2021 Extension 2 Mathematics – Short Answer Questions

Attempt Questions 11 – 13

Answer the question on paper and then take a photo and insert in the relevant google docs on the Year 12 Extension 2 Trial Hapara Workspace.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

Question 11 a)

Let $z = \dfrac{1+i}{1-i}$ and $w = \dfrac{\sqrt{2}}{1-i}$.

(i) Write each of z and w in modulus-argument form. **2**

(ii) On the same Argand diagram, sketch the points z, w and $z + w$. **2**

(iii) Deduce the exact value of $\tan\left(\dfrac{3\pi}{8}\right)$. **2**

Question 11 b)

Given $1 < a < 2$, sketch, on an Argand diagram, the region represented by **2**

$$|z - a - i| < 1.$$

Question 11 c)

i. Show that $\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i$ is a root of the equation $z^3 = i$. **2**

ii. On an Argand diagram, neatly plot all three roots of i. **2**

Question 11 d)

A particle moves along a straight line according to the equation **3**

$$\ddot{x} = -16(x - 2)$$

where x is the displacement in metres and t is the time in seconds.

If the particle is at rest when $x = 7$, find the velocity of the particle when $x = 6$ and the particle is moving towards the origin.

Question 12 (14 marks)

Question 12 a)

(i) Find numbers a, b and c such that **2**

$$\frac{9x - 6}{x^3 + 8} \equiv \frac{a}{x + 2} + \frac{bx + c}{x^2 - 2x + 4}.$$

(ii) Hence evaluate $\displaystyle\int_0^1 \frac{9x - 6}{x^3 + 8}\, dx$. **3**

Question 12 b)

Let $P(x) = x^4 + 16x^3 + 108x^2 + 400x + 800$. $P(x)$ has roots $a + 2ib$ and $3a + ib$, where a, b are real numbers.

(i) Find all the roots of $P(x)$ with integer values for a and b **2**

(ii) Factorise $P(x)$ into its quadratic factors with real coefficients **2**

Question 12 c)

Given that $|\underset{\sim}{a}| = 3$, $|\underset{\sim}{b}| = 2$ and $\underset{\sim}{a} \cdot \underset{\sim}{b} = 4$, calculate the length of $2\underset{\sim}{a} - 3\underset{\sim}{b}$. **2**

Question 13 (14 marks)

Question 13 a)

(i) If $z = e^{i\theta}$, show that $z^n + z^{-n} = 2\cos(n\theta)$. **1**

(ii) Hence, or otherwise, determine the values of θ, where $0 \le \theta < 2\pi$ **4**

such that

$$|e^{4i\theta} + 1| = \sqrt{3} .$$

Question 13 b)

A mass of 1 kg moves along a straight line with velocity $v\ ms^{-1}$. It encounters a resistance of $v + v^3$. The particle has initial velocity U, where $U > 0$ and starts from the origin.

At time t the particle has velocity v and displacement x.

(i) Show that the equation of motion is $\ddot{x} = -v(1 + v^2)$. **1**

(ii) Show that $x = \tan^{-1}\left(\dfrac{U - v}{1 + Uv}\right)$. **3**

(iii) Show that $v^2 = \dfrac{U^2}{(1 + U^2)\,e^{2t} - U^2}$. **3**

(iv) Describe the motion of the particle as $t \to \infty$. **2**

2021 Extension 2 Mathematics – Short Answer Questions

Attempt Questions 14 – 16

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Question 14 (8 marks)

Question 14 a)

By rewriting the equation in the form $a^2 + b^2 = 0$, or otherwise, disprove the statement: **3**

$$\exists x \in \mathbb{R} \text{ such that } x^6 + x^4 + 1 = 2x^2.$$

Question 14 b)

Relative to the origin O, the points A, B, C and D have position vectors given respectively by $-4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $4\mathbf{i} + \lambda\mathbf{j} + 6\mathbf{k}$, $4\mathbf{i} - \mathbf{j} - \mathbf{k}$ and $2\mathbf{j} - 6\mathbf{k}$.

(i) Given that the line AC is perpendicular to the line BD, determine the value of λ. **2**

(ii) Hence find the position vector of F, the point of intersection of the lines AC and BD. **3**

Question 15 (16 marks)

Question 15 a)

D is the midpoint of the side BC or triangle ABC. **4**

Using vectors, show that $|\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = 2(|\overrightarrow{AD}|^2 + |\overrightarrow{BD}|^2)$.

Question 15 b)

(i) Using the substitution $t = \tan\frac{x}{2}$, show that **3**

$$\int_0^{\frac{\pi}{2}} \frac{1}{5 + 5\sin x - 3\cos x}\,dx = \int_0^1 \frac{1}{4t^2 + 5t + 1}\,dt$$

(iii) Hence evaluate in the simplest exact form. **2**

$$\int_0^{\frac{\pi}{2}} \frac{1}{5 + 5\sin x - 3\cos x}\,dx$$

Question 15 c)

Show that $z.\bar{z} = |z|^2$ for any complex numbers **2**

Question 15 d)

Let $I_n = \int \frac{\sin(nx)}{\sin x}\,dx$, where $n \geq 1$.

(i) Prove that, for $n \geq 3$, **2**

$$I_n - I_{n-2} = 2\int \cos(n-1)x\;dx.$$

(ii) Hence determine the exact value of **3**

$$\int_{\pi/6}^{\pi/3} \frac{\sin 5x}{\sin x}\,dx$$

Question 16 (13 marks)

Question 16 a)

Using integration by parts, calculate $\int (1 + 2x^2)\, e^{x^2}\, dx$. **3**

Question 16 b)

A particle is moving in a straight line in simple harmonic motion. If the amplitude of the motion is 3 cm and the period of the motion is 4 seconds, calculate the:

(i) maximum velocity of the particle. **1**

(ii) maximum acceleration of the particle. **2**

(iii) speed of the particle when it is 1 cm from the centre of the motion. **2**

Question 16 c)

If $T_1 = 7$ and $T_n = 2T_{n-1} - 1$ for $n \geq 2$ show that: **2**

$$T_n = 3.2^n + 1 \text{ for } n \geq 1$$

Question 16 d)

If a and b are two positive real numbers, prove that

(i)

$\frac{a+b}{2} \geq \sqrt{ab}$ 1

(ii) $(a+b)\left(\frac{1}{a} + \frac{1}{b}\right) \geq 4$ **2**