EXERCISE 4.2

BASIC

Find

1 $\int x^2(x^3+4)^5\,dx$

2 $\int \frac{x^2}{\sqrt{2x^3-1}}\,dx$

3 $\int e^{2x}\sqrt{e^{2x}-1}\,dx$

4 $\int x\cos x^2\,dx$

5 $\int \tan^3 x\sec^2 x\,dx$

6 $\int \frac{\cos x}{\sin^5 x}\,dx$

MEDIUM

7 $\int \sin^{\frac{3}{2}} 2x\cos 2x\,dx$

8 $\int \sqrt{\sin 2x}\cos 2x\,dx$

9 $\int e^x\cos e^x\,dx$

10 $\int \frac{e^{\cos^{-1} x}}{\sqrt{1-x^2}}\,dx$

11 $\int \frac{\cos(\ln|x|)}{x}\,dx$

12 $\int (3x^2+2x)\sqrt{x^3+x^2}\,dx$

13 $\int x^2 e^{x^3}\,dx$

14 $\int \frac{x^3+x^2}{3x^4+4x^3}\,dx$

15 $\int \frac{\sin(\tan^{-1} x)}{1+x^2}\,dx$

16 $\int (e^{t^2}+16)te^{t^2}\,dt$

17 $\int \frac{\operatorname{cosec} x\cot x}{1+\operatorname{cosec}^2 x}\,dx$

CHALLENGING

18 $\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx$

19 $\int \frac{x\sin\left(\sqrt{2x^2-1}\right)}{\sqrt{2x^2-1}}\,dx$

20 $\int \frac{e^{\sqrt{\sin x}}}{\sec x\sqrt{\sin x}}\,dx$

1

$$\int x^2(x^3+4)^5\,dx$$

$$= \frac{1}{3}\int 3x^2(x^3+4)^5\,dx$$

$$= \frac{1}{3}\times\frac{(x^3+4)^6}{6}+c$$

$$= \frac{(x^3+4)^6}{18}+c$$

$$\int x^2(x^3+4)^5\,dx$$

$$= \int x^2\times u^5\times\frac{du}{3x^2}$$

$$= \frac{1}{3}\int u^5\,du$$

$$= \frac{1}{3}\times\frac{u^6}{6}+c$$

$$= \frac{(x^3+4)^6}{18}+c$$

$$u = x^3+4$$

$$\frac{du}{dx} = 3x^2$$

$$dx = \frac{du}{3x^2}$$

2

$$\int \frac{x^2}{\sqrt{2x^3-1}}\,dx$$

$$= \frac{1}{6}\int 6x^2(2x^3-1)^{-\frac{1}{2}}\,dx$$

$$= \frac{1}{6}\times 2(2x^3-1)^{\frac{1}{2}}+c$$

$$= \frac{\sqrt{2x^3-1}}{3}+c$$

$$\int \frac{x^2}{\sqrt{2x^3-1}}\,dx$$

$$= \int \frac{x^2}{u}\times\frac{u\,du}{3x^2}$$

$$= \frac{1}{3}\int du$$

$$= \frac{1}{3}u+c$$

$$= \frac{\sqrt{2x^3-1}}{3}+c$$

$$u^2 = 2x^3-1$$

$$2u\,du = 6x^2\,dx$$

$$dx = \frac{u\,du}{3x^2}$$

3

$$\int e^{2x}\sqrt{e^{2x}-1}\,dx$$

$$= \frac{1}{2}\int 2e^{2x}(e^{2x}-1)^{\frac{1}{2}}\,dx$$

$$= \frac{1}{2}\times\frac{2}{3}(e^{2x}-1)^{\frac{3}{2}}+c$$

$$= \frac{\sqrt{(e^{2x}-1)^3}}{3}+c$$

$$\int e^{2x}\sqrt{e^{2x}-1}\,dx$$

$$= \int e^{2x}\times u\times\frac{u\,du}{e^{2x}}$$

$$= \int u^2\,du$$

$$= \frac{u^3}{3}+c$$

$$= \frac{\sqrt{(e^{2x}-1)^3}}{3}+c$$

$$u^2 = e^{2x}-1$$

$$2u\,du = 2e^{2x}dx$$

$$dx = \frac{u\,du}{e^{2x}}$$

4

$$\int x\cos x^2\,dx$$

$$= \frac{1}{2}\int 2x\cos x^2\,dx$$

$$= \frac{1}{2}\sin x^2 + c$$

$$\int x\cos x^2\,dx$$

$$= \int x \times \cos u \times \frac{du}{2x}$$

$$= \frac{1}{2}\int \cos u\,du$$

$$= \frac{1}{2}\sin u + c$$

$$= \frac{1}{2}\sin x^2 + c$$

$$\boxed{\begin{aligned} u &= x^2 \\ \frac{du}{dx} &= 2x \\ dx &= \frac{du}{2x} \end{aligned}}$$

