Complex Systems Homework 2

Almudena Chapa, Dani Lazaro, Jon Ander Martin

April 12, 2021

1 Introduction

In this paper we will explore Thomas Schelling's model of segregation. In this model there is a certain number of agents, divided into 2 classes. The agents are happy if they have enough neighbors of their same class. The agents relocate using different policies, always looking for a spot where they are happier. In the following sections, we will present different relocation policies and study how each of them affects the distribution of the agents.

In this work, we will apply Schelling's model in a 40×40 bi-dimensional grid. To avoid edge effects, the boundaries of the grid are periodic. The population of agents is $N = 1400$, and their position is assigned randomly at the beginning of each simulation. Note that this leaves 200 empty locations, a 12.5% of the total space. An agent is considered to be "happy" if more than 3 out of its 8 neighbors belong to its same class. All the simulations are performed over 20 epochs. In every epoch, all the unhappy agents make an attempt to move in a random order. The simulation for each of the cases is run 30 times and the results are averaged.

2 Comparison of Policies 1 and 2

The first policy we will compare here is the random move. We will refer to this policy as Policy 1. With this policy, the moving agent examines $q = 100$ random empty locations in the grid. The agent will move to the first location where it will be happy. If no new happy locations are found, the agent will still move to a location where its happiness improves, if any.

The second policy (Policy 2) is the social network recommendation. In this policy, every agent has a random set of n "friends". The friends search a $p \times p$ neighborhood around them, looking for suitable locations for the agent that is moving. The moving agent picks randomly one of the suitable locations and moves there. It is important to state that the friends do not necessarily belong to the same class as the agent that is moving. However, the recommendation is based on the class of the agent that moves. This policy is tested over a range of the parameters: $n = [5, 10, 20]$ and $p = [3, 5]$.

Some cases show a faster convergence than others towards the maximum number of happy agents. Specifically, Policy 2 with $n = 5$ and $p = 3$ shows the slowest convergence (Figure 1). In this case, the search-space for the moving agent is $5 \times 8 = 40$ locations. Considering the percentage of free locations in the whole grid (12.5%), we can expect an average of 5 empty locations in this search. This is a very narrow search, and can explain why the convergence is slower. It is also worth noting that the final value of happy agents for this case is lower. Since the search-space is smaller, the system reaches a state in which there are no new available locations for unhappy agents to move. In fact, in Figure 2b isolated blue agents inside red clusters and vice versa can be appreciated.

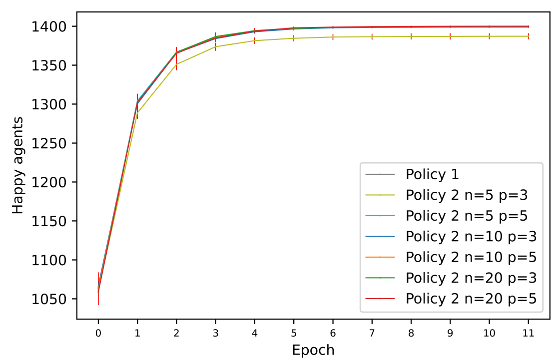
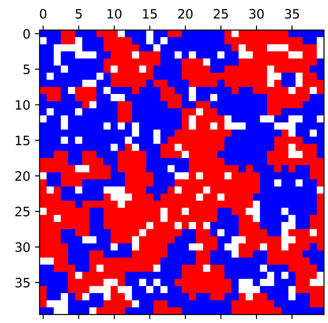


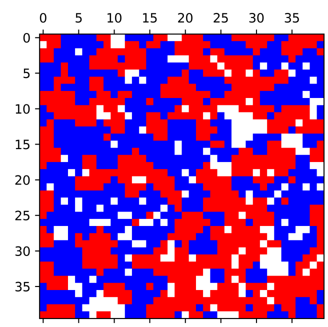
Figure 1: Comparison of Policies 1 and 2. Only 12 epochs are plotted for an easier visualization.

