1$

Question 6: MA HSC 2012 3 **1**

The quadratic equation $x^2+3x-1=0$ has roots α and β.

What is the value of $\alpha\beta+(\alpha+\beta)$?

(A) 4
(B) 2
(C) -4
(D) -2

Question 7: ME1 HSC 2012 3 **1**

A polynomial equation has roots α, β and γ where

$$\alpha+\beta+\gamma=-2,\ \alpha\beta+\alpha\gamma+\beta\gamma=3 \text{ and } \alpha\beta\gamma=1.$$

Which polynomial equation has the roots α, β and γ?

(A) $x^3+2x^2+3x+1=0$
(B) $x^3+2x^2+3x-1=0$
(C) $x^3-2x^2+3x+1=0$
(D) $x^3-2x^2+3x-1=0$

Question 8: ME1 HSC 2022 3 **1**

Let $P(x)$ be a polynomial of degree 5. When $P(x)$ is divided by the polynomial $Q(x)$, the remainder is $2x+5$.

Which of the following is true about the degree of Q?

A. The degree must be 1.
B. The degree could be 1.
C. The degree must be 2.
D. The degree could be 2.

Question 9: ME2 HSC 2019 4 **1**

The polynomial $2x^3+bx^2+cx+d$ has roots 1 and -3, with one of them being a double root.

What is a possible value of b?

(A) -10
(B) -5
(C) 5
(D) 10

Question 10: ME2 HSC 2012 5 **1**

The equation $2x^3 - 3x^2 - 5x - 1 = 0$ has roots α β and γ.

What is the value of $\dfrac{1}{\alpha^3\beta^3\gamma^3}$?

(A) $\dfrac{1}{8}$

(B) $-\dfrac{1}{8}$

(C) 8

(D) -8

Question 11: ME1 HSC 2014 5 **1**

Which group of three numbers could be the roots of the polynomial equation
$x^3 + ax^2 - 41x + 42 = 0$?

(A) $2, 3, 7$

(B) $1, -6, 7$

(C) $-1, -2, 21$

(D) $-1, -3, -14$

Question 12: ME1 HSC 2019 7 **1**

Let $P(x) = qx^3 + rx^2 + rx + q$ where q and r are constants, q $\neq 0$. One of the zeros of $P(x)$ is -1.

Given that α is a zero $P(x)$, $\alpha \neq -1$, which of the following is also a zero?

(A) $-\dfrac{1}{\alpha}$

(B) $-\dfrac{q}{\alpha}$

(C) $\dfrac{1}{\alpha}$

(D) $\dfrac{q}{\alpha}$

Question 13: ME1 HSC 2014 9 **1**

The remainder when the polynomial $P(x) = x^4 - 8x^3 - 7x^2 + 3$ is divided by $x^2 + x$ is $ax + 3$.

What is the value of a?

(A) -14
(B) -11
(C) -2
(D) 5

Question 14: ME1 HSC 2008 1 (a) **2**

The polynomial x^3 is divided by $x + 3$. Calculate the remainder.

Question 15: ME1 HSC 2001 1 (e) **3**

Is $x + 3$ a factor of $x^3 - 5x + 12$? Give reasons for your answer.

Question 16: ME1 HSC 2011 2 (a) **3**

Let $P(x) = x^3 - ax^2 + x$ be a polynomial, where a is a real number.

When $P(x)$ is divided by $x - 3$ the remainder is 12.

Find the remainder when $P(x)$ is divided by $x + 1$.

Question 17: MA HSC 2011 2 (a) (i) **1**

The quadratic equation $x^2 - 6x + 2 = 0$ has roots α and β.

Find $\alpha + \beta$.

Question 18: MA HSC 2011 2 (a) (ii) **1**

The quadratic equation $x^2 - 6x + 2$ has roots α and β.

Find $\alpha\beta$.

Question 19: MA HSC 2011 2 (a) (iii) **1**

The quadratic equation $x^2 - 6x + 2$ has roots α and β.

From part (i), you have shown that $\alpha + \beta = 6$.

From part (ii), you have shown that $\alpha\beta = 2$.

Find $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$.

Question 20: ME1 HSC 2007 2 (c) **3**

The polynomial $P(x) = x^2 + ax + b$ has a zero $x = 2$. When $P(x)$ is divided by $x + 1$, the remainder is 18.

Find the values of a and b.

Question 21: ME1 HSC 2008 2 (c) **3**

The polynomial $p(x)$ is given by $p(x) = ax^3 + 16x^2 + cx - 120$, where a and c are constants.

The three zeros of $p(x)$ are -2, 3 and α.

Find the value of α.

Question 22: ME1 HSC 2010 2 (c) (i) **1**

Let

$$P(x) = (x+1)(x-3)Q(x) + ax + b,$$

where $Q(x)$ is a polynomial and a and b are real numbers. The polynomial $P(x)$ has a factor of $x - 3$.

When $P(x)$ is divided by $x + 1$ the remainder is 8.

Find the values of a and b.

Question 23: ME1 HSC 2010 2 (c) (ii) **2**

Let

$$P(x) = (x+1)(x-3)Q(x) + ax + b,$$

where $Q(x)$ is a polynomial and a and b are real numbers. The polynomial $P(x)$ has a factor of $x-3$.

When $P(x)$ is divided by $x+1$ the remainder is 8.

From part (i), you have found that $a=-2,\ b=6$.

Find the remainder when $P(x)$ is divided by $(x+1)(x-3)$.

Question 24: ME1 HSC 2004 3 (b) (i) **1**

Let $P(x) = (x+1)(x-3)Q(x) + a(x+1) + b,$ where $Q(x)$ is a polynomial and a and b are real numbers.

When $P(x)$ is divided by $(x+1)$ the remainder is -11.
When $P(x)$ is divided by $(x-3)$ the remainder is 1.

What is the value of b?

Question 25: ME2 HSC 2008 3 (b) (i) **1**

Let $p(z) = 1 + z^2 + z^4$.

Show that $p(z)$ has no real zeros.

Question 26: ME1 HSC 2004 3 (b) (ii) **2**

Let $P(x) = (x+1)(x-3)Q(x) + a(x+1) + b,$ where $Q(x)$ is a polynomial and a and b are real numbers.

When $P(x)$ is divided by $(x+1)$ the remainder is -11.
When $P(x)$ is divided by $(x-3)$ the remainder is 1.

From part (i), you have found that the value of b is -11.

What is the remainder when $P(x)$ is divided by $(x+1)(x-3)$?

