EXERCISE 4.1

BASIC

Find

1 $\int \frac{dx}{x^2+6x+25}$

2 $\int \frac{1}{\sqrt{20-8x-x^2}}dx$

3 $\int \frac{1}{\sqrt{x^2-8x+7}}dx$

4 $\int \frac{1}{x^2-3x+3}dx$

MEDIUM

5 $\int \frac{1}{\sqrt{4x^2-16}}dx$

6 $\int \frac{2}{4x^2-4x+17}dx$

7 $\int \frac{2x}{x^4+2x^2+5}dx$

CHALLENGING

8 $\int \frac{\cos x}{\sin^2 x+2\sin x+5}dx$

9 $\int \frac{e^x}{\sqrt{e^{2x}+2e^x-3}}dx$

10 $\int \frac{\cos x-\sin x}{2+\sin 2x}dx$

1

$$\int \frac{dx}{x^2+6x+25}$$

$$= \int \frac{dx}{x^2+6x+9+16}$$

$$= \int \frac{dx}{(x+3)^2+4^2}$$

$$= \frac{1}{4}\tan^{-1}\left(\frac{x+3}{4}\right)+c$$

2

$$\int \frac{dx}{\sqrt{20-8x-x^2}}dx$$

$$= \int \frac{dx}{\sqrt{-(x^2+8x-20)}}$$

$$= \int \frac{dx}{\sqrt{-\left((x+4)^2-36\right)}}$$

$$= \int \frac{dx}{\sqrt{6^2-(x+4)^2}}$$

$$= \sin^{-1}\left(\frac{x+4}{6}\right)+c$$

3

$$\int \frac{dx}{\sqrt{x^2-8x+7}}$$

$$= \int \frac{dx}{\sqrt{x^2-8x+16-9}}$$

$$= \int \frac{1}{\sqrt{(x-4)^2-3^2}}dx$$

$$= \ln\left|x-4+\sqrt{x^2-8x+7}\right|+c$$

4

$$\int \frac{1}{x^2-3x+3}dx$$

$$= \int \frac{1}{x^2-3x+\frac{9}{4}+\frac{3}{4}}dx$$

$$= \int \frac{1}{\left(x-\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x-\frac{3}{2}}{\sqrt{3}/2}\right)+c$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-3}{\sqrt{3}}\right)+c$$

5

$$\int \frac{dx}{\sqrt{4x^2-16}}$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{x^2-2^2}}$$

$$= \frac{1}{2}\ln\left|x+\sqrt{x^2-4}\right|+c$$

6

$$\int \frac{2}{4x^2-4x+17}dx$$

$$= \int \frac{2}{(2x-1)^2+4^2}dx$$

$$= 2\times\frac{1}{4}\tan^{-1}\left(\frac{2x-1}{4}\right)+c$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{2x-1}{4}\right)+c$$

7

$$\int \frac{2x}{x^4 + 2x^2 + 5}\,dx$$

$$= \int \frac{2x}{(x^2+1)^2 + 2^2}\,dx$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{2}\right) + c$$

8

$$\int \frac{\cos x}{\sin^2 x + 2\sin x + 5}\,dx$$

$$= \int \frac{\cos x}{(\sin x + 1)^2 + 2^2}\,dx$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{\sin x + 1}{2}\right) + c$$

9

$$\int \frac{e^x}{\sqrt{e^{2x} + 2e^x - 3}}\,dx$$

$$= \int \frac{e^x}{\sqrt{(e^x+1)^2 - 2^2}}\,dx$$

$$= \ln\left|e^x + 1 + \sqrt{e^{2x} + 2e^x - 3}\right| + c$$

10

$$\int \frac{\cos x - \sin x}{2 + \sin 2x}\,dx$$

$$= \int \frac{\cos x - \sin x}{2 + 2\sin x \cos x}\,dx$$

$$= \int \frac{\cos x - \sin x}{\sin^2 x + 2\sin x \cos x + \cos^2 x + 1}\,dx$$

$$= \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2 + 1}\,dx$$

$$= \tan^{-1}(\sin x + \cos x) + c$$