Lemmas and Theorem (13/16)

- **Theorem 16.** The STRONGLY-CONNECTED-COMPONENTS procedure correctly computes the strongly connected components of the directed graph G provided as its input.

  Proof. The inductive hypothesis is that the first k trees produced in line 3 are strongly connected components.

  Basis: When $k = 0$, trivial.

Lemmas and Theorem (14/16)

Inductive Step: Assume that each of the first k depth-first trees produced in line 3 is a strongly connected component, and we consider the $(k + 1)$st tree produced.

Let the root of this tree be vertex u, and let u be in strongly connected component C.

Because of how we choose roots in the depth-first search in line 3, $u.f = f(C) > f(C')$ for any strongly connected component C' other than C that has yet to be visited.

Lemmas and Theorem (15/16)

By the inductive hypothesis, at the time that the search visits u, all other vertices of C are white.

By the white-path theorem, therefore, all other vertices of C are descendants of u in its depth-first tree.

By the inductive hypothesis and by **Corollary 15**, any edges in G^{T} that leave C must be to strongly connected components that have already been visited.

Lemmas and Theorem (16/16)

- ➡ No vertex in any strongly connected component other than C will be a descendant of u during the depth-first search of G^{T}.
- ➡ The vertices of the depth-first tree in G^{T} that is rooted at u form exactly one strongly connected component.

Homework Assignment #5

Exercise 22.3-13

- TAs will announce the detailed Input/Output format in Moodle.
- Please submit your program to e-Tutor.
- Please submit your README document to Moodle.
- **Due Date: 30 May 2017**.