Question 27: ME1 HSC 2006 4 (a) (i) **1**

The cubic polynomial $P(x) = x^3 + rx^2 + sx + t,$ where r, s and t are real numbers, has three real zeros, 1, α and -α.

Find the value of r.

Question 28: ME1 HSC 2006 4 (a) (ii) **2**

The cubic polynomial $P(x) = x^3 + rx^2 + sx + t$, where r, s and t are real numbers, has three real zeros, 1, α and -α.

From part (i), you have shown that $r = -1$ find the value of $s + t$

Question 29: ME2 HSC 2004 4 (a) (ii) **2**

Let α, β, and γ be the zeros of the polynomial $p(x) = 3x^3 + 7x^2 + 11x + 51$.

From part (i), you found that $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2 = \frac{119}{3}$.

Find $\alpha^2 + \beta^2 + \gamma^2$.

Question 30: ME1 HSC 2002 4 (b) (i) **1**

The polynomial $P(x) = x^3 - 2x^2 + kx + 24$ has roots α, β, γ.

Find the value of $\alpha + \beta + \gamma$.

Question 31: ME1 HSC 2002 4 (b) (ii) **1**

The polynomial $P(x) = x^3 - 2x^2 + kx + 24$ has roots α, β, γ.

Find the value for $\alpha\beta\gamma$.

Question 32: ME1 HSC 2002 4 (b) (iii) **2**

The polynomial $P(x) = x^3 - 2x^2 + kx + 24$ has roots α, β, γ.

From part (i), you have found that $\alpha + \beta + \gamma = 2$.

From (ii), you have found that $\alpha\beta\gamma = -24$.

It is known that two of the roots are equal in magnitude but opposite in sign.

Find the third root and hence find a value of k.

Question 33: ME2 HSC 2002 5 (a) **2**

The equation $4x^3 - 27x + k = 0$ has a double root. Find the possible values of k.

Question 34: ME2 HSC 2008 5 (b) (i) **2**

Let $p(x) = x^{n+1} - (n+1)x + n$ where n is a positive integer.

Show that $p(x)$ has a double zero at $x = 1$.

Question 35: ME2 HSC 2008 5 (b) (iii) **2**

Let $p(x) = x^{n+1} - (n+1)x + n$ where n is a positive integer.

From part (i), you showed that $p(x)$ has a double zero at $x = 1$.

From part (ii), you showed that $p(x) \geq 0$ for $x \geq 0$.

Factorise $p(x)$ when $n = 3$.

Question 36: ME2 HSC 2009 6 (b) (i) **1**

Let $P(x) = x^3 + qx^2 + qx + 1$, where q is real. One zero of $P(x)$ is -1.

Show that if α is a zero of $P(x)$ then $\dfrac{1}{\alpha}$ is a zero of $P(x)$.

Question 37: ME2 HSC 2010 6 (c) (iv) **1**

From part (i), you found that

$$(\cos\theta + i\sin\theta)^5 = \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta.$$

From part (ii), you showed that $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$.

From part (iii), you deduced that $x = \sin\left(\dfrac{\pi}{10}\right)$ is one of the solutions to $16x^5 - 20x^3 + 5x - 1 = 0$.

Find the polynomial $p(x)$ such that $(x-1)p(x) = 16x^5 - 20x^3 + 5x - 1$.

Question 38: ME2 HSC 2010 6 (c) (vi) **1**

From part (i), you found that ◁

$$\left(\cos\theta + i\sin\theta\right)^5 = \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta.$$

From part (ii), you showed that $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta.$

From part (iii), you deduced that $x = \sin\left(\frac{\pi}{10}\right)$ is one of the solutions to $16x^5 - 20x^3 + 5x - 1 = 0.$

From part (iv), you found that the polynomial $p(x)$ such that $(x-1)\,p(x) = 16x^5 - 20x^3 + 5x - 1$ is $p(x) = 16x^4 + 16x^3 - 4x^2 - 4x + 1.$

From part (v), you found that $p(x) = \left(4x^2 + 2x - 1\right)^2.$

Hence find an exact value for $\sin\left(\frac{\pi}{10}\right)$

Question 39: MA HSC 2006 7 (a) (i) **1**

Let α and β be the solutions of $x^2 - 3x + 1 = 0.$

Find $\alpha\beta.$

Question 40: MA HSC 2006 7 (a) (ii) **1**

Let α and β be the solutions of $x^2 - 3x + 1 = 0.$

From (i), you have found that $\alpha\beta = 1.$

Hence find $\alpha + \frac{1}{\alpha}.$

Question 41: ME2 HSC 2001 7 (b) (iii) **1**

Consider the equation $x^3 - 3x - 1 = 0,$ which we denote by (*).

Show that one root of (*) is $2\cos\frac{\pi}{9}.$

(You may assume the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta.$)

Question 42: ME2 HSC 2010 7 (c) (i) **1**

Let $P(x)=(n-1)x^n-nx^{n-1}+1$, where n is an odd integer, $n\geq 3$.

Show that $P(x)$ has exactly two stationary points.

Question 43: ME2 HSC 2010 7 (c) (ii) **1**

Let $P(x)=(n-1)x^n-nx^{n-1}+1$, where n is an odd integer, $n\geq 3$.

From part (i), you showed that $P(x)$ has exactly two stationary points at $x=0$ and $x=1$.

Show that $P(x)$ has a double zero at $x=1$.

Question 44: ME2 HSC 2010 7 (c) (iii) **2**

Let $P(x)=(n-1)x^n-nx^{n-1}+1$, where n is an odd integer, $n\geq 3$.

From part (i), you showed that $P(x)$ has exactly two stationary points at $x=0$ and $x=1$.

From part (ii), you showed that $P(x)$ has a double zero at $x=1$.

Use the graph $y=P(x)$ to explain why $P(x)$ has exactly one real zero other than 1.

Question 45: ME1 HSC 2013 11 (a) **1**

Find the polynomial equation $2x^3-3x^2-11x+7=0$ has roots α, β and γ.

Find $\alpha\beta\gamma$.

Question 46: ME1 HSC 2020 11 (a) (i) **1**

Let $P(x)=x^3+3x^2-13x+6$.

Show that $P(2)=0$.

Question 47: ME1 HSC 2018 11 (a) (i) **1**

Consider the polynomial $P(x)=x^3-2x^2-5x+6$.

Show that $x=1$ is a zero of $P(x)$.

Question 48: ME1 HSC 2018 11 (a) (ii) **2**

Consider the polynomial $P(x) = x^3 - 2x^2 - 5x + 6$.

