Motivation

- Time-demand analysis method requires the *periods* and *execution times* of all the tasks in an application system to determine whether the system is schedulable.
- Before we have completed the design of the application system, some of these parameters may not be known.
- It is desirable to have a schedulability condition similar to Theorem 1 and 2 for the EDF and LST algorithms.

Outline

- Sufficient Schedulability Conditions for the RM and DM Algorithms
  1. ***Schedulable Utilization of the RM Algorithm for Tasks with $D_i = p_i$.***
  2. Schedulable Utilization of RM Algorithms as Functions of Task Parameters
  3. Schedulable Utilization of Fixed Priority Tasks with Arbitrary Relative Deadlines
  4. Schedulable Utilization of the RM Algorithm for Multiframe Tasks

Schedulable Utilization of the RM ($D_i = p_i$)

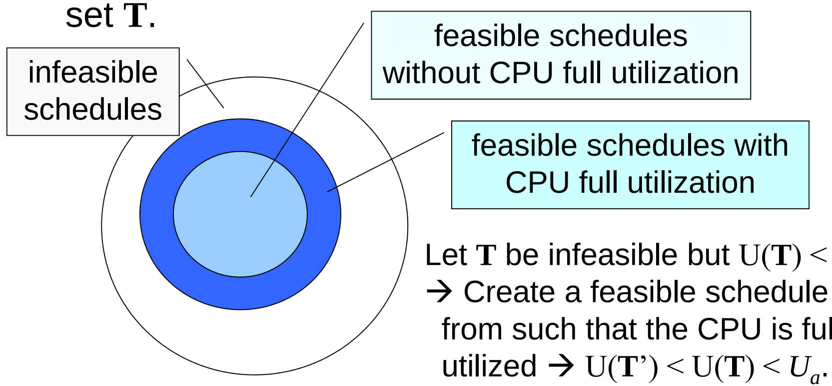
- **Theorem 7.** A system of n independent, preemptable periodic tasks with relative deadlines equal to their respective periods can be feasibly scheduled on a processor according to the RM algorithm if its total utilization U is less than or equal to

$$U_{RM}(n) = n(2^{1/n} - 1)$$

- $U_{RM}(n)$ is the schedulable utilization of the RM algorithm when $D_i = p_i$ for all $1 \le i \le n$.

Achievable Utilization Factor

- The **achievable utilization factor** (**least upper bound of utilization factor**) of a scheduling policy U_a is a real number such that for any process set $\mathbf{T}$, $U(\mathbf{T}) \le U_a$ implies the schedulability of the process set $\mathbf{T}$.

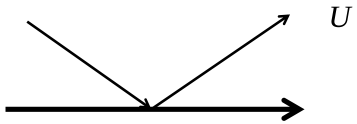


Let $\mathbf{T}$ be infeasible but $U(\mathbf{T}) < U_a$
→ Create a feasible schedule $\mathbf{T}'$ from such that the CPU is fully utilized → $U(\mathbf{T}') < U(\mathbf{T}) < U_a$.

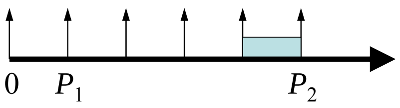
Proof of Theorem 7 (Step 1)

- **Theorem 7-1.** For a set of two processes with a fixed priority assignment, the achievable utilization factor is $2(2^{1/2} - 1)$.

Proof.



$C_1 = P_2 - P_1 \times \lfloor P_2/P_1 \rfloor$

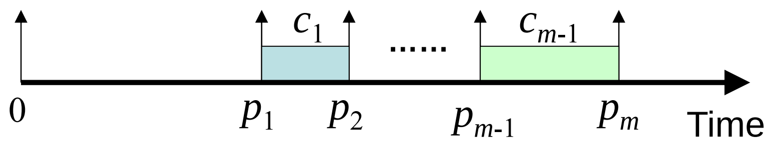


Given P_1 and P_2, the minimum U occurs when $\lfloor P_2/P_1 \rfloor = 1$ & $P_2/P_1 - \lfloor P_2/P_1 \rfloor = 2^{1/2} - 1$.

Proof of Theorem 7 (Step 2)

- **Theorem 7-2.** For a set of m processes with a fixed priority order and the restriction that the ratio between any two request periods is less than 2, the achievable utilization factor is $m(2^{1/m} - 1)$.

Proof.

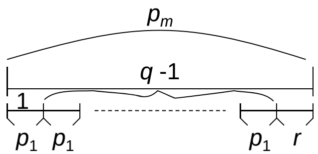


Each process in $\{\tau_1,\ldots,\tau_{m-1}\}$ executes twice within P_m.

Proof of Theorem 7 (Step 3)

- **Theorem 7-3.** For a set of m processes with a fixed priority order, the achievable utilization factor is $m(2^{1/m} - 1)$.

Proof.



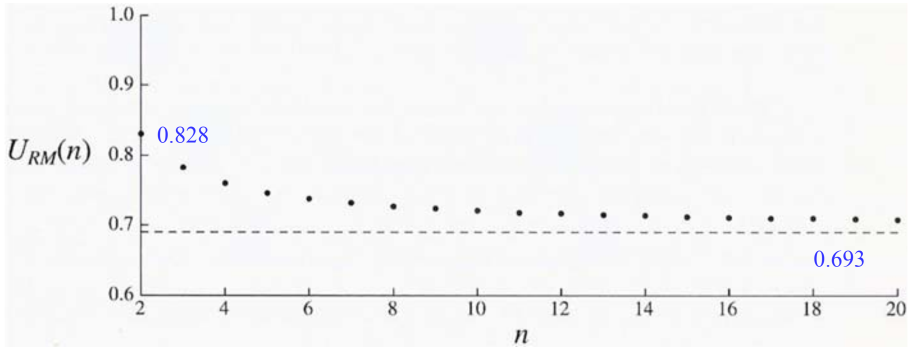
If $p_m = q \times p_i + r$, $q > 1$,

then $p_i = q \times p_i$ and increase c_m till the process set fully utilizes the processor again.

$(c'_m \le c_m + c_i \times (q - 1))$.

Show that U is reduced!

$U_{RM}(n)$ as a Function of n



- As long as the total utilization of a system satisfies $U(n) \le U_{RM}(n)$, it will never miss any deadline.
- We can reach this conclusion without considering the individual values of the phases, periods, and execution times.