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Deep Learning for NLP
Lecture 3: Training as Optimization and (Neural) Language Models

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Ubiquitous Knowledge Processing Lab (UKP Lab)

This lecture

- training as optimization
- backpropagation
- language modeling

Recall

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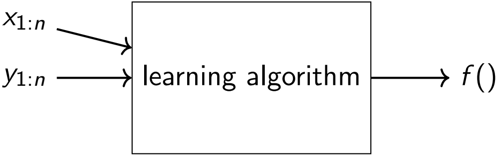

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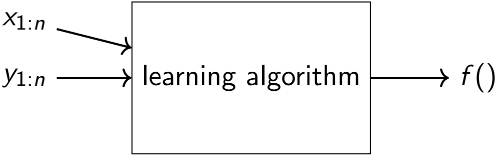

- how to measure if $f()$ works accurately?

Loss Function

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- given a labeled training set $(x_{1:n}; y_{1:n})$, a per-instance loss function L and a parameterized function $f(x; \Theta)$, we define the corpus-wide loss with respect to the parameters as the average loss over all training examples:

$$\mathcal{L}(\theta) = \frac{1}{n} \sum_{i=1}^{n} L(\hat{y}, y_i),$$

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- in this view the training examples are fixed and the values of the parameters determine the loss

Training as Optimization

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Common Loss Functions

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  - per-instance loss:

$$L_{\text{hinge(binary)}}(\hat{y}, y) = \max(0, 1 - y.\hat{y})$$

Common Loss Functions

- Hinge (multi-class)
  - let $\hat{y} = \hat{y}_{[1]}, \hat{y}_{[2]}, ..., \hat{y}_{[n]}$ be the model's output vector, and y be the one-hot vector for the correct output class

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  - if t is the correct class and k is the highest scoring class such that $k \neq t$ then loss is

$$L_{\text{hinge(multiclass)}}(\hat{y}, y) = \max(0, 1 - (\hat{y}_{[t]} - \hat{y}_{[k]}))$$

Common Loss Functions

- Log loss
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  - loss

$$L_{\text{log}}(\hat{y}, y) = \log(1 + \exp(-(\hat{y}_{[t]} - \hat{y}_{[k]}))$$

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  - we assume that the output layer is transformed using sigmoid

Training as Optimization

- the goal of the training algorithm is then to set the values of the parameters such that the value of $\mathcal{L}$ is minimized

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- the above optimization attempts to minimize the loss at all costs, which may result in overfitting the training data

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- training with regularization

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Regularization

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- regularization term considers the parameter values, and scores their complexity
- in practice the regularizers equate complexity with large weights and work to keep the parameter values low

Common Regularization Functions

- L_2 regularization (a.k.a. gaussian prior or weight decay): It keeps the sum of the squares of the parameter values low

$$R_{L_2}(W) = ||W||_2^2 = \sum_{i,j} (W_{[i,j]})^2$$

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- the learner will prefer to decrease the value of one parameter with high weight by 1 than to decrease the value of ten parameters that already have relatively low weights by 0.1 each

Common Regularization Functions

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- the learner will prefer to decrease all the non-zero parameter values toward zero

Common Regularization Functions

- Elastic-Net: combines both L_1 and L_2 regularization

$$R_{\text{elastic-net}}(W) = \gamma_1 R_{L_1}(W) + \gamma_2 R_{L_2}(W)$$

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- dropout: will be discussed later

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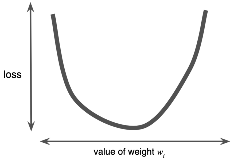
- training = Solving an optimization problem
- how to find parameter values that minimize loss?

Gradient-based Optimization



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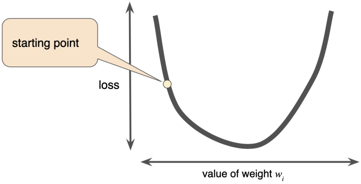

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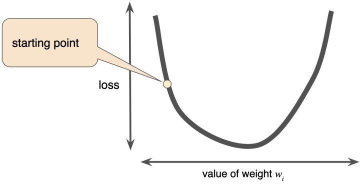

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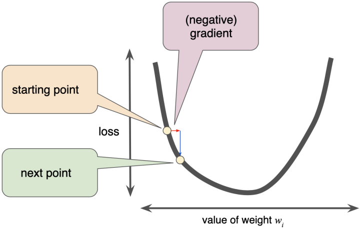

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- we repeatedly compute an estimate of the loss over the training set
- we compute the gradients of the parameters with respect to the loss estimate
- we move the parameter values in the opposite directions of the gradient

(Online) Stochastic Gradient Descent

Algorithm 2.1 Online stochastic gradient descent training.