From part (i), you showed that $x = 1$ is a zero of $P(x)$.

Find the other zeros.

Question 49: ME1 HSC 2020 11 (a) (ii) **2**

Let $P(x) = x^3 + 3x^2 - 13x + 6$.

In part (i), you showed that $P(2) = 0$.

Hence, factor the polynomial $P(x)$ as $A(x)B(x)$, where $B(x)$ is a quadratic polynomial.

Question 50: ME2 HSC 2018 11 (b) **3**

The polynomial $p(x) = x^3 + ax^2 + b$ has a zero at r and a double zero at 4. Find the values of a, b and r.

Question 51: ME1 HSC 2019 11 (d) **2**

Find the polynomial $Q(x)$ that satisfies $x^3 + 2x^2 - 3x - 7 = (x-2)Q(x) + 3$.

Question 52: ME1 HSC 2015 11 (f) (i) **1**

Consider the polynomials $P(x) = x^3 - kx^2 + 5x + 12$ and $A(x) = x - 3$.

Given that $P(x)$ is divisible by $A(x)$, show that $k = 6$.

Question 53: ME1 HSC 2015 11 (f) (ii) **2**

Consider the polynomials $P(x) = x^3 - kx^2 + 5x + 12$ and $A(x) = x - 3$.

From part (i), you showed that $k = 6$, given that $P(x)$ is divisible by $A(x)$.

Find all the zeros of $P(x)$ when $k = 6$.

Question 54: ME1 HSC 2021 11 (h) **2**

The roots of $x^4 - 3x + 6 = 0$ are α, β, γ and δ.

What is the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta}$?

Question 55: ME2 HSC 2014 12 (b) (i) **1**

It can be shown that $4\cos^3\theta - 3\cos\theta = \cos 3\theta$ (Do NOT prove this).
Assume that $x = 2\cos\theta$ is a solution of $x^3 - 3x = \sqrt{3}$.

Show that $\cos 3\theta = \dfrac{\sqrt{3}}{2}$.

Question 56: ME2 HSC 2014 12 (b) (ii) **2**

It can be shown that $4\cos^3\theta - 3\cos\theta = \cos 3\theta$ (Do NOT prove this).
Assume that $x = 2\cos\theta$ is a solution of $x^3 - 3x = \sqrt{3}$.

From part (i), you showed that $\cos 3\theta = \dfrac{\sqrt{3}}{2}$.

Hence, or otherwise, find the three real solutions of $x^3 - 3x = \sqrt{3}$.

Question 57: ME2 HSC 2017 12 (d) (i) **2**

Let $P(x)$ be a polynomial.

Given that $(x-\alpha)^2$ is a factor of $P(x)$, show that

$$P(\alpha) = P'(\alpha) = 0.$$

Question 58: ME2 HSC 2017 12 (d) (ii) **2**

Let $P(x)$ be a polynomial.

From part (i), you showed that $P(\alpha) = P'(\alpha) = 0$.

Given that the polynomial $P(x) = x^4 - 3x^3 + x^2 + 4$ has a factor $(x-\alpha)^2$, find the value of α.

Question 59: ME2 HSC 2017 13 (b) (i) **2**

Let a, b and c be real numbers. Suppose that $P(x) = x^4 + ax^3 + bx^2 + cx + 1$ has roots
$\alpha,\ \dfrac{1}{\alpha},\ \beta,\ \dfrac{1}{\beta}$, where $\alpha > 0$ and $\beta > 0$.

Prove that $a = c$.

Question 60: ME1 HSC 2022 13 (d) **3**

The monic polynomial, P, has degree 3 and roots α, β, γ.

It is given that

$$\alpha^2+\beta^2+\gamma^2=85 \text{ and}$$
$$P'(\alpha)+P'(\beta)+P'(\gamma)=87.$$

Find $\alpha\beta+\beta\gamma+\gamma\alpha$.

Question 61: ME2 HSC 2016 13 (d) (ii) **2**

Suppose $p(x)=ax^3+bx^2+cx+d$ with a, b, c and d real, $a\neq 0$.

From part (i), you deduced that if $b^2-3ac<0$ then $p(x)$ cuts the x-axis only once.

If $b^2-3ac=0$ and $p\left(-\frac{b}{3a}\right)=0$, what is the multiplicity of the root $x=-\frac{b}{3a}$?

Question 62: ME2 HSC 2015 14 (b) (i) **1**

The cubic equation $x^3-px+q=0$ has roots α, β and γ.

It is given that $\alpha^2+\beta^2+\gamma^2=16$ and $\alpha^3+\beta^3+\gamma^3=-9$.

Show that $p=8$.

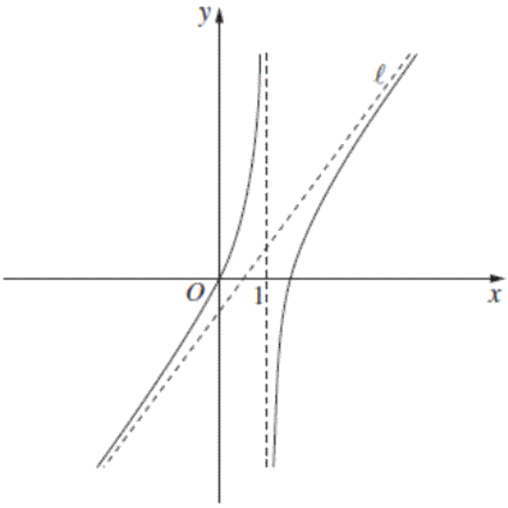
Question 63: MA HSC 2014 14 (b) (i) **1**

The roots of the quadratic equation $2x^2+8x+k=0$ are α and β.

Find the value of $\alpha+\beta$.

Question 64: ME2 HSC 2012 14 (b) (ii) **2**

The diagram shows the graph $y = \frac{x(2x-3)}{x-1}$. The line ℓ is an asymptote.



By writing $\frac{x(2x-3)}{x-1}$ in the form $mx + b + \frac{a}{x-1}$, find the equation of the line ℓ.

Question 65: MA HSC 2014 14 (b) (ii) **2**

The roots of the quadratic equation $2x^2 + 8x + k = 0$ are α and β.

From part (i), you have found that $\alpha + \beta = -4$.

Given that $\alpha^2\beta + \alpha\beta^2 = 6$, find the value of k.

