Ward Clustering on CO2-GDP (1965-2018)

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1.

Dataset

Dataset

1. ## CO2 Emissions _ 1960 - 2018

   https://www.kaggle.com/datasets/kkhandekar/co2-emissions-1960-2018

2. ## GDP per capita (constant LCU)

   https://data.worldbank.org/indicator/NY.GDP.PCAP.KN?end=2021&most_recent_year_desc=false&start=2021&view=map

Data Preparation

Data Cleaning

- Drop unused column
- Discard useless group of country

Join data on Country Name

- CO_2
- GDP

Minor Adjustment

- Round double values
- Fill missing values

2.

Implementation

Implementation

Implemented in **Scala+Spark**

Follows the **MapReduce** paradigm

Main **collections**:

df_annuali : List[Dataframe]
RDD_label_annuali : RDD[List[Int]]
forest : List[Int]
dizionario : List[List[Int]]

Main **functions**:

ward

number_cluster

Spark setting

```scala
val spark: SparkSession = SparkSession
 .builder()
 //.master("local[*]")
 .master("yarn")
 .appName("Ward on annual co2-gdp" )
 .getOrCreate()

import spark.sqlContext.implicits._
```

We use:

- *.master("**local[*]**")* to run **locally**.
- *.master("**yarn**")* to run on a **distributed** cluster.

SQLImplicits:

- A collection of implicit methods for converting common Scala native objects or RDD into DataFrame/Datasets through implicit Encoders.

Ward's minimum variance method

1. **Agglomerative hierarchical clustering** method based on minimizations of the total within-cluster variance.

2. Start from a forest of **n clusters** each containing a single point p such as $p_i = (co2, gdp)$.

3. At each steps we search for **all** the possible merge **combinations** of **two clusters** available in the forest

4. Once **all** the clusters **combinations** are obtained, calculate the **midpoint** among all the points belonging to the clusters under analysis.

```
var forest: List[Int] = List.range(0, original_lenght)
var dizionario: List[List[Int]] = forest.map(List(_))

while(forest.count(_ > -1) > 1) {

val combinazioni = forest.filter(_ != (-
1)).combinations(2).toList

 val error_list = combinazioni.par.map(
      distance(xy_zip, _, dizionario)
 )

 val coppia =
combinazioni(error_list.indexOf(error_list.min))

 forest = forest.updated(coppia(0), -1)
 forest = forest.updated(coppia(1), -1)

 forest = forest :+ forest.length

 dizionario = dizionario :+ coppia
}
```

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   Number of Combinations = $n! / [\,(n-2)! * (2)!\,]$

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```
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```

```
DISTANCE:
val points_new = expand(points, dizionario,dataFrame.length)

val all_x = points_new.map(dataFrame(_)._1)  //CO2
val all_y = points_new.map(dataFrame(_)._2)  //GDP

val ptMedio = Point(
      all_x.sum / points_new_length,
      all_y.sum / points_new_length
      )
```

$$(\bar{X}, \bar{Y}) = \left(\frac{1}{k}\sum_{i=1}^{k} x_i \quad , \quad \frac{1}{k}\sum_{i=1}^{k} y_i \right)$$

Ward's minimum variance method

5. **Squared Error (SE)** calculation between the cluster's points and the midpoint calculated previously

6. Search for the cluster u with the **minimum error**

7. **Delete** from the forest the cluster composing u

8. **Adding** u to the forest

9. **Repeat** until there's only one cluster in the forest (**root of the tree**)

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var forest: List[Int] = List.range(0, original_lenght)
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```

```
val error_list = combinazioni.par.map(
     distance(xy_zip, _, dizionario)
)
```

```
DISTANCE:

val error_square = (all_x zip all_y).map(
punto =>
ptMedio.error_square_fun( Point(punto._1, punto._2) )
).sum

def error_square_fun(other: Point): Double =
   pow(other.x - x, 2) + pow(other.y - y, 2)
}
```

$$SE = \sum_{i=1}^{k} \left((x_i - \bar{X})^2 + (y_i - \bar{Y})^2 \right)$$

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7. **Delete** from the forest the cluster composing u

8. **Adding** u to the forest and to dizionario

9. **Repeat** until there's only one cluster in the forest (**root of the tree**)

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```

Number Cluster function

It returns the **roots** of the clusters created.

