NORTHROP GRUMMAN

PRIVATE / PROPRIETARY LEVEL I COVERSHEET

SUBJECT OR TITLE

DESIGNATION CRITERIA / AUTHORITY

Information must meet one or more of the following:

- Provides Northrop Grumman a competitive edge.
- Unrestricted, outside disclosure may cause adverse effects to Northrop Grumman or to an individual.
- Relates to or describes some aspect of the company's business that is not generally known outside the company.
- Indicates operational direction over a period of time that is not otherwise known outside the company.
- Is important to the technical or financial aspects of a product or the business as a whole and is not generally known outside the company.

Designation Authority:

- Originator

PROCESS / MARKING / IDENTIFICATION

Print Media

- Mark the title/cover page and all pages containing Private/Proprietary Level I information.
- Attach appropriate coversheet.

Electronic Media

- Mark/label removable storage media "NG P/P LI" (tapes, diskettes, CD's, memory sticks, etc.)
- Insert appropriate coversheet.

STORAGE (Print and Electronic Media)

Keep in lockable container, office, or approved open area to which access is controlled during working hours and secured during nonworking hours. Business units may designate open storage limitations.

TRANSMISSION / DISSEMINATION

INTERNAL (Within access-controlled company elements, including subsidiaries.)

Print Media

- Use private/proprietary envelope for company mail.

Electronic Media

- Transmit via the Northrop Grumman Global Network using normal procedures; no additional protection required.

TRANSMISSION / DISSEMINATION (Continued)

EXTERNAL (Outside company elements, including subsidiaries.)

Print Media

- U.S. Mail
- Express Carrier
- Hand carry; maintain in possession of a company employee.

Electronic Media

- Transmit sensitive personal information (see CO No. H407, Safe Harbor Data Protection) providing reasonable protection using one of the following methods:
  - Software encryption.
  - Document password protection. **NOTE**: The password used must meet criteria established in CO No. J104A, Password Requirements.
  - Web-server secure sockets layer (SSL) encrypted tunnel.
  - Northrop Grumman remote access client virtual private network (VPN).
- Transmit business contact personal information (see CO No. H407A, Business Contact Personal Information) using normal procedures; no additional protection required.

DISPOSITION-RETENTION (Print and Electronic Media)

Retain in accordance with CO No. A302, Records Management.

DISPOSITION-DESTRUCTION

Print Media

- Place in a burn barrel or shred.

Electronic Media

- Electronically delete file and reuse the media.

NOTE: A business-type strip shredder is sufficient as a minimum shredding process for Level I waste paper. Shredder remains may be collected and disposed of with regular waste paper. Level I sensitive scrap may be released to a contracted destruction supplier provided the service contract includes an appropriate nondisclosure agreement. The cognizant company element Law Department should be contacted for any additional guidance.

NORTHROP GRUMMAN

PRIVATE / PROPRIETARY LEVEL I COVERSHEET

E07488

FLIGHT DYNAMICS REFERENCE HANDBOOK

by

JURI KALVISTE

APRIL 1988

rev.*06-30-89*

Aircraft Division

Northrop Corporation

CONTENTS

NOMENCLATURE

AIRCRAFT ANGLES

Euler Angles — relating aircraft coordinates to inertial coordinates.

ψ — Aircraft heading angle, horizontal angle between some reference direction (e.g., north) and the projection of the aircraft x axis on the horizontal plane; positive rotation is from north to east.

θ — Aircraft pitch angle, vertical angle between the aircraft x axis and the horizontal plane; positive rotation is up.

ϕ — Aircraft roll angle, the angle between the aircraft x-z plane and the vertical plane containing the aircraft x axis; positive rotation is clockwise, abour the x axis, looking forward.

Flight path angles — relating flight path coordinates to inertial coordinates.

also refferred to as chi (χ)

σ — Flight path heading angle, horizontal angle between some reference direction (e.g., north), and the projection of the velocity vector on the horizontal plane; positive rotation is from north to east.

γ — Flight path elevation angle, vertical angle between the velocity vector and the horizontal plane; positive rotation is up.

μ — Flight path bank angle, the angle between the plane formed by the velocity vector and the lift vector, and the vertical plane containing the velocity vector; positive rotation is clockwise, about the velocity vector, looking forward.

Aerodynamic angles — relating aircraft coordinates to flight path coordinates.

α — Angle of attack, the angle between the aircraft x axis and the projection of the velocity vector on the aircraft x-z plane; positive rotation is from the z axis toward the x axis.

β — Sideslip angle, the angle between the velocity vector and the aircraft x-z plane; positive direction is when the velocity vector is to the right of the x-z plane, when looking forward.

LINEAR VELOCITIES

U — Velocity component along aircraft x axis; positive direction is forward; ft/sec.

V — Velocity component along aircraft y axis; positive direction is to the right ft/sec.

W — Velocity component along aircraft z axis; positive direction is down; ft/sec.

V_T — Total aircraft velocity; ft/sec.

V_{XZ} — Velocity component in aircraft x-z plane; ft/sec.

V_H — Velocity component in inertial horizontal plane; ft/sec.

$\dot{X}_E$ — Velocity component along inertial x axis; ft/sec.

$\dot{Y}_E$ — Velocity component along inertial y axis; ft/sec.

$\dot{Z}_E$ — Velocity component along inertial z axis; ft/sec.

$\dot{h}$ — Rate of change of altitude; positive up; ft/sec.

ANGULAR VELOCITIES

P — Aircraft angular rate component about aircraft x axis; rad/sec.

Q — Aircraft angular rate component about aircraft y axis; rad/sec.

R — Aircraft angular rate component about aircraft z axis; rad/sec.

Ω — Aircraft total angular rate; rad/sec.

ACCELERATIONS

X — Acceleration due to force along aircraft x axis; positive along positive x axis; ft/sec^2.

Y — Acceleration due to force along aircraft y axis; positive along positive y axis; ft/sec^2.

Z — Acceleration due to force along aircraft z axis; positive along positive z axis; ft/sec^2.

X_S — Acceleration due to force along aircraft x stability axis; positive along positive x stability axis; ft/sec^2.

Y_S — Acceleration due to force along aircraft y stability axis; positive along positive y stability axis; ft/sec^2.

Z_S — Acceleration due to force along aircraft z stability axis; positive along positive z stability axis; ft/sec^2.

L — Acceleration due to lift force along aircraft stability z axis; positive along negative stability z axis; ft/sec^2.

D — Acceleration due to drag force along aircraft stability x axis; positive along negative stability x axis; ft/sec^2.

N — Acceleration due to normal force along aircraft z axis; positive along negative z axis; ft/sec^2.

C — Acceleration due to chord force along aircraft x axis; positive along negative x axis; ft/sec^2.

A — Acceleration due to axial force along aircraft x axis; positive along negative x axis; ft/sec^2.

n_x — Load factor along aircraft x axis; positive along positive x axis; g's.

n_y — Load factor along aircraft y axis; positive along positive y axis; g's.

n_z — Load factor along aircraft z axis; positive along negative z axis; g's.

n_L — Load factor along lift vector; positive along negative stability z axis; g's.

g — Acceleration due to gravity, ft/sec^2.

DIRECTION COSINES

l — Cosine of angle between inertial coordinate z axis and aircraft coordinate x axis.

m — Cosine of angle between inertial coordinate z axis and aircraft coordinate y axis.

n — Cosine of angle between inertial coordinate z axis and aircraft coordinate z axis.

l' — Cosine of angle between velocity vector and aircraft coordinate x axis.

m' — Cosine of angle between velocity vector and aircraft coordinate y axis.

n' — Cosine of angle between velocity vector and aircraft coordinate z axis.

AIRCRAFT PARAMETERS

m — Aircraft mass; slugs.

I_{xx} — Moment of inertia about aircraft x axis; slug-ft^2.

I_{yy} — Moment of inertia about aircraft y axis; slug-ft^2.

I_{zz} — Moment of inertia about aircraft z axis; slug-ft^2.

I_{xy} — Product of inertia about aircraft x and y axes; slug-ft^2.

I_{xz} — Product of inertia about aircraft x and z axes; slug-ft^2.

I_{yz} — Product of inertia about aircraft y and z axes; slug-ft^2.

b — Reference wing span; feet.

$\bar{c}$ — Mean aerodynamic wing chord; feet.

