

DIVERSIFICATION AND CAPM

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Uncertainty

Given:

- States $S=\{1, 2, \dots, S\}$
- Probabilities $\pi_1, \pi_2, \dots, \pi_S, \sum_s \pi_s = 1$ (Objective?
Subjective?)
- Payoffs/Outcomes R_1, R_2, \dots, R_S
- Lottery/Gamble L: Hold outcomes fixed, lottery is $L=(\pi_1, \pi_2, \dots, \pi_S)$
- Utility: $u(R)$
- Expected Utility: $U(L) = \sum_s \pi_s u(R_s)$

Risk aversion

- Certainty equivalent = the answer to the question “how much wealth, received for certain, is equivalent (in the consumer's eyes) to a given lottery?”
 - Example: A lottery over $(0, \$100)$ with $(0.4, 0.6)$ is equivalent to you as $\$?$ for certain.
- A risk averse consumer is one for whom the certainty equivalent of any gamble is less than the expected value of that gamble
 - Economists' point of view: A risk-averse consumer is one for whom the expected utility of any lottery is lower than the utility of the expected value of that lottery. Then risk aversion=concavity of u .

Financial Securities

- Keep states and probabilities same
- Introduce multiple sets of Payoffs:
 - $(R_{11}, R_{12}, \dots, R_{1S})$, $(R_{21}, R_{22}, \dots, R_{2S})$..., etc.
- How can you think of prices in this setup?

FANCY Theory: Arrow-debreu Securities

- AD securities: R_{is} iff $i=s$. Consumer j holds x_i^j units of asset i .
- Increase in utility from holding one more unit of asset i :

$$\frac{\partial E[u^j(R'x^j)]}{\partial x_i^j} = \sum_s \pi_s R_i(s) u^{j'}(R'(s)x^j) = \pi_i u^{j'}(x_i^j)$$

- Therefore

$$\frac{\pi_i u^{j'}(x_i^j)}{\pi_k u^{j'}(x_k^j)} = \frac{p_i}{p_k} \quad , \text{ or}$$

$$\frac{u^{j'}(x_i^j)}{u^{j'}(x_k^j)} = \frac{\cancel{p_i}}{\cancel{p_k}} \frac{\cancel{\pi_i}}{\cancel{\pi_k}}$$

Theory: Equilibrium in Complete Markets

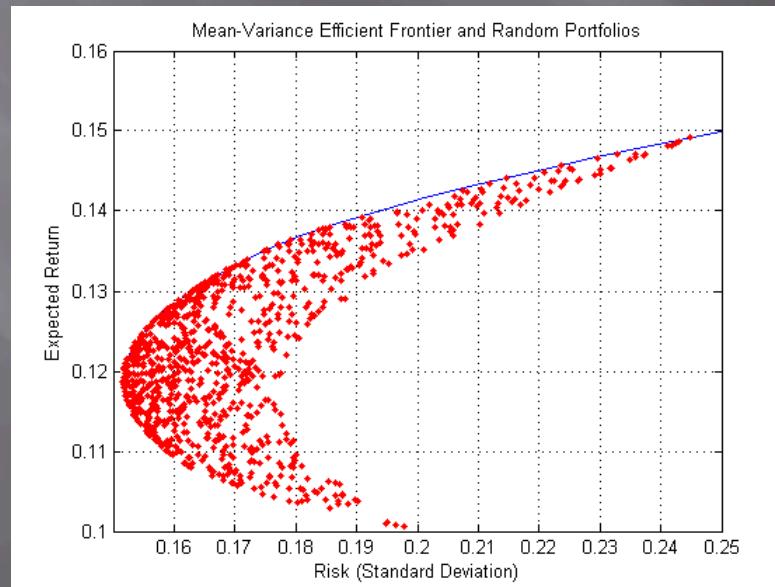
- Properties of equilibrium:
 1. State price probabilities are inversely related to aggregate wealth.
 2. Representative agent can be constructed.
 3. Final wealth/consumption is perfectly rank-correlated among individuals and with aggregate wealth.

