EXERCISE 2.5

BASIC

1 Find the square root of $3 + 4i$, using formal simultaneous equations.

2 Find the square root of $3 + 4i$, using the formula.

3 Find the square root of $3 + 4i$ using the shortcuts for simultaneous equations.

4 Find the square roots of $5 - 12i$ using the shortcuts for simultaneous equations.

5 Find the square roots of $2i$ using the shortcuts for simultaneous equations.

MEDIUM

6 Find the square roots of $2 - 2\sqrt{3}i$ using the shortcuts for simultaneous equations.

7 Find the square roots of $4 - 3i$ using the shortcuts for simultaneous equations.

8 The two square roots of $4i$ are $\pm(\sqrt{2} + \sqrt{2}i)$. Find the solutions of $z^2 + 4z + 4 - i = 0$

9 **a** Find the square roots of $-6 + 8i$

b Hence, or otherwise, solve the equation $z^2 - \sqrt{2}iz + 1 - 2i = 0$

SOLUTIONS - EXERCISE 2.5

1 $a^2 - b^2 = 3 \qquad (1)$

$2ab = 4 \qquad (2)$

$$b = \frac{2}{a} \qquad (2 \text{ rearranged})$$

$$a^2 - \frac{4}{a^2} = 3 \qquad \left(\text{substituting into } (1)\right)$$

$$a^4 - 3a^2 - 4 = 0$$

$$(a^2 - 4)(a^2 + 1) = 0$$

$a^2 - 4 = 0$ or $a^2 + 1 = 0$

$\therefore a = \pm 2$ are the only solutions, since a must be real (not $\pm i$)

Substituting into (2) we get $a = 2, b = 1$ and $a = -2, b = -1$ which gives the square roots as $\pm(2 + i)$.

2 Half of 4 is 2. Find pairs of numbers whose product is 2: ± 1 and ± 2 is the only pair.

Since the real part is positive then $a^2 - b^2$ is positive, so we place the number with the largest absolute value first (otherwise the difference would be negative).

In our heads: $2^2 - 1^2 3$. Thus 2 and 1 are the values for a and b, so $2 + i$ is a solution.

Therefore $-2 - i$ is also a solution.

The square roots of $3 + 4i$ are $\pm(2 + i)$.

3 Taking the coefficient of i and halving it we get -6, so we need to find a pair of numbers that have a product of -6. We have either ± 2 and ∓ 3 or ± 1 and ∓ 6.

Now we check which pair has a difference of their squares being $+5$. $3^2 - 2^2 = 9 - 4 = 5$, so the square roots are $\pm(3 - 2i)$.

4 Taking the coefficient of i and halving it we get $-\sqrt{3}$, so we need to find a pair of numbers that have a product of $\sqrt{3}$.

Trying $\pm\sqrt{3}$ and ∓ 1 we get a difference of squares of 2, which is what we want, so the square roots of $2 - 2\sqrt{3}i$ are $\pm(\sqrt{3} - i)$

5

$$a = \pm\sqrt{\frac{\sqrt{3^2 + 4^2} + 3}{2}} = \pm 2, b = \pm(+)\sqrt{\frac{\sqrt{3^2 + 4^2} - 3}{2}} = \pm 1$$

$\therefore \pm(2 + i)$ are the solutions.

6 Taking the coefficient of i and halving it we get 1, so we are looking for a pair of numbers with a product of 1. We have ± 1 and ± 1.

Now the real part is zero, so the difference of the squares is zero, so 1 and 1 (and their negatives) are the pairs we want.

So the square roots of $2i$ are $\pm(1+i)$.

7 We have a problem here – the coefficient of i isn't even so we cannot use our shortcut yet. We will double it and put it over 2 so we can use our shortcut.

$$4-3i=\frac{8-6i}{2}$$

Taking the coefficient of i in the numerator and halving it we get 3, so we need to find a pair of numbers that have a product of 3. We have ± 1 and ∓ 3. $3^2-1^2=8$, so the square roots of $8-6i$ are $\pm(3-i)$. So we have square rooted the numerator, but we must also square root the denominator.

So the square roots of $4-3i$ are $\pm\frac{3-i}{\sqrt{2}}$.

8

$$z=\frac{-4\pm\sqrt{4^2-4(1)(4-i)}}{2(1)}$$

$$=\frac{-4\pm\sqrt{4i}}{2}$$

$$=\frac{-4\pm\left(\sqrt{2}+\sqrt{2}i\right)}{2}$$

$$=\frac{-4-\sqrt{2}-\sqrt{2}i}{2},\frac{-4+\sqrt{2}+\sqrt{2}i}{2}$$

9 **a**

$|-6+8i|=\sqrt{(-6)^2+8}=10, \mathrm{Re}(-6+8i)$

$=-6$

The square roots of $-6+8i$ are

$$\pm\left(\sqrt{\frac{10-6}{2}}+\sqrt{\frac{10+6}{2}}i\right)=\pm\left(2+2\sqrt{2}i\right)$$

b

$$z=\frac{\sqrt{2}i\pm\sqrt{\left(-\sqrt{2}i\right)^2-4(1)(1-2i)}}{2(1)}$$

$$=\frac{\sqrt{2}i\pm\sqrt{-2-4+8i}}{2}$$

$$=\frac{\sqrt{2}i\pm\sqrt{-6+8i}}{2}$$

$$=\frac{\sqrt{2}i\pm\left(2+2\sqrt{2}i\right)}{2}$$

$$=-2-\sqrt{2}i,2+3\sqrt{2}i$$