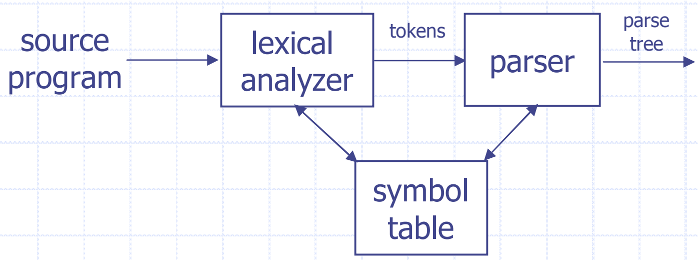
# 編譯器設計

# Chapter 4:
# Syntax Analysis

## Syntax Analysis

- The role of the syntax analyzer
  - To accept a string of tokens from the scanner
  - To verify the string can be generated by the grammar




- Three general types of parsers for grammars
  - Universal parsing methods
    - e.g. Cocke-Younger-Kasami, Earley's
  - Top-down
  - Bottom-up

# Syntax Error Handling

- The goals of the error handler in a parser
  - It should report the presence of errors clearly and accurately
  - It should recover from each error quickly enough to be able to detect subsequent errors
  - If should not significantly slow down the processing of correct programs
- Many errors could be classified [Ripley et al 1978]:
  - 60% punctuation errors
  - 20% operator and operand errors
  - 15% keyword errors
  - 5% others



# Error Recovery Strategies

- Panic mode
  - The simplest method and can be used by most parsers
  - On discovering an error, the parser discards input symbols one at a time until one of synchronizing tokens is found
    - The synchronizing tokens are usually delimiters, e.g. ; or end
- Phrase level
  - On discovering an error, a parser may perform local correction on the remaining input
    - e.g. replacing a comma by a semicolon, inserting a missing ;
- Error production
  - Augment the grammar with productions that generate the erroneous constructs
- Global correction
  - Make as few changes as possible in processing the input

# Context-Free Grammars

- The syntax of programming language constructs can be described by *context-free grammars*
  - Or BNF (Backus-Naur Form)
- Grammars offer significant advantages
  - A grammar gives a precise syntactic specification of a programming language
  - From certain classes of grammars we can automatically construct an efficient parser
    - The parser construction process can reveal syntactic ambiguities and difficult-to-parse constructs
  - A properly designed grammar facilitates the translation into target code
  - New language constructs can be added easily



# Context-Free Grammars

- A context-free grammar G can be denoted by G = $(V_N, V_T, P, S)$
  - $V_N$: nonterminals
    - Syntactic variables that denotes sets of strings
  - $V_T$: terminals
    - The basic symbols (i.e. tokens) from which strings are formed
  - $P$: productions
    - The manner in which the terminals and nonterminals can be combined for form strings, e.g.

      *expr* → *expr op expr*

      *expr* → id

      *op* → + | − | ∗ | /
  - $S$: start symbol

# Why Not Using Regular Languages

- Regular languages are not power enough to represent certain constructs in program languages, e.g. $\{^n\}^n$
  - i.e. $\{^n\}^n$ is not a regular language
- Recall the ways to prove a language is regular
  - Find a regular grammar (or regular expressions) that generates the language
  - Find an NFA that recognizes the language
- However, they can not prove $\{^n\}^n$ is not regular
  - The pumping lemma of regular languages will be used
  - Prove by contradiction

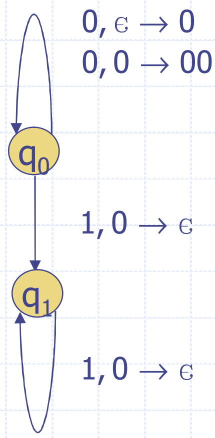


# Pumping Lemma

- If $A$ is a regular language and string $s \in A$, where $\exists n \in N$ such that $|s| \geq n$, then $s = xyz$ where
  - $xy^iz \in A$, $i \geq 0$,
  - $|y| > 0$, and
  - $|xy| \leq n$
- Example: prove $L = \{0^n1^n \mid n \geq 0\}$ is not regular
  - Assume L is regular. Let $s = wyz$, $s \in L$
  - If y consists of 0's only, then $xy^iz$ has more 0's than 1's
  - If y consists of 1's only, then $xy^iz$ has more 1's than 0's
  - If y consists of 0's and 1's, then $xy^iz \neq 0^n1^n$

# Pushdown Automata

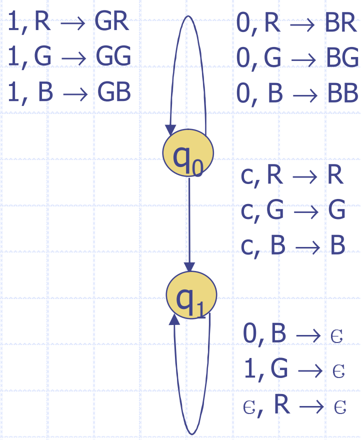
- $M = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$
  - $K$ : Finite set of states
  - $\Sigma$ : Input alphabet
  - $\Gamma$ : Pushdown alphabet
  - $\delta$ : Mapping $K \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow K \times \Gamma^*$
  - $q_0$ : Initial state ($q_0 \in K$)
  - $Z_0$ : Initial pushdown symbol ($Z_0 \in \Gamma$)
  - $F$ : Set of final states
- Example: $L = \{0^n 1^n \mid n \geq 0\}$
  - $M = (\{q_0, q_1\}, \{0, 1\}, \{0, 1\}, \delta, q_0, \epsilon, \phi)$
    - $\delta(q_0, 0, \epsilon) = (q_0, 0)$
    - $\delta(q_0, 0, 0) = (q_0, 00)$
    - $\delta(q_0, 1, 0) = (q_1, \epsilon)$
    - $\delta(q_1, 1, 0) = (q_1, \epsilon)$





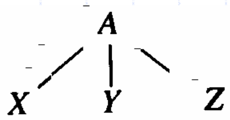

# Pushdown Automata

- Example: $L = \{w c w^R \mid w \in \{0, 1\}^*\}$
  - $M = (\{q_0, q_1\}, \{0, 1, c\}, \{R, G, B\}, \delta, q_0, R, \phi)$
    - $\delta(q_0, 0, R) = (q_0, BR)$
    - $\delta(q_0, 0, G) = (q_0, BG)$
    - $\delta(q_0, 0, B) = (q_0, BB)$
    - $\delta(q_0, 1, R) = (q_0, GR)$
    - $\delta(q_0, 1, G) = (q_0, GG)$
    - $\delta(q_0, 1, B) = (q_0, GB)$
    - $\delta(q_0, c, R) = (q_1, R)$
    - $\delta(q_0, c, G) = (q_1, G)$
    - $\delta(q_0, c, B) = (q_1, B)$
    - $\delta(q_1, 0, B) = (q_1, \epsilon)$
    - $\delta(q_1, 1, G) = (q_1, \epsilon)$
    - $\delta(q_1, \epsilon, R) = (q_1, \epsilon)$



# Parse Trees

- A parse tree pictorially shows how the start symbol of a grammar derives a string in the language
  - e.g. A → XYZ



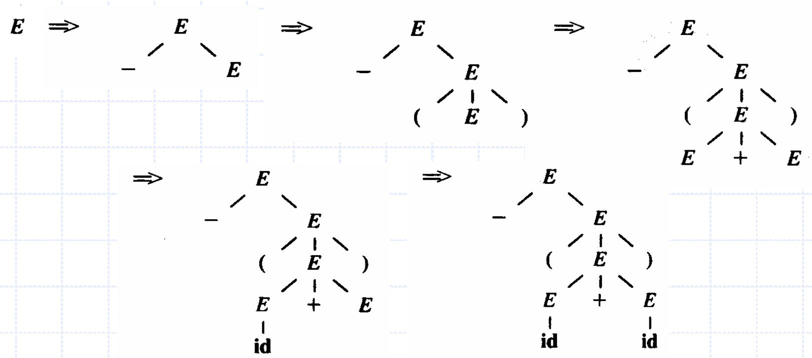

- Formally, given a context-free grammar, a parse tree is a tree with the following properties
  - The root is labeled by the start symbol
  - Each leaf is labeled by a token or by $\epsilon$
  - Each interior node is labeled by a nonterminal
  - If $A$ is the nonterminal labeling some interior node and $X_1, X_2, \ldots, X_n$ are the labels of the children of that node from left to right, then $A \rightarrow X_1 X_2 \ldots X_n$ is a production



# Parse Trees and Derivations

- There are several ways to view the process by which a grammar defines a language
  - Building parse trees
  - Derivations
- Derivations give a precise description of the top-down construction of a parse tree
  - Example: $E \rightarrow E + E \mid E * E \mid (E) \mid -E \mid \text{id}$
  - $w = -(\text{id} + \text{id})$
  - Derivation:

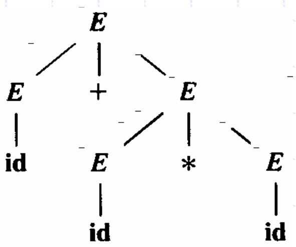
$$
\begin{aligned}
E &\Rightarrow -E \\
&\Rightarrow -(E) \\
&\Rightarrow -(E + E) \\
&\Rightarrow -(\text{id} + E) \\
&\Rightarrow -(\text{id} + \text{id})
\end{aligned}
$$

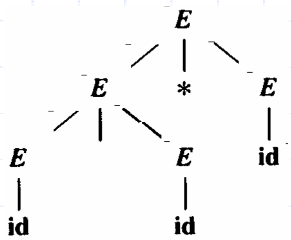


# Parse Trees and Derivations

- Every parse tree has associated with it a unique leftmost and a unique rightmost derivation
  - However, not every sentence has exactly one leftmost or rightmost derivation
  - e.g. id + id * id

$$
\begin{aligned}
E &\Rightarrow E + E\\
&\Rightarrow \text{id} + E\\
&\Rightarrow \text{id} + E * E\\
&\Rightarrow \text{id} + \text{id} * E\\
&\Rightarrow \text{id} + \text{id} * \text{id}
\end{aligned}
\qquad\qquad
\begin{aligned}
E &\Rightarrow E * E\\
&\Rightarrow E + E * E\\
&\Rightarrow \text{id} + E * E\\
&\Rightarrow \text{id} + \text{id} * E\\
&\Rightarrow \text{id} + \text{id} * \text{id}
\end{aligned}
$$