CAPM

- With general utilities asset pricing would require us to know a lot about utilities.
- A nice simplification is possible if we specify at the outset that our consumer likes the mean but dislikes the variance of random returns, .e., the mean is a good, the variance is a bad.

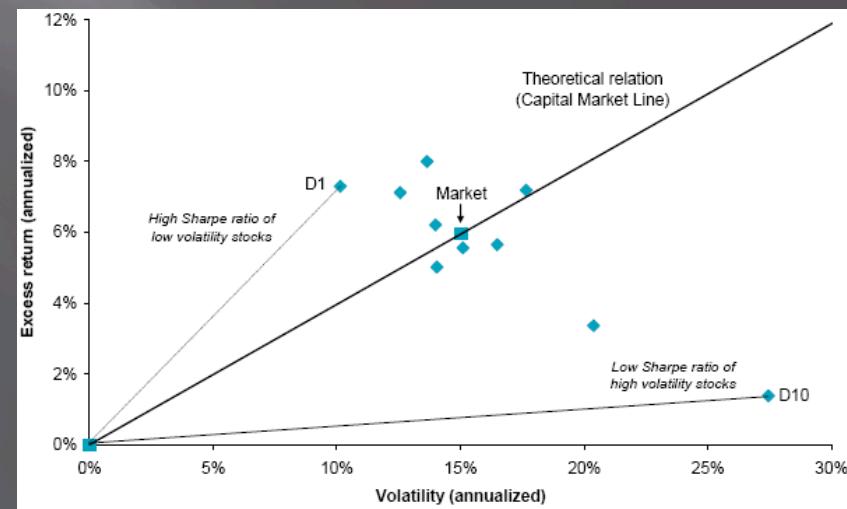
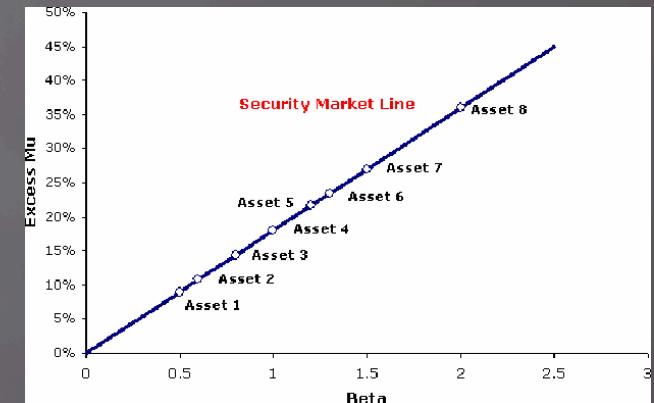
General CAPM

- ❑ Assume a set of risky assets.
- ❑ Agents trade off mean returns and variance of returns.
- ❑ What combination of assets will yield the highest mean for a given st. dev., or, which is often easier to compute, the lowest st. dev. for a given mean. It turns out that this will lead to:



CAPM

- Theory: *In equilibrium, the expected return on risky securities is solely determined by their covariance with aggregate risk ("beta")*
- Equivalent: the "market portfolio" will give you maximum expected reward for its risk (risk=return variance), or *the Sharpe ratio of the market portfolio is maximal*
- Sharpe ratio = Expected return on portfolio minus riskfree rate / return standard deviation



Experiments

I. The Experiments

- Three markets (one riskfree, shortsales allowed), several periods.
- Three states; determine liquidation value at end of each period; known probabilities.

Security	State		
	X	Y	Z
A	170	370	150
B	160	190	250
Notes	100	100	100

(Note: complete markets...)