S — Wing reference area; ft^2.

l_x — Accelerometer moment arm along the x axis from c.g. accelerometer location; feet.

l_y — Accelerometer moment arm along the y axis from c.g. accelerometer location; feet.

l_z — Accelerometer moment arm along the z axis from c.g. accelerometer location; feet.

TRANSFORMATION OF ANGLES

BETWEEN

FLIGHT PATH ANGLES (σ, γ, μ),

EULER ANGLES (ψ, θ, ϕ), AND

AERODYNAMIC ANGLES (α, β)

Flight path angles in terms of Euler angles and aerodynamic angles.

$$\sin\gamma = (\sin\theta\,\cos\alpha - \cos\theta\,\cos\phi\,\sin\alpha)\cos\beta - \cos\theta\,\sin\phi\,\sin\beta$$

$$\cos\gamma = \sqrt{1 - \sin^2\gamma}$$ $\Rightarrow 1 - \cos^2\gamma = \sin^2\gamma$

$$\sin(\sigma - \psi)\cos\gamma = -\sin\phi\,\sin\alpha\,\cos\beta + \cos\phi\,\sin\beta$$

$$\cos(\sigma - \psi)\cos\gamma = (\cos\theta\,\cos\alpha + \sin\theta\,\cos\phi\,\sin\alpha)\cos\beta + \sin\theta\,\sin\phi\,\sin\beta$$

$$\sin\sigma = \sin(\sigma - \psi)\cos\psi + \cos(\sigma - \psi)\sin\psi$$

$$\cos\sigma = \cos(\sigma - \psi)\cos\psi - \sin(\sigma - \psi)\sin\psi$$

$$\begin{aligned}\sin\sigma\,\cos\gamma = {} & [-\sin\phi\,\sin\alpha\,\cos\beta + \cos\phi\,\sin\beta]\cos\psi \\ & + [(\cos\theta\,\cos\alpha + \sin\theta\,\cos\phi\,\sin\alpha)\cos\beta + \sin\theta\,\sin\phi\,\sin\beta]\sin\psi\end{aligned}$$

$$\begin{aligned}\cos\sigma\,\cos\gamma = {} & [(\cos\theta\,\cos\alpha + \sin\theta\,\cos\phi\,\sin\alpha)\cos\beta + \sin\theta\,\sin\phi\,\sin\beta]\cos\psi \\ & - [-\sin\phi\,\sin\alpha\,\cos\beta + \cos\phi\,\sin\beta]\sin\psi\end{aligned}$$

$$\sin\mu\,\cos\gamma = \cos\theta\,\sin\phi\,\cos\beta + (\sin\theta\,\cos\alpha - \cos\theta\,\cos\phi\,\sin\alpha)\sin\beta$$

$$\cos\mu\,\cos\gamma = \cos\theta\,\cos\phi\,\cos\alpha + \sin\theta\,\sin\alpha$$

TRANSFORMATION OF ANGLES

BETWEEN

FLIGHT PATH ANGLES (σ, γ, μ),

EULER ANGLES (ψ, θ, ϕ), AND

AERODYNAMIC ANGLES (α, β)

Euler angles in terms of flight path angles and aerodynamic angles.

$$\sin\theta = \cos\gamma \cos\mu \sin\alpha + \sin\gamma \cos\alpha \cos\beta + \cos\gamma \sin\mu \cos\alpha \sin\beta$$
$$\cos\theta = \sqrt{1 - \sin^2\theta}$$

$$\sin(\psi - \sigma)\cos\theta = \sin\mu \sin\alpha - \cos\mu \cos\alpha \sin\beta$$
$$\begin{aligned}\cos(\psi - \sigma)\cos\theta = &- \sin\gamma \cos\mu \sin\alpha + \cos\gamma \cos\alpha \cos\beta \\ &- \sin\gamma \sin\mu \cos\alpha \sin\beta\end{aligned}$$

$$\sin\psi = \sin(\psi - \sigma)\cos\sigma + \cos(\psi - \sigma)\sin\sigma$$
$$\cos\psi = \cos(\psi - \sigma)\cos\sigma - \sin(\psi - \sigma)\sin\sigma$$

$$\begin{aligned}\sin\psi \cos\theta = &[\sin\mu \sin\alpha - \cos\mu \cos\alpha \sin\beta]\cos\sigma \\ &+ [- \sin\gamma \cos\mu \sin\alpha + \cos\gamma \cos\alpha \cos\beta \\ &\quad - \sin\gamma \sin\mu \cos\alpha \sin\beta]\sin\sigma \\ \cos\psi \cos\theta = &[- \sin\gamma \cos\mu \sin\alpha + \cos\gamma \cos\alpha \cos\beta \\ &\quad - \sin\gamma \sin\mu \cos\alpha \sin\beta]\cos\sigma \\ &- [\sin\mu \sin\alpha - \cos\mu \cos\alpha \sin\beta]\sin\sigma\end{aligned}$$

$$\sin\phi \cos\theta = \cos\gamma \sin\mu \cos\beta - \sin\gamma \sin\beta$$
$$\cos\phi \cos\theta = \cos\gamma \cos\mu \cos\alpha - \sin\gamma \sin\alpha \cos\beta - \cos\gamma \sin\mu \sin\alpha \sin\beta$$

TRANSFORMATION OF ANGLES

BETWEEN

FLIGHT PATH ANGLES (σ, γ, μ),

EULER ANGLES (ψ, θ, ϕ), AND

AERODYNAMIC ANGLES (α, β)

Aerodynamic angles in terms of flight path angles and Euler angles.

$$\sin\beta = [\sin\theta\,\sin\phi\,\cos(\sigma - \psi) + \cos\phi\,\sin(\sigma - \psi)]\cos\gamma - \cos\theta\,\sin\phi\,\sin\gamma$$

$$\cos\beta = \sqrt{1 - \sin^2\beta}$$

$$\sin\alpha\,\cos\beta = [\sin\theta\,\cos\phi\,\cos(\sigma - \psi) - \sin\phi\,\sin(\sigma - \psi)]\cos\gamma - \cos\theta\,\cos\phi\,\sin\gamma$$

$$\cos\alpha\,\cos\beta = \cos\theta\,\cos(\sigma - \psi)\cos\gamma + \sin\theta\,\sin\gamma$$

TRANSLATIONAL EQUATIONS

IN BODY AXIS RECTANGULAR COORDINATES (U, V, W)

Accelerations due to forces are in body axis coordinates, (X, Y, Z).

$$\dot{U} = R\,V - Q\,W + X - g \sin\theta$$
$$\dot{V} = P\,W - R\,U + Y + g \cos\theta \sin\phi$$
$$\dot{W} = Q\,U - P\,V + Z + g \cos\theta \cos\phi$$

Accelerations due to forces are in stability axis coordinates, (D, Y, L).

$$\dot{U} = R\,V - Q\,W - D \cos\alpha + L \sin\alpha - g \sin\theta$$
$$\dot{V} = P\,W - R\,U + Y + g \cos\theta \sin\phi$$
$$\dot{W} = Q\,U - P\,V - L \cos\alpha - D \sin\alpha + g \cos\theta \cos\phi$$

Accelerations due to forces are in terms of body axis load factor, (n_X, n_Y, n_Z).

$$\dot{U} = R\,V - Q\,W + g\,(n_X - \sin\theta)$$
$$\dot{V} = P\,W - R\,U + g\,(n_Y + \cos\theta \sin\phi)$$
$$\dot{W} = Q\,U - P\,V - g\,(n_Z - \cos\theta \cos\phi)$$

TRANSLATIONAL EQUATIONS

IN BODY AXIS SPHERICAL COORDINATES (α, β, V_T)

Accelerations due to forces are in body axis coordinates, (X, Y, Z).

