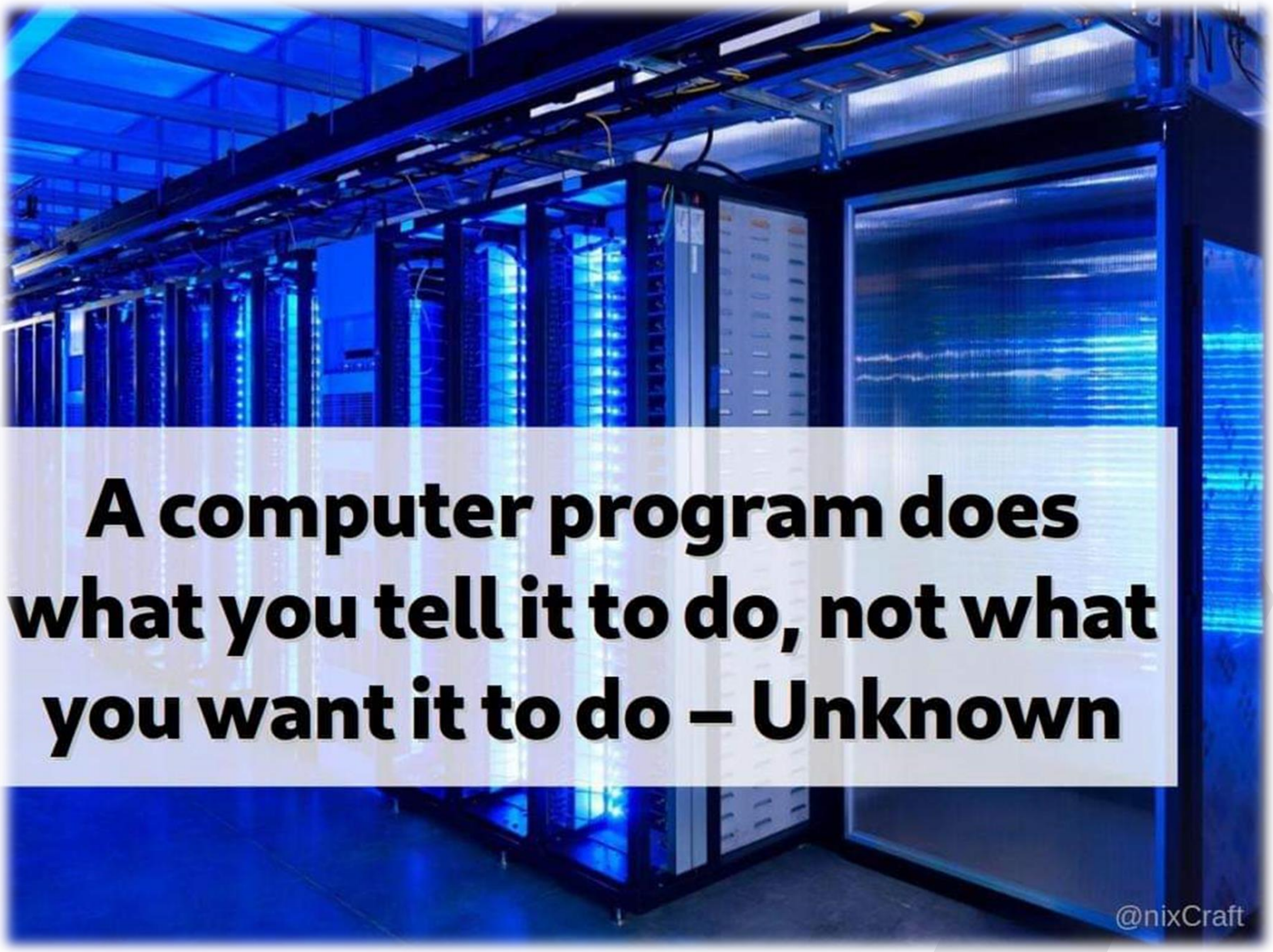
A computer program does what you tell it to do, not what you want it to do – Unknown
@nixCraft

# Automata and machines

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# Industrial embedded systems

What they do

- Monitor physical properties of the system/plant (via *sensors*)
- Might perform some control, or part of, control algos
- Via *actuators*

Control can be

- Continuous in time
- Discrete in time

➔ Control theory

# Industrial controls in a nutshell

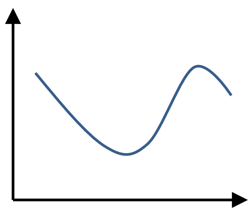
In their generic form,

$$F: \{ S, I \} \rightarrow \{ O \}$$

computed …when?

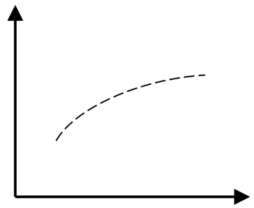
If continuous

- Physical properties and actuators are continuous in time
- *F(t)* continuous
- Combinatorial logic/analogic systems



If discrete

- Computed at pre-determined instance in time
- Event-driven (e.g., timeout, interrupt)
- Sequential logics/digital systems



# Finite state automations for discrete controls

E.g., an elevator, reacts to multiple events

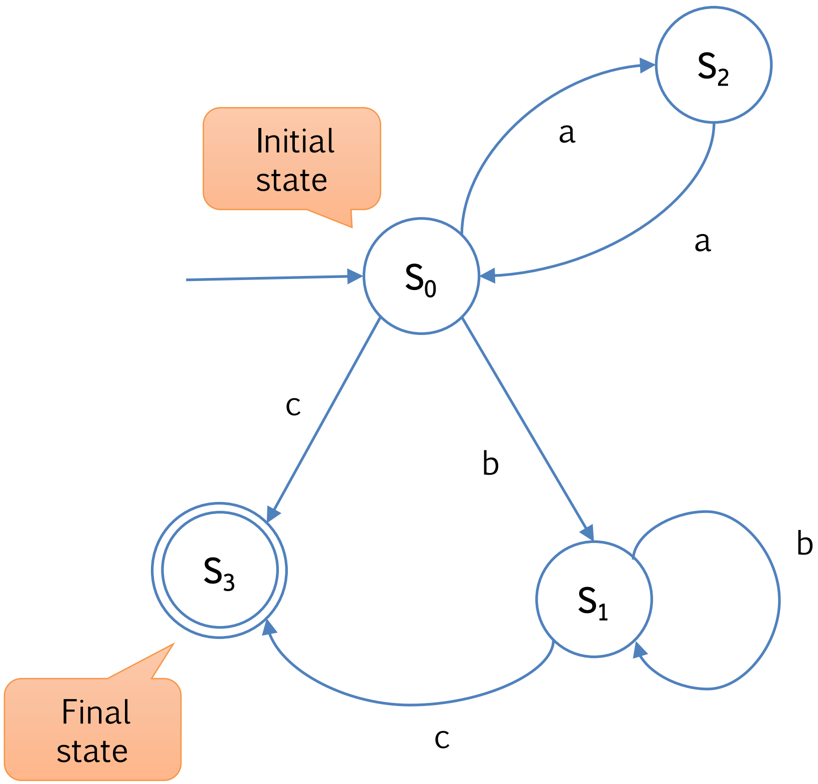
- Typically in idle state
- If you are press the button, the door opens
- You select the floor, doors close
- Then, it reaches the floor (feat. velocity control)
- Then, it opens the door, which subsequently closes after X seconds