Example Of This Experiment: 011126

Draw Type	Subject Type (#)	Signup Reward (franc)	Endowments			Cash (franc)	Loan (franc)	Exchange Rate \$/franc
			A	B	Notes			
D	18	125	5	4	0	400	2200	0.04
	18	125	2	8	0	400	2310	0.04

Navigation

Marketplace

A

B

Notes

[View Transaction Table](#)

Order Form

Order Type: Simple

Market: **A**

Price: **\$195.00**

Units: **1**

Total Value: **\$195.00**

buy **sell** **cancel**

Messaging

Messaging

Received Messages

mc: Markets will be called in 10 minutes
mc: Markets will be called in 9 minutes
mc: Markets will be called in 8 minutes
mc: Markets will be called in 7 minutes

Message History

Markets -- Actual

Marketplace closes in 01:56:25

Cash on hand: \$500.0

A (5)

Item: A

B (4)

Item: B

Notes (0)

Item: Notes

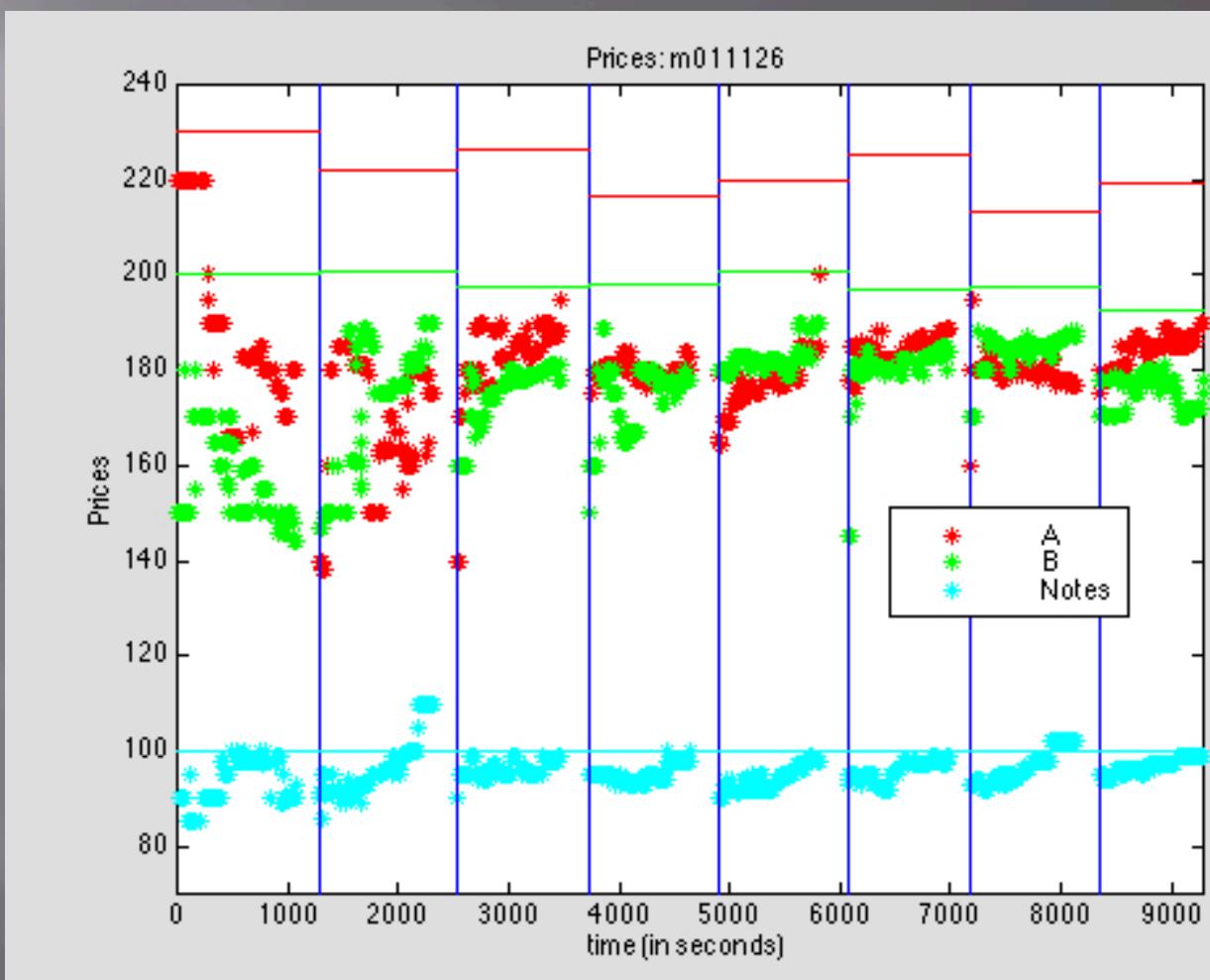


CAPM PREDICTIONS

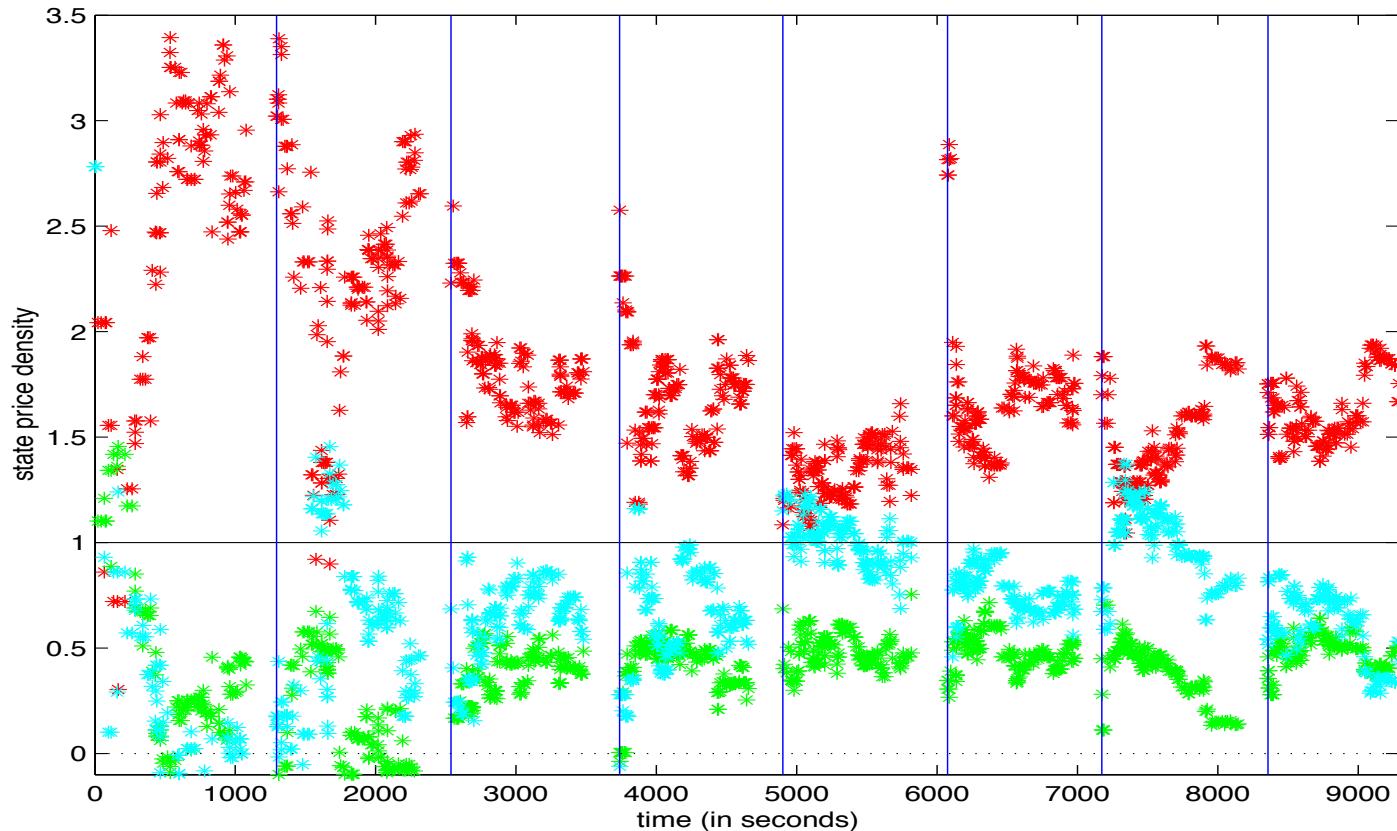
- The expected returns of each of securities A and B are positively related to the betas of the securities
- The market portfolio is mean-variance efficient.
- The final portfolio of each individual should have risky securities in the same proportion as in the market portfolio.