In short, Policy 1, despite being a “know nothing case”, shows a good performance because it has a very large search space with $q = 100$. It is very likely that among those 100 locations the agent will find a happy position. On the other hand, although in Policy 2 only suitable locations are offered, the search space is limited to whether there are empty spaces in the friends’ neighborhood. This fact is critical in Policy 2 with $n = 5$ and $p = 3$. However, as the number of friends and the scope of their search increases, so does the probability of finding empty cells and therefore, this limitation is less noticeable.

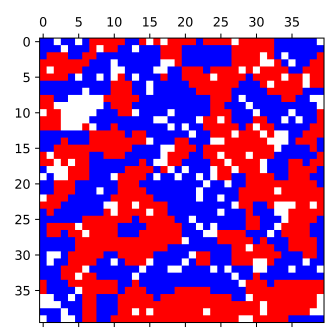
Regarding segregation maps, it does not matter the new location is found. In the end it all comes down to being surrounded by 3 agents of your own class. Therefore, regardless of the policy, similar maps are obtained. This is why the 3 maps in Figure 2 look similar. The consequence is that, according to this model, segregation ends up emerging when a moving agent looks into the class of its new neighbors, independently of the criteria it uses to look for suitable locations.



(a) Policy 1



(b) Policy 2, $n = 5$, $p = 3$



(c) Policy 2, $n = 10$, $p = 5$

Figure 2: Final class distribution maps.

3 User defined policies

3.1 Diverse communities

This policy is a variation of Policy 1. The difference lies in the way the q empty cells are chosen. Instead of making a random selection, all the empty cells are sorted in descending order of community diversity. In a neighborhood, the more equal the number of agents of each class are, the more diverse is the community. Therefore, the agent is offered the first q locations where the most diversity of classes exists. The interest of this policy is to see the effect on segregation of agents trying to find happiness in neighborhoods as diverse as possible.

This new policy has been tested as in Policy 1 for $q = 100$. At least a slight improvement in the convergence rate of happiness of Policy 1 can be expected. If the neighborhood of an empty cell is highly populated -with no empty cells around- and the community is diverse enough, any agent will be happy regardless of its class. Nonetheless, as the diversity decreases, so does the likelihood that the agent will be happy, making it dependent on the dominant class and the class of the agent, as in the random case. Moreover, to see to what extent q can be reduced without worsening the results obtained with Policy 1, several values of q have been tested.

Regarding the final class distribution map, it is worth noting that although agents seek diversity, segregation is inevitable for them to be happy (Figure 3). The main difference with respect to the segregation maps obtained with other policies is that the empty cells are more scattered over the map, fragmenting the large clusters. The reason is very simple, and it is that as you move towards the interior of a cluster the diversity decreases. Therefore, these empty cells go to the end of the queue of locations offered to the agent, being less likely to be occupied.

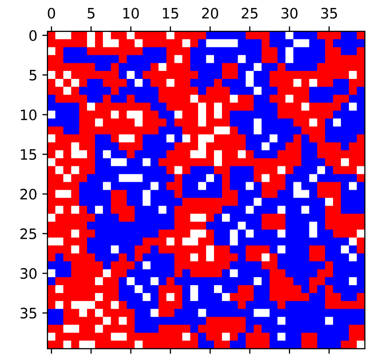


Figure 3: Final class distribution map (Diverse communities $q = 5$).

On the other hand, Figure 4 shows the expected improvement on happiness convergence rate. First, when more diverse communities are available it is easier to make agents happy regardless of the class. However, as the epochs advance, this advantage diminishes, as fewer and fewer diverse communities remain. Finally, when epoch 5 is reached, similar results to the random case are obtained. Furthermore, it is clear that the higher the q, the more likely it is that the agent will find a place where it will be happy. However, it is remarkable that its value can be reduced to 5 in this policy and obtain practically the same results as Policy 1 with $q = 100$.