Input:
- Function $f(\boldsymbol{x};\Theta)$ parameterized with parameters Θ.
- Training set of inputs $\boldsymbol{x_1},\ldots,\boldsymbol{x_n}$ and desired outputs $\boldsymbol{y_1},\ldots,\boldsymbol{y_n}$.
- Loss function L.

while stopping criteria not met **do**
    Sample a training example $\boldsymbol{x_i}, \boldsymbol{y_i}$
    Compute the loss $L(f(\boldsymbol{x_i};\Theta), \boldsymbol{y_i})$
    $\hat{\boldsymbol{g}} \leftarrow$ gradients of $L(f(\boldsymbol{x_i};\Theta), \boldsymbol{y_i})$ w.r.t Θ
    $\Theta \leftarrow \Theta - \eta_t \hat{\boldsymbol{g}}$
return Θ

(Taken from: Neural Network Methods for Natural Language Processing, Yoav Goldberg)

(Minibatch) Stochastic Gradient Descent

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Algorithm 2.2 Minibatch stochastic gradient descent training.

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- Loss function L.

1: **while** stopping criteria not met **do**
2:     Sample a minibatch of m examples $\{(\boldsymbol{x_1},\boldsymbol{y_1}),\dots,(\boldsymbol{x_m},\boldsymbol{y_m})\}$
3:     $\hat{\boldsymbol{g}} \leftarrow 0$
4:     **for** $i = 1$ to m **do**
5:         Compute the loss $L(f(\boldsymbol{x_i};\Theta),\boldsymbol{y_i})$
6:         $\hat{\boldsymbol{g}} \leftarrow \hat{\boldsymbol{g}} +$ gradients of $\frac{1}{m}L(f(\boldsymbol{x_i};\Theta),\boldsymbol{y_i})$ w.r.t Θ
7:     $\Theta \leftarrow \Theta - \eta_t \hat{\boldsymbol{g}}$
8: **return** Θ

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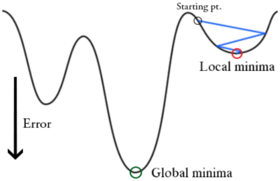
- it's not expensive: while computing loss function gradient on all training set can be computationally expensive
- it converges faster to a good solution than full-batch learning, in which we use all training set to compute gradient
- smaller mini-batch sizes lead often to better solutions (generalize better)

Stochastic Gradient Descent (SGD)

- gradient descent does not always lead to best solutions



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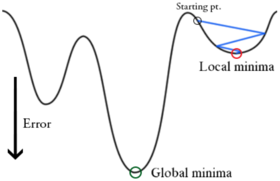

- why?

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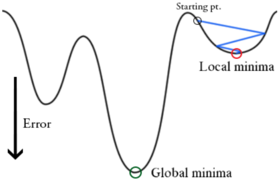
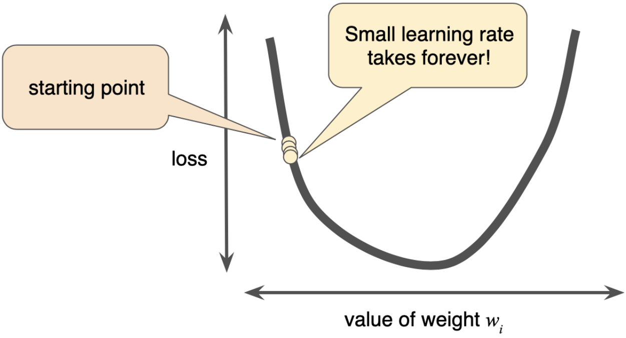


- why?
- SGD is sensitive to the learning rate and initial parameter values (starting point)

Small Learning Rate



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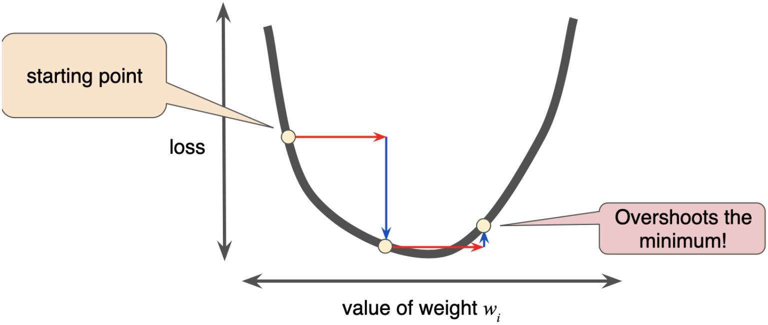
Large Learning Rate