This behavior is controlled by a **finite state automations/machine**

# Finite State Automations/Machines

Problem

- Identify even sequences of *a* (even empty), followed by one, or more, or no, *b*, ended by *c*




Given an <u>alphabet</u> *V*,

*…that identifies a language (we'll see)..*

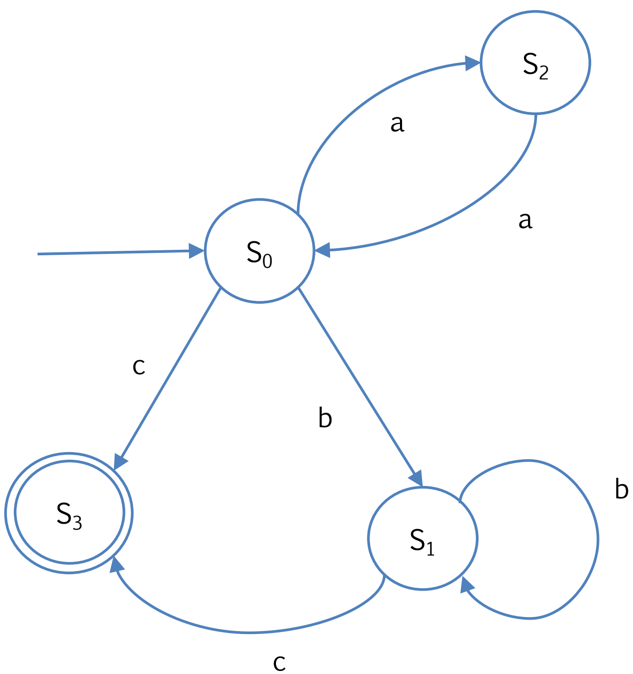
define FSA as

- $S$ : a non-empty states set
- $s_0 \in S$ : initial state
- $S_f \subseteq S$ : final states set
- $t: S \times V \rightarrow S$ : states transaction func

# FSMs and languages

Let V* = *{ v, w, … }* contain all the combinations of words using V symbols

- Including the empty word $\varepsilon$
- For instance, ac, aabbc, abbabbbc belong to V*
- (note that, we can associate words in be to inputs, or combination of them)




A **language** L is a subset of V*

(abbabbbc does **not** belong

to L, as previously defined)

*“Identify even sequences of a (even empty),*

*followed by one, or more, or no, b, ended by c”*

# State transaction function

- $t(s_0, b) = s_1 \quad | \quad s_0 \xrightarrow{b} s_1$
- $s_y$ is <u>reachable</u> by $s_x$ if there exists a path from $s_x$ to $s_y$
  - a combination of alpabet symbols **l** (letters in our case)

$$t: S \times V \rightarrow S$$

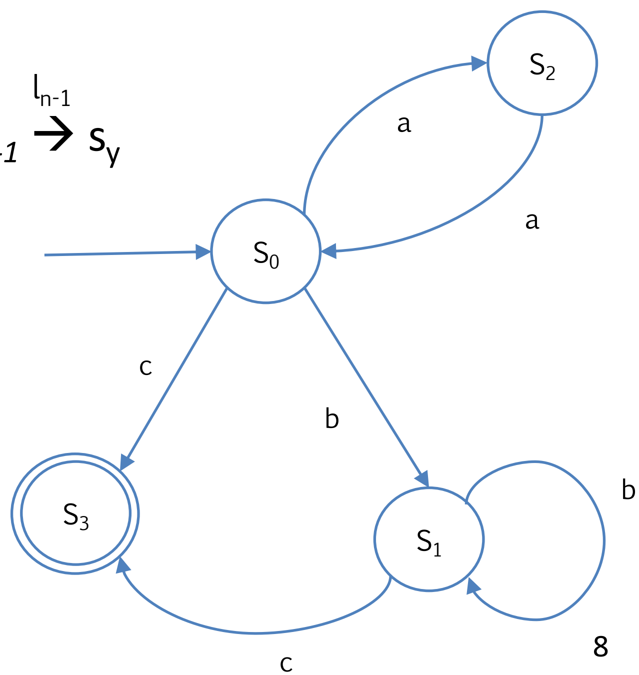
$$\rightarrow^* : \; S \times V^* \times S : \; s_x \xrightarrow{w^*} s_y$$

iff

$$w = l_1 l_2 \ldots l_n \qquad \exists \; s_1, s_2, \ldots, s_n : s_x \xrightarrow{l_1} s_1 \xrightarrow{l_2} s_2 \ldots s_{n-1} \xrightarrow{l_{n-1}} s_y$$

$$s_2 \xrightarrow{w^*} s_1$$

$$w = aaab \qquad \exists \; s_1, s_2, \ldots, s_n : s_2 \xrightarrow{a} s_0 \xrightarrow{a} s_2 \xrightarrow{a} s_0 \xrightarrow{b} s_1$$