$$\dot{\alpha} = Q - (P\cos\alpha + R\sin\alpha)\tan\beta + \frac{Z\cos\alpha - X\sin\alpha}{V_T\cos\beta} + \frac{g}{V_T\cos\beta}(\cos\theta\cos\phi\cos\alpha + \sin\theta\sin\alpha)$$

$$\dot{\beta} = P\sin\alpha - R\cos\alpha + \frac{1}{V_T}[Y\cos\beta - (X\cos\alpha + Z\sin\alpha)\sin\beta] + \frac{g}{V_T}(\cos\theta\sin\phi\cos\beta + \sin\theta\sin\beta\cos\alpha - \cos\theta\cos\phi\sin\beta\sin\alpha)$$

$$\dot{V}_T = Y\sin\beta + (X\cos\alpha + Z\sin\alpha)\cos\beta + g[(\cos\theta\cos\phi\sin\alpha - \sin\theta\cos\alpha)\cos\beta + \cos\theta\sin\phi\sin\beta]$$

$$\dot{\alpha} = Q - (P\cos\alpha + R\sin\alpha)\tan\beta + \frac{Z\cos\alpha - X\sin\alpha}{V_T\cos\beta} + \frac{g}{V_T\cos\beta}\cos\gamma\cos\mu$$

$$\dot{\beta} = P\sin\alpha - R\cos\alpha + \frac{1}{V_T}[Y\cos\beta - (X\cos\alpha + Z\sin\alpha)\sin\beta] + \frac{g}{V_T}\cos\gamma\sin\mu$$

$$\dot{V}_T = Y\sin\beta + (X\cos\alpha + Z\sin\alpha)\cos\beta - g\sin\gamma$$

TRANSLATIONAL EQUATIONS

IN BODY AXIS SPHERICAL COORDINATES (α, β, V_T)

Accelerations due to forces are in stability axis coordinates, (D, Y, L).

$$\dot{\alpha} = Q - (P\cos\alpha + R\sin\alpha)\tan\beta$$

$$- \frac{L}{V_T\cos\beta} + \frac{g}{V_T\cos\beta}(\cos\theta\cos\phi\cos\alpha + \sin\theta\sin\alpha)$$

$$\dot{\beta} = P\sin\alpha - R\cos\alpha + \frac{1}{V_T}(Y\cos\beta + D\sin\beta)$$

$$+ \frac{g}{V_T}(\cos\theta\sin\phi\cos\beta + \sin\theta\sin\beta\cos\alpha - \cos\theta\cos\phi\sin\beta\sin\alpha)$$

$$\dot{V}_T = Y\sin\beta - D\cos\beta$$
$$+ g\,[(\cos\theta\cos\phi\sin\alpha - \sin\theta\cos\alpha)\cos\beta + \cos\theta\sin\phi\sin\beta]$$

$$\dot{\alpha} = Q - (P\cos\alpha + R\sin\alpha)\tan\beta - \frac{L}{V_T\cos\beta} + \frac{g}{V_T\cos\beta}\cos\gamma\cos\mu$$

$$\dot{\beta} = P\sin\alpha - R\cos\alpha + \frac{1}{V_T}(Y\cos\beta + D\sin\beta) + \frac{g}{V_T}\cos\gamma\sin\mu$$

$$\dot{V}_T = Y\sin\beta - D\cos\beta - g\sin\gamma$$

TRANSLATIONAL EQUATIONS

IN BODY AXIS SPHERICAL COORDINATES (α, β, V_T)

Accelerations due to forces are in terms of body axis load factor , (n_x, n_y, n_z).

$$\dot{\alpha} = Q - (P\cos\alpha + R\sin\alpha)\tan\beta$$

$$- \frac{g}{V_T\cos\beta}\left[(n_z - \cos\theta\cos\phi)\cos\alpha + (n_x - \sin\theta)\sin\alpha\right]$$

$$\dot{\beta} = P\sin\alpha - R\cos\alpha + \frac{g}{V_T}\{(n_y + \cos\theta\sin\phi)\cos\beta$$

$$-[(n_x - \sin\theta)\cos\alpha - (n_z - \cos\theta\cos\phi)\sin\alpha]\sin\beta\}$$

$$\dot{V}_T = g\{(n_y + \cos\theta\sin\phi)\sin\beta$$

$$+[(n_x - \sin\theta)\cos\alpha - (n_z - \cos\theta\cos\phi)\sin\alpha]\cos\beta\}$$

TRANSLATIONAL EQUATIONS

IN TERMS OF EULER AND AERODYNAMIC ANGLE DIRECTION COSINES

$$\dot{l}' = m' \, R - n' \, Q + [(X + g\, l) - l' \, \dot{V}_T] \,/\, V_T$$
$$\dot{m}' = n' \, P - l' \, R + [(Y + g\, m) - m' \, \dot{V}_T] \,/\, V_T$$
$$\dot{n}' = l' \, Q - m' \, P + [(Z + g\, n) - n' \, \dot{V}_T] \,/\, V_T$$
$$\dot{V}_T = (X + g\, l)\, l' + (Y + g\, m)\, m' + (Z + g\, n)\, n'$$
$$1 = (l')^2 + (m')^2 + (n')^2$$

$$\dot{l} = m\, R - n\, Q$$
$$\dot{m} = n\, P - l\, R$$
$$\dot{n} = l\, Q - m\, P$$
$$1 = l^2 + m^2 + n^2$$

$$l' = \cos\alpha \, \cos\beta$$
$$m' = \sin\beta \qquad \sin\beta = m'$$
$$n' = \sin\alpha \, \cos\beta \qquad \tan\alpha = n' / l'$$

$$l = -\sin\theta \qquad \sin\theta = -\, l$$
$$m = \cos\theta \, \sin\phi \qquad \tan\phi = m / n$$
$$n = \cos\theta \, \cos\phi$$

$$A_{n'} = \left(\frac{1}{\cos\alpha \, \cos\beta} \right) A_\alpha$$

$$A_{m'} = (A_\beta + A_\alpha \tan\alpha \, \tan\beta) \, \frac{1}{\cos\beta}$$

$$A \equiv \mathcal{L},\ \mathcal{M},\ \mathcal{N},\ X,\ Y,\ Z$$

TRANSLATIONAL EQUATIONS

IN INERTIAL AXIS RECTANGULAR COORDINATES (X_E, Y_E, Z_E)

Accelerations due to forces are in body axis coordinates, (X, Y, Z).

$$\begin{aligned}\ddot{X}_E = {} & X\,(\cos\theta\,\cos\psi) \\ & + Y\,(\sin\phi\,\sin\theta\,\cos\psi - \cos\phi\,\sin\psi) \\ & + Z\,(\cos\phi\,\sin\theta\,\cos\psi + \sin\phi\,\sin\psi)\end{aligned}$$

$$\begin{aligned}\ddot{Y}_E = {} & X\,(\cos\theta\,\sin\psi) \\ & + Y\,(\sin\phi\,\sin\theta\,\sin\psi + \cos\phi\,\cos\psi) \\ & + Z\,(\cos\phi\,\sin\theta\,\sin\psi - \sin\phi\,\cos\psi)\end{aligned}$$

$$\begin{aligned}\ddot{Z}_E = {} & -X\,(\sin\theta) \\ & + Y\,(\sin\phi\,\cos\theta) \\ & + Z\,(\cos\phi\,\cos\theta) \\ & + g\end{aligned}$$

TRANSLATIONAL EQUATIONS

IN INERTIAL AXIS RECTANGULAR COORDINATES (X_E, Y_E, Z_E)

Accelerations due to forces are in stability axis coordinates, (D, Y, L).

$$
\begin{aligned}
\ddot{X}_E = & - (D \cos\alpha - L \sin\alpha)(\cos\theta \cos\psi) \\
& + Y (\sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi) \\
& - (L \cos\alpha + D \sin\alpha)(\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi)
\end{aligned}
$$

$$
\begin{aligned}
\ddot{Y}_E = & - (D \cos\alpha - L \sin\alpha)(\cos\theta \sin\psi) \\
& + Y (\sin\phi \sin\theta \sin\psi + \cos\phi \cos\psi) \\
& - (L \cos\alpha + D \sin\alpha)(\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi)
\end{aligned}
$$

$$
\begin{aligned}
\ddot{Z}_E = & (D \cos\alpha - L \sin\alpha)(\sin\theta) \\
& + Y (\sin\phi \cos\theta) \\
& - (L \cos\alpha + D \sin\alpha)(\cos\phi \cos\theta) \\
& + g
\end{aligned}
$$

$$
\begin{aligned}
\ddot{X}_E = & - D [(\cos\theta \cos\psi) \cos\alpha + (\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi) \sin\alpha] \\
& + L [(\cos\theta \cos\psi) \sin\alpha - (\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi) \cos\alpha] \\
& + Y (\sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi)
\end{aligned}
$$

$$
\begin{aligned}
\ddot{Y}_E = & - D [(\cos\theta \sin\psi) \cos\alpha + (\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi) \sin\alpha] \\
& + L [(\cos\theta \sin\psi) \sin\alpha - (\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi) \cos\alpha] \\
& + Y (\sin\phi \sin\theta \sin\psi + \cos\phi \cos\psi)
\end{aligned}
$$

$$
\begin{aligned}
\ddot{Z}_E = & D (\sin\theta \cos\alpha - \cos\phi \cos\theta \sin\alpha) \\
& - L (\sin\theta \sin\alpha + \cos\phi \cos\theta \cos\alpha) \\
& + Y (\sin\phi \cos\theta) \\
& + g
\end{aligned}
$$