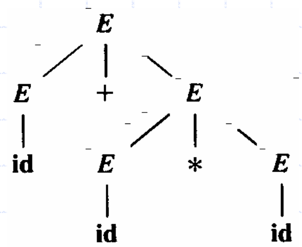





# Ambiguity

- A grammar that produces more than one parse tree for some sentence is said to be *ambiguous*
  - i.e. more than one leftmost or rightmost derivation for the same sentence
- Two options to deal with ambiguity
  - *Disambiguating rules* are used to throw away undesirable parse trees
  - To eliminate ambiguity
    - Precedence and associativity
    - Rewriting the grammar



# Eliminating Ambiguity

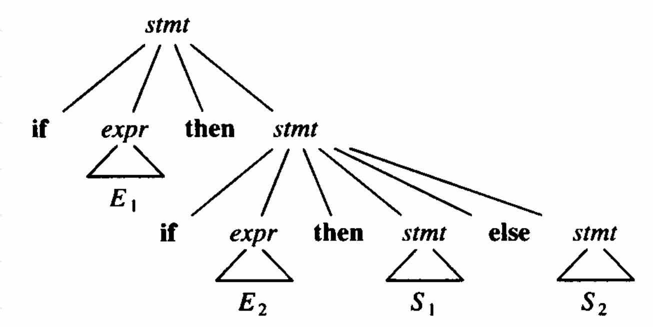
- Example (Dangling Else):

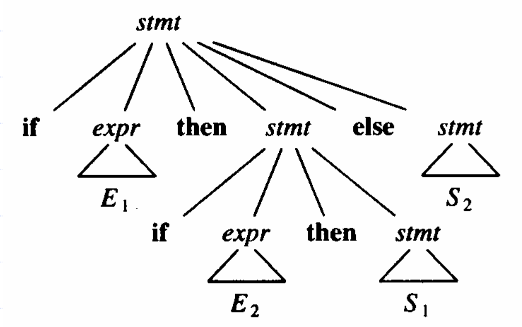
  *stmt* → if *expr* then *stmt*

  *stmt* → if *expr* then *stmt* else *stmt*

  *stmt* → other

  - if $E_1$ then if $E_2$ then $S_1$ else $S_2$





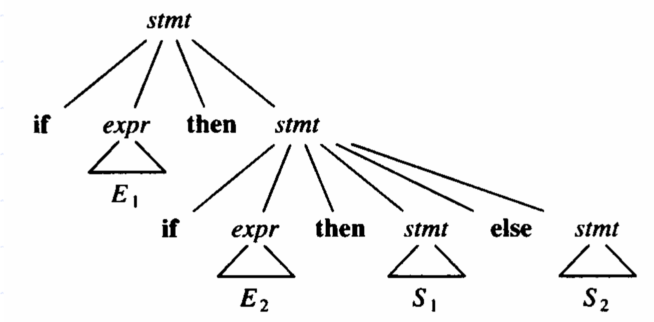

# Eliminating Ambiguity

- Rewrite the grammar:

  *stmt* → matched_stmt
  | unmatched_stmt

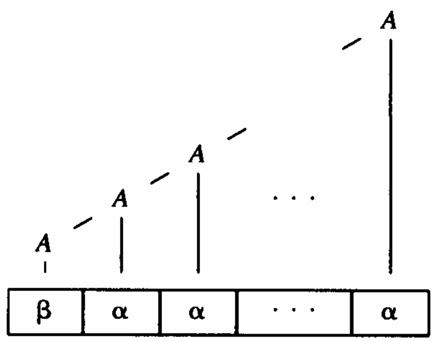
  matched_stmt → if *expr* then *matched_stmt* else *matched_stmt*
  | other

  *unmatched_stmt* → if *expr* then *stmt*
  | if *expr* then *matched_stmt* else *unmatched_stmt*



# Left Recursion

- A grammar is *left recursive* if it has a nonterminal $A$ such that there is a derivation $A \stackrel{+}{\Rightarrow} A\alpha$
  - e.g. $A \rightarrow A\alpha \mid \beta$
- It is possible that a parser to loop forever if the grammar is left recursive






# Eliminating Left Recursion

- A left-recursive pair of production $A \rightarrow A\alpha \mid \beta$ can be changed to non-left-recursive productions

  $A \rightarrow \beta A'$

  $A' \rightarrow \alpha A' \mid \epsilon$

  without changing the set of strings derivable from $A$
- Example:

| | | |
|---|---|---|
| $E \rightarrow E + T \mid T$ | $\Rightarrow$ | $E \rightarrow TE'$ |
| $T \rightarrow T * F \mid F$ | | $E' \rightarrow +TE' \mid \epsilon$ |
| $F \rightarrow (E) \mid \text{id}$ | | $T \rightarrow FT'$ |
| | | $T' \rightarrow *FT' \mid \epsilon$ |
| | | $F \rightarrow (E) \mid \text{id}$ |

# Eliminating Left Recursion

## Algorithm

- Input. Grammar G with no cycles on ϵ-productions
- Output. An equivalent grammar with no left recursion
- Method.

Arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$
for i = 1 to n do
  for j = 1 to i-1 do
    replace each production of the form $A_i \rightarrow A_j\gamma$ by
      the production $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$,
      where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \mid \delta_k$ are all the current $A_j$-productions
    eliminate the immediate left recursion among the $A_i$-productions
  end
end



# Eliminating Left Recursion

## Example

$S \rightarrow A\mathrm{a} \mid \mathrm{b}$
$A \rightarrow A\mathrm{c} \mid S\mathrm{d} \mid \epsilon$

- Arrange the nonterminals in order $S, A$
- i = 1
  - No immediate left recursion among the $S$-productions
- i = 2
  - Substitute the $S$-productions in $A \rightarrow S\mathrm{d}$ to obtain
    $A \rightarrow A\mathrm{c} \mid A\mathrm{ad} \mid \mathrm{bd} \mid \epsilon$
  - Eliminate the left recursion and yield
    $S \rightarrow A\mathrm{a} \mid \mathrm{b}$
    $A \rightarrow \mathrm{bd}A' \mid A'$
    $A \rightarrow \mathrm{c}A' \mid \mathrm{ad}A' \mid \epsilon$

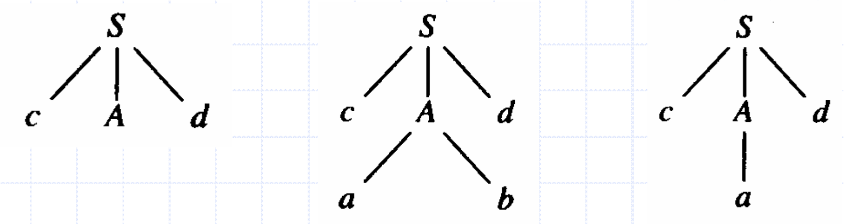
# Left Factoring

- Left factoring is a grammar transformation
  - Usually applied when it is not clear which of two alternative productions to use to expand a nonterminal *A*
  - Rewrite the *A*-productions to defer the decision
  - e.g. *stmt* $\rightarrow$ if *expr* then *stmt* else *stmt*
    *stmt* $\rightarrow$ if *expr* then *stmt*
- Algorithm
  - For each nonterminal *A* find the longest prefix $\alpha$ common to two or more of its alternatives
  - Replace $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \ldots \mid \alpha\beta_n \mid \gamma$ by
    $A \rightarrow \alpha A' \mid \gamma$
    $A' \rightarrow \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$



# Top-Down Parsing

- Top-down parsing an be viewed as an attempt to find a leftmost derivation for an input string
  - Equivalently, it can be viewed as an attempt to construct a parse tree for the input starting from the root and creating the nodes of the parse tree in preorder
- Example
  $S \rightarrow cAd$
  $A \rightarrow ab \mid a$



# Top-Down Parsing

- Recursive-Descent Parsing
  - Start with the root, labeled with the starting symbol, and repeatedly perform the following steps
    - At node *n* labeled with nonterminal *A*, select one of the productions for *A* and construct children at *n* for the symbols on the right side of the production
    - When a terminal is added that does not match the input string, then backtrack
    - find the next node
- Predictive Parsing
  - Recursive-descent parsing without backtracking



# Designing a Predictive Parser

- A *predictive parser* is a program consisting a procedure for every nonterminal. Each procedure does two things
  - For all *A*-productions it decides which production to use by looking at the lookahead symbol, say a, i.e.
    - The production $A \rightarrow \alpha$ is used if

      $a \in FIRST(\alpha)$

      where FIRST($\alpha$) is the set of terminals that begins the strings derived from $\alpha$, or
    - The production $A \rightarrow \epsilon$ is used if

      $a \notin FIRST(\alpha)$
  - The procedure uses a production by mimicking the right side:
    - A nonterminal results a call to the procedure for the nonterminal
    - A token matching the lookahead symbol results in the next input toke being read

# Designing a Predictive Parser

- Example

  *type* → *simple* | ^id | array [ *simple* ] of type

  *simple* → integer | char | num dotdot num

```
procedure match (t : token);
begin
  if lookahead = t then
    lookahead : = nextoken
  else error
end;
```

```
procedure type ;
begin
  if lookahead ∈ {integer, char, num} then
    simple
  else if lookahead = '^' then begin
    match('^'); match (id)
  end
  else if lookahead = array then begin
    match (array); match('['); simple;
    match(']'); match (of); type
  end
  else error
end;
```

```
procedure simple ;
begin
  if lookahead = integer then
    match (integer)
  else if lookahead = char then
    match (char)
  else if lookahead = num then begin
    match(num); match(dotdot); match(num)
  end
  else error
end;
```