# Exercise

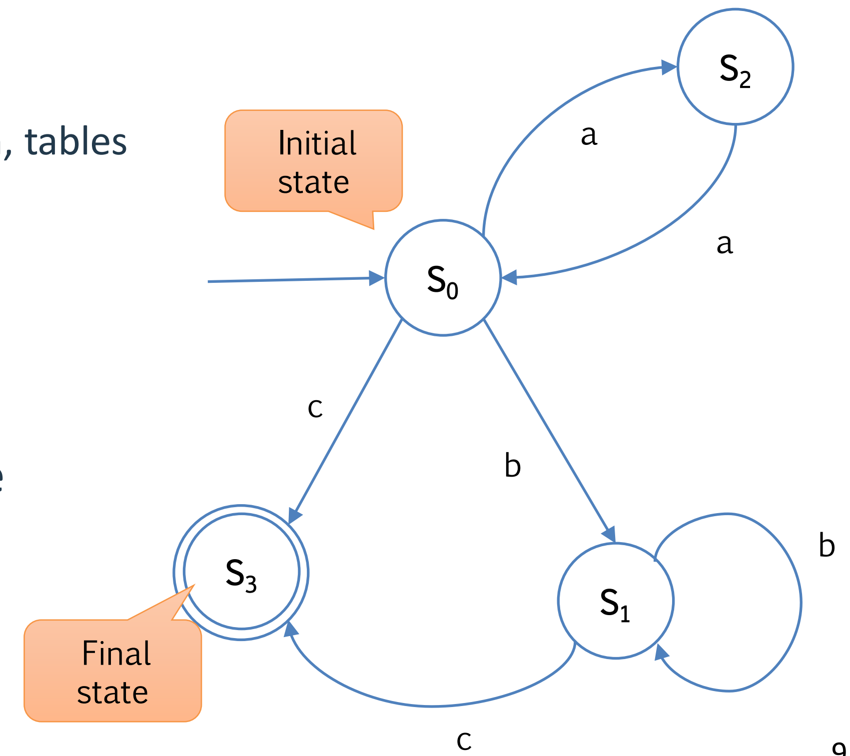
Let's code!

- Implement the automata that understands whether a words is from L

*“Identify even sequences of a (even empty),*

*followed by one, or more, or no, b, ended by c”*

- Use the language that you want
  - You just need IFs,CASE-SWITCH, recursion, tables
  - Receive the target word from stdin




What’s missing?

- In case of error => default error state
- Typically implicit in state diagrams

# Grammars

› A standard way of representing languages (Noam Chomsky, 1950)

$$G = < VT, VN, P, S >$$

*VT and VN disjoint*

*$VT \cap VN = \emptyset$*

› VT : terminal symbols $\subseteq$ V

› VN : non-terminal symbols $\subseteq$ V (aka: syntax categories)

*VT and VN are V*

*$VT \cup VN = V$*

› P : production rules $P \subseteq VN \times (VN \cup VT)$

› S $\in$ VN: initial symbol

A language $L_G$ generated by grammar G is the set of V* elements

derived by start symbol S through productions in P

# Backus-Naur Form

- Productions rules have form

$$\alpha ::= \beta \ , \alpha \in VN \ \beta \in V$$

- x ∈ VN have the form <name>
- | specifies an option

```
VT = { il, gatto, topo, sasso, mangia, beve }

VN = { <frase>, <soggetto>, <verbo>, <compl-ogg>, <articolo>, <nome> }

S = <frase>

P = {
 <frase> ::= <soggetto> <verbo> <compl-ogg>
 <soggetto> ::= <articolo><nome>
 <articolo> ::= il
 <nome> ::= gatto | topo | sasso
 <verbo> ::= mangia | beve
 <compl-ogg> ::= <articolo> <nome>
}
```

Automata states

# Another example

- Natural numbers

```
VT = { 0, 1, …, 9 }

VN = { <num>, <cifra>, <cifra-non-nulla> }

S = <num>

P = {
 <num> ::= <cifra> | <cifra-non-nulla> {<cifra>}
 <cifra> ::= 0 | <cifra-non-nulla>
 <cifra-non-nulla> ::= 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
}
```

"Recursion"
Extended BNF

- **Challenge**: extend it with sign (+, -)!

# Another example: solution

- Natural numbers

```
VT = { 0, 1, …, 9, +, - }

VN = { <int>, <num>, <cifra>, <cifra-non-nulla> }

S = <num>

P = {
 <int> ::= [+|-] <num>
 <num> ::= <cifra> | <cifra-non-nulla> {<cifra>}
 <cifra> ::= 0 | <cifra-non-nulla>
 <cifra-non-nulla> ::= 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
}
```

- **Challenge**: extend it with sign (+, -)!

# In reality....

- We only give production rules: VN, VT, S are implicitly defined..

```
P = {
 <frase> ::= <soggetto> <verbo> <compl-ogg>
 <soggetto> ::= <articolo><nome>
 <articolo> ::= il
 <nome> ::= gatto | topo | sasso
 <verbo> ::= mangia | beve
 <compl-ogg> ::= <articolo> <nome>
}
```

```
Let's
code!
```

Want to try?

- Implement a machine that recognizes whether a sentence (aka: **a word of the Language L**) is legal for that language
- (“our” words are symbols of L)

# Chomsky classification

- 4 types of grammars, with increasing constraints on production rules structures

**Type 0**

- No restriction on productions
- Phrases can even become shorter!

# Type 1 grammars/languages

- *Context-sensitive*
- Production must be in the form

$$x\ A\ y \rightarrow x\ \alpha\ y$$

where

$$x, y, \alpha \in (VT \cup VN)^*,\ A \in VN,\ \alpha \neq \varepsilon$$

- A can be replaced with α only if in the context of (surrounded by) x and y
- Phrases never get shortened
- $\alpha \rightarrow \beta$ con $|\beta| \geq |\alpha|$

# Type 2 grammars/languages

- *Context-free*
- Production must be in the form

$$A \rightarrow \alpha$$

where

$$\alpha \in (VT \cup VN)^*, A \in VN$$

- $\alpha$ can be $\epsilon$
- A can always be replaced with $\alpha$

# Type 3 grammars/languages

- *Regular*
- Production must be in the **linear** form

| | |
|---|---|
| $A \rightarrow \alpha$ | $A \rightarrow \alpha$ |
| $A \rightarrow B\ \alpha$ | $A \rightarrow \alpha\ B$ |

where

$\alpha \in VT^*$, $A, B \in VN$

- $\alpha$ can be $\epsilon$
- Either left, or right linear: not both in the same grammar

# …and..?

We can build specific machines to recognize/process specific grammar Types

- Type 0 => Turing machine (if L(G) is recognizable)
- Type 1 => **Turing machine** with constrained tape length
- Type 2 => Finite state automations with stack (**Push down automations**)
- Type 3 => **Finite state automations**

# Hierarchy of machine types

- Base (combinatorial) machine
- Finite state machines – FSM
- FSM with stack (PDA)
- Turing machine

# Base combinatorial machine

`< I, O, mfn >`

`I` : (finite) set of Input symbols

`O` : (finite) set of output symbols

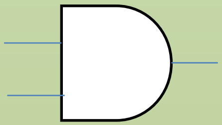
`mfn`: `I` → `O` machine function

- E.g., Logical ports, gates
- Suitable for continuous control
- Non suitable if you need state/memory
  - Need to model all possible cases!

