

Deep Learning for NLP Lecture 7: Recurrent Neural Networks

Dr. Mohsen Mesgar

Ubiquitous Knowledge Processing Lab (UKP Lab)

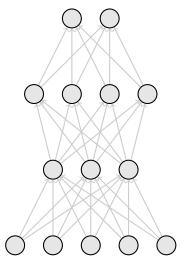
This lecture



- ► RNNs
- Vanishing and explosion
- ► GRUs
- ► LSTMs
- Application of RNNs in NLP



Recall: MultiLayer Perceptron (MLP)



$$\hat{\mathbf{y}} = f(\mathbf{h}_2 \mathbf{W}^{(3)} + \mathbf{b}^{(3)}) = [\hat{y}_1, \hat{y}_2] \\
\mathbf{W}^{(3)} \in \mathbb{R}^{|\mathbf{h}_2| \times |\hat{\mathbf{y}}|}, \mathbf{b}^{(3)} \in \mathbb{R}^{1 \times |\mathbf{h}_3|}$$

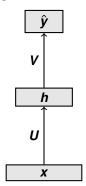
$$egin{aligned} oldsymbol{h}_2 &= fig(oldsymbol{h}_1oldsymbol{W}^{(2)} + oldsymbol{b}^{(2)}ig) \ oldsymbol{W}^{(2)} &\in \mathbb{R}^{|oldsymbol{h}_1| imes |oldsymbol{h}_2|}, oldsymbol{b}^{(2)} &\in \mathbb{R}^{1 imes |oldsymbol{h}_2|} \end{aligned}$$

$$\begin{aligned} & \boldsymbol{h}_1 = f\big(\boldsymbol{x}\boldsymbol{W}^{(1)} + \boldsymbol{b}^{(1)}\big), \\ & \boldsymbol{W}^{(1)} \in \mathbb{R}^{|\boldsymbol{x}| \times |\boldsymbol{h}_1|}, \boldsymbol{b}^{(1)} \in \mathbb{R}^{1 \times |\boldsymbol{h}_1|} \end{aligned}$$

$$\mathbf{x} = [x_1, x_2, x_3, x_4], \mathbf{x} \in \mathbb{R}^{1 \times |\mathbf{x}|}$$



Recall: Feedforward



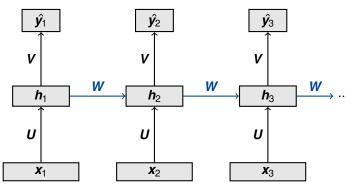
Motivation



- Text is a sequence of symbols.
- Symbols show characters, words etc.
- input = "a persian cat sat on the mat". \longrightarrow input = $(x_1 = a, x_2 = persian, x_3 = cat, x_4 = sat, x_5 = on, x_6 = the, x_7 = mat)$
- As such symbols are related to each other in text, so should symbols' representations.
- How can we relate representations of symbols to each other?

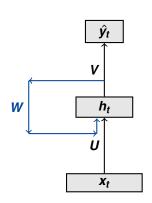


Recurrent









Recurrent



▶ input = a sequence of vectors = $[x_1, x_2, x_3, ..., x_n]$, $x_t \in \mathbb{R}^{1 \times |x|}$

$$\mathbf{h}_{t} = f(\mathbf{x}_{t}U + \mathbf{h}_{t-1}W + \mathbf{b}_{h})$$

- ▶ $\mathbf{U} \in \mathbb{R}^{|\mathbf{x}| \times |h|}$, |h| is the dimensionality of hidden states $\mathbf{W} \in \mathbb{R}^{|h| \times |h|}$
- **W** ∈ K|''|∧|'
- $lackbox{b}_h \in \mathbb{R}^{1 \times |h|}$

$$\hat{\pmb{y}}_{\pmb{t}} = g\big(\pmb{h}_{\pmb{t}} \pmb{V} + \pmb{b}_{\hat{y}}\big)$$

- $m{V} \in \mathbb{R}^{|h| imes |\hat{y}|}$
- $\mathbf{b}_{\hat{y}} \in \mathbb{R}^{1 \times |\hat{y}|}$

Training



- ▶ input = $[x_1,...,x_n]$, \longrightarrow output = $[y_1,...,y_n]$
- ▶ The loss of sequence prediction is the mean of step losses,

$$\ell = \frac{1}{n} \sum_{t=1}^n \ell_t(y_t, \hat{y}_t).$$

- Use backprop to compute gradients.
- Use SGD to update parameters.