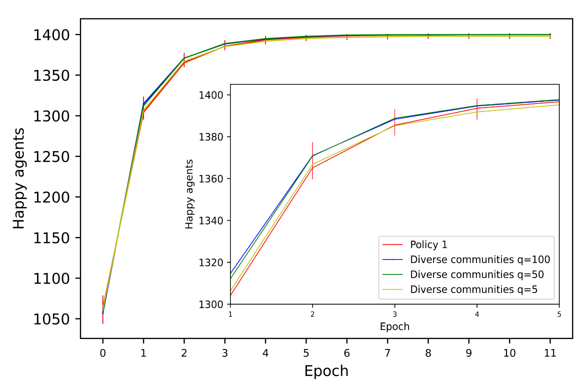


Figure 4: Comparison of diverse communities with Policy 1.

3.2 Same class social network

This policy is a variant of policy 2. With this variation we will restrain an agent's friendship to the agents of its class. The agents will always move to an empty spot that belongs to one agent of the same class. Thus, the happiness of the already happy agents will be enhanced by the presence of new agents of the same class in their neighborhood. If the agent doesn't find a happy spot among the neighborhoods of his same class friends, it will not move. This variation will be tested for the same parameter-combinations used on the original policy 2 ($n = [5, 10, 20]$ and $p = [3, 5]$). Initially, one might think that this policy promotes segregation, as the agents are forced to move next to same class friends.

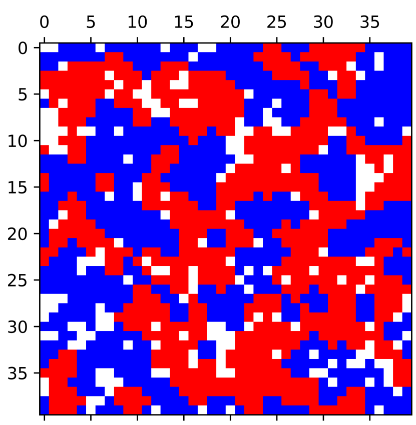
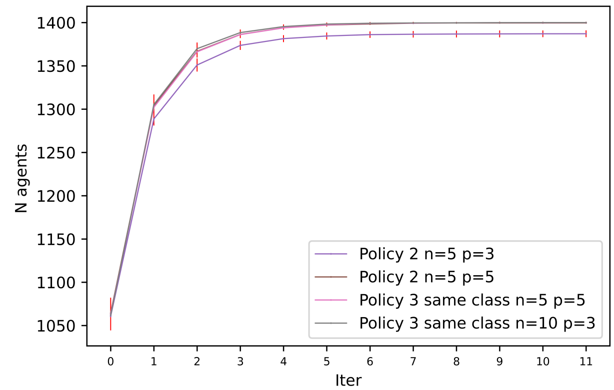
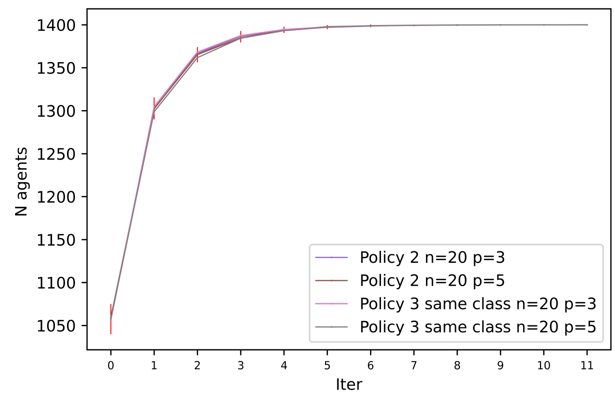


Figure 5: Final class distribution map ($n = 10$, $p = 3$).

