GRA6227 Business Optimisation - Individual Exam 2018

Mathematical models

1 a)

Parameters: $c_{i,j}$ Variable cost for unit produced at i and shipped to j

cap_i Capacity at factory i

d_j Demand at market i

Variables: $X_{i,j}$ Quantity produced at i and shipped to j

Min $\sum_i \sum_j c_{i,j} \cdot X_{i,j}$ (1)

s.t $\sum_j X_{i,j} \leq cap_i$ for all i (2)

$\sum_i X_{i,j} = d_j$ for all j (3)

$X_{i,j} \geq 0$ for all i and j (4)

Data: cap_i = 100, 140, 110 for i = A, B, C

d_j = 55, 60, 35, 50 for j = 1, 2, 3, 4

$c_{i,j}$ =

7, 4, 9, 6	for i = A and j = 1, 2, 3, 4
3, 8, 5, 4	for i = B and j = 1, 2, 3, 4
6, 5, 4, 11	for i = C and j = 1, 2, 3, 4

(1) is objective function. (2) are capacity constraints. (3) are demand constraints. (4) are non-negativity constraints.

1 b)

New parameters: f_i Fixed costs for factory i

New variables: Y_i The 0/1 decision to keep factory i

New objective function:

Min $\sum_i f_i \cdot Y_i + \sum_i \sum_j c_{i,j} \cdot X_{i,j}$ (1')

Modified constraints:

s.t $\sum_j X_{i,j} \leq cap_i \cdot Y_i$ for all i (2')

$Y_{i,j} = 0/1$ for all i (5)

Keep the following from 1 a): (3) and (4)

(1') is objective function where fixed costs are included.

In (2') the production at a factory is forced to be zero if the factory is not operating.

(5) define the binary variables.

Data: f_i = 80, 90, 100 for i = A, B, C

1 c)

New parameters: m_i Minimum quantity for factory i

New variables (same as in b)): Y_i The 0/1 decision to keep factory i

New constraints:

s.t $\sum_j X_{i,j} \le cap_i \cdot Y_i$ for all i (2') same as in b)

$\sum_j X_{i,j} \ge m_i \cdot Y_i$ for all i (6)

$Y_{i,j} = 0/1$ for all i (5) same as in b)

Keep the following from 1 a): (1), (3) and (4)

Data:

m_i = 70, 80, 75 for i = A, B, C

(6) will ensure that total quantity shipped from each factory is either zero or equal to or greater than the minimum level.

1 d)

New parameters: $n_{i,j}$ Minimum quantity between factory i and market j

New variables: $Z_{i,j}$ The 0/1 decision to ship from factory i to market j

s.t $X_{i,j} \leq cap_i \cdot Z_{i,j}$ for all i and j (7)

$X_{i,j} \geq n_{i,j} \cdot Z_{i,j}$ for all i and j (8)

$Z_{i,j} = 0/1$ for all i and j

New data:

$n_{i,j}$ =

20, 20, 20, 20 for i = A and j = 1, 2, 3, 4
20, 20, 20, 20 for i = B and j = 1, 2, 3, 4
20, 20, 20, 20 for i = C and j = 1, 2, 3, 4

(7) will ensure that the binary variable is 1 if the quantity shipped from factory to market is above zero.

(8) will ensure that total quantity shipped from factory i to market j is either zero or equal to or greater than the minimum level.