Some Properties of RNNs

- Hidden state is known also as memory.
- In principle the hidden state represents information from the first step until the current step conditioned on current input symbol.
- So RNNs can capture left-to-right order of input symbols





- input = $(x1 = a, x2 = persian, x3 = cat) \longrightarrow$ output = $(y_1 = DET, y_2 = ADJ, y_3 = NOUN)$
- Assume following embeddings for input:

$$\mathbf{x}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\mathbf{x}_2 = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}$$

$$\mathbf{x}_2 = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}$$

- $\mathbf{x}_3 = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}$
- Assume following 1-hot vectors for output:

$$\mathbf{y}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$y_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

Example



- The sequence prediction task can be solved by RNNs
- ▶ Let |h| = 2, f be ReLU, and g be softmax
- We initialize the RNN's parameters by random values

▶
$$U = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 0.5 & 1 \end{bmatrix}$$

▶ $W = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

▶ $V = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & \frac{1}{3} & 1 \end{bmatrix}$

▶ $b_h, b_{\hat{y}}$ zero vectors

▶ $h_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}$

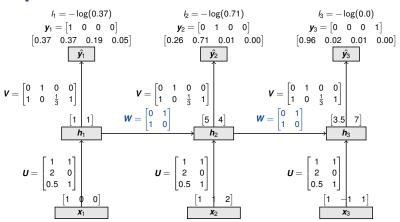
$$\mathbf{W} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & \frac{1}{3} & 1 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Example





$$\begin{split} \hat{\pmb{y}_t} &= \texttt{softmax} \big(\pmb{h_t} \pmb{V} + \pmb{b_{\hat{y}}} \big) \\ \textit{loss} &= \frac{-1}{3} \big(\log(0.37) + \log(0.71) + \log(0.0) \big) \end{split}$$

 $h_t = \text{ReLU}(x_t U + h_{t-1} W + b_h)$

TECHNISCHE UNIVERSITÄT DARMSTADT

Updating Parameters

- Computation of gradients is similar as standard MLP.
- Keep in mind the parameters of RNNs are shared through time steps.
- Sometime backprop for RNNs is called backprop through time (BPTT).
- No need to go through its details as DL frameworks compute gradients.
- If you would like to compute gradients brute-force, you can do that numerically.
 - for each weight w, compute $\frac{Loss(w+h)-Loss(w)}{h}$,
 - weight update after gradient computation is as in SGD $w \longleftarrow w \alpha \frac{\partial \text{Loss}}{\partial w}$



Updating Parameters

- RNNs can be seen as a very very deep neural model with sparse and skip connections
- The output of last steps are calculated based on the hidden state vectors at early steps
- Recall 1:

$$\mathbf{z}_{t}^{(h)} = \mathbf{x}_{t}U + \mathbf{h}_{t-1}W + \mathbf{b}_{h}$$

$$\mathbf{h}_{t} = f(\mathbf{z}_{t}^{(h)})$$

Recall 2:

$$egin{aligned} oldsymbol{z_t^{(\hat{y})}} &= oldsymbol{h_t}V + oldsymbol{b}_{\hat{y}} \ \hat{oldsymbol{y}}_t &= oldsymbol{g}ig(oldsymbol{z_t^{(\hat{y})}}ig) \end{aligned}$$

TECHNISCHE UNIVERSITÄT DARMSTADT

Updating Parameters

Recall 1:

$$egin{aligned} oldsymbol{z_t^{(h)}} &= oldsymbol{x_t} U + oldsymbol{h_{t-1}} W + oldsymbol{b_h} \ oldsymbol{h_t} &= fig(oldsymbol{z_t^{(h)}}ig) \end{aligned}$$

Recall 2:

$$egin{aligned} oldsymbol{z_t^{(\hat{y})}} &= oldsymbol{h_t}V + oldsymbol{b}_{\hat{y}} \ \hat{oldsymbol{y}}_t &= gig(oldsymbol{z_t^{(\hat{y})}}ig) \end{aligned}$$