# Transition Diagrams for Predictive Parsers

- Features
  - There is one diagram (procedure) for each nonterminal
  - The labels of edges are tokens (terminals) and nonterminals
  - A transition on a token is taken if that token is the next input symbol
  - A transition on a nonterminal *A* is a call of the procedure of *A*
- To construct the transition diagram from a grammar
  - Eliminate left recursion from the grammar
  - Left factor the grammar
  - For each nonterminal *A*, do the following
    - Create an initial state and a final (return) state
    - For each production $A \rightarrow X_1X_2\ldots X_n$, create a path from the initial to the final state, with edges labeled $X_1, X_2, \ldots, X_n$

# Transition Diagrams for Predictive Parsers

- Example

  $E \rightarrow TE'$

  $E' \rightarrow +TE' \mid \epsilon$

  $T \rightarrow FT'$

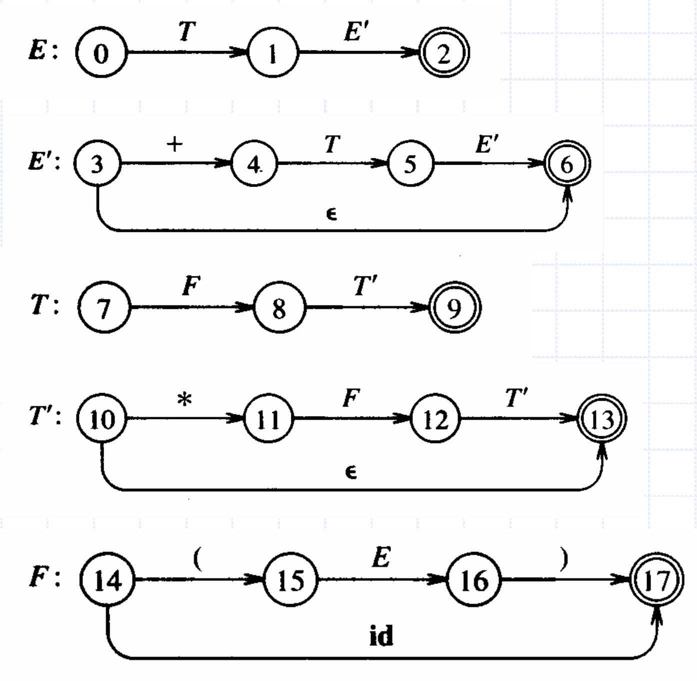
  $T' \rightarrow *FT' \mid \epsilon$

  $F \rightarrow (E) \mid \text{id}$

  - $w$ = id + id * id

```
E( ) {
  T( ); E'( );
}

E'( ) {
  if (lookahead == '+') {
    T( ); E'( );
  }
}
```

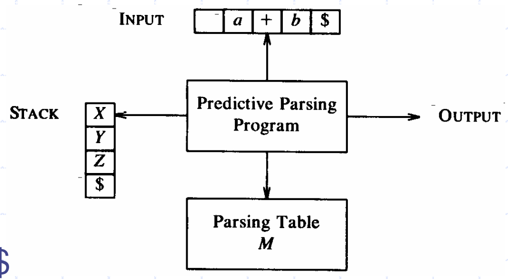





# Nonrecursive Predictive Parser

- The key problem during predictive parsing is that of determining the production to be applied for a nonterminal
- A table-drive predictive parser has an input buffer, a stack, a parsing table, and an output stream
- The action of the parser is determined by $X$, the symbol on top of the stack, and a, the current input symbol
  - If $X$ = a = '\$', the parser halts and announces successful completion of parsing
  - If $X$ = a ≠ '\$', the parser pops $X$ off the stack and advances the input pointer to the next input symbol
  - If $X$ is a nonterminal, the program consults entry M[X, a] of the parsing table M.
    - If, for example, M[X, a] = $\{X \rightarrow UVW\}$, the parser replaces $X$ on top of the stack by $WVU$ (with $U$ on top).
    - If M[X, a] = error, the parser calls an error recovery routine.

# Nonrecursive Predictive Parser




- Algorithm
  - Input. A string $w\$$ and a parsing table M
  - Output. A leftmost derivation of $w$
  - Method. Initially, stack = $S

    set $ip$ to point to the first symbol of $w\$$
    repeat
    let $X$ be the top stack symbol and a the symbol pointed by ip
    if $X$ is a termonal or $ then
    if $X$ = a then
    pop $X$ off the stack and advance $ip$
    else /* $X$ is a nonterminal */
    if M[$X$, a] = $X \rightarrow Y_1\ Y_2 \ldots Y_k$, then
    pop $X$ from the stack
    push $Y_k, Y_{k-1}, \ldots, Y_1$ onto the stack, with $Y_1$ on the top
    Output the production $X \rightarrow Y_1\ Y_2 \ldots Y_k$
    else $error$( )
    until $X$ = $ /* stack is empty */



# Nonrecursive Predictive Parser

- Example
  - $E \rightarrow TE'$
  - $E' \rightarrow +TE' \mid \epsilon$
  - $T \rightarrow FT'$
  - $T' \rightarrow *FT' \mid \epsilon$
  - $F \rightarrow (E) \mid \text{id}$
- Parsing Table
  - Fig. 4.15

| NONTERMINAL | INPUT SYMBOL id | + | * | ( | ) | $ |
|---|---|---|---|---|---|---|
| $E$ | $E \rightarrow TE'$ | | | $E \rightarrow TE'$ | | |
| $E'$ | | $E' \rightarrow +TE'$ | | | $E' \rightarrow \epsilon$ | $E' \rightarrow \epsilon$ |
| $T$ | $T \rightarrow FT'$ | | | $T \rightarrow FT'$ | | |
| $T'$ | | $T' \rightarrow \epsilon$ | $T' \rightarrow *FT'$ | | $T' \rightarrow \epsilon$ | $T' \rightarrow \epsilon$ |
| $F$ | $F \rightarrow \text{id}$ | | | $F \rightarrow (E)$ | | |

| Stack | Input | Output | Leftmost Derivation |
|---|---|---|---|
| $\$E$ | id + id * id$ | | $E \Rightarrow$ |
| $\$E'T$ | id + id * id$ | $E \rightarrow TE'$ | $TE' \Rightarrow$ |
| $\$E'T'F$ | id + id * id$ | $T \rightarrow FT'$ | $FT'E' \Rightarrow$ |
| $\$E'T'\text{id}$ | id + id * id$ | $F \rightarrow \text{id}$ | $\text{id}\,T'E' \Rightarrow$ |
| $\$E'T'$ | + id * id$ | | $\text{id}\,T'E' \Rightarrow$ |
| $\$E'$ | + id * id$ | $T' \rightarrow \epsilon$ | $\text{id}\,E' \Rightarrow$ |
| $\$E'T+$ | + id * id$ | $E' \rightarrow +TE'$ | $\text{id} + T\,E' \Rightarrow$ |
| $\$E'T$ | id * id$ | | |
| $\$E'T'F$ | id * id$ | $T \rightarrow FT'$ | $\text{id} + FT'E' \Rightarrow$ |
| $\$E'T'\text{id}$ | id * id$ | $F \rightarrow \text{id}$ | $\text{id} + \text{id}\,T'E' \Rightarrow$ |
| $\$E'T'$ | * id$ | | |
| $\$E'T'F*$ | * id$ | $T' \rightarrow *FT'$ | $\text{id} + \text{id} * FT'\,E' \Rightarrow$ |
| $\$E'T'F$ | id$ | | |
| $\$E'T'\text{id}$ | id$ | $F \rightarrow \text{id}$ | $\text{id} + \text{id} * \text{id}\ T'\,E' \Rightarrow$ |
| $\$E'T'$ | $ | | |
| $\$E'$ | $ | $T' \rightarrow \epsilon$ | $\text{id} + \text{id} * \text{id}\ E' \Rightarrow$ |
| $ | $ | $E' \rightarrow \epsilon$ | $\text{id} + \text{id} * \text{id}$ |

# FIRST and FOLLOW

- The construction of a predictive parser is aided by two functions associated with a grammar *G*
- FIRST($\alpha$)
  - The set of terminals that begins the strings derived from $\alpha$
  - To compute FIRST($X$) for all grammar symbols $X$, apply the following rules until no more terminals or $\epsilon$ can be added
    - If $X$ is a terminal, then FIRST($X$) = $\{X\}$
    - If $X \rightarrow \epsilon$ is a production, then add $\epsilon$ to FIRST($X$)
    - If $X \rightarrow Y_1\ Y_2\ \ldots\ Y_k$ is a production, then
      - Place a in FIRST($X$) if
        $\exists i\ (1 \le i \le k) \ni a \in \text{FIRST}(Y_i)$ and $\epsilon \in \text{FIRST}(Y_j)\ \forall j\ (1 \le i \le j)$
      - Place $\varepsilon$ in FIRST($X$) if
        $\forall i\ (1 \le i \le k)\ \epsilon \in \text{FIRST}(Y_i)$



# FIRST and FOLLOW

- FOLLOW($A$)
  - The set of terminals a that can appear immediately to the right of $A$ in some sentential form, i.e.

    a $\in$ FOLLOW($A$) if there exists a derivation $S \Rightarrow \alpha A a \beta$
  - To compute FOLLOW($A$) for all nonterminals $A$, apply the following rules until nothing can be added
    - Place \$ in FOLLOW($S$)
    - If there is production $A \rightarrow \alpha B \beta$, then
      add FIRST($\beta$) – $\{\epsilon\}$ to FOLLOW($B$)
    - If there is production $A \rightarrow \alpha B$, then
      add FOLLOW($A$) to FOLLOW($B$)
    - If there is production $A \rightarrow \alpha B \beta$ and $\epsilon \in$ FIRST($\beta$), then
      add FOLLOW($A$) to FOLLOW($B$)

# FIRST and FOLLOW

- Example

  $E \to TE'$

  $E' \to +TE' \mid \epsilon$

  $T \to FT'$

  $T' \to *FT' \mid \epsilon$

  $F \to (E) \mid \text{id}$

- FIRST
  - FIRST(E) = FIRST(T) = FIRST(F) = {(, id}
  - FIRST(E′) = {+, ϵ}
  - FIRST(T′) = {*, ϵ}
- FOLLOW
  - FOLLOW(E) = FOLLOW(E′) = {), $}
  - FOLLOW(T) = FOLLOW(T′) = {+, ), $}
  - FOLLOW(F) = = {+, *, ), $}