Example: logical AND

`I = { {0,1} × {0,1} }`

`O = {0,1}`



**`mfn`** defined by a table

| | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

# Finite state machine

`< I, O, S, mfn, sfn >`

`I` : (finite) set of Input symbols

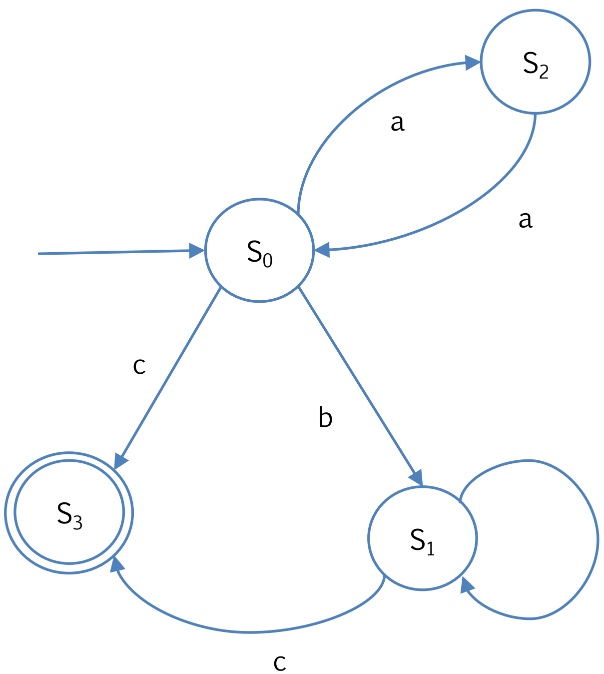
`O` : (finite) set of output symbols

`S` : (finite) set of states

`mfn: I x S` → `O` machine function

`sfn: I x S` → `S` state function

- Partly already seen
- Has memory
- Memory is a limitation



# Finite state machine with stack

**< I, O, A, S, mfn, sfn >**

I : (finite) set of Input symbols

A : (finite) set of stack alphabet symbols

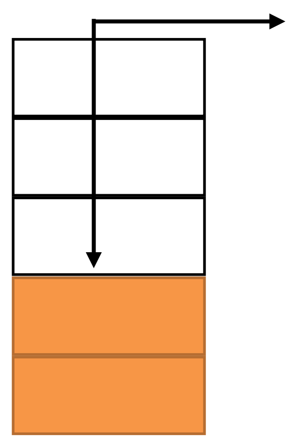
O : (finite) set of output symbols

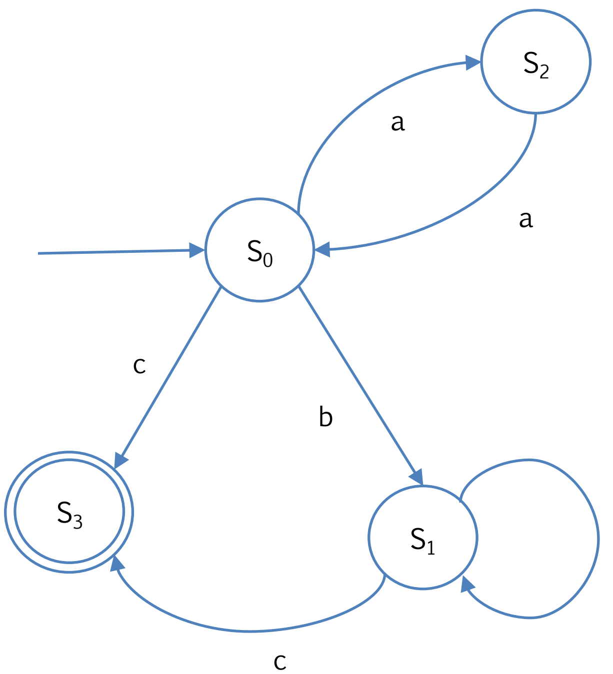
S : (finite) set of states

mfn: I x S x A → O machine function

sfn: I x S x A → S state function

- Also known as Push-Down Automata (PDA)
- Uses a stack
- We'll see them…





# Turing machine

**`< A, S, mfn, sfn, dfn >`**

`A` : (finite) set of in/out symbols

`S` : (finite) set of states

`mfn: A x S → A` machine function

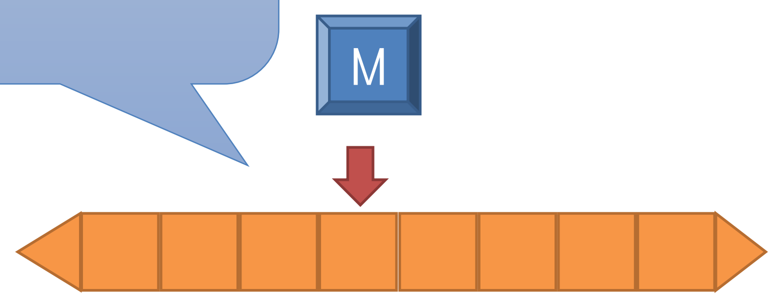
`sfn: A x S → S` state function (inc. `HALT`)

`dfn: A x S → { left, right, none }`

direction function

› Unlimited memory

Possible operations:
- Read from tape
- Write (**mfn**) to tape
- Change internal status (**sfn**)
- Move tape in any (**dfn**) direction




**Church-Turing thesis**

*A function on the natural numbers can be calculated by an effective method if and only if it is computable by a Turing machine*