TRANSLATIONAL EQUATIONS

IN INERTIAL AXIS RECTANGULAR COORDINATES (X_E, Y_E, Z_E)

Accelerations due to forces are in terms of body axis load factor, (n_x, n_y, n_z).

$$\begin{aligned}\ddot{X}_E = g\,[&n_x\,(\cos\theta\,\cos\psi)\\ &+ n_y\,(\sin\phi\,\sin\theta\,\cos\psi - \cos\phi\,\sin\psi)\\ &- n_z\,(\cos\phi\,\sin\theta\,\cos\psi + \sin\phi\,\sin\psi)]\end{aligned}$$

$$\begin{aligned}\ddot{Y}_E = g\,[&n_x\,(\cos\theta\,\sin\psi)\\ &+ n_y\,(\sin\phi\,\sin\theta\,\sin\psi + \cos\phi\,\cos\psi)\\ &- n_z\,(\cos\phi\,\sin\theta\,\sin\psi - \sin\phi\,\cos\psi)]\end{aligned}$$

$$\begin{aligned}\ddot{Z}_E = g\,[&- n_x\,(\sin\theta)\\ &+ n_y\,(\sin\phi\,\cos\theta)\\ &- n_z\,(\cos\phi\,\cos\theta)\\ &+ 1]\end{aligned}$$

TRANSLATIONAL EQUATIONS

IN INERTIAL AXIS SPHERICAL COORDINATES (σ, γ, V_T, μ)

Accelerations due to forces are in body axis coordinates, (X, Y, Z).

$$\begin{aligned}\dot{\sigma} = \frac{1}{V_T \cos\gamma} [& - X (\cos\alpha \sin\beta \cos\mu - \sin\alpha \sin\mu) \\ & + Y (\cos\beta \cos\mu) \\ & - Z (\sin\alpha \sin\beta \cos\mu + \cos\alpha \sin\mu)]\end{aligned}$$

$$\begin{aligned}\dot{\gamma} = \frac{1}{V_T} [& \ X (\cos\alpha \sin\beta \sin\mu + \sin\alpha \cos\mu) \\ & - Y (\cos\beta \sin\mu) \\ & + Z (\sin\alpha \sin\beta \sin\mu - \cos\alpha \cos\mu) \\ & - g \cos\gamma]\end{aligned}$$

$$\begin{aligned}\dot{V}_T = & \ X (\cos\alpha \cos\beta) \\ & + Y (\sin\beta) \\ & + Z (\sin\alpha \cos\beta) \\ & - g \sin\gamma\end{aligned}$$

$$\begin{aligned}\dot{\mu} = & \ (P \cos\alpha + R \sin\alpha) \sec\beta + \dot{\sigma} \sin\gamma \\ & + \frac{1}{V_T} (X \sin\alpha - Z \cos\alpha - g \cos\mu \cos\gamma) \tan\beta\end{aligned}$$

TRANSLATIONAL EQUATIONS

IN INERTIAL AXIS SPHERICAL COORDINATES (σ, γ, V_T, μ)

Accelerations due to forces are in stability axis coordinates, (D, Y, L).

$$\dot{\sigma} = [\mathrm{L} \sin\mu + (\mathrm{Y} \cos\beta + \mathrm{D} \sin\beta) \cos\mu] / (\mathrm{V_T} \cos\gamma)$$

$$\dot{\gamma} = [\mathrm{L} \cos\mu - (\mathrm{Y} \cos\beta + \mathrm{D} \sin\beta) \sin\mu - \mathrm{g} \cos\gamma] / \mathrm{V_T}$$

$$\dot{\mathrm{V}}_\mathrm{T} = - \mathrm{D} \cos\beta + \mathrm{Y} \sin\beta - \mathrm{g} \sin\gamma$$

$$\dot{\mu} = (\mathrm{P} \cos\alpha + \mathrm{R} \sin\alpha) \sec\beta + \dot{\sigma} \sin\gamma + (\mathrm{L} - \mathrm{g} \cos\mu \cos\gamma) \tan\beta / \mathrm{V_T}$$

$$\begin{aligned} \dot{\mu} = {} & (\mathrm{P} \cos\alpha + \mathrm{R} \sin\alpha) \sec\beta \\ & + [\mathrm{L} \sin\mu + (\mathrm{D} \sin\beta + \mathrm{Y} \cos\beta) \cos\mu] \tan\gamma / \mathrm{V_T} \\ & + (\mathrm{L} - \mathrm{g} \cos\gamma \cos\mu) \tan\beta / \mathrm{V_T} \end{aligned}$$

$$\dot{\mu} = (\mathrm{P} \cos\alpha + \mathrm{R} \sin\alpha) \cos\beta + (\mathrm{Q} - \dot{\alpha}) \sin\beta + \dot{\sigma} \sin\gamma$$

TRANSLATIONAL EQUATIONS

IN INERTIAL AXIS SPHERICAL COORDINATES (σ, γ, V_T, μ)

Accelerations due to forces are in terms of body axis load factor, (n_X, n_Y, n_Z).

$$\begin{aligned}\dot{\sigma} = \frac{g}{V_T \cos\gamma} [& - n_X (\cos\alpha \sin\beta \cos\mu - \sin\alpha \sin\mu) \\ & + n_Y (\cos\beta \cos\mu) \\ & + n_Z (\sin\alpha \sin\beta \cos\mu + \cos\alpha \sin\mu)]\end{aligned}$$

$$\begin{aligned}\dot{\gamma} = \frac{g}{V_T} [& \; n_X (\cos\alpha \sin\beta \sin\mu + \sin\alpha \cos\mu) \\ & - n_Y (\cos\beta \sin\mu) \\ & - n_Z (\sin\alpha \sin\beta \sin\mu - \cos\alpha \cos\mu) \\ & - \cos\gamma]\end{aligned}$$

$$\begin{aligned}\dot{V}_T = g\,[& n_X (\cos\alpha \cos\beta) \\ & + n_Y (\sin\beta) \\ & - n_Z (\sin\alpha \cos\beta) \\ & - \sin\gamma]\end{aligned}$$

$$\begin{aligned}\dot{\mu} = & (P \cos\alpha + R \sin\alpha) \sec\beta + \dot{\sigma} \sin\gamma \\ & + \frac{g}{V_T} (n_X \sin\alpha + n_Z \cos\alpha - \cos\mu \cos\gamma) \tan\beta\end{aligned}$$

TRANSFORMATION OF ACCELERATIONS DUE TO FORCES

X, Y, Z	- in body axis coordinates
C, Y, N	- in body axis coordinates, chord, side, normal
X_S, Y_S, Z_S	- in stability axis coordinates
D, Y, L	- in stability axis coordinates, drag, side, lift
A = C	- axial = chord
$Y = Y_S$	- side

$$X = -C = X_S \cos\alpha - Z_S \sin\alpha = -D\cos\alpha + L\sin\alpha$$

$$Z = -N = Z_S \cos\alpha + X_S \sin\alpha = -L\cos\alpha - D\sin\alpha$$

$$X_S = -D = X\cos\alpha + Z\sin\alpha = -C\cos\alpha - N\sin\alpha$$

$$Z_S = -L = Z\cos\alpha - X\sin\alpha = -N\cos\alpha + C\sin\alpha$$

$$D = -X_S = -X\cos\alpha - Z\sin\alpha = C\cos\alpha + N\sin\alpha$$

$$L = -Z_S = -Z\cos\alpha + X\sin\alpha = N\cos\alpha - C\sin\alpha$$

$$C = -X = -X_S\cos\alpha + Z_S\sin\alpha = D\cos\alpha - L\sin\alpha$$

$$N = -Z = -Z_S\cos\alpha - X_S\sin\alpha = L\cos\alpha + D\sin\alpha$$

LOAD FACTOR COMPUTED FROM ACCELERATIONS DUE TO FORCES

X, Y, Z	- in body axis coordinates
C, Y, N	- in body axis coordinates, chord, side, normal
X_S, Y_S, Z_S	- in stability axis coordinates
D, Y, L	- in stability axis coordinates, drag, side, lift
A = C	- axial = chord
$Y = Y_S$	- side

