Example 1

- Does the system $\mathbf{T} = \{(1.0, 0.25), (1.25, 0.1), (1.5, 0.3), (1.75, 0.07), (2.0, 0.1)\}$ schedulable?

Example 2

- Does the system $\mathbf{T} = \{(0.3, 1.3, 0.1), (1.0, 1.5, 0.3), (1.75, 0.1), (2.0,0.1), (7.0, 2.45)\}$ schedulable?

Schedulable Utilization of the RM

- Since $U(n) \leq U_{RM}(n)$ is not a necessary condition, a system of tasks may nevertheless be schedulable even when its total utilization exceeds the schedulable bound!

- **Example:** Total utilization of $\mathbf{T} = \{(3, 1), (5, 1.5), (7, 1.25), (9, 0.5)\}$ is 0.85, which is larger than $U_{RM}(4) = 0.756$, but this system is schedulable according to the RM algorithm!

Outline

- Sufficient Schedulability Conditions for the RM and DM Algorithms
  1. Schedulable Utilization of the RM Algorithm for Tasks with $D_i = p_i$.
  2. ***Schedulable Utilization of RM Algorithms as Functions of Task Parameters***
  3. Schedulable Utilization of Fixed Priority Tasks with Arbitrary Relative Deadlines
  4. Schedulable Utilization of the RM Algorithm for Multiframe Tasks

Enhanced Schedulable Utilization

- When some of the task parameters are known, this information allows us to improve the schedulable utilization of the RM algorithm:
  - The Utilization of Individual Tasks
  - The Number n_h of Disjoint Subsets Each Containing Simply Periodic Tasks
  - Some Functions of the Periods of the Tasks

Task Utilizations

- **Corollary 8.** n independent, preemptable periodic tasks with relative deadlines equal to their respective periods are schedulable rate-monotonically if their utilizations $u_1, u_2, \ldots, u_n$ satisfy the inequality

$$(1+u_1)(1+u_2)...(1+u_n) \leq 2$$

Subsets of Simply Periodic Tasks (1/2)

- **Example.** We can partition the system $\mathbf{T}$ of tasks with periods 4, 7, 8, 14, 16, 28, 32, 56, and 64 into two subsets $\mathbf{Z}_1$ and $\mathbf{Z}_2$. $\mathbf{Z}_1$ contains the tasks with periods 4, 8, 16, 32, and 64; and $\mathbf{Z}_2$ contains tasks with periods 7, 14, 28, and 56. Let $U(\mathbf{Z}_1)$ and $U(\mathbf{Z}_2)$ denote the total utilization of the tasks in $\mathbf{Z}_1$ and $\mathbf{Z}_2$, respectively.
- ➡ If $U(\mathbf{Z}_1)+U(\mathbf{Z}_2) \leq 0.828$, all these tasks are schedulable rate-monotonically.

References

Kuo, T. W. and A. K. Mok, "Load Adjustment in Adaptive Real-Time Systems," Proceedings of the IEEE Real-Time Systems Symposium, December 1991.

Subsets of Simply Periodic Tasks (2/2)

- **Theorem 9.** If a system $\mathbf{T}$ of independent, preemptable periodic tasks, whose relative deadlines are equal to their respective periods, can be partitioned into n_h disjoint subsets, $\mathbf{Z}_1, \mathbf{Z}_2, ..., \mathbf{Z}_{n_h}$, each of which contains simply periodic tasks, then the system is schedulable rate-monotonically if the total utilizations $U(\mathbf{Z}_i)$, for $i = 1, 2, \ldots, n_h$, of the tasks in the subsets satisfy the inequality

$$(1+U(\mathbf{Z}_1))(1+U(\mathbf{Z}_2))...(1+U(\mathbf{Z}_{n_h})) \leq 2$$

Task Periods (1/2)

- For a given system $\mathbf{T}$ of tasks, $\mathbf{T'}$ is an accelerated set of $\mathbf{T}$ if following properties hold:
  1. There is a task T_i' in $\mathbf{T'}$ if and only if there is a task T_i in $\mathbf{T}$.
  2. The execution time of T_i' is equal to the execution time of T_i.
  3. The period of T_i' is shorter than the period of T_i.

Task Periods (2/2)

- **Theorem 10.** A system $\mathbf{T}$ of independent, preemptive periodic tasks whose relative deadlines are equal to their respective periods is schedulable according the RM algorithm if it has an accelerated set $\mathbf{T'}$ which is simply periodic and has a total utilization equal to or less than 1.

Outline

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Observation

1. A system of n tasks with a total utilization $U_{RM}(n)$ may not be schedulable rate-monotonically when the relative deadlines of some tasks are shorter than their periods.
2. If the relative deadlines of the tasks are larger than the respective task periods, we expect the schedulable utilization of the RM algorithm to be larger than $U_{RM}(n)$.

➡ We now consider the case where the relative deadline D_i of every task is equal to δ times its period p_i for some $0 <$δ.