# A Universal Turing Machine

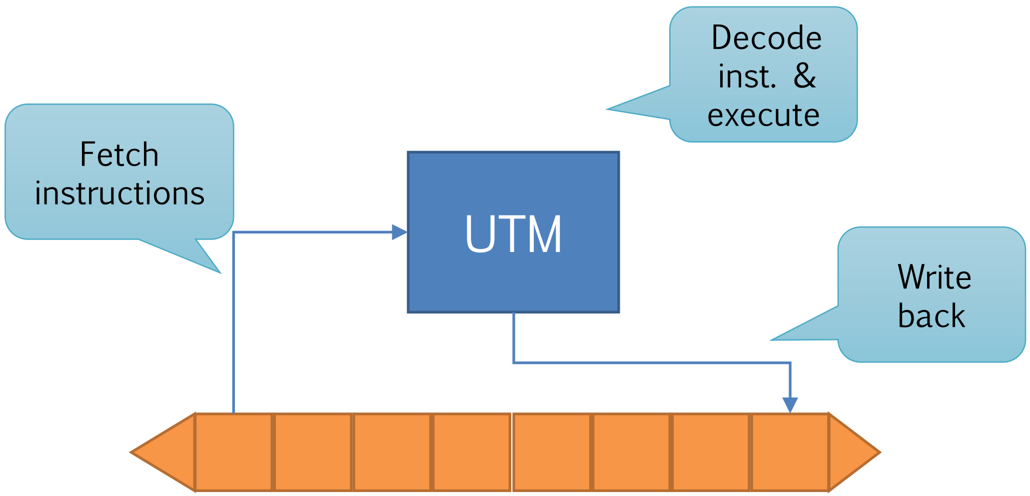
- In TM, the algorithm is inside the machine M, we write results in the tape

What if istruction as well is **<u>in</u>** the tape?

- We have a programmable machine, with a memory
- …..does this remind something?

Which are the catch? What do we miss?

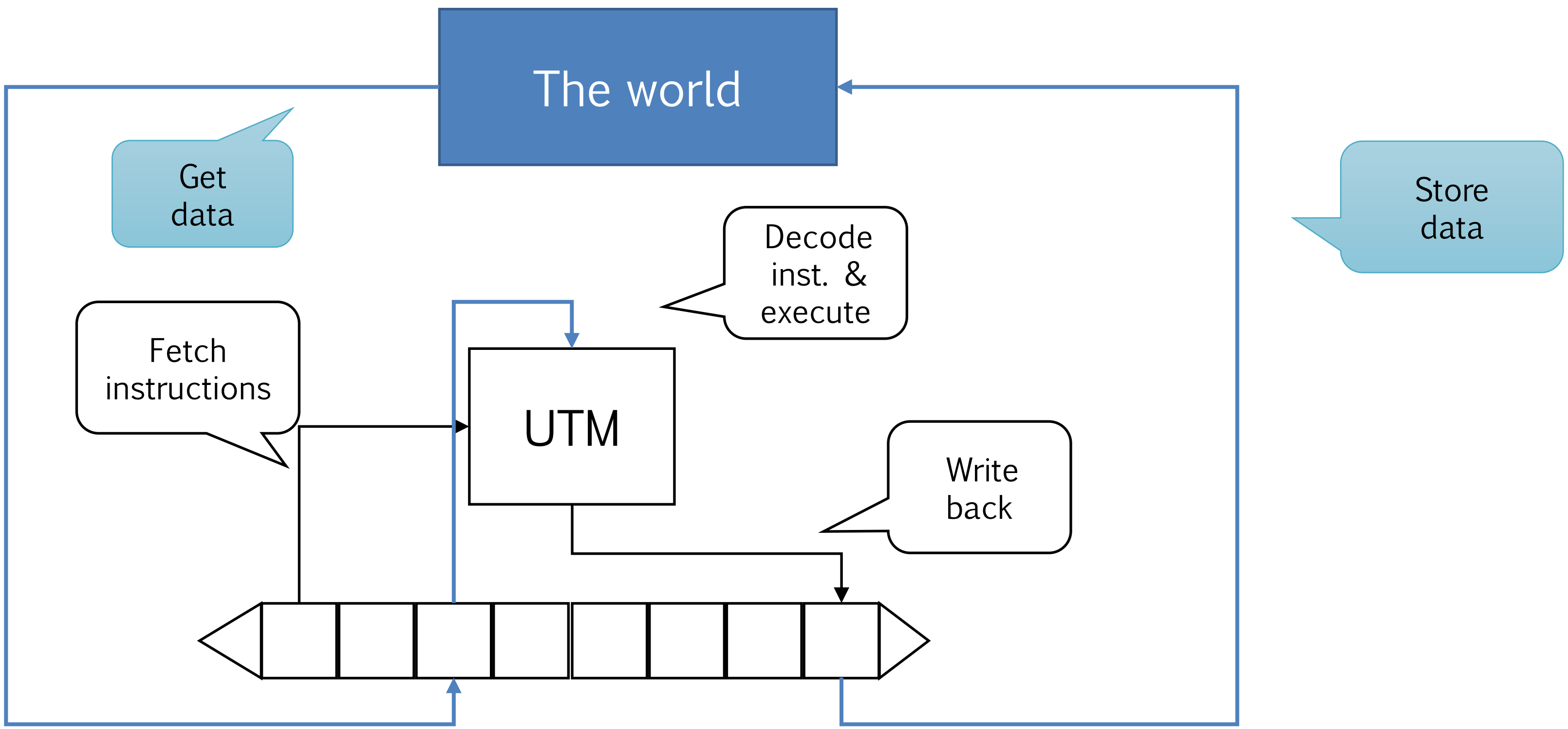
- Ok, the infinite tape makes it infeasible
- ..but what else?



# The Van Neumann Machine

We also need to model the interaction with the environment!

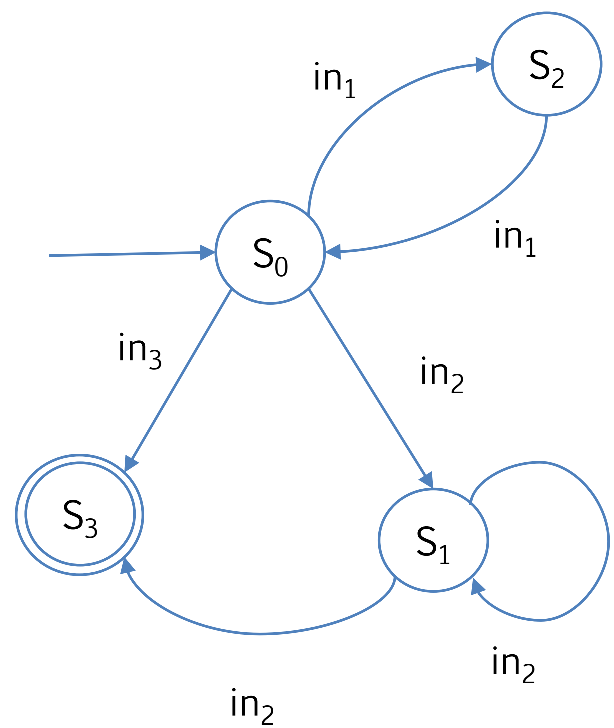
- Aka: I/O (HD/SSD is also I/O)
- Where data comes from!
- It is a real machine: we can **build** it