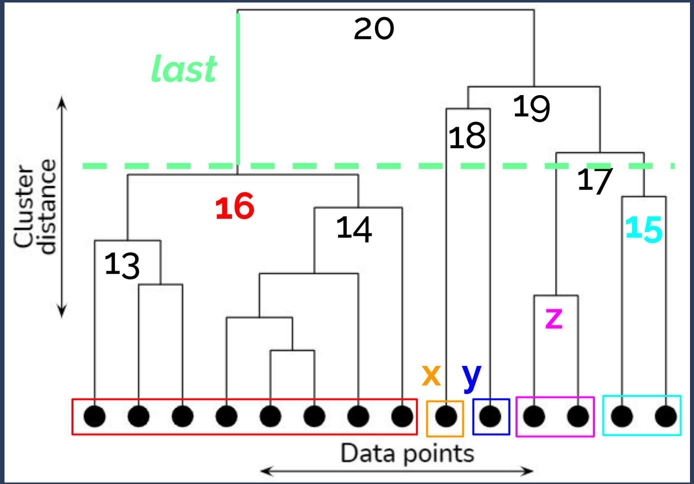
The **dendrogram**'s root is the couple (*dizionario.last*).

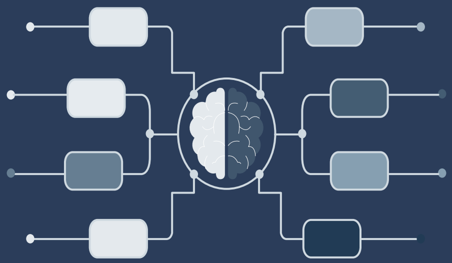
dizionario.last(0) has the longest branch of the couple.

Discard the values lower than ***last*** because I'm interested only in the ones included between the two in the **root**.

Flat and discard the values higher than ***last***.

```
def number_cluster (dizionario: List[List[ Int]]): List[ Int]
= {
 val last = dizionario.last( 0)
 val out = last :: dizionario.drop(last +
1).flatten.filter(_ < last)
 out
}
```



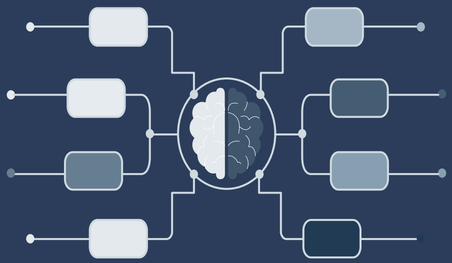


Implementation - Parallel

- **Data** parallelism:

```scala
// Mapping della lista di combinazioni con l'errore quadratico associato
val error_list = combinazioni.par.map( distance(xy_zip, _, dizionario))
```

```scala
// Applicazione dell'algoritmo ward sui dataframe annuali
val label_annuali = input_ward_annuali.par.map(t =>  ward(t._1, t._2, t._3, t._4))
```



Implementation - Distributed

- **RDD** usage:

We **convert** into RDD *input_ward_annuali*, a structure that contains the input for the annual Ward function.

We apply the Ward function on this RDD with a **map**, taking as input these values.

```
val RDD_inputWardAnnuali = spark.sparkContext.parallelize(input_ward_annuali)

// Applicazione dell'algoritmo ward sui dataframe annuali
val RDD_label_annuali = RDD_inputWardAnnuali.map(t =>  ward(t._1, t._2, t._3, t._4))
```

3.

Performances

Performances - Local

With *.master("**local[*]**")*

data_gdp_co2.csv **(1965-2018)**

	Sequential	Parallel
Execution Time (s)	258	68

Scalability Test

Strong

We run the application on a different number of workers (with the same configuration) **without changing** the dataset.

Weak

We run the application on a different number of workers (with the same configuration) **scaling** the dataset.

Performances - Google Cloud Platform

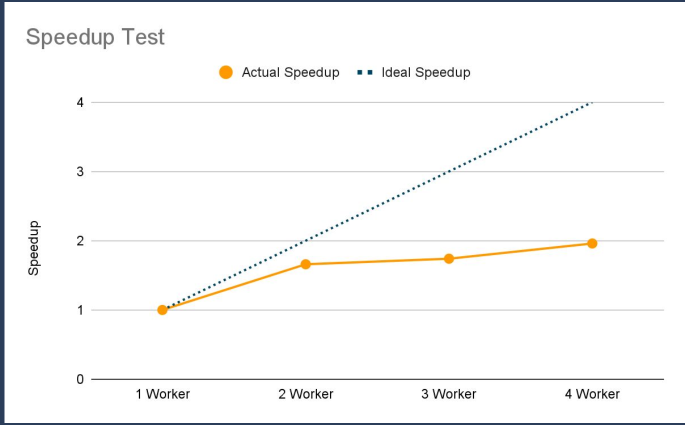
Strong scaling

data_gdp_co2.csv **(1965-2018)**

Master: **n2-standard-8** (8 vCPU, 32 GB di memoria)
Worker: **n1-custom** (6 vCPU, 22.5 GB di memoria)