5

$$\int \tan^3 x\sec^2 x\,dx$$

$$= \int \sec^2 x\,(\tan x)^3\,dx$$

$$= \frac{\tan^4 x}{4} + c$$

$$\int \tan^3 x\sec^2 x\,dx$$

$$= \int u^3 \times \sec^2 x \times \frac{du}{\sec^2 x}$$

$$= \int u^3\,du$$

$$= \frac{u^4}{4} + c$$

$$= \frac{\tan^4 x}{4} + c$$

$$\boxed{\begin{aligned} u &= \tan x \\ du &= \sec^2 x\,dx \\ dx &= \frac{du}{\sec^2 x} \end{aligned}}$$

6

$$\int \frac{\cos x}{\sin^5 x}\,dx$$

$$= \int \cos x\,(\sin x)^{-5}\,dx$$

$$= \frac{(\sin x)^{-4}}{-4} + c$$

$$= -\frac{1}{4\sin^4 x} + c$$

$$\int \frac{\cos x}{\sin^5 x}\,dx$$

$$= \int \frac{\cos x}{u^5} \times \frac{du}{\cos x}$$

$$= \int u^{-5}\,du$$

$$= \frac{u^{-4}}{-4} + c$$

$$= -\frac{1}{4\sin^4 x} + c$$

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x\,dx \\ dx &= \frac{du}{\cos x} \end{aligned}}$$

7

$$\int \sin^{\frac{3}{2}} 2x \cos 2x \, dx$$
$$= \frac{1}{2}\int 2\cos 2x\,(\sin 2x)^{\frac{3}{2}}\,dx$$
$$= \frac{1}{2} \times \frac{2}{5}(\sin 2x)^{\frac{5}{2}} + c$$
$$= \frac{\sin^{\frac{5}{2}} 2x}{5} + c$$

$$\int \sin^{\frac{3}{2}} 2x \cos 2x \, dx$$
$$= \int u^{\frac{3}{2}} \times \cos 2x \times \frac{du}{2\cos 2x}$$
$$= \frac{1}{2}\int u^{\frac{3}{2}}\,du$$
$$= \frac{1}{2} \times \frac{2}{5} u^{\frac{5}{2}} + c$$
$$= \frac{\sin^{\frac{5}{2}} 2x}{5} + c$$

$$u = \sin 2x$$
$$du = 2\cos 2x\,dx$$
$$dx = \frac{du}{2\cos 2x}$$

8

$$\int \sqrt{\sin 2x} \cos 2x \, dx$$
$$= \frac{1}{2}\int (2\cos 2x)(\sin 2x)^{\frac{1}{2}}\,dx$$
$$= \frac{1}{2} \times \frac{2}{3}(\sin 2x)^{\frac{3}{2}} + c$$
$$= \frac{\sqrt{\sin^3 2x}}{3} + c$$

$$\int \sqrt{\sin 2x} \cos 2x \, dx$$
$$= \int u \times \cos 2x \times \frac{u\,du}{\cos 2x}$$
$$= \int u^2\,du$$
$$= \frac{u^3}{3} + c$$
$$= \frac{\sqrt{\sin^3 2x}}{3} + c$$

$$u^2 = \sin 2x$$
$$2u\,du = 2\cos 2x\,dx$$
$$dx = \frac{u\,du}{\cos 2x}$$

9

$$\int e^x \cos e^x \, dx$$
$$= \sin e^x + c$$

$$\int e^x \cos e^x \, dx$$
$$= \int e^x \cos u \times \frac{du}{e^x}$$
$$= \int \cos u\,du$$
$$= \sin u + c$$
$$= \sin e^x + c$$

$$u = e^x$$
$$du = e^x dx$$
$$dx = \frac{du}{e^x}$$

10

$$\int \frac{e^{\cos^{-1}x}}{\sqrt{1-x^2}}\,dx$$
$$= -\int \frac{-1}{\sqrt{1-x^2}} \times e^{\cos^{-1}x}\,dx$$
$$= -e^{\cos^{-1}x} + c$$

$$\int \frac{e^{\cos^{-1}x}}{\sqrt{1-x^2}}\,dx$$
$$= \int \frac{e^u}{\sqrt{1-x^2}} \times \left(-\sqrt{1-x^2}\,du\right)$$
$$= -\int e^u\,du$$
$$= -e^u + c$$
$$= -e^{\cos^{-1}x} + c$$

$$\boxed{\begin{aligned} u &= \cos^{-1}x \\ du &= \frac{-1}{\sqrt{1-x^2}}dx \\ dx &= -\sqrt{1-x^2}du \end{aligned}}$$