Prices

- Prices of the two securities are virtually the same.
- Expected payoff of A is higher, i.e., expected return of A is higher.
- Precisely because $\text{Beta}(A) > \text{Beta}(B)$



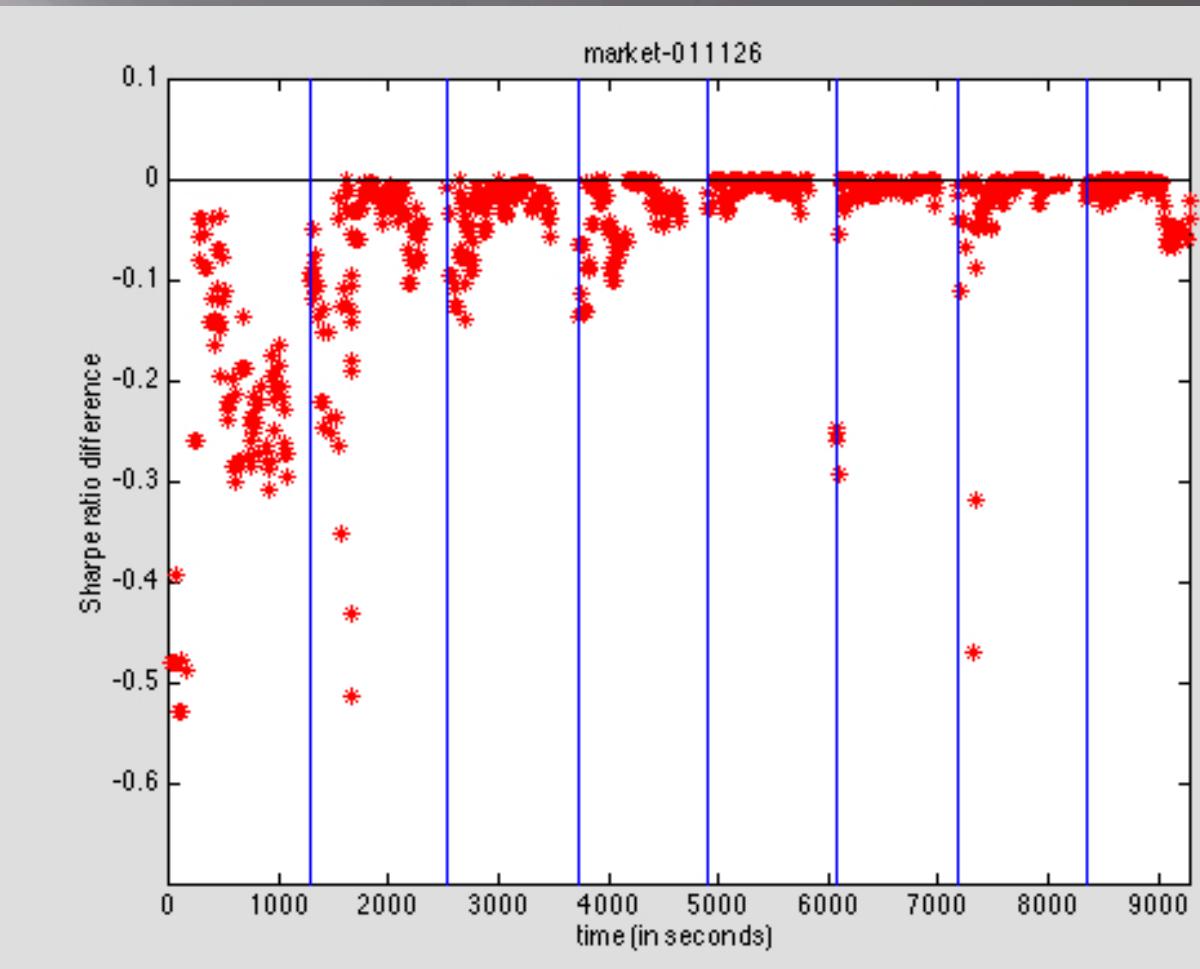
State Prices (Relative To Probabilities)