Question 66: ME2 HSC 2013 15 (b) (i) **2**

The polynomial $P(x) = ax^4 + bx^3 + cx^2 + e$ has remainder -3 when divided by $x-1$.
The polynomial has a double root at $x = -1$.

Show that $4a + 2c = -\frac{9}{2}$.

Question 67: ME2 HSC 2013 15 (b) (ii) **1**

The polynomial $P(x) = ax^4 + bx^3 + cx^2 + e$ has remainder -3 when divided by $x-1$.
The polynomial has a double root at $x = -1$.

From part (i), you showed that $4a + 2c = -\frac{9}{2}$.

Hence, or otherwise, find the slope of the tangent to the graph $y = P(x)$ when $x = 1$.

FIRST EDUCATION

2024 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1 - Solutions

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Total marks: 102

Solution 1: ME1 HSC 2013 1 **1**

If $P(x)$ has a factor of $x-2$,
$\therefore\ P(2)=0$

$P(2)=8-16-12+k=0$
$\therefore k=20.$

The answer is (C).

Solution 2: ME1 HSC 2015 1 **1**

$$
\begin{aligned}
P(x) &= x^3-6x \\
P(-3) &= (-3)^3-6(-3) \\
&= -27+18 \\
&= -9
\end{aligned}
$$

$\therefore$ Answer is (A).

Solution 3: ME1 HSC 2017 1 **1**

$$
\begin{aligned}
P(x) &= x^3-5x^2+11x-10 \\
P(2) &= (2)^3-5(2)^2+11(2)-10 \\
&= 8-20+22-10 \\
&= 0
\end{aligned}
$$

$\therefore x-2$ is a factor.

$\therefore$ Answer is (A).

Solution 4: ME2 HSC 2016 2 **1**

All polynomials except (B) satisfy $p(1)=0$.

For (A), $p'(x)=5x^4-4x^3-2x$
$p'(1)=-1\neq 0$

For (C), $p'(x)=5x^4-4x^3-1$
$p'(1)=0$

For (D), $p'(x)=5x^4-3x^2-1$
$p'(1)=1\neq 0$

$\therefore$ Answer is (C).

Solution 5: ME1 HSC 2016 2 **1**

$$P(x) = 2x^3 - 10x^2 + 6x + 2$$
$$P(2) = 2(2)^3 - 10(2)^2 + 6(2) + 2$$
$$= -10$$

$\therefore$ Answer is (B).

Solution 6: MA HSC 2012 3 **1**

$$\alpha\beta = \frac{c}{a}$$
$$= \frac{-1}{1}$$
$$= -1$$

$$\alpha + \beta = -\frac{b}{a}$$
$$= -\frac{3}{1}$$
$$= -3$$

$$\therefore \alpha\beta + (\alpha + \beta) = -1 + -3$$
$$= -4$$

Answer (C).

Solution 7: ME1 HSC 2012 3 **1**

Polynomial with α, β and γ

$x^3 - \sum \alpha . x^2 + \sum \alpha\beta . x - \text{Product of } \alpha.$

The answer is (B).

Solution 8: ME1 HSC 2022 3 **1**

$P(x)$ degree 5.

$\therefore$ Remainder up to degree 4.

If remainder is $2x + 5$ (linear), degree of $Q(x)$ not necessarily 2.

eg. $Q(x) = x^3$ and $P(x) = x^5 + 2x + 5$

$$= x^3\left(x^2 + 0x + 0\right) + 0x^2 + 2x + 5$$

$\therefore$ Degree could be 2.

$\therefore$ Answer is D.

Solution 9: ME2 HSC 2019 4 **1**

Roots can either be $1, 1, -3$ or $1, -3, -3$.

$$\text{Sum of roots} = -\frac{b}{2}$$

$$\therefore -\frac{b}{2} = 1+1-3 \text{ or } \qquad -\frac{b}{2} = 1-3-3$$
$$b = 2 \qquad\qquad b = 10$$

$\therefore$ Answer is (D).

Solution 10: ME2 HSC 2012 5 **1**

$$\begin{aligned}\frac{1}{\alpha^3\beta^3\gamma^3} &= \frac{1}{(\alpha\beta\gamma)^3} \\ &= \frac{1}{\left(-\frac{d}{a}\right)^3} \\ &= \frac{1}{\left(-\frac{-1}{2}\right)^3} \\ &= 8\end{aligned}$$

$\therefore$ Answer is C.

Solution 11: ME1 HSC 2014 5 **1**

Let the roots of the polynomial be α, β, γ.

$$\begin{aligned}\alpha\beta\gamma &= -\frac{d}{a} \\ &= -42\end{aligned}$$

$\therefore$ Can only be (B) or (D).

$$\begin{aligned}\alpha\beta + \alpha\gamma + \beta\gamma &= \frac{c}{a} \\ &= -41\end{aligned}$$

$\therefore$ Roots are $-1, -3, -14$.

$\therefore$ Answer is (B).

Solution 12: ME1 HSC 2019 7 **1**

$P(x) = qx^3 + rx^2 + rx + q$

Let the roots of $P(x)$ be $-1, \alpha, \beta$.

$$(-1)\alpha\beta = -\frac{q}{q}$$
$$\alpha\beta = 1$$
$$\therefore \beta = \frac{1}{\alpha}$$

$\therefore$ Answer is (C).

Solution 13: ME1 HSC 2014 9 **1**

$$\begin{aligned} P(x) &= x^4 - 8x^3 - 7x^2 + 3 \\ &= (x^2 + x)Q(x) + ax + 3 \\ &= x(x+1)Q(x) + ax + 3 \end{aligned}$$

$$\begin{aligned} P(-1) &= (-1)^4 - 8(-1)^3 - 7(-1)^2 + 3 \\ &= 1 + 8 - 7 + 3 \\ &= 5 \end{aligned}$$

$$a(-1) + 3 = 5$$
$$\therefore a = -2$$

$\therefore$ Answer is (C).

Solution 14: ME1 HSC 2008 1 (a) **2**

$P(x) = x^3$

$$\begin{aligned} R(x) &= P(-3) \\ &= (-3)^3 \\ &= -27 \end{aligned}$$

Solution 15: ME1 HSC 2001 1 (e) **3**

Let $P(x) = x^3 - 5x + 12$

$$\begin{aligned} P(-3) &= (-3)^3 - 5(-3) + 12 \\ &= -27 + 15 + 12 \\ &= 0 \end{aligned}$$

$\therefore x + 3$ is a factor of $x^3 - 5x + 12$ by the factor theorem.