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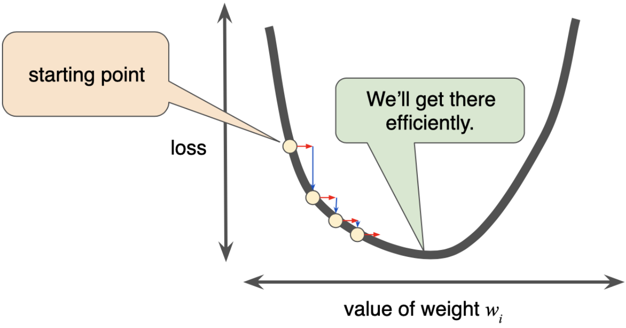
Adaptive Learning Rate



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Stochastic Gradient Descent (SGD)

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- Use adaptive learning rate algorithms
  - AdaGrad [Duchi et al., 2011],
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- found solution is often not as good as that by SGD $\rightarrow$ First train with Adam, fine-tune with SGD
- they use different initial parameter values in different runs of experiments and report the average of scores

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- chain rule: Assume $y = f(g(x))$

$$\frac{\partial y}{\partial x} = \frac{\partial f}{\partial g} \times \frac{\partial g}{\partial x} = \frac{\partial f}{\partial g}\frac{\partial g}{\partial x}$$

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  - forward pass $\rightarrow$ use current parameter values to compute the loss value
  - backward pass $\rightarrow$ use the gradient of the loss to update the parameter values

Backpropagation: Forward Pass

$$h^{(1)}$$

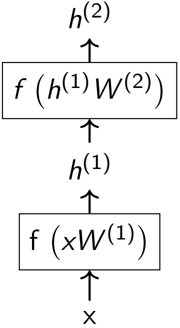
$$\uparrow$$

$$\boxed{\mathrm{f}\left(xW^{(1)}\right)}$$

$$\uparrow$$

$$\mathrm{x}$$

Backpropagation: Forward Pass



Backpropagation: Forward Pass

$$\hat{y}$$

$$\uparrow$$

$$\boxed{f\left(h^{(2)} W^{(3)}\right)}$$

$$\uparrow$$

$$h^{(2)}$$

$$\uparrow$$

$$\boxed{f\left(h^{(1)} W^{(2)}\right)}$$

$$\uparrow$$

$$h^{(1)}$$

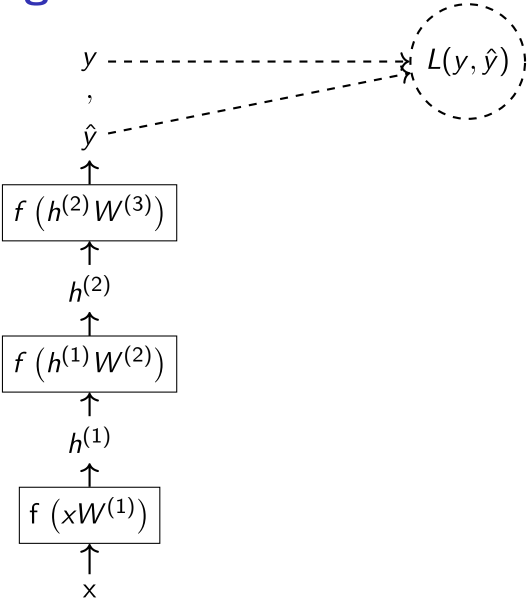
$$\uparrow$$

$$\boxed{\mathrm{f}\left(xW^{(1)}\right)}$$

$$\uparrow$$

$$\mathrm{x}$$

Backpropagation: Forward Pass



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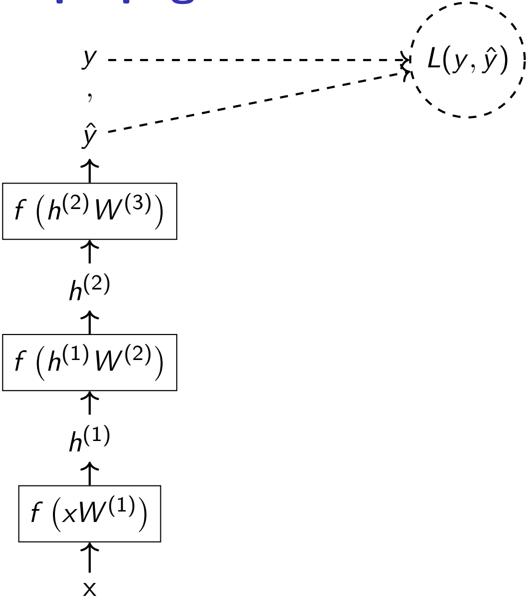