State pricing: **X** becomes most expensive; **Y** cheapest; **Z** in-between

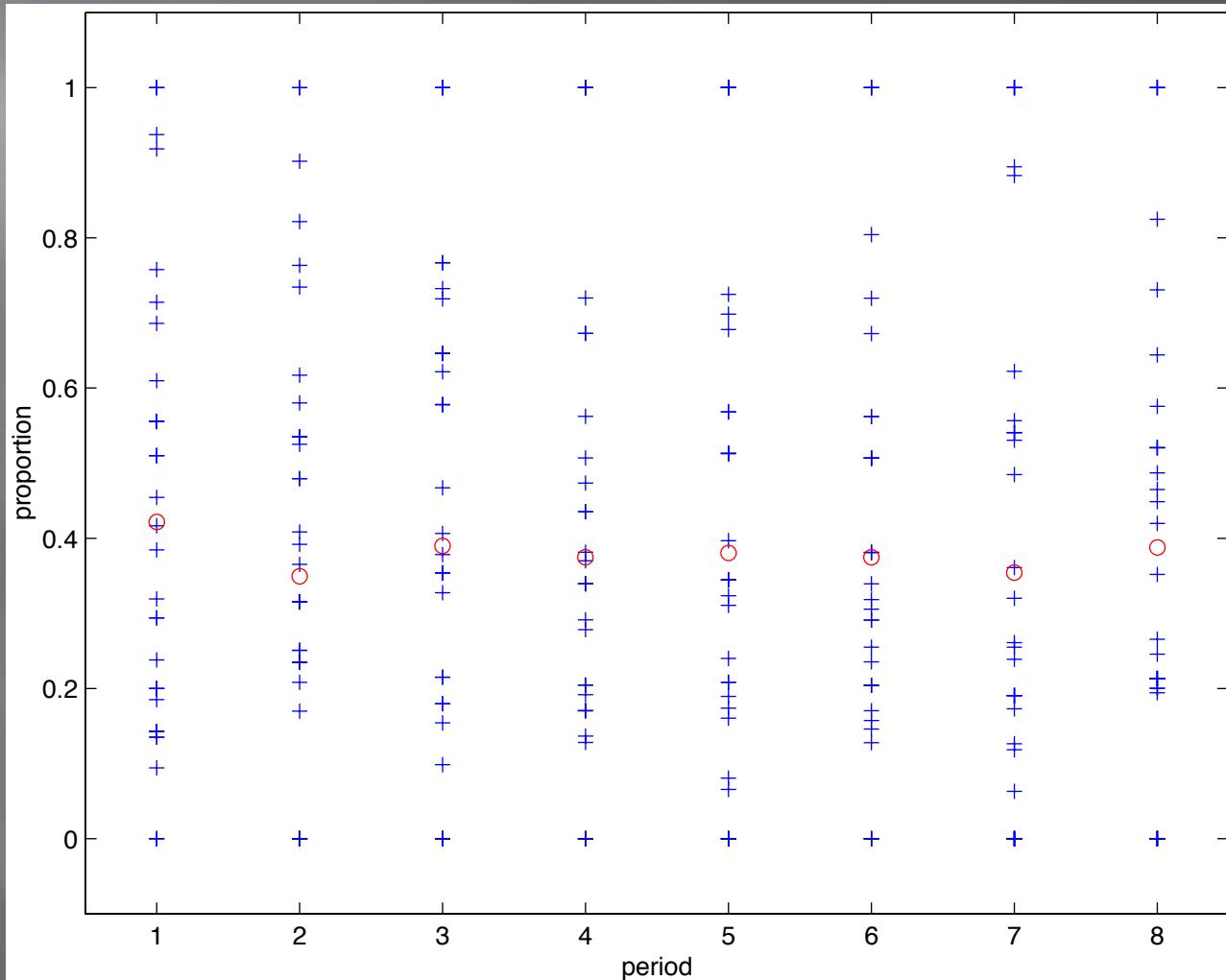
Efficiency of market portfolio

- Compute Sharpe ratio of market portfolio with each transaction.
- Take the difference between Sharpe of the market and the highest Sharpe ratio.
- CAPM predicts this difference should be 0.

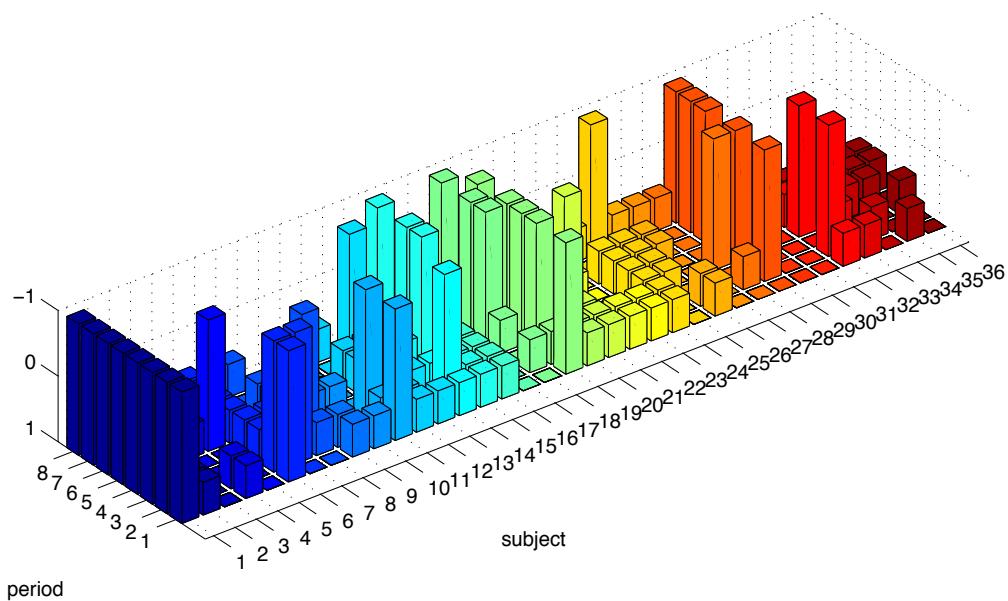


INDIVIDUAL Behavior

- For each of the 8 periods plot each subject's final holdings of asset A.
- Red dot indicates the proportion of A in the market portfolio.
- Individuals are all over the place.
Allocations do not improve with time.



INDIVIDUAL Behavior



AD holdings: no improvement in poor rank correlations among final wealths

Lessons

- Despite the modest risks, experimental financial markets can provide significant and useful insights.
- “Rationality” imposes little restriction on demands, other than “increase demand for a security as its price goes down”
- While demands are heterogeneous, they possess a common factor for all agents – and this is the mean-variance optimal demand. Individual demands can have other factors that have much higher weight than the mean variance one. But, the above guarantees that in the aggregate prices look like CAPM prices.