If we have only two steps:

$$\frac{\partial \text{Loss}}{\partial \mathbf{W}} = \sum_{t=1}^{2} \frac{\partial \ell_{t}}{\partial \mathbf{W}} = \frac{\partial \ell_{1}}{\partial \mathbf{W}} + \frac{\partial \ell_{2}}{\partial \mathbf{W}}$$

$$\frac{\partial \ell_{1}}{\partial \mathbf{W}} = \frac{\partial \ell_{1}}{\partial \hat{\mathbf{y}}_{1}} \frac{\partial \hat{\mathbf{y}}_{1}}{\partial \mathbf{z}_{1}^{(\hat{\mathbf{y}})}} \frac{\partial \mathbf{z}_{1}^{(\hat{\mathbf{y}})}}{\partial \mathbf{h}_{1}} \frac{\partial \mathbf{h}_{1}}{\partial \mathbf{z}_{1}^{(h)}} \frac{\partial \mathbf{z}_{1}^{(h)}}{\partial \mathbf{W}} \propto \frac{\partial \mathbf{h}_{1}}{\partial \mathbf{z}_{1}^{(h)}}$$

$$\frac{\partial \ell_{2}}{\partial \mathbf{W}} = \frac{\partial \ell_{2}}{\partial \hat{\mathbf{y}}_{2}} \frac{\partial \hat{\mathbf{y}}_{2}}{\partial \mathbf{z}_{2}^{(\hat{\mathbf{y}})}} \frac{\partial \mathbf{z}_{2}^{(\hat{\mathbf{y}})}}{\partial \mathbf{h}_{2}} \frac{\partial \mathbf{h}_{2}}{\partial \mathbf{z}_{2}^{(h)}} \mathbf{h}_{1} \frac{\partial \mathbf{h}_{1}}{\partial \mathbf{z}_{1}^{(h)}} \frac{\partial \mathbf{z}_{1}^{(h)}}{\partial \mathbf{W}} \propto \frac{\partial \mathbf{h}_{2}}{\partial \mathbf{z}_{2}^{(h)}} \frac{\partial \mathbf{h}_{1}}{\partial \mathbf{z}_{2}^{(h)}}$$



Updating Parameters

If we have three steps:

$$\begin{split} \frac{\partial \mathsf{Loss}}{\partial \mathbf{W}} &= \sum_{t=1}^{3} \frac{\partial \ell_{t}}{\partial \mathbf{W}} = \frac{\partial \ell_{1}}{\partial \mathbf{W}} + \frac{\partial \ell_{2}}{\partial \mathbf{W}} + \frac{\partial \ell_{3}}{\partial \mathbf{W}} \\ &\qquad \qquad \frac{\partial \ell_{1}}{\partial \mathbf{W}} \propto \frac{\partial \mathbf{h}_{1}}{\partial \mathbf{z}_{1}^{(h)}} \\ &\qquad \qquad \frac{\partial \ell_{2}}{\partial \mathbf{W}} \propto \frac{\partial \mathbf{h}_{2}}{\partial \mathbf{z}_{2}^{(h)}} \frac{\partial \mathbf{h}_{1}}{\partial \mathbf{z}_{1}^{(h)}} \\ &\qquad \qquad \frac{\partial \ell_{3}}{\partial \mathbf{W}} \propto \frac{\partial \mathbf{h}_{3}}{\partial \mathbf{z}_{3}^{(h)}} \frac{\partial \mathbf{h}_{2}}{\partial \mathbf{z}_{2}^{(h)}} \frac{\partial \mathbf{h}_{1}}{\partial \mathbf{z}_{1}^{(h)}} \end{split}$$



Updating Parameters

If we have *n* steps:

$$\begin{split} \frac{\partial \mathsf{Loss}}{\partial \mathbf{W}} &= \sum_{t=1}^{3} \frac{\partial \ell_t}{\partial \mathbf{W}} = \frac{\partial \ell_1}{\partial \mathbf{W}} + \frac{\partial \ell_2}{\partial \mathbf{W}} + \frac{\partial \ell_3}{\partial \mathbf{W}} \\ & \frac{\partial \ell_1}{\partial \mathbf{W}} \propto \frac{\partial \mathbf{h}_1}{\partial \mathbf{z}_1^{(h)}} \\ & \frac{\partial \ell_2}{\partial \mathbf{W}} \propto \frac{\partial \mathbf{h}_2}{\partial \mathbf{z}_2^{(h)}} \frac{\partial \mathbf{h}_1}{\partial \mathbf{z}_1^{(h)}} \\ & \frac{\partial \ell_3}{\partial \mathbf{W}} \propto \frac{\partial \mathbf{h}_3}{\partial \mathbf{z}_3^{(h)}} \frac{\partial \mathbf{h}_2}{\partial \mathbf{z}_2^{(h)}} \frac{\partial \mathbf{h}_1}{\partial \mathbf{z}_1^{(h)}} \\ & \dots \\ & \frac{\partial \ell_n}{\partial \mathbf{W}} \propto \frac{\partial \mathbf{h}_n}{\partial \mathbf{z}_n^{(h)}} \dots \frac{\partial \mathbf{h}_3}{\partial \mathbf{z}_3^{(h)}} \frac{\partial \mathbf{h}_2}{\partial \mathbf{z}_3^{(h)}} \frac{\partial \mathbf{h}_2}{\partial \mathbf{z}_3^{(h)}} \frac{\partial \mathbf{h}_1}{\partial \mathbf{z}_1^{(h)}} \end{split}$$