Solution 16: ME1 HSC 2011 2 (a) **3**

$P(3)=12$

$$27-9a+3=12$$
$$\therefore a=2$$
$$\therefore P(x)=x^3-2x^2+x$$

$$\begin{aligned} P(-1)&=-1-2-1 \\ &=-4 \end{aligned}$$

$\therefore$ The remainder when $P(x)$ is divided by $x+1$ is -4

Solution 17: MA HSC 2011 2 (a) (i) **1**

$$\begin{aligned} \alpha+\beta &= \frac{-b}{a} \\ &= -\frac{-6}{1} \\ &= 6 \end{aligned}$$

Solution 18: MA HSC 2011 2 (a) (ii) **1**

$$\begin{aligned} \alpha\beta &= \frac{c}{a} \\ &= \frac{2}{1} \\ &= 2 \end{aligned}$$

Solution 19: MA HSC 2011 2 (a) (iii) **1**

$$\begin{aligned} \frac{1}{\alpha}+\frac{1}{\beta} &= \frac{\alpha+\beta}{\alpha\beta} \\ &= \frac{6}{2} \quad \left(\text{from parts (i) and (ii)}\right) \\ &= 3 \end{aligned}$$

Solution 20: ME1 HSC 2007 2 (c) **3**

$P(x) = x^2 + ax + b$

Now $P(2) = 0$

$2^2 + 2a + b = 0$

$4 + 2a + b = 0$

$2a + b = -4 \quad \ldots(1)$

Also $P(-1) = 18$

$(-1)^2 + a(-1) + b = 18$

$1 - a + b = 18$

$-a + b = 17$

$b = 17 + a \quad \ldots(2)$

Substituting $b = 17 + a$ into (1):

$2a + 17 + a = -4$

$3a = -21$

$a = -7$

$b = 17 - 7$

$= 10$

$\therefore a = -7$ and $b = 10$.

Solution 21: ME1 HSC 2008 2 (c) **3**

$$\text{Sum of roots} = -\frac{b}{a}$$
$$= -\frac{16}{a}$$

$$\therefore -2+3+\alpha = -\frac{16}{a}$$
$$\alpha = -\frac{16}{a} - 1 \qquad \ldots(1)$$

$$\text{Product of roots} = -\frac{d}{a}$$
$$= \frac{120}{a}$$

$$\therefore -2\times 3\times \alpha = \frac{120}{a}$$

$$-6\alpha = \frac{120}{a}$$
$$\alpha = -\frac{20}{a} \qquad \ldots(2)$$

Equating (1) and (2) above,

$$-\frac{16}{a} - 1 = -\frac{20}{a}$$
$$-\frac{16}{a} + \frac{20}{a} = 1$$
$$\frac{4}{a} = 1$$
$$a = 4$$

Substituting $a = 4$ into (2),

$$\therefore \alpha = -\frac{20}{4}$$
$$\alpha = -5$$

Solution 22: ME1 HSC 2010 2 (c) (i) **1**

$$P(3) = 0$$
$$0 + 3a + b = 0$$
$$3a + b = 0$$
$$b = -3a \qquad \text{...(1)}$$

$$P(-1) = 8$$
$$0 - a + b = 8$$
$$-a - 3a = 8 \qquad \text{(from part (1))}$$
$$\therefore a = -2,\ b = 6$$

Solution 23: ME1 HSC 2010 2 (c) (ii) **2**

The remainder is $-2x+6$.

Solution 24: ME1 HSC 2004 3 (b) (i) **1**

$$P(x)=(x+1)(x-3)Q(x)+a(x+1)+b$$

Now $P(-1)=-11$

$$\therefore(-1+1)(-1+3)Q(x)+a(-1+1)+b=-11$$
$$0+0+b=-11$$
$$\therefore b=-11$$

Solution 25: ME2 HSC 2008 3 (b) (i) **1**

$p(z)=1+z^2+z^4$.

If z were a real number, then z^2 and z^4 are non-negative and hence $p(z)\neq 0$.

$\therefore\ p(z)$ has no real zeros.

Solution 26: ME1 HSC 2004 3 (b) (ii) **2**

$P(-1)=-11$ by the remainder theorem

$$(-1+1)(-1-3)Q(x)+a(-1+1)+b=-11$$
$$0+0+b=-11$$
$$b=-11$$

$P(3)=1$ by the remainder theorem

$$(3+1)(3-3)Q(x)+a(3+1)+b=1$$
$$0+4a+b=1$$
$$4a-11=1$$
$$4a=12$$
$$a=3$$

$\therefore$ the remainder is

$$\begin{aligned}a(x+1)+b&=3(x+1)-11\\&=3x+3-11\\&=3x-8\end{aligned}$$

Solution 27: ME1 HSC 2006 4 (a) (i) **1**

$$\begin{aligned}\alpha+\beta+\gamma&=\text{-}r\\ r&=\text{-}(1+\alpha+(\text{-}\alpha))\\&=\text{-}1.\end{aligned}$$

Solution 28: ME1 HSC 2006 4 (a) (ii) **2**

$$\alpha\beta + \alpha\gamma + \beta\gamma = \alpha - \alpha - \alpha^2$$
$$= s$$
$$\therefore s = -\alpha^2$$

$$\alpha\beta\gamma = -a^2$$
$$= -t$$
$$\therefore t = a^2$$

$$\therefore s + t = 0$$

Solution 29: ME2 HSC 2004 4 (a) (ii) **2**

$$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$$
$$= \left(-\frac{7}{3}\right)^2 - 2\left(\frac{11}{3}\right)$$
$$= -\frac{17}{9}$$

Solution 30: ME1 HSC 2002 4 (b) (i) **1**

$$\alpha + \beta + \gamma = \frac{-b}{a}$$
$$= 2.$$

Solution 31: ME1 HSC 2002 4 (b) (ii) **1**

$$\alpha\beta\gamma = \frac{-d}{a}$$
$$= -24$$

Solution 32: ME1 HSC 2002 4 (b) (iii) **2**

Let $\alpha = -\beta$,
Then the roots are $\beta, -\beta$ and γ.