# How to implement a FSM

# A generic FSM

- Till now, we only saw machines that can recognize a **word** from a language
  - I say "word", you might want to understand "sentence"
- Let's now see how a machine can actually **produce** an output



# The Machine of Mealy

- When crossing an edge, produce an output

**`< I, O, S, mfn, sfn >`**

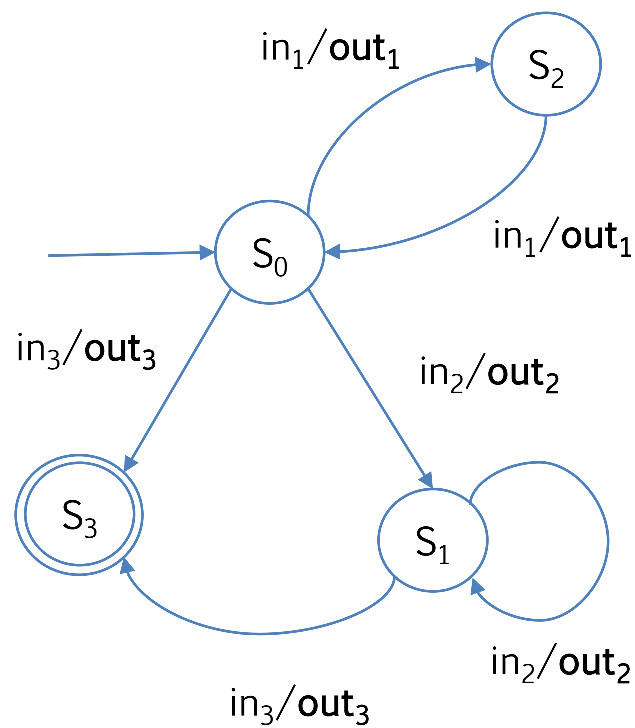
`I` : (finite) set of Input symbols

`O` : (finite) set of output symbols

`S` : (finite) set of states ($s_0$ initial state)

`mfn: I x S` → `O` machine/output function

`sfn: I x S` → `S` state transition function



# The Machine of Moore

› When in a state an edge, produce an output

**`< I, O, S, mfn, sfn >`**

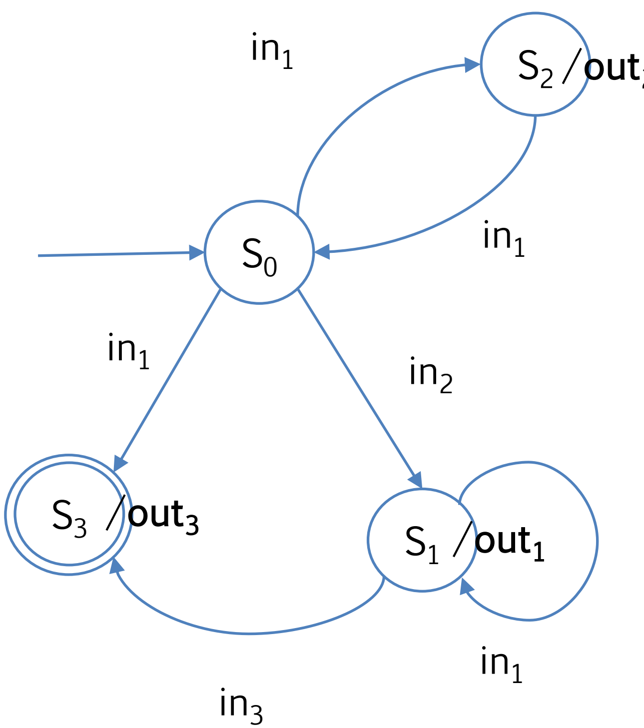
`I` : (finite) set of Input symbols

`O` : (finite) set of output symbols

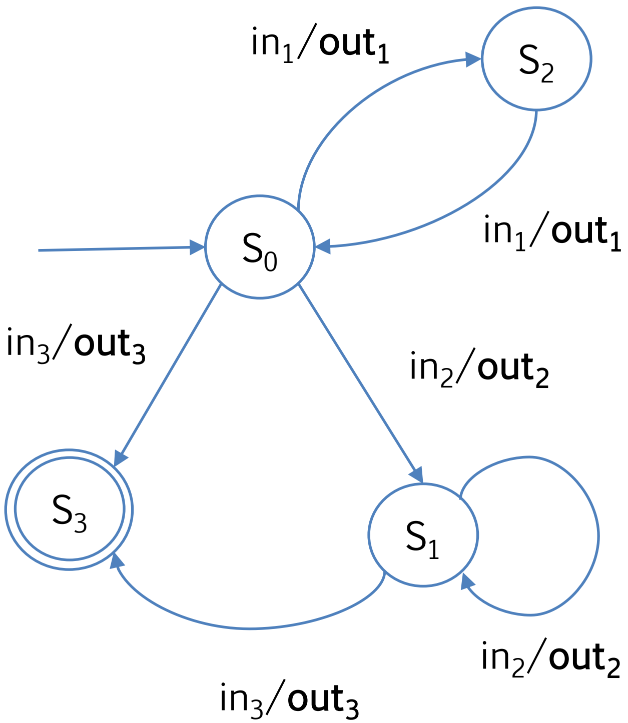
`S` : (finite) set of states ($s_0$ initial state)

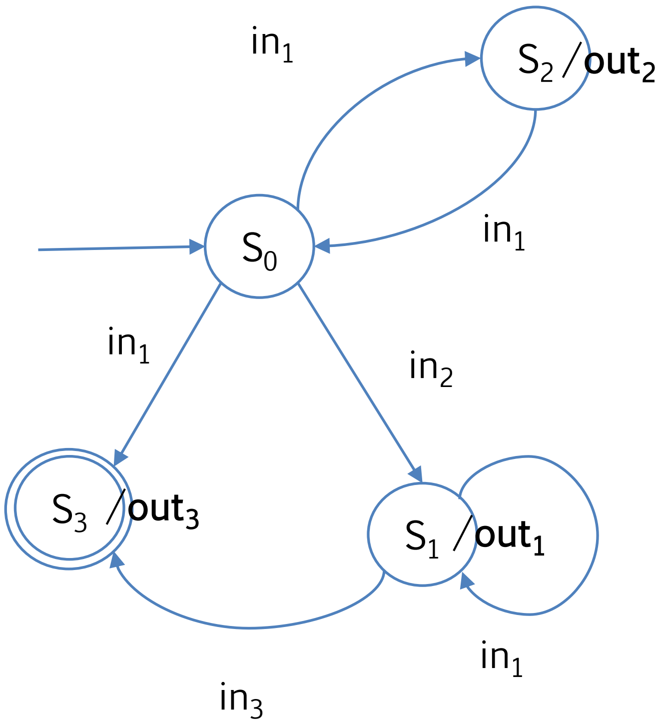
`mfn: S` → `O` machine/output function

`sfn: I x S` → `S` state transition function



# What's the difference?



# What’s the difference?

Mathematically equivalent

- One can be transformed in another

..but..