Exploding Gradients

Given:

$$\frac{\partial \ell_n}{\partial \mathbf{W}} \propto \frac{\partial \mathbf{h_n}}{\partial \mathbf{z_n^{(h)}}} ... \frac{\partial \mathbf{h_3}}{\partial \mathbf{z_3^{(h)}}} \frac{\partial \mathbf{h_2}}{\partial \mathbf{z_2^{(h)}}} \frac{\partial \mathbf{h_1}}{\partial \mathbf{z_1^{(h)}}}$$

if all the gradients in the chain are greater than one then their multiplications explodes

$$\frac{\partial \ell_n}{\partial \mathbf{W}} = \text{NaN}$$



Vanishing Gradients

Given:

$$\frac{\partial \ell_n}{\partial \mathbf{W}} \propto \frac{\partial \mathbf{h_n}}{\partial \mathbf{z_n^{(h)}}} ... \frac{\partial \mathbf{h_3}}{\partial \mathbf{z_3^{(h)}}} \frac{\partial \mathbf{h_2}}{\partial \mathbf{z_2^{(h)}}} \frac{\partial \mathbf{h_1}}{\partial \mathbf{z_1^{(h)}}}$$

if one of the gradients in the chain is close to zero or all gradients are less than one then their multiplications vanishes

$$\frac{\partial \ell_n}{\partial \mathbf{W}} = 0.0$$



Vanishing and Exploding Gradients

- Why are such gradients a problem?
- In case of exploding gradients, the learning is very unstable
- ▶ The last steps become independent from the early steps
- The prediction at each step is conditioned only on a few previous steps
- These problems also happen in MLPs with many hidden layer where we use the Sigmoid activation function



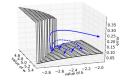
Vanishing and Exploding Gradients

Some sings that show a model and its training may suffer from these problems:

- a very high loss on training set or no learning
- large changes in loss on each update due to the models instability
- loss becomes NaN during training
- model weights grow exponentially during training (explosion)
- the model does not learn during training
- training stops very early and any further training does not decrease the loss
- the weights closer to the last steps would change more than those at early steps
- weights shrink exponentially and become very small
- the model weights become 0 in the training phase.

Simple Remedies for Vanishing/Exploding Gradients

- ► For activation of hidden layers use ReLU, and initialize W with the identity matrix(Le et al., 2015)
- Gradient clipping (Pascanu et al., 2013)



GRUs and LSTMs

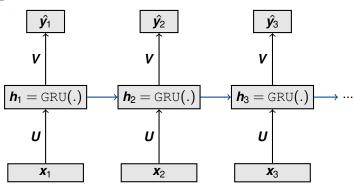


Gated Recurrent Units (GRUs)

- ► GRUs are introduced by Cho et al., (2014)
- more advanced method for hidden state representation
- The key idea behind GRUs is to enable hidden states to capture long distance dependencies

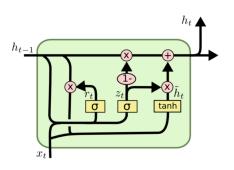
GRUs





GRU





- reset gate $r_t = \sigma(\mathbf{x}_t \mathbf{U}^{(r)} + \mathbf{h}_{t-1} \mathbf{W}^{(r)})$
- ▶ new memory content could be $\tilde{\textbf{h}}_{t} = anh(\textbf{x}_{t}\textbf{U} + \textbf{h}_{t-1}\textbf{W} \odot \textbf{r}_{t})$
- update gate $z_t = \sigma(\mathbf{x}_t \mathbf{U}^{(z)} + \mathbf{h}_{t-1} \mathbf{W}^{(z)})$
- ▶ final memory encodes a combination of current content and its content in the previous time step $\mathbf{h_t} = (1 z_t) \odot \mathbf{h_{t-1}} + z_t \odot \tilde{\mathbf{h_t}}$