# Construction of Predictive Parsing Table

- Algorithm
  - Input. Grammar *G*
  - Output. Parsing table M
  - Method.
    1. For each production $A \to \alpha$, do steps 2 to 4
    2. For each terminal a ∈ FIRST(α), then
       add $A \to \alpha$ to M[*A*, a]
    3. If ϵ ∈ FIRST(α), then
       add $A \to \alpha$ to M[*A*, b] for each terminal b ∈ FOLLOW(*A*)
    4. If ϵ ∈ FIRST(α) and $ ∈ FOLLOW(*A*), then
       add $A \to \alpha$ to M[*A*, $]
    5. Make each undefined entry of M be *error*

# Construction of Predictive Parsing Table

- Example

$E \rightarrow TE'$
$E' \rightarrow +TE' \mid \epsilon$
$T \rightarrow FT'$
$T' \rightarrow *FT' \mid \epsilon$
$F \rightarrow (E) \mid \text{id}$

- FIRST($E$) = {(, id}
- FIRST($T$) = {(, id}
- FIRST($F$) = {(, id}
- FIRST($E'$) = {+, $\epsilon$}
- FIRST($T'$) = {*, $\epsilon$}

- FOLLOW($E$) ={), $}
- FOLLOW($E'$) = {), $}
- FOLLOW($T$) = {+, ), $}
- FOLLOW($T'$) = {+, ), $}
- FOLLOW($F$) = = {+, *, ), $}

| Nonter-minal | Input Symbol | | | | | |
|---|---|---|---|---|---|---|
| | id | + | * | ( | ) | $ |
| $E$ | $E \rightarrow TE'$ | | | $E \rightarrow TE'$ | | |
| $E'$ | | $E' \rightarrow +TE'$ | | | $E' \rightarrow \epsilon$ | $E' \rightarrow \epsilon$ |
| $T$ | $T \rightarrow FT'$ | | | $T \rightarrow FT'$ | | |
| $T'$ | | $T' \rightarrow \epsilon$ | $T' \rightarrow *FT'$ | | $T' \rightarrow \epsilon$ | $T' \rightarrow \epsilon$ |
| $F$ | $F \rightarrow \text{id}$ | | | $F \rightarrow (E)$ | | |



# Construction of Predictive Parsing Table

- Example

$S \rightarrow iEtSS' \mid a$
$S' \rightarrow eS \mid \epsilon$
$E \rightarrow b$

- FIRST(S) = {i, a}
- FIRST(S') = {e, $\epsilon$}
- FIRST(E) = {b}

- FOLLOW(S) ={e, $}
- FOLLOW(S') = {e, $}
- FOLLOW(E) = {t}

| Nonter-minal | Input Symbol | | | | | |
|---|---|---|---|---|---|---|
| | a | b | e | i | t | $ |
| $S$ | $S \rightarrow a$ | | | $S \rightarrow iEtSS'$ | | |
| $S'$ | | | $S' \rightarrow eS$<br>$S' \rightarrow \epsilon$ | | | $S' \rightarrow \epsilon$ |
| $E$ | | $E \rightarrow b$ | | | | |

# LL(1) Grammars

- A grammar whose predictive parsing table has no multiply-defined entries is said to be *LL*(1)
  - 1st *L*: scanning the input from left to right
  - 2nd *L*: producing a leftmost derivation
  - 1: using one input symbol for lookahead at each step
- Properties
  - Not ambiguous
  - Not left recursive
  - Where $A \to \alpha \mid \beta$ are two distinct productions in *G*, the following conditions hold:
    - For no terminal a do both $\alpha$ and $\beta$ derive strings beginning with a
    - At most one of $\alpha$ and $\beta$ can derive the empty string
    - If $\beta$ derives the empty string then $\alpha$ does not derive any string beginning with a terminal in FOLLOW(*A*)



# Error Handling in Predicting Parsing

- An error is detected when
  - The terminal on top of the stack does not match the next input symbol, or
  - Nonterminal *A* is on top of the stack, a is the next input symbol, and M[*A*, a] is empty
- Panic-mode error recovery
  - Based on the idea of skipping symbols on the input until a token in a selected set of synchronizing tokens appears
  - Some heuristics of choosing the synchronizing tokens
    - Place all symbols in FOLLOW(*A*) into the synchronizing set of *A*
    - Add symbols in FIRST(*A*) to the synchronizing set of *A*
    - If a nonterminal can generate $\epsilon$, then the production deriving $\epsilon$ can be used as a default
    - If a terminal on top of the stack cannot be matched, then pop the terminal

# Error Handling in Predicting Parsing

- Example
  - $E \rightarrow TE'$
  - $E' \rightarrow +TE' \mid \epsilon$
  - $T \rightarrow FT'$
  - $T' \rightarrow *FT' \mid \epsilon$
  - $F \rightarrow (E) \mid \text{id}$
- Parsing table
  - Fig. 4.18
- Actions
  - M[A, a] = $\phi$
    input symbol is skipped
  - M[A, a] = sync
    nonterminal is popped
  - Token on stack isn't matched
    pop the token

| Stack | Input | Output |
|---|---|---|
| $E | + id * + id$ | error, skip + |
| $E | id * + id$ | id ∈ FIRST(E) |
| $E'T | id * + id$ | |
| $E'T'F | id * + id$ | |
| $E'T'id | id * + id$ | |
| $E'T' | * + id$ | |
| $E'T'F* | * + id$ | |
| $E'T'F | + id$ | error, M[F,+] = sync |
| $E'T' | + id$ | F has been popped |
| $E' | + id$ | |
| $E'T+ | + id$ | |
| $E'T | id$ | |
| $E'T'F | id$ | |
| $E'T'id | id$ | |
| $E'T' | $ | |
| $E' | $ | |
| $ | $ | |



# Bottom-Up Parsing

- Attempt to construct a parse tree for an input string beginning at the leaves (the bottom) and working up towards to the start symbol
  - Example. Consider the grammar

    $S \rightarrow aABe$

    $A \rightarrow Abc \mid b$

    $B \rightarrow d$

The sentence abbcde can be reduced to S:

abbcde
aAbcde
aAde
aABe
S

Rightmost Derivation

S ⇒
aABe ⇒
aAde ⇒
aAbcde ⇒
abbcde

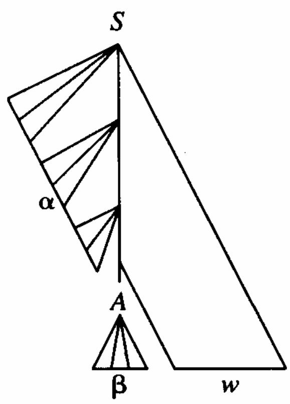
# Bottom-Up Parsing

- A general style of bottom-up syntax analysis, known as *shift-reduce parsing,* will be introduced
  - The process of constructing a parse tree can be thought of as "reducing" a string *w* to the start symbol
  - It is equivalent to performing a rightmost derivation in reverse
  - At each reduction step, a particular substring matching the right side of a production is replaced by the symbol on the left
    - The matched substring can be called a *handle*
    - The reduction of a handle represents one step along the rightmost derivation in reverse
  - Formally, a handle of $\gamma$ is
    - A production $A \rightarrow \beta$, and
    - A position of $\gamma$ where the substring $\beta$ may be found
    - e.g. $A \rightarrow b$ at position 2 is a handle of abbcde
      $A \rightarrow Abc$ at position 2 is a handle of aAbcde



# Handle Pruning

- Suppose $A \rightarrow \beta$ is a handle
  - It represents a leftmost complete subtree consisting of a node and all its children
  - Reducing $\beta$ to $A$ in $\alpha\beta w$ can be thought of "pruning the handle"
    - i.e. removing the children of $A$
- A rightmost derivation in reverse can be obtained by handle pruning



# Stack Implementation of Shift-Reduce Parsing

- Two problems for parsing by handle pruning
  - To locate the substring to be reduced
  - To determine what production to choose in case there is more than one production with that substring
- A convenient way to implement a shift-reduce parser is to use a stack to hold grammar symbols
  - Actions
    - Shift
      - the next input symbol is shifted onto the top of the stack
    - Reduce
      - The handle is at the top of the stack
      - Remove the handle and place the left side nonterminal on the stack
    - Accept
      - Announce successful completion of parsing
    - error



# Stack Implementation of Shift-Reduce Parsing

- Example. (1) $E \rightarrow E + E$
  (2) $E \rightarrow E * E$
  (3) $E \rightarrow (E)$
  (4) $E \rightarrow \text{id}$

| Stack | Input | Action | Stack | Input | Action |
|---|---|---|---|---|---|
| $ | id + id * id$ | shift | $ | id + id * id$ | shift |
| $id | + id * id$ | reduce by (4) | $id | + id * id$ | reduce by (4) |
| $*E* | I + id * id$ | shift | $*E* | + id * id$ | shift |
| $*E* + | id * id$ | shift | $*E* + | id * id$ | shift |
| $*E* + id | * id$ | reduce by (4) | $*E* + id | * id$ | reduce by (4) |
| $*E* + *E* | * id$ | reduce by (1) | $*E* + *E* | * id$ | shift |
| $*E* | * id$ | shift | $*E* + *E* * | id$ | shift |
| $*E* * | id$ | shift | $*E* + *E* * id | $ | reduce by (4) |
| $*E* * id | $ | reduce by (4) | $*E* + *E* * *E* | $ | reduce by (2) |
| $*E* * *E* | $ | reduce by (2) | $*E* + *E* | $ | reduce by (1) |
| $*E* | $ | accept | $*E* | $ | accept |

## Conflicts During Shift-Reduce Parsing

- For some grammars, a shift-reduce parser can't decide
  - whether to shift or to reduce (a *shift/reduce conflict*), or
  - which of several reductions to make (a *reduce/reduce conflict*)
  - Example 1

| Stack | Input | Action | Stack | Input | Action |
|---|---|---|---|---|---|
| $\$E + E$ | $*$ id\$ | shift | $\$E + E$ | $*$ id\$ | reduce by $E \to E + E$ |
| $\$E + E\ *$ | id\$ | | $\$E$ | $*$ id\$ | |

