EXERCISE 4.3

BASIC

Find:

1 $\displaystyle\int \frac{x^2 - 2x - 2}{x + 1}\,dx$

2 $\displaystyle\int \frac{2x - 3}{x + 1}\,dx$

3 $\displaystyle\int \frac{2x^2 + 19}{x^2 + 9}\,dx$

4 $\displaystyle\int \frac{e^{-x}}{e^{-x} - 1}\,dx$

5 $\displaystyle\int \frac{2x}{x^2 + 4x + 20}\,dx$

6 $\displaystyle\int \frac{3x + 2}{x^2 + 6x + 10}\,dx$

7 $\displaystyle\int \frac{2}{x(x - 1)}\,dx$

8 $\displaystyle\int \frac{2}{x(x^2 + 4)}\,dx$

MEDIUM

9 $\displaystyle\int \frac{e^{2x} + e^x}{e^{2x} + 1}\,dx$

10 $\displaystyle\int \frac{x^4 + 1}{x^2 + 2}\,dx$

11 $\displaystyle\int \frac{x^3}{x^2 + 2}\,dx$

CHALLENGING

12 $\displaystyle\int \frac{2\cos x\,(\sin x + 1)}{(\sin x + 3)(\sin x - 1)}\,dx$

13 $\displaystyle\int \frac{\cos x}{\sin^2 x - 3\sin x + 2}\,dx$

14 $\displaystyle\int \frac{8x}{1 + e^2 x}\,dx$

1

$$\int \frac{x^2 - 2x - 2}{x + 1} dx$$

$$= \int \frac{x(x+1) - 3(x+1) + 1}{x + 1} dx$$

$$= \int \left(x - 3 + \frac{1}{x+1}\right) dx$$

$$= \frac{x^2}{2} - 3x + \ln|x+1| + c$$

2

$$\int \frac{2x - 3}{x + 1} dx$$

$$= \int \frac{2(x+1) - 5}{x + 1} dx$$

$$= \int \left(2 - \frac{5}{x+1}\right) dx$$

$$= 2x - 5\ln|x+1| + c$$

3

$$\int \frac{2x^2 + 19}{x^2 + 9} dx$$

$$= \int \frac{2(x^2+9) + 1}{x^2 + 9} dx$$

$$= \int \left(2 + \frac{1}{x^2 + 3^2}\right) dx$$

$$= 2x + \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c$$

4

$$\int \frac{e^x + 1}{e^x - 1} dx$$

$$= \int \frac{e^x - 1 + 2}{e^x - 1} dx$$

$$= \int \left(1 + \frac{2}{e^x - 1}\right) dx$$

$$= \int \left(1 + 2\frac{e^{-x}}{1 - e^{-x}}\right) dx$$

$$= x + 2\ln\left|1 - e^{-x}\right| + c$$

5

$$\int \frac{2x}{x^2 + 4x + 20} dx$$

$$= \int \frac{2x + 4 - 4}{x^2 + 4x + 4 + 16} dx$$

$$= \int \frac{2x + 4}{x^2 + 4x + 20} dx - 4\int \frac{1}{(x+2)^2 + 4^2} dx$$

$$= \ln|x^2 + 4x + 20| - \tan^{-1}\left(\frac{x+2}{4}\right) + c$$

6

$$\int \frac{3x + 2}{x^2 + 6x + 10} dx$$

$$= \int \frac{\frac{3}{2}(2x+6) - 7}{x^2 + 6x + 9 + 1} dx$$

$$= \frac{3}{2}\int \frac{2x + 6}{x^2 + 6x + 10} dx - 7\int \frac{1}{(x+3)^2 + 1^2} dx$$

$$= \frac{3}{2}\ln|x^2 + 6x + 10| - 7\tan^{-1}(x+3) + c$$

7

$$\int \frac{2}{x(x-1)} dx$$

$$= 2\int \left(\frac{1}{x-1} - \frac{1}{x}\right) dx$$

$$= 2\ln\left|x - 1\right| - 2\ln\left|x\right| + c$$

8

$$\int \frac{2}{x(x^2+4)} dx$$

$$= \frac{1}{2}\int \left(\frac{1}{x} - \frac{x}{x^2+4}\right) dx$$

$$= \frac{1}{2}\int \left(\frac{1}{x} - \frac{1}{2} \times \frac{2x}{x^2+4}\right) dx$$

$$= \frac{1}{2}\ln|x| - \frac{1}{4}\ln|x^2 + 4| + c$$

9

$$\int \frac{e^{2x}+e^{x}}{e^{2x}+1}\,dx$$

$$= \int \left(\frac{1}{2}\times\frac{2e^{2x}}{e^{2x}+1}+\frac{e^{x}}{(e^{x})^2+1}\right)dx$$

$$= \frac{1}{2}\ln\left|e^{2x}+1\right| + \tan^{-1}(e^{x}) + c$$

10

$$\int \frac{x^4+1}{x^2+2}\,dx$$

$$= \int \frac{x^2(x^2+2)-2(x^2+2)+5}{x^2+2}\,dx$$

$$= \int \left(x^2-2+\frac{5}{x^2+2}\right)dx$$

$$= x^3-2x+\frac{5}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+c$$

11

$$\int \frac{x^3}{x^2+2}\,dx$$

$$= \int \frac{x(x^2+2)-2x}{x^2+2}\,dx$$

$$= \int \left(x-\frac{2x}{x^2+2}\right)dx$$

$$= \frac{x^2}{2}-\ln\left|x^2+2\right|+c$$

12

$$\int \frac{2\cos x\,(\sin x+1)}{(\sin x+3)(\sin x-1)}\,dx$$

$$= \int \frac{2\sin x\cos x+2\cos x}{\sin^2 x+2\sin x-3}\,dx$$

$$= \ln\left|\sin^2 x+2\sin x-3\right|+c$$

13

$$\int \frac{\cos x}{\sin^2 x-3\sin x+2}\,dx$$

$$= \int \frac{\cos x}{(\sin x-2)(\sin x-1)}\,dx$$

$$= \int \left(\frac{\cos x}{\sin x-2}-\frac{\cos x}{\sin x-1}\right)dx$$

$$= \ln\left|\sin x-2\right|-\ln\left|\sin x-1\right|+c$$

$$= \ln\left|\frac{\sin x-2}{\sin x-1}\right|+c$$

14

$$\int \frac{8x}{1+e^2x}\,dx$$

$$= \int \frac{\frac{8}{e^2}(1+e^2x)-\frac{8}{e^2}}{1+e^2x}\,dx$$

$$= \int \left(\frac{8}{e^2}-\frac{8}{e^4}\times\frac{e^2}{1+e^2x}\right)dx$$

$$= \frac{8x}{e^2}-\frac{8\ln\left|1+e^2x\right|}{e^4}+c$$