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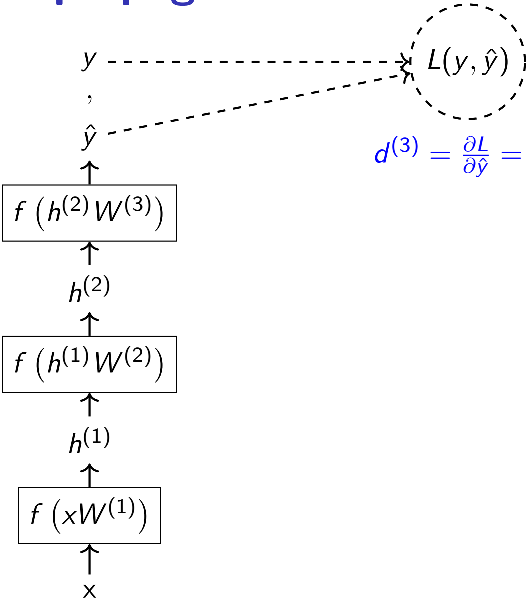

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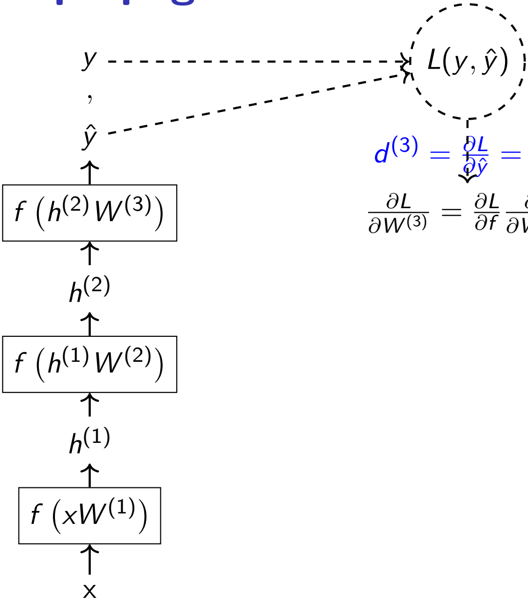

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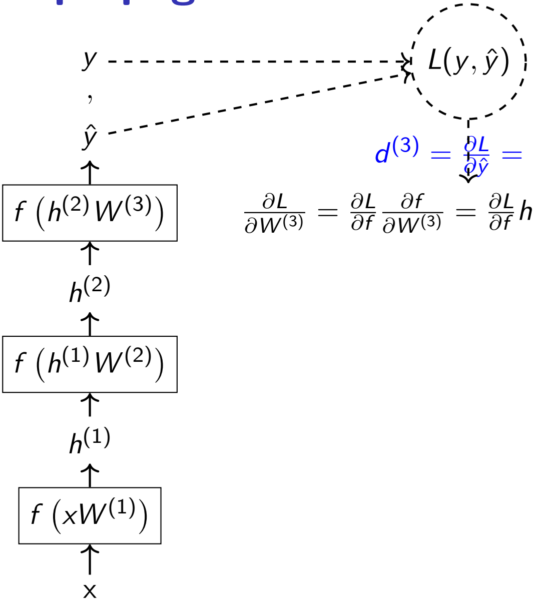

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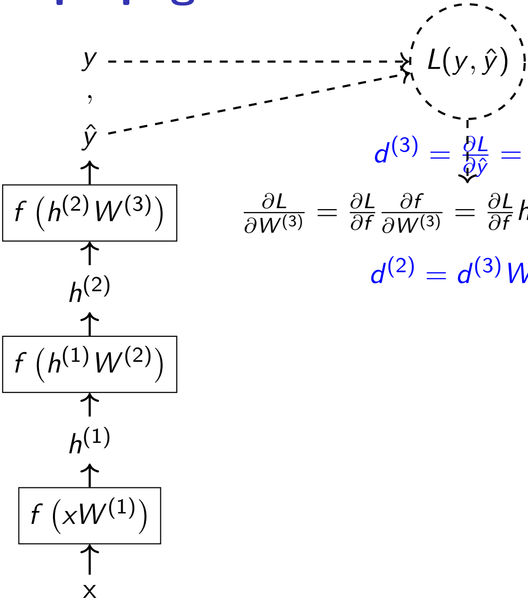

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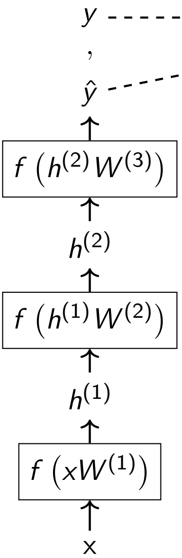