It is given that $\alpha + \beta + \gamma = 2 \;\Rightarrow\; \beta - \beta + \gamma = 2$

$$0 + \gamma = 2$$
$$\gamma = 2.$$

$$\alpha\beta\gamma = -24$$
$$\therefore (-\beta)\beta\gamma = -24$$
$$\therefore -\beta^2\gamma = -24$$
$$\therefore -2\beta^2 = -24$$
$$\therefore -\beta^2 = -12 \qquad ...(1)$$

$$\frac{c}{a} = k$$
$$= \alpha\beta + \alpha\gamma + \beta\gamma$$
$$= (-\beta)\beta + (-\beta)\gamma + \beta\gamma$$
$$= -\beta^2 - \beta\gamma + \beta\gamma$$
$$= -\beta^2$$
$$= -12 \qquad \text{(from (1))}$$

Solution 33: ME2 HSC 2002 5 (a) **2**

Let $P(x) = 4x^3 - 27x + k$

$P'(x) = 12x^2 - 27$

If $P(x)$ has a double root, say α, then α is a single root of $P'(x)$.

$$P'(\alpha) = 12\alpha^2 - 27$$
$$P'(\alpha) = 0$$
$$\therefore 12\alpha^2 = 27$$
$$\alpha^2 = \frac{9}{4}$$
$$\alpha = \pm\frac{3}{2}$$

$$P\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^3 - 27\left(\frac{3}{2}\right) + k$$
$$= -27 + k$$

$$P\left(\frac{3}{2}\right) = 0$$
$$k = 27$$

$$P\left(-\frac{3}{2}\right) = 4\left(-\frac{3}{2}\right)^3 - 27\left(-\frac{3}{2}\right) + k$$
$$= 27 + k$$

$$P\left(-\frac{3}{2}\right) = 0$$
$$k = -27$$

$\therefore$ Possible values of $k = \pm 27$.

Solution 34: ME2 HSC 2008 5 (b) (i) **2**

$$
\begin{aligned}
p(x) &= x^{n+1} - (n+1)x + n \\
p(1) &= 1^{n+1} - (n+1)(1) + n \\
&= 1 - n - 1 + n \\
&= 0
\end{aligned}
$$

$\therefore p(x)$ has a zero at $x = 1$.

$$
\begin{aligned}
p'(x) &= (n+1)x^{n} - (n+1) \\
p'(1) &= (n+1)(1^{n}) - (n+1) \\
&= n + 1 - (n+1) \\
&= 0
\end{aligned}
$$

$\therefore p'(x)$ has a zero at $x = 1$.

$$
\begin{aligned}
p''(x) &= n(n+1)x^{n-1} \\
p''(1) &= n(n+1) \neq 0
\end{aligned}
$$

$\therefore p(x)$ has a double zero at $x = 1$.

Solution 35: ME2 HSC 2008 5 (b) (iii) **2**

When $n = 3$,

$$
\begin{aligned}
p(x) &= x^4 - 4x + 3 \\
&= (x-1)^2 q(x) && (\text{since } x = 1 \text{ is a double root}) \\
&= (x^2 - 2x + 1)(x^2 + 2x + 3) && (\text{by inspection}) \\
&= (x-1)^2 (x^2 + 2x + 3)
\end{aligned}
$$

Solution 36: ME2 HSC 2009 6 (b) (i) **1**

Let roots be $-1, \alpha, \beta$

$$-1 \times \alpha \times \beta = -\frac{d}{a} = -1$$

$$\therefore \beta = \frac{1}{\alpha}$$

Solution 37: ME2 HSC 2010 6 (c) (iv) **1**

By inspection,

$$16x^5 - 20x^3 + 5x - 1 = (x-1)(16x^4 + 16x^3 - 4x^2 - 4x + 1)$$

Solution 38: ME2 HSC 2010 6 (c) (vi) **1**

$$16x^5 - 20x^3 + 5x - 1 = (x-1)(4x^2 + 2x - 1)^2 = 0$$

$$\therefore x = 1, \ \frac{-1 \pm \sqrt{5}}{4}$$

$$\therefore \sin\frac{\pi}{10} = \frac{-1+\sqrt{5}}{4}$$

Solution 39: MA HSC 2006 7 (a) (i) **1**

$$\begin{aligned} \alpha\beta &= \frac{c}{a} \\ &= \frac{1}{1} \\ &= 1 \end{aligned}$$

Solution 40: MA HSC 2006 7 (a) (ii) **1**

Since $\alpha\beta = 1$,

$$\beta = \frac{1}{\alpha}$$

$$\begin{aligned} \alpha + \beta &= \alpha + \frac{1}{\alpha} \\ &= -\frac{b}{a} \\ &= -\frac{-3}{1} \\ &= 3. \end{aligned}$$

Solution 41: ME2 HSC 2001 7 (b) (iii) **1**

If $x = 2\cos\frac{\pi}{9}$

$$\begin{aligned} x^3 - 3x - 1 &= \left(2\cos\frac{\pi}{9}\right)^3 - 3\left(2\cos\frac{\pi}{9}\right) - 1 \\ &= 8\cos^3\frac{\pi}{9} - 6\cos\frac{\pi}{9} - 1 \\ &= 2\left(4\cos^3\frac{\pi}{9} - 3\cos\frac{\pi}{9}\right) - 1 \\ &= 2\left(\cos 3\left(\frac{\pi}{9}\right)\right) - 1 \quad \text{using the identity } \cos 3\theta = 4\cos^3\theta - 3\cos\theta \\ &= 2\left(\frac{1}{2}\right) - 1 \\ &= 0 \end{aligned}$$

$\therefore \ x = 2\cos\frac{\pi}{9}$ is a root of (*).

Solution 42: ME2 HSC 2010 7 (c) (i) **1**

$$P(x) = (n-1)x^n - nx^{n-1} + 1$$
$$P'(x) = n(n-1)x^{n-1} - n(n-1)x^{n-2}$$
$$P'(x) = n(n-1)x^{n-2}(x-1)$$

Now $P'(x) = 0$ when $x = 0$ or $x = 1$
Hence there are exactly two turning points.

Solution 43: ME2 HSC 2010 7 (c) (ii) **1**

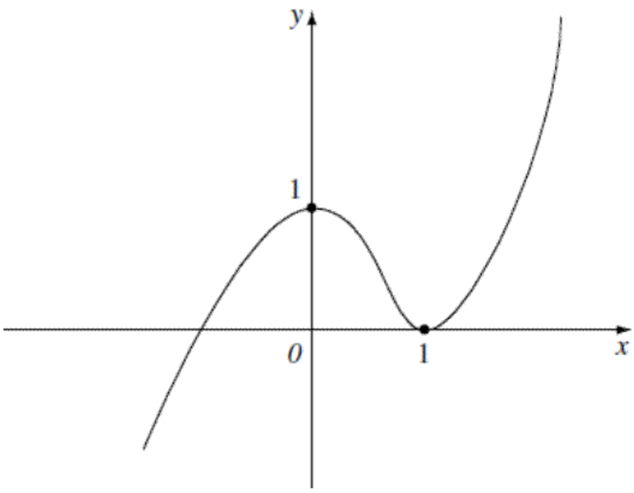
$$\begin{aligned} P(1) &= (n-1)\times 1^n - n\times 1^{n-1} + 1 \\ &= 0 \end{aligned}$$
$P'(1) = 0$ $\quad$ (shown in (i))

$\therefore$ $P(x)$ has a double root when $x = 1$.