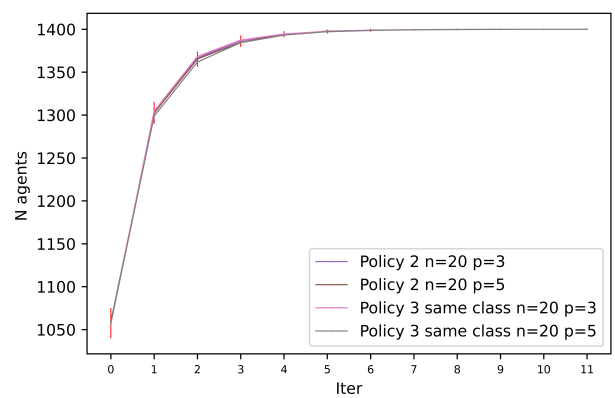
The results (Figure 5) prove that segregation does not get worse with this variation. After analyzing the plots in Figure 6 it can be concluded that this variation does not really bring anything new to the table. The distribution of the agents is similar and the convergence speed are akin. It might be work remarking that the variation of Policy 2 $[n = 10, p = 3]$ has the fastest convergence.



(a) $n = 5$



(b) $n = 10$



(c) $n = 20$

Figure 6: Comparison of same class social network with Policy 2.

3.3 Selfish social network

This policy is a modification of Policy 2. Each agent will also ask its group of n "friends" to look for new locations to move. However, in this case the friends will say where *they* would move. Thus, they will base their recommendation on their own class, instead of the class of the moving agent. Since our model only has 2 classes, there is a 50% chance that a friend belongs to the moving agent's opposite class. This could cause the moving agent to be unhappy in its new location.

As with Policy 2, this policy is also tested with $n = [5, 10, 20]$ and $p = [3, 5]$. The results are compared to Policy 1 and the first two cases of Policy 2 ($n = 5$, $p = [3, 5]$), since they are the ones that show variations in their results.

The final class distribution map does not show any significant difference with respect to the previous cases (Figure 7). The different classes appear clustered and there is segregation visible. There are some cases in which a few agents can be surrounded by agents of the opposite class (agent at $(10, 21)$ in Figure 7) because of the new policy. Nevertheless, that is the only appreciable difference in this map.

There are more differences visible in Figure 8. As expected, the convergence is in general slower than in Policies 1 and 2. This can be easily explained if we consider the 50% chance of an agent moving into a position where an agent of the other class would be happy. Different parameters for this policy also affect the convergence speed of the happiness. The lower n and p are, the slower the convergence. The reason is the same as explained in Section 2: lower values in these parameters imply a smaller search space. In this case the differences are even more prominent because of the additional handicap derived from the friends' preference.

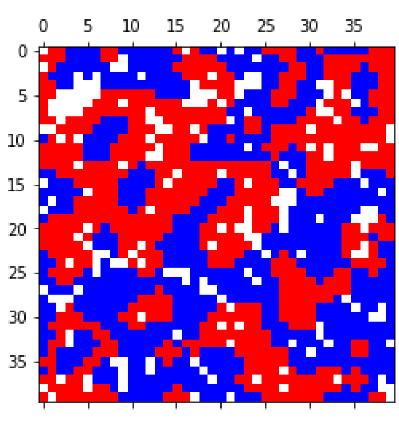


Figure 7: Final class distribution map ($n = 10$, $p = 5$).

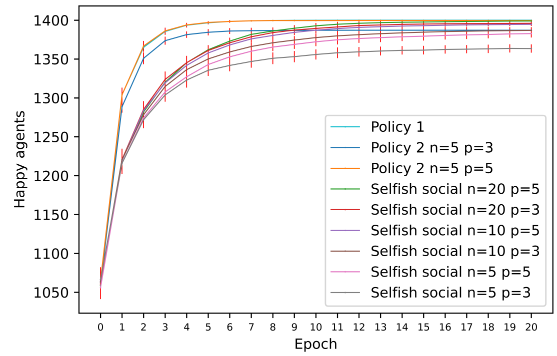


Figure 8: Comparison of selfish social network policy with Policies 1 and 2.1.

The final stable values are also affected by this policy. There is a moment at which the agents reach a steady-state where happiness cannot improve any further. Two situations lead to this: unhappy agents might not have empty locations to move into because their friends have all their neighborhood occupied, or they move to locations where they are also unhappy and keep cycling through those locations. As it happens with the convergence speed, this effect is also magnified with this policy.

Nevertheless, the remarkable result of this policy is the fact that segregation still emerges at the end of the simulation, even in the case with the smallest search-space.