GRU: Extreme Cases



- reset gate $r_t \in 0, 1$
- ▶ update gate $z_t \in 0, 1$
- $If z_t = 0$

$$h_t = (1 - z_t) \odot h_{t-1} + z_t \odot \tilde{h}_t$$

- $h_t = h_{t-1}$
- zero gradients over different time steps
- no vanishing gradients
- $If z_t = 1$

$$h_t = (1 - z_t) \odot h_{t-1} + z_t \odot \tilde{h}_t$$

$$h_t = \tilde{h}_t$$

$$\tilde{\boldsymbol{h}}_{t} = \tanh(\boldsymbol{x}_{t}\boldsymbol{U} + \boldsymbol{h}_{t-1}\boldsymbol{W} \odot \boldsymbol{r}_{t})$$

$$If r_t = 0$$

$$h_t = \tanh(x_t U)$$

- forget past
- $If r_t = 1$

$$h_t = \tanh(x_t U + h_{t-1} W)$$

a standard RNN gate

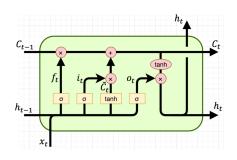


Long Short-Term Memory (LSTMs)

- ▶ LSTMs were introduced by Hochreiter and Schmidhuber (1997)
- LSTMs contains more parameters than what GRU has

LSTM Unit



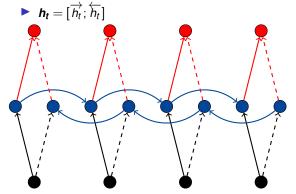


- ▶ Input gate (=write gate) $i_t = \sigma(\mathbf{x}_t \mathbf{U}^{(i)} + \mathbf{h}_{t-1} \mathbf{W}^{(i)})$
- ▶ Forget gate (=reset gate) $f_t = \sigma(\mathbf{x_t} \mathbf{U}^{(t)} + \mathbf{h}_{t-1} \mathbf{W}^{(t)})$
- Output gate (=read gate) $o_t = \sigma(\mathbf{x}_t \mathbf{U}^{(o)} + \mathbf{h}_{t-1} \mathbf{W}^{(o)})$
- New memory cell is $\tilde{c}_t = \tanh(x_t U + h_{t-1} W)$
- Final memory cell is $\mathbf{c_t} = f_t \odot \mathbf{c}_{t-1} + i_t \odot \tilde{\mathbf{c}_t}$
- ▶ Final hidden state is $h_t = o_t \odot tanh(c_t)$



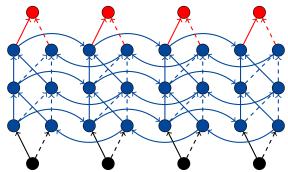
Bidirectional RNNs (BiRNNs)

- ► We use two RNNs with two different sets of parameters
 - one RNN for processing input symbols from left-to-right
 - one RNN for processing input symbols from right-to-left
- Final representations of each step is the concatenation of the outputs of these RNNs.



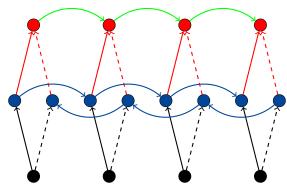


Deep BiRNNs





RNNs With Output Connections



TECHNISCHE UNIVERSITÄT DARMSTADT

Applications of RNNs in NLP

- Part-of-Speech (POS) tagging
 - input: a sequence of words
 - output: a sequence of POS tags (NOUN, VERB, ...)
- Named Entity Recognition (NER)
 - input: a sequence of words
 - output: a sequence of NER tags (B-PER, I-PER, O, B-LOC, I-LOC,...)
- Language Modeling (LM)
 - input: a sequence of words
 - output: a sequence of words y_t = x_{t-1}
- Sentence Classification
 - input: a sequence of words
 - output: one label for the whole sequence
 - trick: use the hidden state of the last step to represent the whole sentence

Summary



- RNNs are used mostly for sequence prediction.
- RNNs face with vanishing and exploding gradient issues.
- Vanishing and Exploding Gradients in RNNs
- LSTMs and GRUs
- Applications of RNNs



Thank You!