$$n_X = \frac{X}{g} = -\frac{C}{g} = \frac{1}{g}\left(X_S \cos\alpha - Z_S \sin\alpha\right) = \frac{1}{g}\left(-D\cos\alpha + L\sin\alpha\right)$$

$$n_Z = -\frac{Z}{g} = \frac{N}{g} = \frac{1}{g}\left(-Z_S \cos\alpha - X_S \sin\alpha\right) = \frac{1}{g}\left(L\cos\alpha + D\sin\alpha\right)$$

$$n_L = \frac{L}{g} = \frac{-Z_S}{g} = \frac{1}{g}\left(-Z\cos\alpha + X\sin\alpha\right) = \frac{1}{g}\left(N\cos\alpha - C\sin\alpha\right)$$

$$n_y = \frac{Y}{g}$$

$$n_L = n_Z \cos\alpha + n_X \sin\alpha$$

$$n_D = n_Z \sin\alpha - n_X \cos\alpha$$

LOAD FACTOR AT ANY STATION

Load factor at aircraft c.g.

$$n_{x_{c.g.}} = \frac{X}{g}$$

$$n_{y_{c.g.}} = \frac{Y}{g}$$

$$n_{z_{c.g.}} = -\frac{Z}{g}$$

Load factor at new station.

$$n_{x_{new}} = n_{x_{c.g.}} - \frac{l_x}{g}(Q^2 + R^2) - \frac{l_y}{g}(\dot{R} - P\,Q) + \frac{l_z}{g}(\dot{Q} + P\,R)$$

$$n_{y_{new}} = n_{y_{c.g.}} - \frac{l_y}{g}(R^2 + P^2) - \frac{l_z}{g}(\dot{P} - Q\,R) + \frac{l_x}{g}(\dot{R} + P\,Q)$$

$$n_{z_{new}} = n_{z_{c.g.}} + \frac{l_z}{g}(Q^2 + P^2) + \frac{l_x}{g}(\dot{Q} - R\,P) - \frac{l_y}{g}(\dot{P} + Q\,R)$$

c.g. shift

l_x - measured from c.g. to new location, positive forward - ft.

l_y - measured from c.g. to new location, positive toward right wing - ft.

l_z - measured from c.g. to new location, positive down - ft.

TRANSFORMATION OF BODY AXIS VELOCITIES

BETWEEN

RECTANGULAR COORDINATES (U, V, W)

AND

SPHERICAL COORDINATES (V_T, α, β)

Rectangular coordinates to spherical coordinates.

$$V_{xz} = \sqrt{U^2 + W^2}$$

$$V_T = \sqrt{U^2 + V^2 + W^2}$$

$$\sin\alpha = \frac{W}{\sqrt{U^2 + W^2}} = \frac{W}{V_{xz}}$$

$$\cos\alpha = \frac{U}{\sqrt{U^2 + W^2}} = \frac{U}{V_{xz}}$$

$$\sin\beta = \frac{V}{\sqrt{U^2 + V^2 + W^2}} = \frac{V}{V_T}$$

$$\cos\beta = \frac{\sqrt{U^2 + W^2}}{\sqrt{U^2 + V^2 + W^2}} = \frac{V_{xz}}{V_T}$$

Spherical coordinates to rectangular coordinates.

$$U = V_T \cos\beta \cos\alpha$$
$$V = V_T \sin\beta$$
$$W = V_T \cos\beta \sin\alpha$$

TRANSFORMATION OF TRANSLATIONAL EQUATIONS

BETWEEN

RECTANGULAR COORDINATES ($\dot{U}$, $\dot{V}$, $\dot{W}$)

AND

SPHERICAL COORDINATES ($\dot{V}_T$, $\dot{\alpha}$, $\dot{\beta}$)

Rectangular coordinates to shperical coordinates.

$$\dot{\alpha} = \frac{1}{V_T \cos\beta} (\dot{W} \cos\alpha - \dot{U} \sin\alpha)$$

$$\dot{\beta} = \frac{1}{V_T} (\dot{V} \cos\beta - \dot{U} \cos\alpha \sin\beta - \dot{W} \sin\alpha \sin\beta)$$

$$\dot{V}_T = \dot{U} \cos\alpha \cos\beta + \dot{V} \sin\beta + \dot{W} \sin\alpha \cos\beta$$

Spherical coordinates to rectangular coordinates.

$$\dot{U} = \dot{V}_T \cos\beta \cos\alpha - \dot{\alpha} V_T \cos\beta \sin\alpha - \dot{\beta} V_T \sin\beta \cos\alpha$$

$$\dot{V} = \dot{V}_T \sin\beta + \dot{\beta} V_T \cos\beta$$

$$\dot{W} = \dot{V}_T \cos\beta \sin\alpha + \dot{\alpha} V_T \cos\beta \cos\alpha - \dot{\beta} V_T \sin\beta \sin\alpha$$

TRANSFORMATION OF VELOCITIES

BETWEEN

BODY AXES (U, V, W)

AND

INERTIAL AXES ($\dot{X}_E$, $\dot{Y}_E$, $\dot{Z}_E$)

Body axes to inertial axes.

$$\begin{aligned}\dot{X}_E = {} & U\ (\cos\theta\ \cos\psi) \\ & + V\ (\sin\phi\ \sin\theta\ \cos\psi\ -\ \cos\phi\ \sin\psi) \\ & + W\ (\cos\phi\ \sin\theta\ \cos\psi\ +\ \sin\phi\ \sin\psi)\end{aligned}$$

$$\begin{aligned}\dot{Y}_E = {} & U\ (\cos\theta\ \sin\psi) \\ & + V\ (\sin\phi\ \sin\theta\ \sin\psi\ +\ \cos\phi\ \cos\psi) \\ & + W\ (\cos\phi\ \sin\theta\ \sin\psi\ -\ \sin\phi\ \cos\psi)\end{aligned}$$

$$\begin{aligned}\dot{Z}_E = {} & U\ (-\ \sin\theta) \\ & + V\ (\sin\phi\ \cos\theta) \\ & + W\ (\cos\phi\ \cos\theta) = -\ \dot{h}\end{aligned}$$

Inertial axes to body axes.

$$\begin{aligned}U = {} & \dot{X}_E\ (\cos\theta\ \cos\psi) \\ & + \dot{Y}_E\ (\cos\theta\ \sin\psi) \\ & + \dot{Z}_E\ (-\ \sin\theta)\end{aligned}$$

$$\begin{aligned}V = {} & \dot{X}_E\ (\sin\phi\ \sin\theta\ \cos\psi\ -\ \cos\phi\ \sin\psi) \\ & + \dot{Y}_E\ (\sin\phi\ \sin\theta\ \sin\psi\ +\ \cos\phi\ \cos\psi) \\ & + \dot{Z}_E\ (\sin\phi\ \cos\theta)\end{aligned}$$

$$\begin{aligned}W = {} & \dot{X}_E\ (\cos\phi\ \sin\theta\ \cos\psi\ +\ \sin\phi\ \sin\psi) \\ & + \dot{Y}_E\ (\cos\phi\ \sin\theta\ \sin\psi\ -\ \sin\phi\ \cos\psi) \\ & + \dot{Z}_E\ (\cos\phi\ \cos\theta)\end{aligned}$$

TRANSFORMATION OF INERTIAL VELOCITIES

BETWEEN

RECTANGULAR COORDINATES ($\dot{X}_E$, $\dot{Y}_E$, $\dot{Z}_E$)

AND

SPHERICAL COORDINATES (V_T, γ, σ)

Rectangular coordinates to spherical coordinates.

