編譯器設計

Languages and Their Representations

Alphabets and Languages

- Webster defines a *language* as
  - "the body of words and methods of combining words used and understood by a considerable community"
- The definition is not precise
  - A formal language will be defined
- An *alphabet*:
  - any finite set of symbols, e.g.
    - Latin alphabet {A, B, C, …, Z}
    - Greek alphabet {α, β, γ, …, ω}
    - binary alphabet {0, 1}

Alphabets and Languages

- A *sentence* over an alphabet
  - any string of finite length composed of symbols from the alphabet
  - Synonyms for sentence are *string* and *word*
- The empty sentence ϵ
  - the sentence consisting of no symbols
- If V is an alphabet, then
  - V^* denotes the set of all sentences composed of symbols of V, including the empty sentence
  - $V^+ = V^* - \{\epsilon\}$
  - If $V = \{0,1\}$, then

    $V^* = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \ldots\}$

Alphabets and Languages

- A *language*
  - any set of sentences over an alphabet
  - e.g. {0, 1} is a language
- Three questions are raised
  - How do we represent a language?
    - It's simple if the language is finite
    - How to represent an infinite language with a finite representation
  - Does there exist a finite representation for every language?
  - What can be said about the structures of those languages for which there exist finite representation?

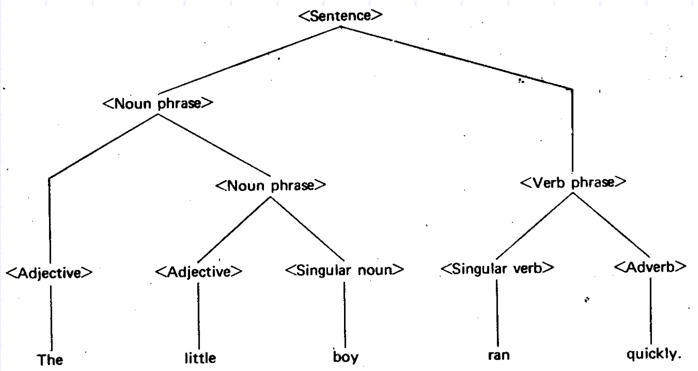
Representations of Languages

- Two ways to represent a language
  - To give an algorithm which determines if a sentence is in the language or not
    - To give a procedure which halts with the answer “yes” for sentences in the language and either does not terminate or else halts with the answer “no” for sentences not in the language
  - To give a grammar that generates sentences in the language

Grammars

- Example: “The little boy ran quickly”

<sentence> → <noun phrase><verb phrase>
<noun phrase> → <adjective><noun phrase>
<noun phrase> → <adjective><noun>
<verb phrase> → <verb><adverb>
<adjective> → The
<adjective> → little
<noun> → boy
<verb> → ran
<adverb> → quickly



Formal Notation of a Grammar

- Four concepts
  - Nonterminals (or Variables)
    - e.g. <sentence>, <adjective>, <verb phrase>, etc.
  - Terminals
    - e.g. words such as The, little, boy, etc.
  - Productions

    relationships between strings of variables and terminals
    - e.g. <sentence>→<noun phrase><verb phrase>
  - Start Symbol

    distinguished symbol that generates exactly those strings of terminals that are deemed in the language
    - e.g. <sentence>

Formal Notation of a Grammar

- A grammar G can be denoted by (V_N, V_T, P, S)
  - V_N: nonterminals
  - V_T: terminals ($V_N \cap V_T = \phi$, $V_N \cup V_T = V$)
  - P : productions
    - $\alpha \rightarrow \beta \in P$
  - S : start symbol

Derivation

- Derivation by a production
  - If $\alpha \rightarrow \beta \in P$ and $\gamma, \delta \in V$, then

    $\gamma\alpha\delta \Rightarrow \gamma\beta\delta$
  - i.e. $\gamma\alpha\delta$ directly derives $\gamma\beta\delta$
- Derivation by productions
  - If $\alpha_1, \alpha_2, \ldots, \alpha_m$ are strings in V^*, and

    $\alpha_1 \Rightarrow \alpha_2 \Rightarrow \ldots \Rightarrow \alpha_m$

    then we say

    $\alpha_1 \stackrel{*}{\Rightarrow} \alpha_m$

Derivation

- The language generated by G is defined

  $$L(G) = \{w \mid w \in V_T^* \wedge S \stackrel{*}{\Rightarrow} w\}$$
  - That is, a string is in $L(G)$ if
    - The string consists solely of terminals
    - The string can be derived from S
  - Grammars G_1 and G_2 are *equivalent* if
    - $L(G_1) = L(G_2)$
  - Example $G = (V_N, V_T, P, S)$

    $V_N = \{S\}$, $V_T = \{0, 1\}$, P = $\{S\rightarrow 0S1, S\rightarrow 01\}$

    $$S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 0^3S1^3 \Rightarrow \ldots \Rightarrow 0^{n-1}S1^{n-1} \Rightarrow 0^n1^n$$

    $\therefore$ $L(G) = \{0^n1^n\}$
  - A string of terminals and nonterminals α is called a *sentential form* if $S \stackrel{*}{\Rightarrow} \alpha$

Types of Grammars

- Let $G = (V_N, V_T, P, S)$ be a grammar
  - Type 0 grammar
  - Type 1 grammar (context-sensitive grammar)
    - For every production $\alpha \rightarrow \beta$ in P, $|\alpha| \leqq |\beta|$
    - e.g. P = {S→aSBC, S→aBC, CB→BC, aB→ab, bB→bb, bC→bc, cC→cc}
  - Type 2 grammar (context-free grammar)
    - For every production $\alpha \rightarrow \beta$ in P, $|\alpha| = 1$ and $\beta \neq \epsilon$
    - e.g. P = {S→0S1, S→01}
  - Type 3 grammar (regular grammar)
    - Every production in P is of the form

      A →aB, or

      A →a