Worker	Time (ms)	Speedup
1	230'236	1
2	138'447	1,66
3	132'149	1,74
4	117'461	1,96

Speedup = t_1 / t_i



Performances - Google Cloud Platform

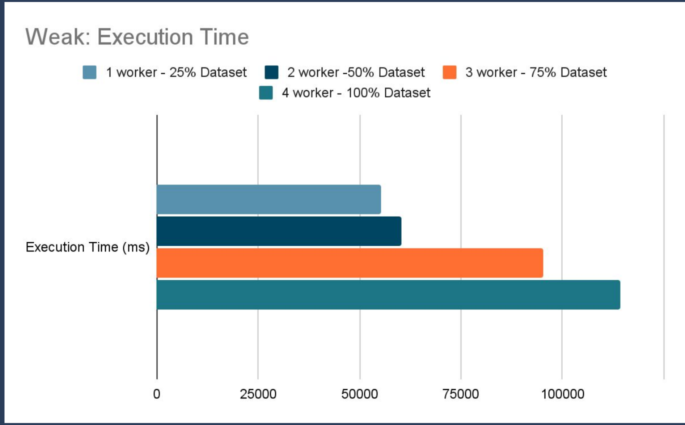
Weak scaling

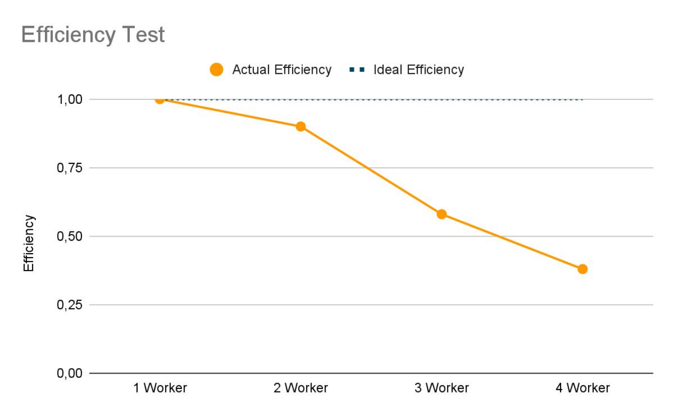
data_gdp_co2.csv (1967-2018)

Master: **n2-standard-8** (8 vCPU, 32 GB di memoria)
Worker: **n1-custom** (6 vCPU, 22.5 GB di memoria)

Dataset Size	Worker	Time (ms)	Efficiency
25%	1	55'417	1
50%	2	60'296	0,9
75%	3	95'255	0,58
100%	4	114'125	0,38

Efficiency = t_1 / t_i





Increasing Complexity

Change Dataset:

- Old Dataset = 214 Countries per year
  - Number of Starting Combination = 22791
- New Dataset = 264 Countries per year
  - Number of Starting Combination = 34716
  - Grouping Countries
  - Cannot be showed on the map

Number of Combinations = *n! / [(n-2)! * 2!]*

Old: 214! / [(214-2)! * 2!] = 22791

New: 264! / [(264-2)! * 2!] = 34716

Performances - Google Cloud Platform

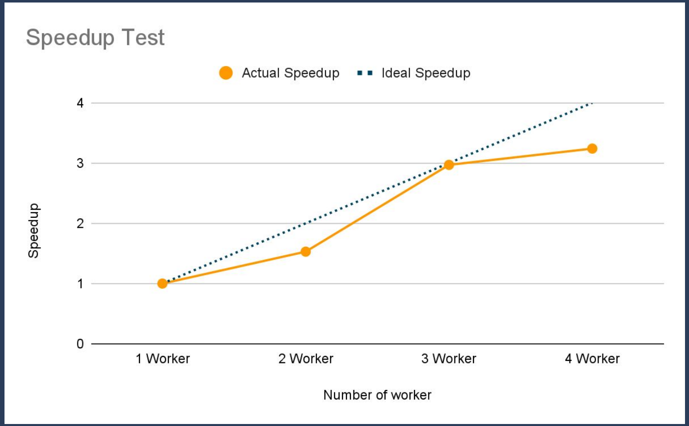
Strong scaling

data_gdp_co2_with_groups.csv (1990-2013)

Master: **n2-standard-8** (8 vCPU, 32 GB di memoria)
Worker: **n1-custom** (6 vCPU, 22.5 GB di memoria)