$$V_H = \sqrt{\dot{X}_E^2 + \dot{Y}_E^2}$$, horizontal velocity

$$V_T = \sqrt{\dot{X}_E^2 + \dot{Y}_E^2 + \dot{Z}_E^2}$$, total velocity

$$\sin\sigma = \frac{\dot{Y}_E}{V_H}$$, σ = heading angle

$$\cos\sigma = \frac{\dot{X}_E}{V_H}$$

$$\sin\gamma = \frac{-\dot{Z}_E}{V_T} = \frac{\dot{h}}{V_T}$$, γ = elevation angle

$$\cos\gamma = \frac{V_H}{V_T}$$

Spherical coordinates to rectangular coordinates.

$$\dot{X}_E = V_T \cos\gamma \cos\sigma$$
$$\dot{Y}_E = V_T \cos\gamma \sin\sigma$$
$$\dot{Z}_E = -V_T \sin\gamma$$

TRANSFORMATION OF INERTIAL ACCELERATIONS

BETWEEN

RECTANGULAR COORDINATES ($\ddot{X}_E$, $\ddot{Y}_E$, $\ddot{Z}_E$)

AND

SPHERICAL COORDINATES ($\dot{\sigma}$, $\dot{\gamma}$, $\dot{V}_T$)

Rectangular coordinates to spherical coordinates.

$$\dot{\sigma} = \frac{1}{V_T \cos\gamma} \left(\ddot{Y}_E \cos\sigma - \ddot{X}_E \sin\sigma \right)$$

$$\dot{\gamma} = \frac{1}{V_T} \left(- \ddot{X}_E \sin\gamma \cos\sigma - \ddot{Y}_E \sin\gamma \sin\sigma - \ddot{Z}_E \cos\gamma \right)$$

$$\dot{V}_T = \ddot{X}_E \cos\gamma \cos\sigma + \ddot{Y}_E \cos\gamma \sin\sigma - \ddot{Z}_E \sin\gamma$$

Spherical coordinates to rectangular coordinates.

$$\ddot{X}_E = \dot{V}_T \cos\gamma \cos\sigma - \dot{\gamma} V_T \sin\gamma \cos\sigma - \dot{\sigma} V_T \cos\gamma \sin\sigma$$

$$\ddot{Y}_E = \dot{V}_T \cos\gamma \sin\sigma - \dot{\gamma} V_T \sin\gamma \sin\sigma + \dot{\sigma} V_T \cos\gamma \cos\sigma$$

these are derivatives of $\dot{X}_E$ $\dot{Y}_E$ $\dot{Z}_E$ eq$^{\underline{n}}$s on p. 27

$$\ddot{Z}_E = - \dot{V}_T \sin\gamma - \dot{\gamma} V_T \cos\gamma$$

TRANSFORMATION OF RATES

BETWEEN

BODY AXIS RATES (P, Q, R)

AND

AERODYNAMIC ANGLE RATES ($\dot{\alpha}$, $\dot{\beta}$, $\dot{\mu}$)

Body axes to aerodynamic angle rates.

$$
\begin{aligned}
\dot{\alpha} = {} & Q - (P \cos\alpha + R \sin\alpha) \tan\beta \\
& + [Z \cos\alpha - X \sin\alpha \\
& + g (\cos\theta \cos\phi \cos\alpha + \sin\theta \sin\alpha)] / (V_T \cos\beta)
\end{aligned}
$$

$$
\begin{aligned}
\dot{\beta} = {} & P \sin\alpha - R \cos\alpha \\
& + \{Y \cos\beta - (X \cos\alpha + Z \sin\alpha) \sin\beta \\
& + g [\cos\theta \sin\phi \cos\beta + (\sin\theta \cos\alpha - \cos\theta \cos\phi \sin\alpha) \sin\beta]\} / V_T
\end{aligned}
$$

$$
\begin{aligned}
\dot{\mu} = {} & (P \cos\alpha + R \sin\alpha) \sec\beta + \dot{\sigma} \sin\gamma \\
& + (X \sin\alpha - Z \cos\alpha - g \cos\mu \cos\gamma) \tan\beta / V_T
\end{aligned}
$$

Aerodynamic angle rates to body axis rates.

$$
\begin{aligned}
P = {} & \dot{\mu} \cos\alpha \cos\beta + \dot{\beta} \sin\alpha - \dot{\sigma} \sin\gamma \cos\alpha \cos\beta \\
& + [Z \sin\beta - Y \sin\alpha \cos\beta \\
& + g (\cos\theta \cos\phi \sin\beta - \cos\theta \sin\phi \sin\alpha \cos\beta)] / V_T
\end{aligned}
$$

$$
\begin{aligned}
Q = {} & \dot{\mu} \sin\beta + \dot{\alpha} - \dot{\sigma} \sin\gamma \sin\beta \\
& + [X \sin\alpha - Z \cos\alpha \\
& - g (\cos\theta \cos\phi \cos\alpha + \sin\theta \sin\alpha)] \cos\beta / V_T
\end{aligned}
$$

$$
\begin{aligned}
R = {} & \dot{\mu} \sin\alpha \cos\beta - \dot{\beta} \cos\alpha - \dot{\sigma} \sin\gamma \sin\alpha \cos\beta \\
& + [- X \sin\beta + Y \cos\alpha \cos\beta \\
& + g (\sin\theta \sin\beta + \cos\theta \sin\phi \cos\alpha \cos\beta)] / V_T
\end{aligned}
$$

ROTATIONAL ACCELERATION EQUATIONS

IN

BODY AXIS COORDINATES ($\dot{P}$, $\dot{Q}$, $\dot{R}$)

Standard form.

$$\dot{P} = \frac{1}{I_{xx}} \left[(\dot{Q} - P\,R)\, I_{xy} + (\dot{R} + P\,Q)\, I_{xz} + Q\,R\,(I_{yy} - I_{zz}) + (Q^2 - R^2)\, I_{yz} \right] + \mathcal{L}$$

$$\dot{Q} = \frac{1}{I_{yy}} \left[(\dot{R} - P\,Q)\, I_{yz} + (\dot{P} + Q\,R)\, I_{xy} + P\,R\,(I_{zz} - I_{xx}) + (R^2 - P^2)\, I_{xz} \right] + \mathcal{M}$$

$$\dot{R} = \frac{1}{I_{zz}} \left[(\dot{P} - Q\,R)\, I_{xz} + (\dot{Q} + P\,R)\, I_{yz} + P\,Q\,(I_{xx} - I_{yy}) + (P^2 - Q^2)\, I_{xy} \right] + \mathcal{N}$$

For $I_{xy} = 0$ and $I_{yz} = 0$.

$$\dot{P} = \frac{1}{I_{xx}} \left[(\dot{R} + P\,Q)\, I_{xz} + Q\,R\,(I_{yy} - I_{zz}) \right] + \mathcal{L} \quad , \qquad \mathcal{L} = \frac{\bar{q}\,S\,b}{I_{xx}}\, C_l + \mathcal{L}_T$$

$$\dot{Q} = \frac{1}{I_{yy}} \left[P\,R\,(I_{zz} - I_{xx}) + (R^2 - P^2)\, I_{xz} \right] + \mathcal{M} \quad , \qquad \mathcal{M} = \frac{\bar{q}\,S\,\bar{c}}{I_{yy}}\, C_m + \mathcal{M}_T$$

$$\dot{R} = \frac{1}{I_{zz}} \left[(\dot{P} - Q\,R)\, I_{xz} + P\,Q\,(I_{xx} - I_{yy}) \right] + \mathcal{N} \quad , \qquad \mathcal{N} = \frac{\bar{q}\,S\,b}{I_{zz}}\, C_n + \mathcal{N}_T$$

Body axis rates, P, Q, R - rad/sec

Body axis accelerations, $\dot{P}$, $\dot{Q}$, $\dot{R}$ - rad/sec^2

Accelerations due to moments due to thrust, $\mathcal{L}_T$, $\mathcal{M}_T$, $\mathcal{N}_T$ - rad/sec^2

ROTATIONAL ACCELERATION EQUATIONS

IN

BODY AXIS COORDINATES ($\dot{P}$, $\dot{Q}$, $\dot{R}$)

Implicit solution for $\dot{P}$, $\dot{Q}$ and $\dot{R}$.

$$\dot{P} = \mathcal{L}' = \frac{I_{xy}}{I_{xx}}\,\dot{Q} + \frac{I_{xz}}{I_{xx}}\,\dot{R} + \mathcal{L}_I$$

$$\dot{Q} = \mathcal{M}' = \frac{I_{yz}}{I_{yy}}\,\dot{R} + \frac{I_{xy}}{I_{yy}}\,\dot{P} + \mathcal{M}_I$$

$$\dot{R} = \mathcal{N}' = \frac{I_{xz}}{I_{zz}}\,\dot{P} + \frac{I_{yz}}{I_{zz}}\,\dot{Q} + \mathcal{N}_I$$