11

$$\int \frac{\cos(\ln|x|)}{x}\,dx$$
$$= \int \frac{1}{x}\cos(\ln|x|)\,dx$$
$$= \sin(\ln|x|) + c$$

$$\int \frac{\cos(\ln|x|)}{x}\,dx$$
$$= \int \frac{\cos u}{x} \times x\,du$$
$$= \int \cos u\,du$$
$$= \sin u + c$$
$$= \sin(\ln|x|) + c$$

$$\boxed{\begin{aligned} u &= \ln|x| \\ du &= \frac{1}{x}dx \\ dx &= x\,du \end{aligned}}$$

12

$$\int (3x^2+2x)\sqrt{x^3+x^2}\,dx$$
$$= \int (3x^2+2x)(x^3+x^2)^{\frac{1}{2}}\,dx$$
$$= \frac{2(x^3+x^2)^{\frac{3}{2}}}{3} + c$$
$$= \frac{2\sqrt{(x^3+x^2)^3}}{3} + c$$

$$\int (3x^2+2x)\sqrt{x^3+x^2}\,dx$$
$$= \int (3x^2+2x) \times u \times \frac{2u\,du}{3x^2+2x}$$
$$= 2\int u^2\,du$$
$$= 2 \times \frac{u^3}{3} + c$$
$$= \frac{2(x^3+x^2)^{\frac{3}{2}}}{3} + c$$
$$= \frac{2\sqrt{(x^3+x^2)^2}}{3} + c$$

$$\boxed{\begin{aligned} u^2 &= x^3+x^2 \\ 2u\,du &= (3x^2+2x)dx \\ dx &= \frac{2u\,du}{3x^2+2x} \end{aligned}}$$

13

$$\int x^2 e^{x^3}\,dx$$
$$= \frac{1}{3}\int 3x^2 e^{x^3}\,dx$$
$$= \frac{1}{3}e^{x^3} + c$$

$$\int x^2 e^{x^3}\,dx$$
$$= \int x^2 \times e^u \times \frac{du}{3x^2}$$
$$= \frac{1}{3}\int e^u\,du$$
$$= \frac{1}{3}e^u + c$$
$$= \frac{1}{3}e^{x^3} + c$$

$$\boxed{\begin{aligned} u &= x^3 \\ du &= 3x^2dx \\ dx &= \frac{du}{3x^2} \end{aligned}}$$

14

$$\int \frac{x^3+x^2}{3x^4+4x^3}\,dx$$

$$= \frac{1}{12}\int \frac{12x^3+12x^2}{3x^4+4x^3}\,dx$$

$$= \frac{1}{12}\ln|3x^4+4x^3| + c$$

$$\int \frac{x^3+x^2}{3x^4+4x^3}\,dx$$

$$= \int \frac{x^3+x^2}{u} \times \frac{du}{12x^3+12x^2}$$

$$= \frac{1}{12}\int \frac{1}{u}\,du$$

$$= \frac{1}{12}\ln|u| + c$$

$$= \frac{1}{12}\ln|3x^4+4x^3| + c$$

$$\boxed{\begin{aligned} u &= 3x^4+4x^3 \\ du &= (12x^3+12x^2)dx \\ dx &= \frac{du}{12x^3+12x^2} \end{aligned}}$$

15

$$\int \frac{\sin(\tan^{-1} x)}{1+x^2}\,dx$$

$$= \int \frac{1}{1+x^2}\sin(\tan^{-1} x)\,dx$$

$$= -\cos(\tan^{-1} x) + c$$

$$= -\frac{1}{\sqrt{1+x^2}} + c$$

$$\int \frac{\sin(\tan^{-1} x)}{1+x^2}\,dx$$

$$= \int \frac{\sin u}{1+x^2} \times (1+x^2)du$$

$$= \int \sin u\,du$$

$$= -\cos u + c$$

$$= -\cos(\tan^{-1} x) + c$$

$$= -\frac{1}{\sqrt{1+x^2}} + c$$

$$\boxed{\begin{aligned} u &= \tan^{-1} x \\ du &= \frac{1}{1+x^2}\,dx \\ dx &= (1+x^2)du \end{aligned}}$$

16

$$\int (e^{t^2}+16)te^{t^2}\,dt$$

$$= \frac{1}{2}\int 2te^{t^2}\left(e^{t^2}+16\right)dt$$

$$= \frac{1}{2} \times \frac{\left(e^{t^2}+16\right)^2}{2} + c$$

$$= \frac{1}{4}\left(e^{t^2}+16\right)^2 + c$$

$$\int (e^{t^2}+16)te^{t^2}\,dt$$

$$= \int u \times te^{t^2} \times \frac{du}{2te^{t^2}}$$

$$= \frac{1}{2}\int u\,du$$

$$= \frac{1}{2} \times \frac{u^2}{2} + c$$

$$= \frac{1}{4}\left(e^{t^2}+16\right)^2 + c$$

$$\boxed{\begin{aligned} u &= e^{t^2}+16 \\ du &= 2te^{t^2}dt \\ dt &= \frac{du}{2te^{t^2}} \end{aligned}}$$

