Theorem 4. *A system of independent, preemptable sporadic jobs is schedulable according to the EDF algorithm if the total density of all active jobs in the system is no greater than 1 at all times.*

Proof. We prove the theorem by contradiction. Suppose that a job misses its deadline at time t, and there is no missed deadline before t. Let t_{-1} be the latest time instant before t at which either the system idles or some job with a deadline after t executes. Suppose that k jobs execute in the time interval $(t_{-1}, t]$. We call these jobs $J_1, J_2, \dots, J_k$ and order them in increasing order of their deadlines. (Ties in deadlines are broken arbitrarily.) J_k is the job that misses its deadline at t. Because the processor remains busy in $(t_{-1}, t]$ executing jobs of equal or higher priorities than J_k and J_k misses its deadline at time t, we must have $\sum_{i=1}^{k} e_i > t - t_{-1}$.

We let the number of job releases and completions during the time interval (t_{-1}, t) be l, and t_i be the time instant when the ith such event occurs. In terms of this notation, $t_{-1} = t_1$ and, for the sake of convenience, we also use t_{l+1} to denote t. These time instants partition the interval $(t_{-1}, t]$ into l disjoint subintervals, $(t_1, t_2], (t_2, t_3], \dots (t_l, t_{l+1}]$. The active jobs in the system and their total density remain unchanged in each of these subintervals. Let $\mathbf{X}_i$ denote the subset containing all the jobs that are active during the subinterval $(t_i, t_{i+1}]$ for $1 \leq i \leq l$ and Δ_i denote the total density of the jobs in $\mathbf{X}_i$.

The total time demanded by all the jobs that execute in the time interval (t_{-1}, t) is $\sum_{i=1}^{k} e_i$. We can rewrite the sum as

$$\sum_{i=1}^{k} \frac{e_i}{d_i - r_i}(d_i - r_i) = \sum_{j=1}^{l}(t_{j+1} - t_j) \sum_{J_k \in \mathbf{X}_j} \frac{e_k}{d_k - r_k} = \sum_{j=1}^{l} \Delta_j (t_{j+1} - t_j)$$

Since $\Delta_j \leq 1$ for all $j = 1, 2, \dots, l-1$, we have

$$\sum_{i=1}^{k} e_i \leq \sum_{j=1}^{l}(t_{j+1} - t_j) = t_{l+1} - t_1 = t - t_{-1}$$

This leads to a contradiction.

□