- Mealy can potentially have different outs, to different inputs/transitions
  - Less states, if output depends on inputs one can add an edge to the machine
- Moore potentially keeps the output stable for all the state
  - Moore requires more states, in case out depends on input and not only on state

# Exercise

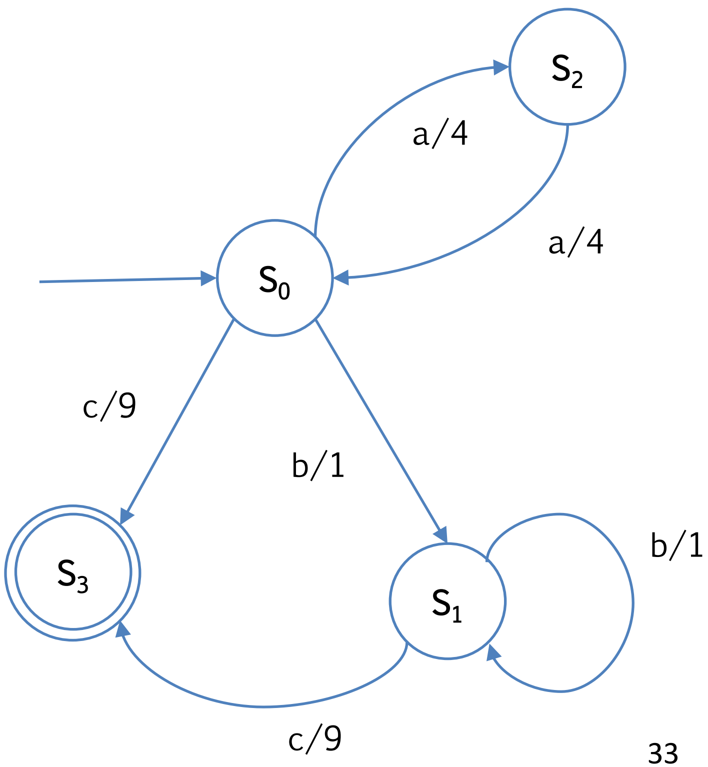
Let's code!

- Implement the automata that understands whether a words is from L

*"Identify even sequences of a (even empty),*

*followed by one, or more, or no, b, ended by c"*

- ..and writes the corresponding number (I choose them randomly)
- Mealy? Moore? You choose
  - Here, I show Mealy




Hint

- If not already done, use tables for state/output transactions

# What else?

Several tools to support the design

› Matlab Stateflow, UML

Several grammar interpreters to rely the burden of writing FSM code

› GNU Bison

› YACC

# Event driven Systems

# Event driven systems

A system that reacts from external stimula

- Istantly?
- Aka: Cyber-Physical Systems (CPS)

Can be

- Synchronous
- Asynchronous

# Synchronous (Active polling)

- Infinite loop

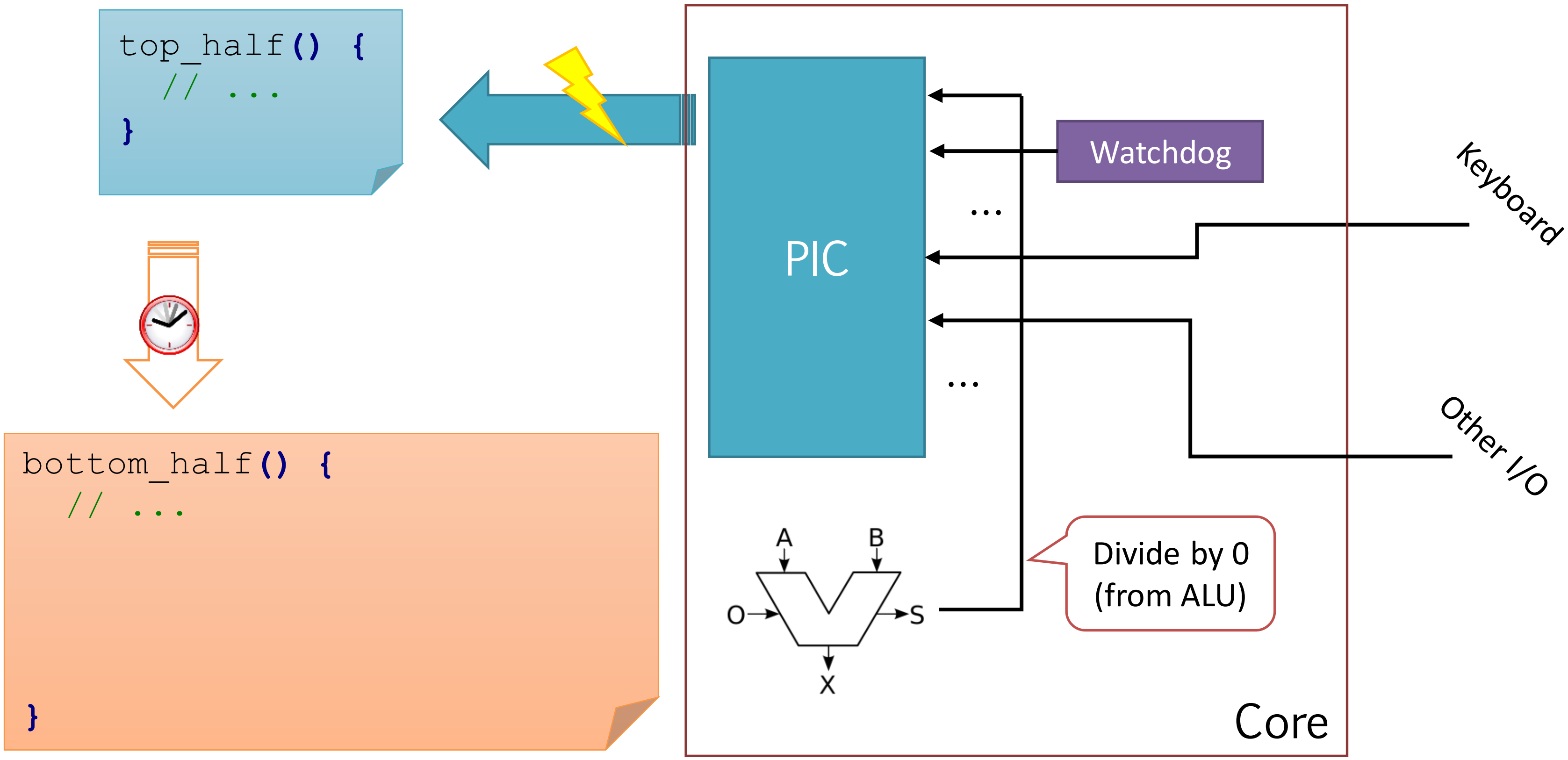
```c
char c;
while (c != SOME_VALUE)
  c = readC();

// We can go, now
```

