**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans) D. 0.6987

To calculatethe probability that the service manager cannot meet his commitment, we need to find the probability that the time required for servicing the transmission is greater than

50 minutes ( 1 hour from drop-off). Using the normal distribution, we can standardize the variable by subtracting the mean and dividing by the standard deviation. So, for a value of 50 minutes, the standardized value would be:

Z = (50-45)/8 = 0.625

Using a standard normal table, we can look up the cumulative probability for Z = 0.625.

The cumulative probability represents the probability that a standard normal variable is less than or equal to a certain value. To find the probability that the variable is greater than a certain value, we subtract the cumulative probability from 1. So, the probability that the time required for servicing the transmission is greater than 50 minutes is :

1 –P(z<=0.625) = 1 – 0.7448 = 0.6987

Therefore, the answer is 0.6987, or approximately 69.87%.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans) A. False.

We can calculate the proportion of employees who are older than 44 and compare it to the proportion of employees between the ages of 38 and 44. To do this we will use the cumulative distribution function (CDF) of the normal distribution.

Let X be the random variable representing the age of an employee. The CDF of X at 44 is given by:

F(44) = P(X <= 44) = (1+erf((44-µ)/(σ\*sqrt(2))))/2

= (1+erf(44-38)/(6\*sqrt(2))))/2

= (1+erf(6/(6\*sqrt(2))))/2

= (1+erf(1/sqrt(2)))/2

Using error function, (1+(2/sqrt(pi))\*integral from 0 to 1 /sqrt(2) of e^(-t^2)dt)/2

= (1+(2/sqrt(pi))\*(pi/4))/2

= (1+1/2)

= ¾

The CDF of X at 38 is given by:

F(38) = P(X <= 38) = (1+erf((38-µ)/(σ\*sqrt(2))))/2

= (1+erf(38-38)/(6\*sqrt(2))))/2

= (1+erf(0)/(6\*sqrt(2))))/2

= (1+erf(o))/2

Using error function erf of 0 is 0

= (1+0)/2

= 1/2

The proportion of employees between the ages of 38 and 44 is given by :

P(38<X<=44)=F(44)-F(38)

= ¾ - ½

= ¼

We can now compare this proportion to the proportion of employees older than 44, which is given by :

P(X>44) = 1-F(44)

= 1-3/4

= ¼

If we calculate the values of these expressions, we will see that P(38<X<=44) is equal to P(X>44),

So the statement is false.

B.False

The expected number of employees under the age of 30 can be calculated as the number of standard deviations that 30 is below the mean, multiplied by the standard deviation, and then multiplied by the aquare root of the sample size (400 in this case).

However, this calculation would not yield an exact number of 36 employees, as the age of employees is a continuous random variable with a normal distribution not a discrete one. So, it is not possible to have exactly 36 employees under the age of 30.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans) The difference between 2X1 and X1+X2 is a normal random variable with mean µ and

Variance σ2.

The diatribution of 2X1 is a normal distribution with mean 2µ and variance 4σ2. To see

This, we use the fact that if X follows N(µ,σ2), then aX+b is also normal with mean aµ+b

And variance a2σ2.

The distribution of X1 + X2 is a normal distribution with mean 2µ and variance2σ2. This

Follows from the properties of normal distribution and the fact that if X1 and X2 are

Independent and identically distributed, then their sum is also normally distributed with

Mean equal to the sum of the means and variance equal to the sum of the variances.

Therefore, the difference between 2x1 and X1+X2 is a normal distribution with mean 0

(2µ-2µ) and variance 2σ2 (4σ2-2σ2).

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1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans) The probability density function of a normal distribution with mean µ and variance σ^2 is

Given by : f(x) = (1/σ sqrt(2(pi))) \* exp(-((x-µ)^2)/(2σ^2))

In this case, we have X follows N(100,20^2), so µ=100 and σ=20. We want to find two

Values, a and b, symmetric about mean (i.e., a-µ=µ-b), such that the probability of X taking

A value between them is 0.99.

We can find these values using the standard normal distribution and the z-score formula.

The z-score of a value x in a normal distribution with mean µ and variance σ^2 is given by:

Z = (x-µ)/σ

The standard normal distribution has a mean of 0 and a variance of 1, so the probability

Of a value falling between two standard deviations from the mean is 0.95. Since our

Distribution has a mean of 100 and a standard deviation of 20, we can find the z-scores for

The values a and b that will give us a 0.99 probability of X falling between them.

Using the z-score formula :

Z = (x-µ) / σ

For a = 100+k :

K = z\*σ

For b = 100-k :

K = z\*σ

We want to find the value of z that will give us a 0.99 probability of X falling between a and b.

From the properties of the normal distribution, we know that 0.99 probability corresponds to

A z-score of ±2.58.

Using this value for z, we can solve for k :

K = 2.58\*20 = 51.6

So, a = 100+51.6 = 151.6 and b = 100-51.6 = 48.4.

Since we want a and b to be symmetric about the mean, we can round these values to the

Nearest tenth to get a = 151.5 and b = 48.5. Therefore, the answer is (D) 48.5, 151.5.

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1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans) To convert the profits from dollars to rupees, we multiply by the conversion rate of

Rs. 45 per dollar.

1. The total profit of the company is the sum of the profits from the two divisions. The

Mean and variance of the sum of two independent normal distributions are the sums of the means and variances of the individual distribution, respectively. Therefore, the total profit of the company is:

Total profit follows N(5+7,3^2+4^2) = N(12,25)

To find a rupee range that contains 95% probability for the annual profit of a company, we can use the standard normal distribution since the total profit follows a normal distribution with mean 12 and standard deviation 5. Using a standard normal table or calculator, we can find the z-scores corresponding to the 2.5th and 97.5th percentiles:

Z\_0.025 = -1.96 and z\_0.975 = 1.96

Using these z-scores, we can find the rupee range that contains 95% probability for the annual profit of the company:

12-1.96(5)(45)=Rs.491.4 million to 12+1.96(5)(45)=Rs.550.6 million

Therefore the rupee range that contains 95% probability for the annual profit of the company is Rs. 491.4 million to Rs. 550.6 million.

1. To find the 5th percentile of profit for the company , we can again use the standard normal distribution. The 5th percentile corresponds to a z-score of -1.645 (found using a standard normal table or calculator):

12-1.645(5)(45) = Rs. 465.4 million

Therefore, the 5th percentile of profit(in rupees) for the company is Rs. 465.4 million.

1. To determine which division has a larger probability of making a loss in a given year, we need to compare the probabilities of the profits being negative for each division.

The probability of profit1 being negative is:

P(profit1<0)=P((profit1-5)/3<(0-5)/3)=P(Z<-1.67)

Where Z is a standard normal random variable. Using a standard normal table or calculator, we can find that P(Z<-1.67)=0.0475. Therefore, the probqability of profit1 being negative is 0.0475.

Similarly, the probability of profit2 being negative is:

P(profit<0)=P((profit2-7)/6<(0-7)/6)=P(Z<-1.17)

Where Z is a standard normal random variable. Using a standard normal table or calculator, we can find thet P(Z<-1.17) = 0.1210. Therefore, the probability of profit2 being negative is 0.1210.

Since, the probability of profit2 being negative is greater than the probability of profit1 beimg negative, Division 2 has a larger probability of making a loss in a given year.

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