17

$$\int \frac{\operatorname{cosec} x \cot x}{1+\operatorname{cosec}^2 x}\,dx$$

$$= -\int \frac{-\operatorname{cosec} x \cot x}{1+(\operatorname{cosec} x)^2}\,dx$$

$$= -\tan^{-1}(\operatorname{cosec} x) + c$$

$$\boxed{\begin{aligned} u &= \operatorname{cosec} x \\ du &= -\operatorname{cosec} x \cot x\,dx \\ dx &= -\frac{du}{\operatorname{cosec} x \cot x} \end{aligned}}$$

$$\int \frac{\operatorname{cosec} x \cot x}{1+\operatorname{cosec}^2 x}\,dx$$

$$= \int \frac{\operatorname{cosec} x \cot x}{1+u^2} \times \left(-\frac{du}{\operatorname{cosec} x \cot x}\right)$$

$$= -\int \frac{1}{1+u^2}\,du$$

$$= -\tan^{-1} u + c$$

$$= -\tan^{-1}(\operatorname{cosec} x) + c$$

18

$$\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx$$

$$= \int \frac{\cos^2 x}{\sqrt{\sin x}} \times \cos x\,dx$$

$$= \int \frac{1-\sin^2 x}{\sqrt{\sin x}} \times \cos x\,dx$$

$$= \int \cos x\left((\sin x)^{-\frac{1}{2}} - (\sin x)^{\frac{3}{2}}\right)dx$$

$$= 2(\sin x)^{\frac{1}{2}} - \frac{2}{5}(\sin x)^{\frac{5}{2}} + c$$

$$= 2\sqrt{\sin x} - \frac{2\sqrt{\sin^5 x}}{5} + c$$

$$\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx$$

$$\boxed{\begin{aligned} u^2 &= \sin x \\ 2u\,du &= \cos x\,dx \\ dx &= \frac{2u\,du}{\cos x} \end{aligned}}$$

$$= \int \frac{\cos^3 x}{u} \times \frac{2u\,du}{\cos x}$$

$$= 2\int \cos^2 x\,du$$

$$= 2\int (1-\sin^2 x)\,du$$

$$= 2\int (1-u^4)\,du$$

$$= 2u - \frac{2u^5}{5} + c$$

$$= 2\sqrt{\sin x} - \frac{2\sqrt{\sin^5 x}}{5} + c$$

19

$$\int \frac{x\sin\left(\sqrt{2x^2-1}\right)}{\sqrt{2x^2-1}}\,dx$$

$$= \frac{1}{2}\int \left(\frac{1}{2}(2x^2-1)^{-\frac{1}{2}} \times 4x\right)\sin\left(\sqrt{2x^2-1}\right)dx$$

$$= -\frac{1}{2}\cos\sqrt{2x^2-1} + c$$

$$\int \frac{x\sin\left(\sqrt{2x^2-1}\right)}{\sqrt{2x^2-1}}\,dx$$

$$= \int \frac{x\sin u}{u} \times \frac{u\,du}{2x}$$

$$= \frac{1}{2}\int \sin u\,du$$

$$= -\frac{1}{2}\cos u + c$$

$$= -\frac{1}{2}\cos\sqrt{2x^2-1} + c$$

$$\boxed{\begin{aligned} u &= (2x^2-1)^{\frac{1}{2}} \\ du &= \frac{1}{2}(2x^2-1)^{-\frac{1}{2}}(4x)dx \\ dx &= \frac{u\,du}{2x} \end{aligned}}$$

20

$$\int \frac{e^{\sqrt{\sin x}}}{\sec x\sqrt{\sin x}}\,dx$$

$$= 2\int \frac{\cos x}{2\sqrt{\sin x}}e^{\sqrt{\sin x}}\,dx$$

$$= 2e^{\sqrt{\sin x}} + c$$

$$\int \frac{e^{\sqrt{\sin x}}}{\sec x\sqrt{\sin x}}\,dx$$

$$\boxed{\begin{aligned} u^2 &= \sin x \\ 2u\,du &= \cos x\,du \\ dx &= \frac{2u\,du}{\cos x} \end{aligned}}$$

$$= \int \frac{e^u}{\sec x\sqrt{2x^2-1}} \times \frac{2u\,du}{\cos x}$$

$$= 2\int e^u\,du$$

$$= 2e^u + c$$

$$= 2e^{\sqrt{\sin x}} + c$$