Also note that $P(x)$ does not have a triple root at $x = 1$:
$$P''(x) = n(n-1)^2 x^{n-2} - n(n-1)(n-2)x^{n-3}$$
$$P''(1) \neq 0$$

Solution 44: ME2 HSC 2010 7 (c) (iii) **2**

$P(0) = 1$ and $P(1) = 0$
$$\underbrace{P(x) \to \infty \text{ as } x \to \infty \text{ and } P(x) \to -\infty \text{ as } x \to -\infty}_{\text{Since the degree of } P(x) \text{ is odd}}$$



The curve cuts the x-axis in only one place (other than $x = 1$).
$\therefore$ There is exactly one real zero of $P(x)$ other than 1.

Solution 45: ME1 HSC 2013 11 (a) **1**

$$\alpha\beta\gamma = -\frac{7}{2}$$

Solution 46: ME1 HSC 2020 11 (a) (i) **1**

$$
\begin{aligned}
P(x) &= x^3 + 3x^2 - 13x + 6. \\
P(2) &= (2)^3 + 3(2)^2 - 13(2) + 6 \\
&= 8 + 12 - 26 + 6 \\
&= 0
\end{aligned}
$$

Solution 47: ME1 HSC 2018 11 (a) (i) **1**

$$
\begin{aligned}
P(x) &= x^3 - 2x^2 - 5x + 6 \\
P(1) &= (1)^3 - 2(1)^2 - 5(1) + 6 \\
&= 1 - 2 - 5 + 6 \\
&= 0
\end{aligned}
$$

$\therefore x = 1$ is a zero of $P(x)$.

Solution 48: ME1 HSC 2018 11 (a) (ii) **2**

Since $x = 1$ is a zero of $P(x)$, then $(x-1)$ is a factor.

$$
\begin{aligned}
P(x) &= x^3 - 2x^2 - 5x + 6 \\
&= (x-1)(x^2 - x - 6) \\
&= (x-1)(x-3)(x+2)
\end{aligned}
$$

$\therefore$ Other zeroes are -2 and 3.

Solution 49: ME1 HSC 2020 11 (a) (ii) **2**

$P(x) = (x-2)(x^2 + 5x - 3) \quad$ (by inspection)

Solution 50: ME2 HSC 2018 11 (b) **3**

$p(x)$ has roots $r, 4, 4$.

Sum of the roots two at a time $= \dfrac{c}{a}$

$$4r + 4r + 16 = 0$$
$$8r = -16$$
$$\therefore r = -2$$

Sum of the roots $= -\dfrac{b}{a}$

$$-2 + 4 + 4 = -a$$
$$\therefore a = -6$$

Product of the roots $= -\dfrac{d}{a}$

$$(-2)(4)(4) = -b$$
$$\therefore b = 32$$

ALTERNATIVE SOLUTION

$$p(x) = x^3 + ax^2 + b$$
$$p'(x) = 3x^2 + 2ax$$

Double zero at 4 means $p(4) = 0$ and $p'(4) = 0$.

$$64 + 16a + b = 0 \quad ...(1)$$
$$48 + 8a = 0 \quad ...(2)$$

From (2), $\therefore a = -6$

Sub into (1), $64 - 96 + b = 0$

$$\therefore b = 32$$

Roota at r means $p(r) = 0$.

$$r^3 + ar^2 + b = 0$$
$$r^3 - 6r^2 + 32 = 0$$

Testing factors of 32, $r = -2$ satisfies the above equation.

$\therefore r = -2.$

Solution 51: ME1 HSC 2019 11 (d) **2**

$$x^3 + 2x^2 - 3x - 7 = (x-2)(x^2 + 4x + 5) + 3$$

$$\therefore Q(x) = x^2 + 4x + 5$$

Solution 52: ME1 HSC 2015 11 (f) (i) **1**

$P(x) = x^3 - kx^2 + 5x + 12$

Since $(x-3)$ is a factor of $P(x)$, $\therefore P(3) = 0$

$$
\begin{aligned}
(3)^3 - k(3)^2 + 5(3) + 12 &= 0 \\
9k &= 54 \\
\therefore k &= 6
\end{aligned}
$$

Solution 53: ME1 HSC 2015 11 (f) (ii) **2**

When $k = 6$,

$$
\begin{aligned}
P(x) &= x^3 - 6x^2 + 5x + 12 \\
&= (x-3)(x^2 - 3x - 4) \qquad \text{(by inspection)} \\
&= (x-3)(x-4)(x+1)
\end{aligned}
$$

$\therefore$ Zeros of $P(x)$ are $3, 4, -1$.

Solution 54: ME1 HSC 2021 11 (h) **2**

$$
\begin{aligned}
\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} &= \frac{\beta\gamma\delta + \alpha\gamma\delta + \alpha\beta\delta + \alpha\beta\gamma}{\alpha\beta\gamma\delta} \\
&= \frac{-\frac{d}{a}}{\frac{e}{a}} \\
&= \frac{3}{6} \\
&= \frac{1}{2}
\end{aligned}
$$

Solution 55: ME2 HSC 2014 12 (b) (i) **1**

$x^3 - 3x = \sqrt{3}$

Substituting $x = 2\cos\theta$,

$$
\begin{aligned}
8\cos^3\theta - 6\cos\theta &= \sqrt{3} \\
4\cos^3\theta - 3\cos\theta &= \frac{\sqrt{3}}{2} \\
\therefore \cos 3\theta &= \frac{\sqrt{3}}{2}
\end{aligned}
$$

Solution 56: ME2 HSC 2014 12 (b) (ii) **2**

$$\cos 3\theta = \frac{\sqrt{3}}{2}$$
$$3\theta = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}$$
$$\therefore \theta = \frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}$$

$$\therefore x = 2\cos\frac{\pi}{18},\ 2\cos\frac{11\pi}{18},\ 2\cos\frac{13\pi}{18}$$