  - Example 2

*stmt* → if *expr* then *stmt*
| if *expr* then *stmt* else *stmt*
| other

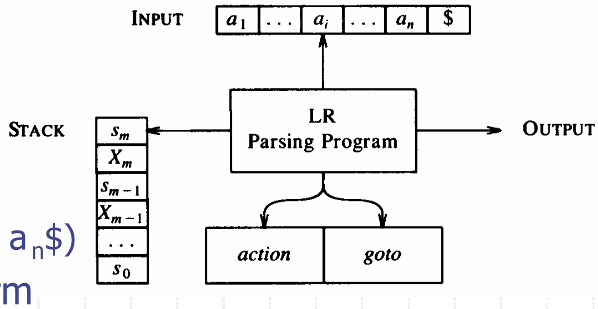
| Stack | Input |
|---|---|
| … if *expr* then *stmt* | else … \$ |



## LR Parsers

- LR(k) parsing
  - an efficient, bottom-up parsing technique
  - L: scanning the input from left to right
  - R: producing a rightmost derivation in reverse
  - k: the number of input symbols of lookahead
- Advantages
  - Can recognize virtually all programming-language constructs for which context-free grammars can be written
  - The most general nonbacktracking shift-reduce parsing
  - The class of grammars that can be parsed using LR methods is a proper superset of the class of grammars that can be parsed with predictive parsers
  - Can detect a syntactic error as soon as possible
- Drawback
  - Too much work to construct an LR parser by hand

# LR Parsing Algorithm




- Configuration
  - The stack contents
  - The unexpended input
    $(s_0\ X_1\ s_1\ X_2\ s_2\ \ldots\ X_m\ s_m,\ a_i\ a_{i+1}\ \ldots\ a_n\$)$
  - It represents the sentential form
    $X_1X_2\ldots\ X_m a_i\ a_{i+1}\ \ldots\ a_n$
- Next move is determined by the input symbol $a_i$ and stack top $s_m$
  - action[$s_m$, $a_i$] = shift $s$
    $(s_0\ X_1\ s_1\ X_2\ s_2\ \ldots\ X_m\ s_m\ a_i\ s,\ a_{i+1}\ \ldots\ a_n\$)$
  - action[$s_m$, $a_i$] = reduce $A \rightarrow \beta$
    $(s_0\ X_1\ s_1\ X_2\ s_2\ \ldots\ X_{m-r}\ s_{m-r}\ A\ s,\ a_i\ \ldots\ a_n\$)$
    where $s$ = goto[$s_{m-r}$, $A$] and $|\beta| = r$
  - action[$s_m$, $a_i$] = accept
  - action[$s_m$, $a_i$] = error



# LR Parsing Algorithm

- Input. String $w$ and an LR parsing table
- Output. A bottom-up parsing for $w$
- Method. Initially, stack = $s_0$ and input = $w\$$

```
set ip to point to the first symbol of w$
repeat forever
  let s be the state on stack top and a the symbol pointed by ip
  if action[s, a] = shift s′ then
    push a and s′ onto the stack and advance the ip
  else if action[s, a] = reduce A → β then
    push 2*|β| symbols off the stack and now s′ is the top state
    push A and then goto[s′, A] on top of the stack
    output A → β
  else if action[s, a] = accept then
    return
  else error()
end
```

# LR Parsing Algorithm

- Example.
  - (1) $E \rightarrow E + T$
  - (2) $E \rightarrow T$
  - (3) $T \rightarrow T * F$
  - (4) $T \rightarrow F$
  - (5) $F \rightarrow (E)$
  - (6) $F \rightarrow \text{id}$
- Parsing table
  - Fig. 4.31

| STATE | action | | | | | | goto | | |
|---|---|---|---|---|---|---|---|---|---|
| | id | + | * | ( | ) | $ | E | T | F |
| 0 | s5 | | | s4 | | | 1 | 2 | 3 |
| 1 | | s6 | | | | acc | | | |
| 2 | | r2 | s7 | | r2 | r2 | | | |
| 3 | | r4 | r4 | | r4 | r4 | | | |
| 4 | s5 | | | s4 | | | 8 | 2 | 3 |
| 5 | | r6 | r6 | | r6 | r6 | | | |
| 6 | s5 | | | s4 | | | | 9 | 3 |
| 7 | s5 | | | s4 | | | | | 10 |
| 8 | | s6 | | | s11 | | | | |
| 9 | | r1 | s7 | | r1 | r1 | | | |
| 10 | | r3 | r3 | | r3 | r3 | | | |
| 11 | | r5 | r5 | | r5 | r5 | | | |

| Stack | Input | Action |
|---|---|---|
| $0 | id * id + id$ | shift 5 |
| $0 id 5 | * id + id$ | reduce by $F \rightarrow \text{id}$ |
| $0 $F$3 | * id + id$ | reduce by $T \rightarrow F$ |
| $0 $T$2 | * id + id$ | shift |
| $0 $T$2 * 7 | id + id$ | shift |
| $0 $T$2 * 7 id 5 | + id$ | reduce by $F \rightarrow \text{id}$ |
| $0 $T$2 * 7 $F$ 10 | + id$ | reduce by $T \rightarrow T * F$ |
| $0 $T$2 | + id$ | reduce by $E \rightarrow T$ |
| $0 $E$1 | + id$ | shift |
| $0 $E$1 + 6 | id$ | shift |
| $0 $E$1 + 6 id 5 | $ | reduce by $F \rightarrow \text{id}$ |
| $0 $E$1 + 6 $F$3 | $ | reduce by $T \rightarrow F$ |
| $0 $E$1 + 6 $T$9 | $ | reduce by $E \rightarrow E + T$ |
| $0 $E$1 | $ | accept |



# Constructing SLR Parsing Tables

- Three techniques for constructing an LR parsing table
  - Simple LR (SLR)
  - Canonical LR (LR)
  - Lookahead LR (LALR)
- LR(0) item
  - A production of G with a dot at some position of the right side,
  - e.g. production $A \rightarrow XYZ$ yields
    - $A \rightarrow \cdot XYZ$
    - $A \rightarrow X \cdot YZ$
    - $A \rightarrow XY \cdot Z$
    - $A \rightarrow XYZ \cdot$

    Production $A \rightarrow \epsilon$ yields
    - $A \rightarrow \cdot$
- Augmented grammar $G'$
  - $G$ with a new start symbol $S'$ and production $S' \rightarrow S$
  - Acceptance occurs when the parser is to reduce by $S' \rightarrow S$

# Closure Operation

- If $I$ is a set of items, then *closure*($I$) is the set of items constructed from $I$:
  - Initially, every item in I is added to *closure*($I$)
  - If $A \to \alpha \cdot B \beta$ is in *closure*($I$) and $B \to \gamma$ is a production, then add the item $B \to \cdot \gamma$ to *closure*($I$)
  - Repeat until no more new items can be added

function *closure*($I$)
  $J = I$
  repeat
    for each item $A \to \alpha \cdot B \beta \in J$
    and each $B \to \gamma$
    such that $B \to \cdot \gamma \notin J$ do
      add $B \to \cdot \gamma$ to $J$
  until no more items can be added to $J$
  return $J$
end

- Example

| | |
|---|---|
| | *closure*($\{E' \to E\}$) = { |
| $E' \to E$ | $E' \to \cdot E$ |
| $E \to E + T$ | $E \to \cdot E + T$ |
| $E \to T$ | $E \to \cdot T$ |
| $T \to T * F$ | $T \to \cdot T * F$ |
| $T \to F$ | $T \to \cdot F$ |
| $F \to (E)$ | $F \to \cdot (E)$ |
| $F \to \mathrm{id}$ | $F \to \cdot \mathrm{id}$ } |



# Goto Operation

- If $I$ is a set of items and $X$ is a grammar symbol, then *goto*($I$, $X$) is defined to be the closure of the set of all items $A \to \alpha X \cdot \beta$ such that $A \to \alpha \cdot X \beta$ is in $I$

Example

| | |
|---|---|
| $E' \to E$ | *goto* ($\{E' \to E \cdot, E \to E \cdot + T\}$, +) = |
| $E \to E + T$ | *closure* ($\{E \to E + \cdot T\}$) = { |
| $E \to T$ | $E \to E + \cdot T$ |
| $T \to T * F$ | $T \to \cdot T * F$ |
| $T \to F$ | $T \to \cdot F$ |
| $F \to (E)$ | $F \to \cdot (E)$ |
| $F \to \mathrm{id}$ | $F \to \cdot \mathrm{id}$ } |

- Viable prefixes
  - The set of prefixes of right sentential forms that can appear on the stack of a shift-reduce parser

# Set-of-Items Construction

procedure *items*($G'$)
  $C = \{closure\{S' \to \cdot S\}\}$
  repeat
    for each set of items $I$ in $C$ and each grammar symbol $X$
    such that $goto(I, X)$ is not empty and not in $C$ do
      add $goto(I, X)$ to $C$
  until no more set of items can be added to $C$
end

Example $G$ =

$E' \to E$
$E \to E + T$
$E \to T$
$T \to T * F$
$T \to F$
$F \to (E)$
$F \to \text{id}$

$I_0$: $E' \to \cdot E$
$E \to \cdot E + T$
$E \to \cdot T$
$T \to \cdot T * F$
$T \to \cdot F$
$F \to \cdot(E)$
$F \to \cdot \text{id}$



# Set-of-Items Construction

$I_0$: $E' \to \cdot E$
$E \to \cdot E + T$
$E \to \cdot T$
$T \to \cdot T * F$
$T \to \cdot F$
$F \to \cdot(E)$
$F \to \cdot \text{id}$

$goto(I_0, E)$ =
$I_1$: $E' \to E \cdot$
$E \to E \cdot + T$

$goto(I_0, T)$ =
$I_2$: $E \to T \cdot$
$T \to T \cdot * F$

$goto(I_0, F)$ =
$I_3$: $T \to F \cdot$

$goto(I_0, \text{"("})$ =
$I_4$: $F \to (\cdot E)$
$E \to \cdot E + T$
$E \to \cdot T$
$T \to \cdot T * F$
$T \to \cdot F$
$F \to \cdot(E)$
$F \to \cdot \text{id}$