Body axis rates, P, Q, R - rad/sec
Body axis accelerations, $\dot{P}$, $\dot{Q}$, $\dot{R}$ - rad/sec^2
Body axis accelerations due to moments,
$\mathcal{L}$, $\mathcal{M}$, $\mathcal{N}$ - rad/sec^2.

$$\mathcal{L}' = G_{11}\,\mathcal{L}_I + G_{12}\,\mathcal{M}_I + G_{13}\,\mathcal{N}_I$$
$$\mathcal{M}' = G_{21}\,\mathcal{L}_I + G_{22}\,\mathcal{M}_I + G_{23}\,\mathcal{N}_I$$
$$\mathcal{N}' = G_{31}\,\mathcal{L}_I + G_{32}\,\mathcal{M}_I + G_{33}\,\mathcal{N}_I$$

$$\mathcal{L}_I = \mathcal{L} + [P\,Q\,I_{xz} - P\,R\,I_{xy} + Q\,R\,(I_{yy} - I_{zz}) + (Q^2 - R^2)\,I_{yz}]/I_{xx}$$
$$\mathcal{M}_I = \mathcal{M} + [Q\,R\,I_{xy} - Q\,P\,I_{yz} + P\,R\,(I_{zz} - I_{xx}) + (R^2 - P^2)\,I_{xz}]/I_{yy}$$
$$\mathcal{N}_I = \mathcal{N} + [R\,P\,I_{yz} - R\,Q\,I_{xz} + P\,Q\,(I_{xx} - I_{yy}) + (P^2 - Q^2)\,I_{xy}]/I_{zz}$$

$$G_{11} = \frac{1 - G_9}{G_0} \qquad G_{12} = \frac{G_6}{G_0\,G_2} \qquad G_{13} = \frac{G_5}{G_0\,G_1}$$

$$G_{21} = \frac{G_6}{G_0\,G_3} \qquad G_{22} = \frac{1 - G_8}{G_0} \qquad G_{23} = \frac{G_4}{G_0\,G_1}$$

$$G_{31} = \frac{G_5}{G_0\,G_3} \qquad G_{32} = \frac{G_4}{G_0\,G_2} \qquad G_{33} = \frac{1 - G_7}{G_0}$$

$$G_1 = I_{xx}\,I_{yy} \qquad G_2 = I_{xx}\,I_{zz} \qquad G_3 = I_{yy}\,I_{zz}$$
$$G_4 = I_{xx}\,I_{yz} + I_{xy}\,I_{xz} \qquad G_5 = I_{yy}\,I_{xz} + I_{xy}\,I_{yz} \qquad G_6 = I_{zz}\,I_{xy} + I_{xz}\,I_{yz}$$
$$G_7 = I_{xy}^2/G_1 \qquad G_8 = I_{xz}^2/G_2 \qquad G_9 = I_{yz}^2/G_3$$
$$G_0 = 1 - (2\,I_{xy}\,I_{xz}\,I_{yz})/(G_1\,I_{zz}) - G_7 - G_8 - G_9$$

ROTATIONAL ACCELERATION EQUATIONS

IN

BODY AXIS COORDINATES ($\dot{P}$, $\dot{Q}$, $\dot{R}$)

Implicit solution for $\dot{P}$, $\dot{Q}$ and $\dot{R}$.

For $I_{xy} = 0$, $I_{yz} = 0$, and rates in deg/sec.

$$\dot{P} = [C_1 Q R + C_2 P Q + C_3 \mathcal{N} + \mathcal{L}] C_4$$

$$\dot{R} = [C_5 P Q + C_6 Q R + C_7 \mathcal{L} + \mathcal{N}] C_4$$

$$\dot{Q} = [C_8 P R + C_9 (R^2 - P^2) + \mathcal{M}] (RAD)$$

$$C_1 = \frac{I_{zz}(I_{yy} - I_{zz}) - I_{xz}^2}{I_{xx} I_{zz} (RAD)^2}, \quad C_2 = \frac{I_{xz}(I_{xx} - I_{yy} + I_{zz})}{I_{xx} I_{zz} (RAD)^2}, \quad C_3 = \frac{I_{xz}}{I_{xx}}$$

$$C_4 = \frac{I_{xx} I_{zz} (RAD)}{I_{xx} I_{zz} - I_{xz}^2}, \quad C_5 = \frac{I_{xx}(I_{xx} - I_{yy}) + I_{xz}^2}{I_{xx} I_{zz} (RAD)^2}$$

$$C_6 = \frac{I_{xz}(I_{yy} - I_{xx} - I_{zz})}{I_{xx} I_{zz} (RAD)^2}, \quad C_7 = \frac{I_{xz}}{I_{zz}}, \quad C_8 = \frac{I_{zz} - I_{xx}}{I_{yy} (RAD)^2}$$

$$C_9 = \frac{I_{xz}}{I_{yy} (RAD)^2}, \quad (RAD) = \frac{180}{\pi} = 57.2957795...$$

Body axis rates, P, Q, R - deg/sec

Body axis accelerations, $\dot{P}$, $\dot{Q}$, $\dot{R}$ - deg/sec^2

$$\mathcal{L}' = (\mathcal{L} + \frac{I_{xz}}{I_{xx}} \mathcal{N}) \frac{I_{xx} I_{zz}}{I_{xx} I_{zz} - I_{xz}^2}$$

$$\mathcal{N}' = (\mathcal{N} + \frac{I_{xz}}{I_{zz}} \mathcal{L}) \frac{I_{xx} I_{zz}}{I_{xx} I_{zz} - I_{xz}^2}$$

ROTATIONAL KINEMATIC EQUATIONS

EULER ANGLES RATES ($\dot{\psi}$, $\dot{\theta}$, $\dot{\phi}$)

Body axis rates to Euler angle rates transformation.

$$\dot{\phi} = \mathrm{P} + \tan\theta\,(\mathrm{Q}\sin\phi + \mathrm{R}\cos\phi)$$
$$\dot{\theta} = \mathrm{Q}\cos\phi - \mathrm{R}\sin\phi$$
$$\dot{\psi} = \sec\theta\,(\mathrm{Q}\sin\phi + \mathrm{R}\cos\phi)$$

Euler angle rates to body axis rates transformation.

$$\mathrm{P} = \dot{\phi} - \dot{\psi}\sin\theta$$
$$\mathrm{Q} = \dot{\psi}\sin\phi\cos\theta + \dot{\theta}\cos\phi$$
$$\mathrm{R} = \dot{\psi}\cos\phi\cos\theta - \dot{\theta}\sin\phi$$

DIRECTION COSINES (l, m, n)

$$l = -\sin\theta$$
$$m = \cos\theta\sin\phi$$
$$n = \cos\theta\cos\phi$$

$$\dot{l} = m\,\mathrm{R} - n\,\mathrm{Q}$$
$$\dot{m} = n\,\mathrm{P} - l\,\mathrm{R}$$
$$\dot{n} = l\,\mathrm{Q} - m\,\mathrm{P}$$

$$1 = l^2 + m^2 + n^2$$

DIRECTION COSINES FOR CONSTANT P, Q, R

$$l = -\sin\theta$$

$$m = \cos\theta \sin\phi$$

$$n = \cos\theta \cos\phi$$

$$\Omega = \sqrt{P^2 + Q^2 + R^2}$$

$$a = P\,d$$

$$b = Q\,d$$

$$c = R\,d$$

$$d = \frac{1}{\Omega^2}\,(l_o\,P + m_o\,Q + n_o\,R)$$

$$l = (m_o\,R - n_o\,Q)\,\frac{1}{\Omega}\sin(\Omega t) + a + (l_o - a)\cos(\Omega t)$$

$$m = (n_o\,P - l_o\,R)\,\frac{1}{\Omega}\sin(\Omega t) + b + (m_o - b)\cos(\Omega t)$$

$$n = (l_o\,Q - m_o\,P)\,\frac{1}{\Omega}\sin(\Omega t) + c + (n_o - c)\cos(\Omega t)$$

From Juri's Fligh Dyn. Ref.)

