Arbitrary Relative Deadlines

- **Theorem 11.** A system of n independent, preemptable periodic tasks with relative deadlines $D_i = \delta p_i$ for all $1 \leqq i \leqq n$ is schedulable rate-monotonically if its total utilization is equal to or less than

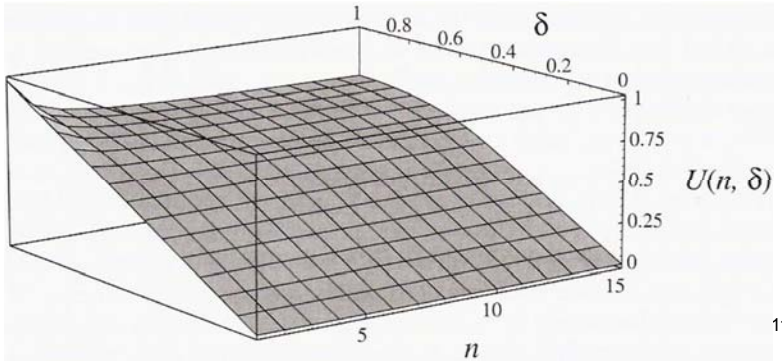
$$U_{RM}(n,\delta) = \delta(n-1)\left[\left(\frac{\delta+1}{\delta}\right)^{1/n-1} - 1\right], \quad \text{for } \delta = 2, 3, \ldots$$

$$= n((2\delta)^{1/n} - 1) + 1 - \delta, \quad \text{for } 0.5 \leqq \delta \leqq 1$$

$$= \delta, \quad \text{for } 0 \leqq \delta \leqq 0.5$$

$U_{RM}(n,\delta)$

n	δ=4.0	δ=3.0	δ=2.0	δ=1.0	δ=0.9	δ=0.8	δ=0.7	δ=0.6	δ=0.5
2	0.944	0.928	0.898	0.828	0.783	0.729	0.666	0.590	0.500
3	0.926	0.906	0.868	0.779	0.749	0.708	0.656	0.588	0.500
4	0.917	0.894	0.853	0.756	0.733	0.698	0.651	0.586	0.500
5	0.912	0.888	0.844	0.743	0.723	0.692	0.648	0.585	0.500
∞	0.892	0.863	0.810	0.693	0.687	0.670	0.636	0.582	0.500



Outline

- Sufficient Schedulability Conditions for the RM and DM Algorithms
  1. Schedulable Utilization of the RM Algorithm for Tasks with $D_i = p_i$.
  2. Schedulable Utilization of RM Algorithms as Functions of Task Parameters
  3. Schedulable Utilization of Fixed Priority Tasks with Arbitrary Relative Deadlines
  4. ***Schedulable Utilization of the RM Algorithm for Multiframe Tasks***

Motivation

- Consider a task that models the transmission of an MPEG compressed video over a network link.
  - Jobs in this task, modeling the transmissions of individual video frames, are released periodically.
  - The size of I-frames can be very large compared with that of B- and P- frames, the execution times of the jobs can vary widely.
  - When modeled as a periodic task, the execution time of the task is equal to the transmission time of an I-frame.
- If we were to determine whether a system of such tasks is schedulable based on the schedulability tests we have learned, we would surely underutilized the processor.
- The multiframe task model is a more accurate model and leads to more accurate schedulability tests.

Multiframe Task Model

- Each task T_i is characterized by a 4-tuple $(p_i, \xi_i, e_i^p, e_i^n)$
  - p_i : period of the task

  Jobs in T_i have either one of two possible maximum execution time
  - e_i^p: peak execution time
  - e_i^n: normal execution time

  Each period which begins at the release time of a job with the peak execution time is called a **peak frame**, and the other periods are called **normal frames**.
  - ξ_i : each peak frame is followed by $\xi_i - 1$ normal frames, which in turn are followed by a peak frame and so on.

References Mok, A. K.-L., and D. Chen, "A Multiframe Model for Real-Time Tasks," Proceedings of IEEE Real-Time Systems Symposium, December 1996.

Example

- The task (33, 6, 1.0, 0.3) can model an MPEG video transmission task.
  - The period of the task is 33ms.
  - The execution time of the job in each peak frame, which models the transmission of an I-frame in the video stream, is never more than 1ms.
  - I-frame is followed by 5 normal frames.
  - The execution times of jobs released in normal frames are never more than 0.3. These jobs model the transmissions of B- and P-frames in the video.
  - They are followed by the transmission of an I-frame, that is, a peak frame, which is in turn followed by 5 normal frames, and so on.

Critical Instant and Load Variation

- Critical Instant

  The response time of a job $J_{i,k}$ in T_i has the maximum possible value if the k-th period, which begins when $J_{i,k}$ is released, is a peak frame and this peak frame begins at the same time as a peak frame in every high-priority task.
- Load Variation

  Given a system of n multiframe tasks, the load variation, denoted by Ξ, of the system is $\min_{1 \le i \le n}(e_i^p / e_i^n)$

Schedulable Utilization

- **Theorem 12.** A system of n independent, preemptable multiframe tasks, whose relative deadlines are equal to the respective periods, is schedulable according to the RM algorithm if their total utilization is no greater than

$$U_{RM}(n, \Xi) = \Xi\, n \left(\left(\frac{\Xi + 1}{\Xi} \right)^{1/n} - 1 \right)$$

$U_{RM}(n,\Xi)$