BASIC

1   Consider the statements:

P: n is a multiple of 9. Q: n is a multiple of 3.

Which of the following is true? There is more than one correct answer.

a) $P \Rightarrow Q$ b) $P \Leftarrow Q$ c) $Q \Rightarrow P$ d) P implies Q

2   If m is a positive integer, add the most relevant quantifier to the start of the statement to make it true as often as possible.

a) __ m, $2m$ is even   b) __ m, $m^2 = 4$

c) __ m, $m^2 \leq 2$ d) __ m, $1 + \cos m \geq 0$

3   If m and n are odd, prove
a mn is odd
b $m + n$ is even

4   If m is a multiple of 4, prove m^2 is a multiple of 16

5   Prove that the sum of any two consecutive numbers equals the difference of their squares.

6   If m is even, prove m^2 is even.

7   Prove that the product of any three consecutive numbers is even.

8   Prove that the sum of any four consecutive numbers is even.

MEDIUM

9   Given $a^k - b^k = (a - b)(a^{k-1} + a^{k-2}b + a^{k-3}b^2 \ldots + b^{k-1})$ prove
a $\dfrac{3^k}{2}$ always has a remainder of 1.
b $3^{2n} - 1$ is divisible by 4

10   If $a + b = 2$ prove $a^2 + 2b = b^2 + 2a$

11   Prove the expression $a^3 - a + 1$ is odd for all positive integer values of a.

12   Prove $n^2 - 1$ is divisible by 3 if n is not a multiple of 3.