p. 18 or 29:

$$\dot{\mu} = (P\cos\alpha + R\sin\alpha)\sec\beta + \dot{\sigma}\sin\gamma + \frac{\tan\beta}{V_T}(X\sin\alpha - Z\cos\alpha - g\cos\mu\cos\gamma)$$

from p. 28, $\dot{\sigma} = \frac{1}{V_T\cos\gamma}(\ddot{Y}_E\cos\sigma - \ddot{X}_E\sin\sigma)$

$$\dot{\mu} = (P\cos\alpha + R\sin\alpha)\sec\beta + \frac{\sin\gamma}{V_T\cos\gamma}(\ddot{Y}_E\cos\sigma - \ddot{X}_E\sin\sigma) + \frac{\tan\beta}{V_T}(X\sin\alpha - Z\cos\alpha - g\cos\mu\cos\gamma)$$

from p. 7, $\cos\mu\cos\gamma = \cos\theta\cos\phi\cos\alpha + \sin\theta\sin\alpha$

$$\dot{\mu} = (P\cos\alpha + R\sin\alpha)\sec\beta + \frac{\tan\gamma}{V_T}(\ddot{Y}_E\cos\sigma - \ddot{X}_E\sin\sigma) + \frac{\tan\beta}{V_T}\left[X\sin\alpha - Z\cos\alpha - g(\cos\theta\cos\phi\cos\alpha + \sin\theta\sin\alpha)\right]$$

```
MUDOT CALCULATION  body
*      inertial
        XDDE = AX*CTHT*CPSI +
     &         AY*(SPHI*STHT*CPSI - CPHI*SPSI) +
     &         AZ*(CPHI*STHT*CPSI - SPHI*SPSI)
        YDDE = AX*CTHT*SPSI +
     &         AY*(SPHI*STHT*SPSI - CPHI*CPSI) +
     &         AZ*(CPHI*STHT*SPSI - SPHI*CPSI)
*
        XMUD = (PBDG*CALFA + RBDG*SALFA)/COSD(BETG)   +
     &         ((1./VRW)*TAND(GMVG)*(YDDE*COS(GAMH) -
     &          XDDE*SIN(GAMH))) * R2D  +
     &         (((AX*SALFA - AZ*CALFA -
     &           G*(CTHT*CPHI*CALFA + STHT*SALFA))
     &           * TAND(BETA)) / VRW) * R2D

*-------New MU calculation code (start)
*
*        CALCULATE MU
*
        CGVSMU = CTHT*SPHI*CBETA + (STHT*CALFA-CTHT*CPHI*SALFA)*SBE
TA
        CGVCMU = CTHT*CPHI*CALFA + STHT*SALFA
        IF ((CGVSMU .NE. 0.0) .OR. (CGVCMU .NE. 0.0))
     &        XMU  = ATAN2D(CGVSMU,CGVCMU)
*
*-------New MU calculation code (end)
```

from page 15

+?

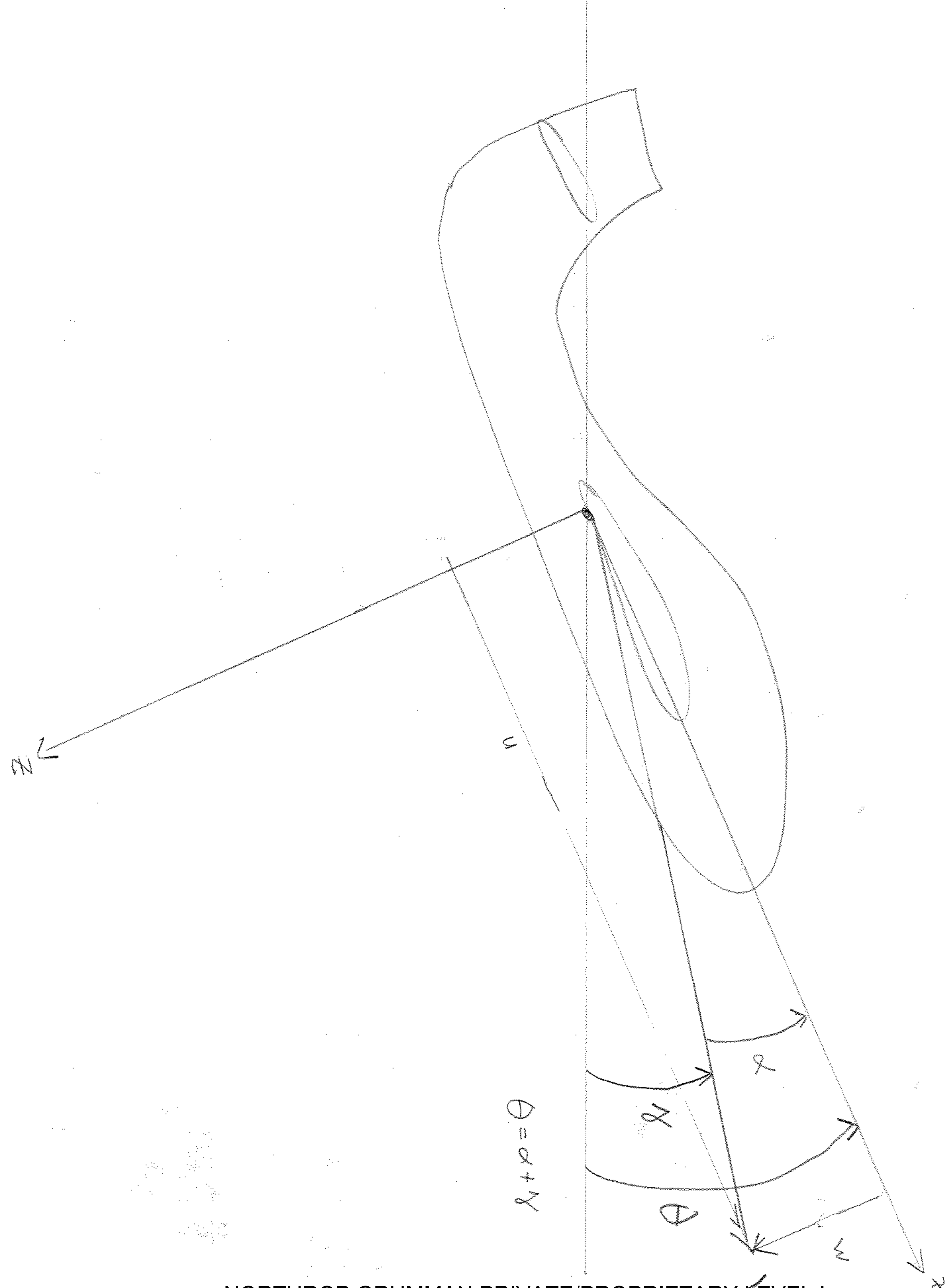
$$\ddot{X}_E = X\cos\theta\cos\psi + Y(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) + Z(\cos\phi\sin\theta\cos\psi - \sin\phi\sin\psi)$$

page ~~28~~ 29 or 18

$$\dot{\mu} = (P\cos\alpha + R\sin\alpha)/\cos\beta_G + \frac{1}{V_T}\tan\gamma\,(\ddot{Y}_E\cos\sigma - \ddot{X}_E\sin\sigma) +$$

$$\frac{1}{V_T}\left[X\sin\alpha - Z\cos\alpha - g(\cos\theta\cos\phi\cos\alpha + \sin\theta\sin\alpha\right]\tan\beta$$

where, from p. 28 $\dot{\sigma} = \frac{1}{V_T\cos\gamma}(\ddot{Y}_E\cos\sigma - \ddot{X}_E\sin\sigma)$


z
u
α
χ
θ = α + χ
θ
m
x

p. 28, has $\begin{bmatrix} \ddot{x}_E \\ \ddot{y}_E \\ \ddot{z}_E \end{bmatrix}$ in terms of $\dot{V}_T, V_T, \gamma, \sigma, \dot{\gamma}, \dot{\sigma}$

p. 28 $\ddot{x}_E = \dot{V}_T \underbrace{\cos\gamma \cos\sigma}_{\Rightarrow p. 7} - \dot{\gamma} V_T \sin\gamma \cos\sigma - \dot{\sigma} V_T \cos\gamma \sin\sigma$

NORTHROP GRUMMAN PRIVATE/PROPRIETARY LEVEL I