OPER 427/527 Intro to Optimization Fall 2017 Project
Due at 2:00pm EST, Dec. 6th, 2017

Suppose Jon Snow, Daenerys Targaryen, Cersei Lannister have agreed the alliance against the white walkers. Tyrion Lannister has put together a map, and modeled the continent of Westeros as a network $G(V, E)$ (see the figure), with V being the set of locations of interest (we call them forts), and E being the set of edges in the network representing roads in Westeros. We assume that for each road we can go both ways. They have elected Sam Tarly, the only one who knows optimization in the entire Westeros, as the chief strategic planner for the wars to come. Sam Tarly has proposed the following two types of optimization models:

Model 1: Optimal coverage by allocating army in forts Suppose the white walkers have completely invaded Westeros, and they can basically show up anywhere at any time. Instead of stopping them, the alliance has decided to protect the forts, i.e., provide coverage to forts as much as possible. Suppose the Westeros alliance can allocate armies up to $N = 4$ different forts.

1. Model 1a: We say that a fort is "covered", if there is an army that is located "close enough" to this fort. We use a parameter $D = 500$ to denote the threshold. Determine the locations where the armies should be allocated to maximize the weighted coverage for all forts of Westeros (weighted by their priority level).

2. Model 1b: Instead of using a threshold for coverage, we simply want to allocate armies in the network to minimize the total weighted response time (weighted by their priority level) in case of any emergency. The response time for each fort is calculated by the distance between the fort and the closest army nearby.

Model 2: Optimal delay by allocating army on roads Suppose all the white walkers have gathered at Castle Black (sorry, the watchers on the wall!). Their goal is to destroy forts of Westeros as soon as possible. Suppose that armies can be allocated on the roads. If an army is allocated on a road, it can delay the white walker's traveling progress by a factor $\alpha = 0.5$: e.g., if an army is allocated between Summerhall and Horn Hill, the distance between these two forts become $500(1 + \alpha) = 750$.

1. Model 2a: Determine the locations of armies on roads to maximize the shortest distance traveled from Castle Black to King's Landing (If you do not like Cersei Lannister, you can replace King's Landing by Sunspear).

2. Model 2b: Determine the locations of armies on roads to maximize the summation of the shortest distance traveled from Castle Black to all other forts of Westeros.

TASKS:

1. Formulate an integer programming formulation for each model, and solve them using the given data set.

2. Discuss different solutions obtained by different models and briefly discuss the pros and cons for each model using some criteria of your choice.

3. Perform a sensitivity analysis on the key parameters and see how the optimal solutions of the models change as you change these parameters. The key parameters include: (1) D, the coverage threshold distance; (2) N, number of armies to allocate; (3) α, the factor of delay in white walkers' travel.

4. Submit Julia code together with your project report.