EXERCISE 2.2

BASIC

1 Simplify:

a $(3+2i)+(4+i)$ **b** $(3+2i)-(4+i)$

2 Simplify:

a $3(2+4i)$ **b** $-2(4-3i)$

3 Simplify:

a $i(3+i)$ **b** $-2i(3-4i)$

4 Simplify $(3+i)(4-2i)$

5 Simplify $(2-i)^2$

6 Simplify $(2+i)^3$

7 Find the conjugate of each complex number:

a $2-2i$ **b** $-6i$

8 Simplify $\dfrac{2-3i}{2i}$

9 Simplify $\dfrac{3-2i}{1-i}$

10 Given $z=3+2i$ find $\text{Re}(2z+iz)$

11 Given $z=a+ib$, find a and b such that $z-2i\bar{z}=3+4i$

12 Find the real numbers a and b such that $(1+2i)(1-3i)=a+ib$

13 Let $z=1+i$ and $w=1-i$. Find, in the form $x+iy$

a $z+iw$ **b** $z^2\overline{w}$ **c** $\frac{z}{w}$

14 If $z=3+i$ and $w=-2+2i$, find:

a $z+w$ **b** $z-w$ **c** $2z$ **d** $-3w$

e iz **f** $z\times w$ **g** $\dfrac{w}{3i}$ **h** $\dfrac{z}{w}$

MEDIUM

15 Given that a and b are real numbers and $\frac{a}{1+i}-\frac{b}{2i}=2$ find the values of a and b.

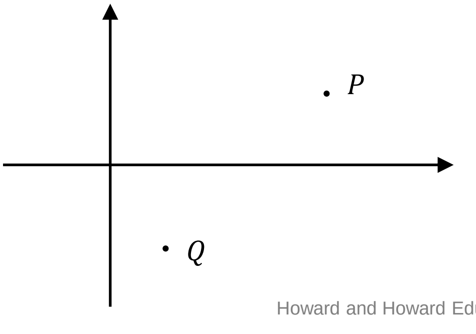
16 The points P and Q represent the complex numbers z and w respectively.

Mark the following points on the diagram.

a the point R representing $\bar{z}$

b the point S representing iw

c the point T representing $w+z$



1 **a** $(3+2i)+(4+i)=7+3i$ **b** $(3+2i)-(4+i)=-1+i$

2 **a** $3(2+4i)=6+12i$ **b** $-2(4-3i)=-8+6i$

3 **a** $i(3+i)=3i+i^2=-1+3i$ **b** $-2i(3-4i)=-6i+8i^2=-8-6i$

4 $(3+i)(4-2i)=12-6i+4i-2i^2=12-2i+2=14-2i$

5 $(2-i)^2=4-4i+i^2=4-4i-1=3-4i$

6 $(2+i)^3=(2)^3+3(2)^2(i)^1+3(2)(i)^2+i^3=8+12i-6-i=2+11i$

7 **a** $\overline{2-2i}=2+2i$ **b** $\overline{-6i}=6i$

8 $\dfrac{2-3i}{2i}\times\dfrac{i}{i}=\dfrac{2i+3}{-2}=\dfrac{-3-2i}{2}$

9 $\dfrac{3-2i}{1-i}\times\dfrac{1+i}{1+i}=\dfrac{3+3i-2i+2}{1^2+1^2}=\dfrac{5+i}{2}$

10 $\mathrm{Re}(2z+iz)=\mathrm{Re}\big(2(3+2i)+i(3+2i)\big)=\mathrm{Re}(6+4i+3i-2)=\mathrm{Re}(4+7i)=4$

11 LHS $=a+ib-2i(a-ib)=a+ib-2ai-2b=(a-2b)+(b-2a)i$

$\therefore a-2b=3\quad(1)\qquad b-2a=4\quad(2)$

$(1)+2(2):\ -3a=11\quad\rightarrow\quad a=-\dfrac{11}{3}$

$\text{sub in }(1):\ -\dfrac{11}{3}-2b=3\ \rightarrow\ b=-\dfrac{10}{3}$

12 $1-3i+2i-6i^2=7-i\quad\therefore a=7,b=-1$

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a

$$\begin{aligned}z+iw&=1+i+i(1-i)\\&=1+i+i+1\\&=2+2i\end{aligned}$$

b

$$\begin{aligned}z^2\overline{w}&=(1+i)^2(1+i)\\&=(1+i)^3\\&=1^3+3(1)^2i+3(1)i^2\\&=1+3i-3-i\\&=-2+2i\end{aligned}$$

c

$$\begin{aligned}\frac{z}{w}&=\frac{1+i}{1-i}\times\frac{1+i}{1+i}\\&=\frac{1+2i-1}{1^2+1^2}\\&=i\end{aligned}$$

14 **a** $z+w=(3+i)+(-2+2i)=1+3i$
b $z-w=(3+i)-(-2+2i)=5-i$
c $2z=2(3+i)=6+2i$
d $-3w=-3(-2+2i)=6-6i$
e $iz=i(3+i)=-1+3i$
f $z\times w=(3+i)(-2+2i)=-6+6i-2i-2=-8+4i$
g $\dfrac{w}{3i}=\dfrac{-2+2i}{3i}\times\dfrac{i}{i}=\dfrac{-2i-2}{-3}=\dfrac{2}{3}+\dfrac{2}{3}i$
h $\dfrac{z}{w}=\dfrac{3+i}{-2+2i}\times\dfrac{-2-2i}{-2-2i}=\dfrac{-6-6i-2i+2}{4+4}=\dfrac{-4-8i}{8}=\dfrac{-1-2i}{2}$

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$$\frac{a}{1+i}\times\frac{1-i}{1-i}-\frac{b}{2i}\times\frac{i}{i}=2$$

$$\frac{a-ai}{1^2+1^2}-\frac{bi}{-2}=2$$

$$\frac{a-ai+bi}{2}=2$$

$$a+(b-a)i=4$$

$$\therefore a=4, \qquad b-a=0\rightarrow b=4$$

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