$goto(I_0, \text{id})$ =
$I_5$: $F \to \text{id} \cdot$

$goto(I_1, \text{"+"})$ =
$I_6$: $E \to E + \cdot T$
$T \to \cdot T * F$
$T \to \cdot F$
$F \to \cdot(E)$
$F \to \cdot \text{id}$

$goto(I_2, \text{"*"})$ =
$I_7$: $T \to T * \cdot F$
$F \to \cdot(E)$
$F \to \cdot \text{id}$

$goto(I_4, E)$ =
$I_8$: $F \to (E \cdot)$
$E \to E \cdot + T$

$goto(I_4, T) = I_2$

$goto(I_4, F) = I_3$

$goto(I_4, \text{"("}) = I_4$

$goto(I_4, \text{id}) = I_5$

$goto(I_6, T)$ =
$I_9$: $E \to E + T \cdot$
$T \to T \cdot * F$

$goto(I_6, F) = I_3$

$goto(I_6, \text{"("}) = I_4$

$goto(I_6, \text{id}) = I_5$

$goto(I_7, F)$ =
$I_{10}$: $T \to T * F \cdot$

$goto(I_7, \text{"("}) = I_4$

$goto(I_7, \text{id}) = I_5$

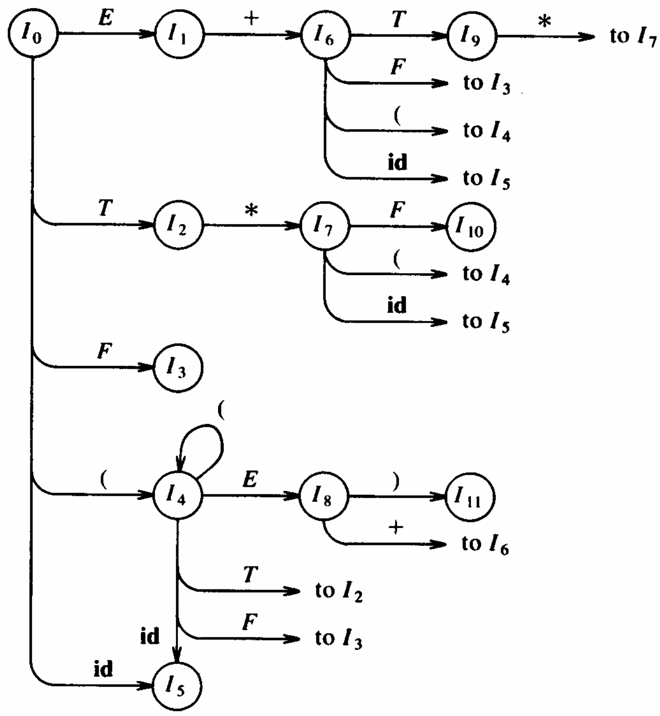
$goto(I_8, \text{")"})$ =
$I_{11}$: $F \to (E) \cdot$

$goto(I_8, \text{"+"}) = I_6$

$goto(I_9, \text{"*"}) = I_7$

# Transition Diagram of DFA for Viable Prefixes

- The *goto* function for the sets of items can be represented as the transition diagram of a DFA *D*
  - Each set of items $I_i$ is a state of D




Rightmost Derivation

$E \Rightarrow$

$E + T \Rightarrow$

$E + F \Rightarrow$

$E$ + id $\Rightarrow$

$T$ + id $\Rightarrow$

$T * F$ + id $\Rightarrow$

$T *$ *id* + id $\Rightarrow$

$F *$ *id* + id $\Rightarrow$

id + id * id



# Transition Diagram of DFA for Viable Prefixes

- *D* recognizes exactly the viable prefixes of *G* if
  - Each state is a final state, and
  - $I_0$ is the initial state
- For every grammar *G*, the *goto* function of the collection of sets of items defines a DFA that recognizes the viable prefix of *G*
  - If each item is treated as a state, an NFA *N* can be formed:
    - There is a transition from $A \rightarrow \alpha \cdot X\beta$ to $A \rightarrow \alpha X \cdot \beta$ label $X$, and
    - There is a transition from $A \rightarrow \alpha \cdot B\beta$ to $B \rightarrow \cdot\gamma$ labeled $\epsilon$
  - Then *closure*($I$) for the set of items $I$ is exactly the $\epsilon$-*closure* of a set of NFA states
  - Thus goto($I$, $X$) gives the transition from $I$ on symbol $X$ in the DFA constructed from *N* by the subset construction
  - The procedure *items*($G'$) is just the subset construction itself applied to the NFA *N*

# Constructing an SLR Parsing Table

- Algorithm
  - Input. An augmented grammar $G'$
  - Output. The SLR parsing table for $G'$
  - Method.
    1. Construct $C = \{I_0, I_1, I_2, \ldots, I_n\}$, the sets of LR(0) items
    2. State $i$ is constructed from $I_i$. Actions for state $i$
       - If $A \to \alpha \cdot a\beta \in I_i$ and goto($I_i$, a) = $I_j$, then
         action[$i$, a] = shift $j$
       - If $A \to \alpha \cdot \in I_i$ , then
         action[$i$, a] = reduce $A \to \alpha$ $\forall a \in$ FOLLOW($A$)
       - If $S' \to S \cdot \in I_i$ , then
         action[$i$, \$] = accept
    3. If goto($I_i$ , $A$) = $I_j$, then
       goto[$i$, $A$] = j
    4. The set of items containing $S' \to \cdot S$ is the initial state



# Constructing an SLR Parsing Table

- Example

(0) $E' \to E$
(1) $E \to E + T$
(2) $E \to T$
(3) $T \to T * F$
(4) $T \to F$
(5) $F \to (E)$
(6) $F \to$ id

- FOLLOW($E$) ={+, ), \$}
- FOLLOW($T$) = {+, *, ), \$}
- FOLLOW($F$) = {+, *, ), \$}

| | *action* | | | | | | *goto* | | |
|---|---|---|---|---|---|---|---|---|---|
| | id | + | * | ( | ) | \$ | *E* | *T* | *F* |
| 0 | s5 | | | s4 | | | 1 | 2 | 3 |
| 1 | | s6 | | | | acc | | | |
| 2 | | r2 | s7 | | r2 | r2 | | | |
| 3 | | r4 | r4 | | r4 | r4 | | | |
| 4 | s5 | | | s4 | | | 8 | 2 | 3 |
| 5 | | r6 | r6 | | r6 | r6 | | | |
| 6 | s5 | | | s4 | | | | 9 | 3 |
| 7 | s5 | | | s4 | | | | | 10 |
| 8 | | s6 | | | s11 | | | | |
| 9 | | r1 | s7 | | r1 | r1 | | | |
| 10 | | r3 | r3 | | r3 | r3 | | | |
| 11 | | r5 | r5 | | r5 | r5 | | | |

# Constructing an SLR Parsing Table

- Some grammars are not SLR, e.g.
  - (0) $S' \to S$
  - (1) $S \to L = R$
  - (2) $S \to R$
  - (3) $L \to *R$
  - (4) $L \to \text{id}$
  - (5) $R \to L$

$I_0$: $S' \to \cdot S$
$S \to \cdot L = R$
$S \to \cdot R$
$L \to \cdot *R$
$L \to \cdot \text{id}$
$R \to \cdot L$

*goto*($I_0$, $S$) =
$I_1$: $S' \to S \cdot$

*goto*($I_0$, $L$) =
$I_2$: $S \to L \cdot = R$
$R \to L \cdot$

*goto*($I_0$, $R$) =
$I_3$: $S \to R \cdot$

*goto*($I_0$, "*") =
$I_4$: $L \to * \cdot R$
$R \to \cdot L$
$L \to \cdot * R$
$L \to \cdot \text{id}$

*goto*($I_0$, id) =
$I_5$: $L \to \text{id} \cdot$

*goto*($I_2$, "=") =
$I_6$: $S \to L = \cdot R$
$R \to \cdot L$
$L \to \cdot * R$
$L \to \cdot \text{id}$

*goto*($I_4$, $R$) =
$I_7$: $L \to *R \cdot$

*goto*($I_4$, $L$) =
$I_8$: $R \to L \cdot$

*goto*($I_6$, $R$) =
$I_9$: $S \to L = R \cdot$



# Constructing an SLR Parsing Table

- Some grammars are not SLR, e.g.
  - (0) $S' \to S$
  - (1) $S \to L = R$
  - (2) $S \to R$
  - (3) $L \to * R$
  - (4) $L \to \text{id}$
  - (5) $R \to L$
- FOLLOW($S$) ={\$}
- FOLLOW($L$) = {=, \$}
- FOLLOW($R$) = {=, \$}

$I_2$: $S \to L \cdot = R$
$R \to L \cdot$

| | *action* | | | | *goto* | | |
|---|---|---|---|---|---|---|---|
| | id | = | * | $ | *S* | *L* | *R* |
| 0 | | | | | | | |
| 1 | | | | | | | |
| 2 | | s6<br>r5 | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | | | | | | | |
| 8 | | | | | | | |

# Constructing Canonical LR Parsing Tables

- Why does SLR sometimes fail
  - $R \rightarrow L \cdot \in I_2 \Rightarrow$ action[2, '='] = reduce $R \rightarrow L$
    - $\because$ = $\in$ FOLLOW($R$)
  - However, there is no sentential form begins $R$ = …
  - Extra information will be incorporated by redefining items
- LR(1) items
  - The general form of an items becomes $[A \rightarrow \alpha \cdot \beta, a]$
    - The lookahead a has no effect when $\beta \neq \epsilon$
    - $[A \rightarrow \alpha \cdot, a]$ calls for a reduction by $A \rightarrow \alpha$ if the next symbol is a
  - Formally, LR(1) is valid for a viable prefix $\gamma$ if there is a derivation $S \overset{*}{\Rightarrow} \delta A w \Rightarrow \delta\alpha\beta w$, where
    - $\gamma = \delta\alpha$, and
    - Either a is the first symbol of $w$, or $w$ is $\epsilon$ and a is \$