Solution 57: ME2 HSC 2017 12 (d) (i) **2**

Let $P(x) = (x-\alpha)^2 Q(x)$

$$\begin{aligned} P'(x) &= 2(x-\alpha)Q(x) + (x-\alpha)^2 Q'(x) \\ &= (x-\alpha)\left[2Q(x) + (x-\alpha)Q'(x)\right] \end{aligned}$$

$$\begin{aligned} P(\alpha) &= (\alpha-\alpha)^2 Q(\alpha) \\ &= 0 \end{aligned}$$

$$\begin{aligned} P'(\alpha) &= (\alpha-\alpha)\left[2Q(\alpha) + (\alpha-\alpha)Q'(\alpha)\right] \\ &= 0 \end{aligned}$$

Solution 58: ME2 HSC 2017 12 (d) (ii) **2**

$$\begin{aligned} P(x) &= x^4 - 3x^3 + x^2 + 4 \\ P'(x) &= 4x^3 - 9x^2 + 2x \\ &= x(4x^2 - 9x + 2) \\ &= x(4x-1)(x-2) \end{aligned}$$

$\therefore$ Roots of $P'(x)$ are $x = 0, \frac{1}{4}, 2.$

$$P(0) = 4$$
$$P\left(\frac{1}{4}\right) = \frac{1029}{256}$$
$$P(2) = 0$$

$\therefore (x-2)^2$ is a factor of $P(x)$

$\therefore \alpha = 2$

Solution 59: ME2 HSC 2017 13 (b) (i) **2**

Sum of roots one at a time:

$$\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}=-a$$

Sum of roots three at a time:

$$\begin{aligned}\alpha\left(\frac{1}{\alpha}\right)\beta+\alpha\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)+\alpha\beta\left(\frac{1}{\beta}\right)+\left(\frac{1}{\alpha}\right)\beta\left(\frac{1}{\beta}\right)&=-c\\&=\beta+\frac{1}{\beta}+\alpha+\frac{1}{\alpha}\\&=-a\end{aligned}$$

$\therefore a=c$

Solution 60: ME1 HSC 2022 13 (d) **3**

Let $P(x)=x^3+bx^2+cx+d$ $\qquad \alpha+\beta+\gamma=-b$

$P'(x)=3x^2+2bx+c$ $\qquad \alpha\beta+\beta\gamma+\gamma\alpha=c$

$$\begin{aligned}\alpha^2+\beta^2+\gamma^2&=85\\(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)&=85\\b^2-2c&=85 \quad ...(1)\end{aligned}$$

$$\begin{aligned}P'(\alpha)+P'(\beta)+P'(\gamma)&=87\\3(\alpha^2+\beta^2+\gamma^2)+2b(\alpha+\beta+\gamma)+3c&=87\\3(85)+2b(-b)+3c&=87\\255-2b^2+3c&=87\\2b^2-3c&=168 \quad ...(2)\end{aligned}$$

$(2)-2\times(1):\quad \therefore c=-2$

$\therefore \alpha\beta+\beta\gamma+\gamma\alpha=-2$

Solution 61: ME2 HSC 2016 13 (d) (ii) **2**

$$p\left(-\frac{b}{3a}\right)=0$$

$$p'\left(-\frac{b}{3a}\right)=3a\left(-\frac{b}{3a}\right)^2+2b\left(-\frac{b}{3a}\right)+c$$
$$=\frac{b^2}{3a}-\frac{2b^2}{3a}+c$$
$$=-\frac{1}{3a}\left(b^2-3ac\right)$$
$$=0$$

$$p''(x)=6ax+2b$$
$$p''\left(-\frac{b}{3a}\right)=6a\left(-\frac{b}{3a}\right)+2b$$
$$=-2b+2b$$
$$=0$$

$$p'''(x)=6a$$
$$p'''\left(-\frac{b}{3a}\right)=6a\neq 0$$

$\therefore\ x=-\dfrac{b}{3a}$ is a triple root.

Solution 62: ME2 HSC 2015 14 (b) (i) **1**

$$\alpha+\beta+\gamma=-\frac{b}{a}=0$$
$$\alpha\beta+\alpha\gamma+\beta\gamma=\frac{c}{a}=-p$$

$$(\alpha+\beta+\gamma)^2=0$$
$$\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\alpha\gamma+\beta\gamma)=0$$
$$16-2p=0$$
$$\therefore\ p=8$$

Solution 63: MA HSC 2014 14 (b) (i) **1**

$$\text{Sum of roots}=\alpha+\beta$$
$$=\frac{-b}{a}$$
$$=\frac{-8}{2}$$
$$=-4$$

Solution 64: ME2 HSC 2012 14 (b) (ii) **2**

$$\frac{x(2x-3)}{x-1} = \frac{(x-1)(2x-1)-1}{x-1}$$
$$= 2x-1-\frac{1}{x-1}$$

As $x \to \infty$, $\frac{1}{x-1} \to 0$

$\therefore$ Asymptote ℓ is $y = 2x-1$.

Solution 65: MA HSC 2014 14 (b) (ii) **2**

$$\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta)$$
$$6 = \alpha\beta(\alpha+\beta)$$
$$\alpha+\beta = -4 \text{ (from part (i))}$$
$$\therefore 6 = \alpha\beta(-4)$$
$$\alpha\beta = -\frac{3}{2}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} = \frac{k}{2}$$
$$\therefore \frac{k}{2} = -\frac{3}{2}$$
$$k = -3$$

Solution 66: ME2 HSC 2013 15 (b) (i) **2**

$$P(x) = ax^4 + bx^3 + cx^2 + e$$
$$P'(x) = 4ax^3 + 3bx^2 + 2cx$$

$P(1) = -3$
$a+b+c+e = -3 \quad ...(1)$

$P(-1) = 0$
$a-b+c+e = 0 \quad ...(2)$

$P'(-1) = 0$
$-4a+3b-2c = 0 \quad ...(3)$

$$(1)-(2): \quad 2b = -3$$
$$b = -\frac{3}{2}$$

$$\text{From } (3), \quad 4a+2c = 3b$$
$$= 3\left(-\frac{3}{2}\right)$$
$$= -\frac{9}{2}$$

$$\therefore 4a+2c = -\frac{9}{2}.$$

Solution 67: ME2 HSC 2013 15 (b) (ii) 1

At $x = 1$,

$$
\begin{aligned}
P'(1) &= 4a + 3b + 2c \\
&= (4a + 2c) + 3b \\
&= -\frac{9}{2} + 3\left(-\frac{3}{2}\right) \\
&= -9
\end{aligned}
$$