- **Pros**: extremely fast and reactive
- **Cons**: waste of resources as one core is busy
  - Possible workaround: insert a sleep

# Asynchronous (Interrupt Service Routine)

› Programmable interrupt controller (hierarchy)



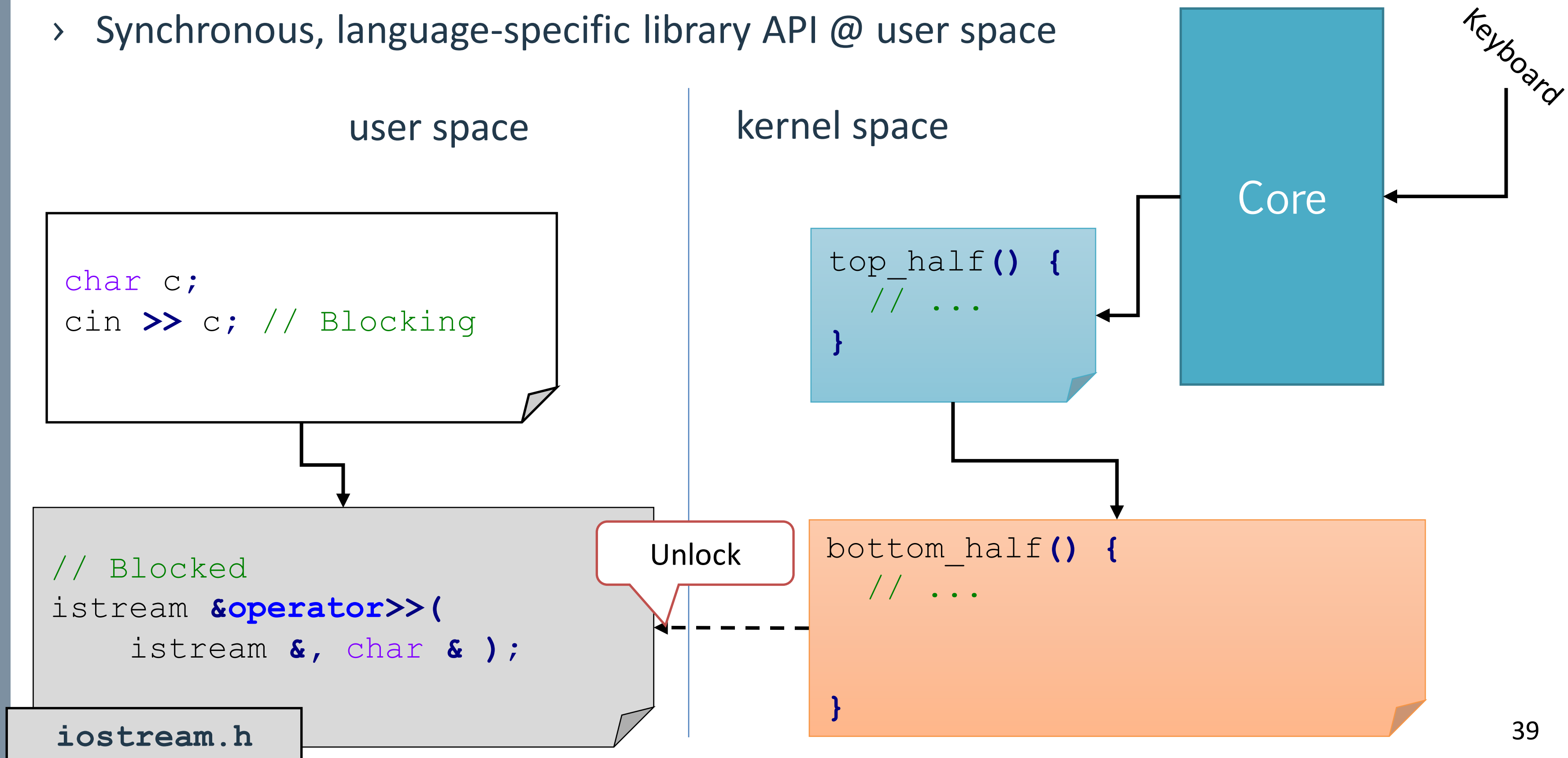

› **Pros**: “pay-as-you-go”

› **Cons**: takes more time to issue a ISP

# …a mix of the two

Keybaroard management in a General-Purpose system

- GNU/Linux
- ISP with bottom-half and top-half @ kernel space
- Synchronous, language-specific library API @ user space



# How to run the examples

Let's
code!

- Find them in `Code/` folder from the course website

For C: compile

- `$ gcc code.c -o code`

Run (Unix/Linux)

```
$ ./code
```

Run (Win/Cygwin)

```
$ ./code.exe
```

# References

**Course website**

- http://hipert.unimore.it/people/paolob/pub/Industrial_Informatics/index.html

**My contacts**

- paolo.burgio@unimore.it
- http://hipert.mat.unimore.it/people/paolob/

**Resources**

- Alessandro Fantechi, «Informatica Industriale», Città Studi Edizioni
- Robert Love, «Linux kernel development», Pearson
- A "small blog"
  - http://www.google.com

# Non-deterministic automata

# Petri nets