# Sets of LR(1) Items

```
function closure (I)
   J = I
   repeat
      for each item [A → α·Bβ, a] ∈ J
      each B → γ in G' and
      each terminal b ∈ FIRST(βa)
      such that [B → ·γ, b] ∉ J do
          add [B → ·γ, b] to J
   until no more items can be added to J
   return J
end
procedure items (G')
   C = {closure ({S' → ·S})}
   repeat
      for each set of item I ∈ C and each grammar symbol X
      such that goto (I, X) ∉ C do
          add goto (I, X) to C
   until no more sets of items can be added to C
end
```

```
function goto (I, X)
   J = set of items [A → αX·β, a]
       such that [A → α·Xβ, a] ∈ i
   return closure (J)
end
```

# Sets of LR(1) Items

Example $G'$ =

$S' \rightarrow S$
$S \rightarrow CC$
$C \rightarrow cC$
$C \rightarrow d$

$I_0$: $S' \rightarrow \cdot S$, \$
$S \rightarrow \cdot\, CC$, \$
$C \rightarrow \cdot\, cC$, c/d
$C \rightarrow \cdot$ d, c/d

*goto* ($I_0$, $S$) =
$I_1$: $S' \rightarrow S\,\cdot$, \$

*goto* ($I_0$, $C$) =
$I_2$: $S \rightarrow C \cdot C$, \$
$C \rightarrow \cdot\, cC$, \$
$C \rightarrow \cdot$ d, \$

*goto* ($I_0$, c) =
$I_3$: $C \rightarrow c \cdot C$, c/d
$C \rightarrow \cdot\, cC$, c/d
$C \rightarrow \cdot$ d, c/d

*goto* ($I_0$, d) =
$I_4$: $C \rightarrow$ d $\cdot$, c/d

*goto* ($I_2$, $C$) =
$I_5$: $S \rightarrow CC\,\cdot$, \$

*goto* ($I_2$, c) =
$I_6$: $C \rightarrow c \cdot C$, \$
$C \rightarrow \cdot\, cC$, \$
$C \rightarrow \cdot$ d, \$

*goto* ($I_2$, d) =
$I_7$: $C \rightarrow$ d $\cdot$, \$

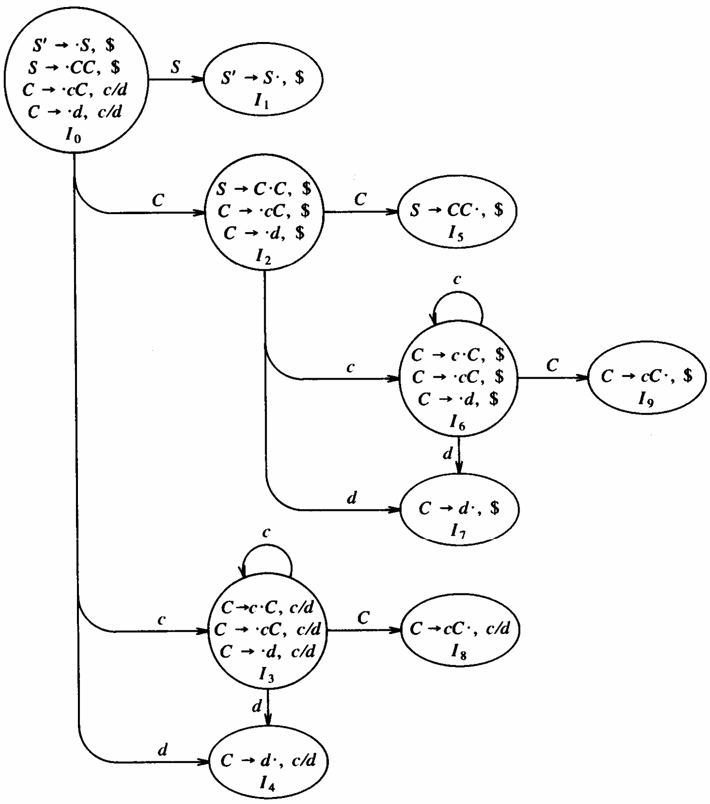
*goto* ($I_3$, $C$) =
$I_8$: $C \rightarrow cC\,\cdot$ , c/d
*goto* ($I_3$, c) = $I_3$
*goto* ($I_3$, d) = $I_4$

*goto* ($I_6$, $C$) =
$I_9$: $C \rightarrow cC\,\cdot$ , \$
*goto* ($I_6$, c) = $I_6$
*goto* ($I_6$, d) = $I_7$



# Sets of LR(1) Items

The *goto* graph



# Construction of the LR Parsing Table

## Algorithm

- Input. An augmented grammar $G'$
- Output. The canonical LR parsing table for $G'$
- Method.
  1. Construct $C = \{I_0, I_1, I_2, \ldots, I_n\}$, the sets of LR(1) items
  2. State $i$ is constructed from $I_i$. Actions for state $i$
     - If $[A \rightarrow \alpha \cdot a\beta, b] \in I_i$ and goto($I_i$, a) = $I_j$, then action[$i$, a] = shift $j$
     - If $[A \rightarrow \alpha \cdot, a] \in I_i$ and , $A \neq S'$ then action[$i$, a] = reduce $A \rightarrow \alpha$
     - If $[S' \rightarrow S \cdot, \$] \in I_i$ , then action[$i$, \$] = accept
  3. If goto($I_i$ , $A$) = $I_j$, then goto[$i$, $A$] = j
  4. The set of items containing $[S' \rightarrow \cdot S, \$]$ is the initial state



# Constructing an SLR Parsing Table

Example $G'$ =

$S' \rightarrow S$

$S \rightarrow CC$

$C \rightarrow cC$

$C \rightarrow d$

| | *action* | | | *goto* | |
|---|---|---|---|---|---|
| | c | d | \$ | $S$ | $C$ |
| 0 | s3 | s4 | | 1 | 2 |
| 1 | | | acc | | |
| 2 | s6 | s7 | | | 5 |
| 3 | s3 | s4 | | | 8 |
| 4 | r3 | r3 | | | |
| 5 | | | r1 | | |
| 6 | s6 | s7 | | | 9 |
| 7 | | | r3 | | |
| 8 | r2 | r2 | | | |
| 9 | | | r2 | | |

# Constructing LALR Parsing Tables

- Lookahead LR (LALR) technique
  - Often used in practice because the tables are much smaller than the LR tables
  - Yet most common syntactic constructs of programming languages can be expressed by an LALR grammar
  - Consider the following sets of LR(1) items

$I_3$: $C \to c \cdot C$, c/d
$\quad C \to \cdot\, cC$, c/d
$\quad C \to \cdot\, d$, c/d

$I_6$: $C \to c \cdot C$, \$
$\quad C \to \cdot\, cC$, \$
$\quad C \to \cdot\, d$, \$

$I_4$: $C \to d\, \cdot$, c/d

$I_7$: $C \to d\, \cdot$, \$

    - The only difference is the lookahead symbols
    - Note the $G'$ generates the language c*dc*d
  - An LALR parsing table can be obtained by merging the sets of items of a LR table with the same *core*



# Construction of the LALR Parsing Table

- Algorithm
  - Input. An augmented grammar $G'$
  - Output. The LALR parsing table for $G'$
  - Method.
    1. Construct $C = \{I_0, I_1, I_2, \ldots, I_n\}$, the sets of LR(1) items
    2. For each core in $C$, find all sets with that core and replace these sets by their union. Let result be $C' = \{J_0, J_1, J_2, \ldots, J_m\}$
    3. State $i$ is constructed from $J_i$. Actions for state $i$
       - If $[A \to \alpha \cdot a\beta, b] \in J$ and goto($J_i$, a) = $J_j$, then action[$i$, a] = shift $j$
       - If $[A \to \alpha \cdot, a] \in J_i$ and , $A \neq S'$ then action[$i$, a] = reduce $A \to \alpha$
       - If $[S' \to S \cdot, \$] \in J_i$ , then action[$i$, \$] = accept
    4. If $J_i = I_1 \cup I_2 \cup \ldots \cup I_k$, then
       $goto(I_1, X) = goto(I_2, X) = \ldots = goto(I_k, X) = J_j$
       $\therefore$ goto[$i$, $X$] = j

# Construction of the LALR Parsing Table

$I_3$: $C \to c \cdot C$, c/d
$C \to \cdot cC$, c/d
$C \to \cdot$ d, c/d

$I_6$: $C \to c \cdot C$, \$
$C \to \cdot cC$, \$
$C \to \cdot$ d, \$

$I_4$: $C \to$ d $\cdot$, c/d

$I_7$: $C \to$ d $\cdot$, \$

$I_8$: $C \to cC \cdot$ , c/d

$I_9$: $C \to cC \cdot$ , \$

| | *action* | | | *goto* | |
|---|---|---|---|---|---|
| | c | d | \$ | *S* | *C* |
| 0 | s36 | s47 | | 1 | 2 |
| 1 | | | acc | | |
| 2 | s36 | s47 | | | 5 |
| 36 | s36 | s47 | | | 89 |
| 47 | r3 | r3 | r3 | | |
| 5 | | | r1 | | |
| | | | | | |
| | | | | | |
| 89 | r2 | r2 | r2 | | |
| | | | | | |



# Error Detection by LALR

- LALR parser may proceed to do some reductions after the LR parser has declared an error
  - LALR parser will never shift another symbol after the LR parser declares an error
  - Example $w$ = ccd\$

| Stack | Input | Action |
|---|---|---|
| \$0 | ccd\$ | shift 36 |
| \$0 c 36 | cd\$ | shift 36 |
| \$0 c 36 c 36 | d\$ | shift 36 |
| \$0 c 36 c 36 d 47 | \$ | reduce by $C \to$ d |
| \$0 c 36 c 36 $C$ 89 | \$ | reduce by $C \to cC$ |
| \$0 c 36 C 89 | \$ | reduce by $C \to cC$ |
| \$0 C 2 | \$ | error |

| Stack | Input | Action |
|---|---|---|
| \$0 | ccd\$ | shift 3 |
| \$0 c 3 | cd\$ | shift 3 |
| \$0 c 3 c 3 | d\$ | shift 3 |
| \$0 c 3 c 3 d 4 | \$ | error |