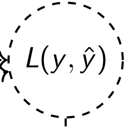

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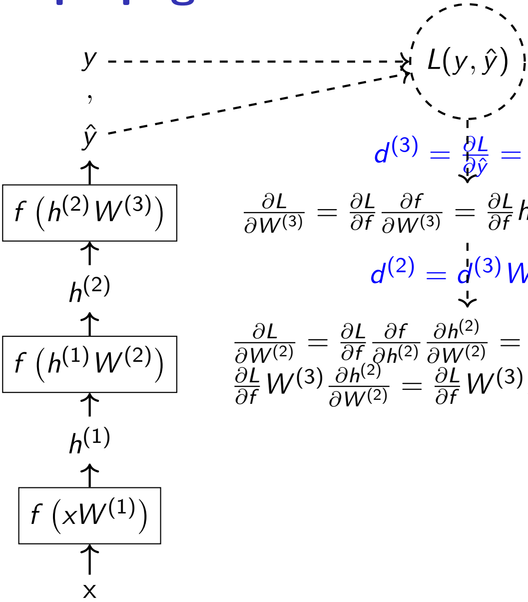

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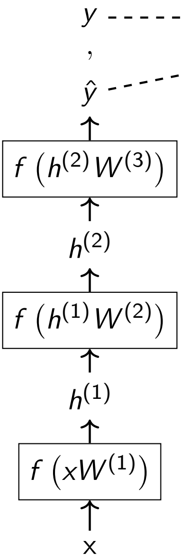

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y, $\hat{y}$ → $L(y, \hat{y})$

$$\boxed{f\left(h^{(2)} W^{(3)}\right)} \rightarrow \hat{y}$$

$$h^{(2)}: \boxed{f\left(h^{(1)} W^{(2)}\right)}$$

$$h^{(1)}: \boxed{f\left(x W^{(1)}\right)}$$

$$x$$

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Backprop + SGD

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- once we have the gradients we can use SGD rule to update the parameter values

A Simple Training Loop in PyTorch

```
optimizer = SGD(model_params, lr)

for epoch in range(num_epochs):
    for x,y in  data_batches:

        y_hat = model(x)
        loss = loss_func(y_hat, y)

        optimizer.zero_grad()
        loss.backward()

        optimizer.step()
```

Language Models (LMs)

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- ideal performance at language modeling is to predict the next token in a sequence with a number of guesses that is the identical to or lower than the number of guesses required by a human expert
- even without achieving human-level performance, language modeling is a crucial component in real-world NLP applications such as conversational AI, machine-translation, text summarization, ...

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- markov-assumption: the future is independent of the past given the present

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- to make it computationally-friendly for computers (What is the problem with the above format?)

$$\log_2 P(w_{1:n}) \approx \sum_{i=1}^{n} \log_2 \left(P(w_i|w_{i-k:i-1})\right)$$

Evaluating LMs

- the perplexity metric over an unseen sentence indicates how well a LM predicts the likelihood of the sentence

$$\text{prep}_{w_{1:n}}(\text{LM}) = 2^{-\frac{1}{n}\sum_{i=1}^{n}\log_2 \text{LM}(w_i|w_{1:i-1})}$$

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- perplexities of two language models are only comparable with respect to the same evaluation dataset

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- example: $w_1 w_2 w_3 =$ the cat sat

$$P(w_3|w_{1:2}) = \frac{\#(\text{the cat sat})}{\#(\text{the cat})}$$

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- additive smoothing: assume $|V|$ is the vocabulary size and $0 < \alpha \leq 1$

$$P(w_i|w_{i-1-k:i-1}) = \frac{\#(w_{i-1-k:i}) + \alpha}{\#(w_{i-1-k:i-1}) + \alpha|V|}$$

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- MLE-based language models suffer from lack of generalization across contexts
- having observed “black car” and “blue car” does not influence our estimates of the sequence “red car” if we haven’t seen it before

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- we focus on the neural LM that was introduced by Bengio et al. (2003)

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$$x = [v(w_1), v(w_2), ..., v(w_k)]$$

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- loss function: cross-entropy loss

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- our LM returns $P(w_{k+1})$ of each word in V
- we pick up the word with the maximum probability to generate the next word
- we add the the predicted word to the context and repeat the above procedure

Summary

- training as optimization of a loss function
- common loss functions and regularization terms
- gradient descent (GD, SGD): A general technique for optimization
- backprop(agation): An algorithm for deriving gradients in neural models, once gradients are determined we can train a model with SGD
- (Neural) Language Models (LMs)

Thank You!