Minimum Spanning Trees

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Outline

- ***Overview***
- Growing a Minimum Spanning Tree
- The Algorithms of Kruskal and Prim

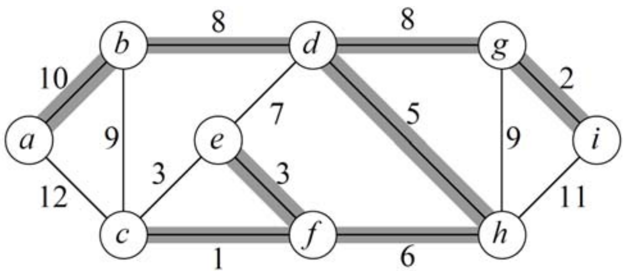
Problem

- A town has a set of houses and a set of roads.
- A road connects 2 and only 2 houses.
- A road connecting houses u and v has a repair cost $w(u, v)$.
- **Goal:** Repair enough (and no more) roads such that
  1. everyone stays connected: can reach every house from all other houses, and
  2. total repair cost is minimum.

Model as a Graph

- Undirected graph $G = (V, E)$.
- Weight $w(u, v)$ on each edge $(u, v) \in E$.
- Find $T \subseteq E$ such that
  - T connects all vertices (T is a spanning tree), and
  - $w(T) = \sum_{(u,v)\in T} w(u, v)$ is minimized.
- A spanning tree whose weight is minimum over all spanning trees is called a minimum spanning tree, or MST.

Example of MST



- In this example, there is more than one MST.
- We can replace edge (e, f) in the MST by (c, e) to get a different spanning tree with the same weight.

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Some Properties of an MST

- It has $|V| - 1$ edges.
- It has no cycles.
- It might not be unique.

Building up the Solution

- We will build a set A of edges.
- Initially, A has no edges.
- As we add edges to A, maintain a loop invariant:
  - Loop invariant: A is a subset of some MST.
- Add only edges that maintain the invariant. If A is a subset of some MST, an edge (u, v) is safe for A if and only if $A \cup \{(u, v)\}$ is also a subset of some MST. So we will add only safe edges.