# Conflicts Caused by Merging States

- Merging states with common cores might cause conflicts
  - No shift-reduce conflicts will be introduced
    - Because shift actions depend only on the core, not the lookahead
  - Reduce-reduce conflicts might be produced by merging

Example $G'$ =

$S' \to S$

$S \to \mathrm{a}A\mathrm{d} \mid \mathrm{b}B\mathrm{d} \mid \mathrm{a}B\mathrm{e} \mid \mathrm{b}A\mathrm{e}$

$A \to \mathrm{c}$

$B \to \mathrm{c}$

$I_0$: $S' \to \cdot S$, \$

$S \to \cdot\, \mathrm{a}A\mathrm{d} \mid \cdot\, \mathrm{b}B\mathrm{d} \mid \cdot\, \mathrm{a}B\mathrm{e} \mid \cdot\, \mathrm{b}A\mathrm{e}$, \$

$goto(I_0, S) = I_1$: $S' \to S\, \cdot$, \$

$goto(I_0, \mathrm{a})$ =

$I_2$: $S \to \mathrm{a} \cdot A\mathrm{d} \mid \mathrm{a} \cdot B\mathrm{e}$, \$

$A \to \cdot\, \mathrm{c}$, d

$B \to \cdot\, \mathrm{c}$, e

$goto(I_0, \mathrm{b})$ =

$I_3$: $S \to \mathrm{b} \cdot B\mathrm{d} \mid \mathrm{b} \cdot A\mathrm{e}$, \$

$A \to \cdot\, \mathrm{c}$, e

$B \to \cdot\, \mathrm{c}$, d

$goto(I_2, \mathrm{c})$ =

$I_4$: $A \to \mathrm{c}\, \cdot$, d

$B \to \mathrm{c}\, \cdot$, e

$goto(I_3, \mathrm{c})$ =

$I_5$: $A \to \mathrm{c}\, \cdot$, e

$B \to \mathrm{c}\, \cdot$, d

merge $I_4$ and $I_5$ =

$I_{45}$: $A \to \mathrm{c}\, \cdot$, d/e

$B \to \mathrm{c}\, \cdot$, d/e



# Using Ambiguous Grammars

- Every ambiguous grammar fails to be LR
  - Certain types of ambiguous grammars are useful in the specification and implementation of languages
    - An ambiguous grammar provides a shorter, more natural specification than any equivalent unambiguous grammar
      e.g. $E \to E + E \mid E * E \mid (E) \mid \mathrm{id}$
    - Ambiguous grammars can be used to isolate commonly occurring syntactic constructs for special case optimization
  - Disambiguating rules must be specified to allow only one parse tree for each sentence
    - e.g. using precedence and associativity to resolve conflicts
    - In this way, the overall language specification still remains unambiguous

# Using Precedence and Associativity

Example $G'$

(0) $E' \to E$

(1) $E \to E + E$

(2) $E \to E * E$

(3) $E \to (E)$

(4) $E \to \text{id}$

$I_0$: $E' \to \cdot E$
$E \to \cdot E + E$
$E \to \cdot E * E$
$E \to \cdot (E)$
$E \to \cdot \text{id}$

$goto(I_0, E) =$
$I_1$: $E' \to E \cdot$
$E \to E \cdot + E$
$E \to E \cdot * E$

$goto(I_0, \text{'('}) =$
$I_2$: $E \to (\cdot E)$
$E \to \cdot E + E$
$E \to \cdot E * E$
$E \to \cdot (E)$
$E \to \cdot \text{id}$

$goto(I_0, \text{id}) =$
$I_3$: $E \to \cdot \text{id}$

$goto(I_1, \text{'+'}) =$
$I_4$: $E \to E + \cdot E$
$E \to \cdot E + E$
$E \to \cdot E * E$
$E \to \cdot (E)$
$E \to \cdot \text{id}$

$goto(I_1, \text{'*'}) =$
$I_5$: $E \to E * \cdot E$
$E \to \cdot E + E$
$E \to \cdot E * E$
$E \to \cdot (E)$
$E \to \cdot \text{id}$

$goto(I_0, E) =$
$I_6$: $E \to (E \cdot)$
$E \to E \cdot + E$
$E \to E \cdot * E$

$goto(I_4, E) =$
$I_7$: $E \to E + E \cdot$
$E \to E \cdot + E$
$E \to E \cdot * E$

$goto(I_5, E) =$
$I_8$: $E \to E * E \cdot$
$E \to E \cdot + E$
$E \to E \cdot * E$

$goto(I_6, \text{')'}) =$
$I_9$: $E \to (E) \cdot$

| | action | | | goto |
|---|---|---|---|---|
| | + | * | … | |
| … | | | | |
| 7 | r2 | s5 | | |
| 8 | r3 | r3 | | |
| 9 | | | | |



# The "Dangling-else" Ambiguity

Example $G'$

(0) $S' \to S$

(1) $S \to \text{i}S\text{e}S$

(2) $S \to \text{i}S$

(3) $S \to \text{a}$

$I_0$: $S' \to \cdot S$
$S \to \cdot \text{i}S\text{e}S$
$S \to \cdot \text{i}S$
$S \to \cdot \text{a}$

$goto(I_0, S) =$
$I_1$: $S' \to S \cdot$

$goto(I_0, \text{i}) =$
$I_2$: $S \to \text{i} \cdot S\text{e}S$
$S \to \text{i} \cdot S$
$S \to \cdot \text{i}S\text{e}S$
$S \to \cdot \text{i}S$
$S \to \cdot \text{a}$

$goto(I_0, \text{a}) =$
$I_3$: $S \to \text{a} \cdot$

$goto(I_2, S) =$
$I_4$: $S \to \text{i}S \cdot \text{e}S$
$S \to \text{i}S \cdot$

$goto(I_4, \text{e}) =$
$I_5$: $S \to \text{i}S\text{e} \cdot S$
$S \to \cdot \text{i}S\text{e}S$
$S \to \cdot \text{i}S$
$S \to \cdot \text{a}$

$goto(I_5, S) =$
$I_6$: $S \to \text{i}S\text{e}S \cdot$

| | action | | | goto |
|---|---|---|---|---|
| | i | e | … | |
| … | | | | |
| 4 | | s5 | | |
| 8 | | | | |
| 9 | | | | |

# Ambiguities from Special-Case Productions

Example $G'$

(0) $E' \to E$
(1) $E \to E \text{ sub } E \text{ sup } E$
(2) $E \to E \text{ sub } E$
(3) $E \to E \text{ sup } E$
(4) $E \to \{E\}$
(5) $E \to \text{c}$

$I_0$: $E' \to \cdot E$
$E \to \cdot E \text{ sub } E \text{ sup } E$
$E \to \cdot E \text{ sub } E$
$E \to \cdot E \text{ sup } E$
$E \to \cdot \{E\}$
$E \to \cdot \text{c}$

$I_7$: $E' \to E \cdot$
$E \to E \cdot \text{ sub } E \text{ sup } E$
$E \to E \text{ sub } E \cdot \text{ sup } E$
$E \to E \cdot \text{ sub } E$
$E \to E \text{ sub } E \cdot$
$E \to E \cdot \text{ sup } E$

$I_8$: $E' \to E \cdot$
$E \to E \cdot \text{ sub } E \text{ sup } E$
$E \to E \cdot \text{ sub } E$
$E \to E \cdot \text{ sub } E$
$E \to E \text{ sup } E \cdot$

$I_{11}$: $E' \to E \cdot$
$E \to E \cdot \text{ sub } E \text{ sup } E$
$E \to E \text{ sub } E \text{ sup } E \cdot$
$E \to E \cdot \text{ sub } E$
$E \to E \cdot \text{ sup } E$
$E \to E \text{ sup } E \cdot$

| | *action* | | | | *goto* |
|---|---|---|---|---|---|
| | sub | sup | } | $ | |
| ... | | | | | |
| 7 | s4<br>r2 | s10<br>r2 | r2 | r2 | |
| 8 | s4<br>r3 | s5<br>r3 | r3 | r3 | |
| 9 | | | | | |
| 11 | s4<br>r1<br>r3 | s5<br>r1<br>r3 | r1<br>r3 | r1<br>r3 | |



# Operator-Precedence Parsing

- Works on a class of grammars: *operator grammars*
  - No production right side is $\varepsilon$ or has two adjacent nonterminals
  - e.g. $E \to EAE \mid (E) \mid -E \mid \text{id}$
    $A \to + \mid - \mid * \mid /$
    is not an operator grammar
  - $E \to E + E \mid E - E \mid E * E \mid E / E \mid (E) \mid -E \mid \text{id}$
    is an operator grammar
- Advantages
  - Easy to implement
- Disadvantages
  - Hard to handle tokens like the minus sign (with 2 precedences)
  - Can't be sure the parser accepts exactly the desired language
  - Only a small class of grammars can be parsed

# Operator-Precedence Parsing

- Precedence relations

| Relation | Meaning |
|---|---|
| a <· b | a "yields precedence to" b |
| a ≐ b | a "has the same precedence as" b |
| a ·> b | a "takes precedence over" b |

- Example $E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid (E) \mid -E \mid$ id

| | id | + | * | $ |
|---|---|---|---|---|
| id | | ·> | ·> | ·> |
| + | <· | ·> | <· | ·> |
| * | <· | ·> | ·> | ·> |
| $ | <· | <· | <· | |



# Operator-Precedence Parsing

- The handle can be found by the steps
  - Scan the string from left until the first ·> is encountered
  - Then scan back backwards over any ≐ until a <· is encountered
  - The handle contains everything within the <· and ·> above, including any intervening or surrounding nonterminals
  - e.g. $w$ = id + id * id

$ <· id ·> + <· id ·> * <· id ·> $

$ <· *E* + <· id ·> * <· id ·> $

$ <· *E* + <· *E* * <· id ·> $

$ <· *E* + <· *E* * *E* ·> $

$ <· *E* + *E* ·> $

$*E*$