Worker	Time (ms)	Speedup
1	178'291	1
2	116'127	1,53
3	59'982	2,97
4	55'018	3,24

Speedup = t_1 / t_i



Performances - Google Cloud Platform

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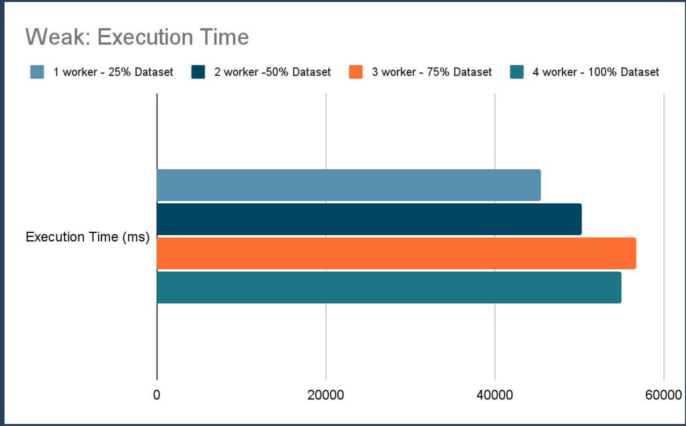
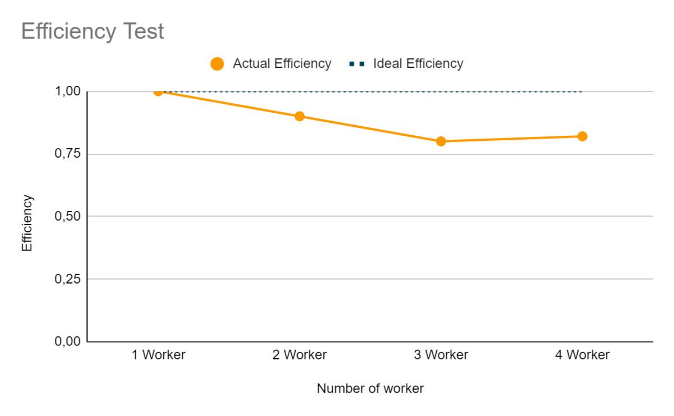
Dataset Size	Worker	Time (ms)	Efficiency
25%	1	45'480	1
50%	2	50'334	0,9
75%	3	56'775	0,80
100%	4	55'018	0,82

Efficiency = t_1 / t_i

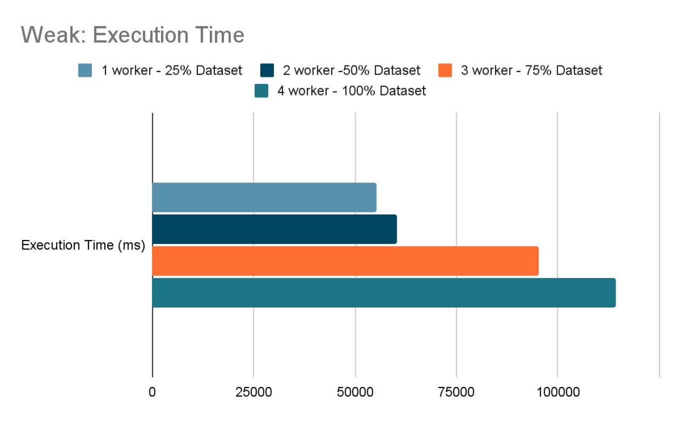




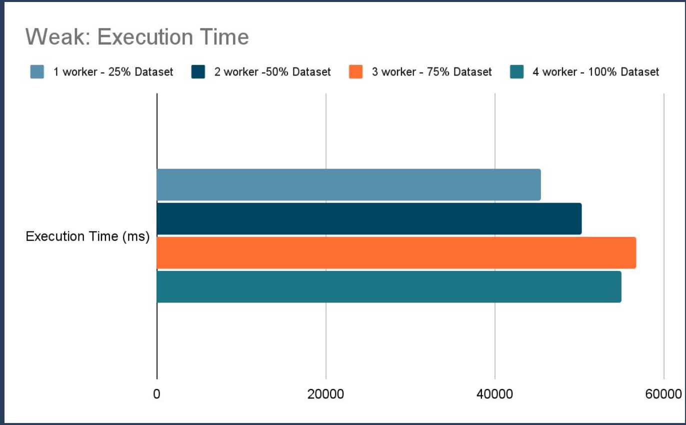
Performances - Google Cloud Platform

Comparison

214 Countries

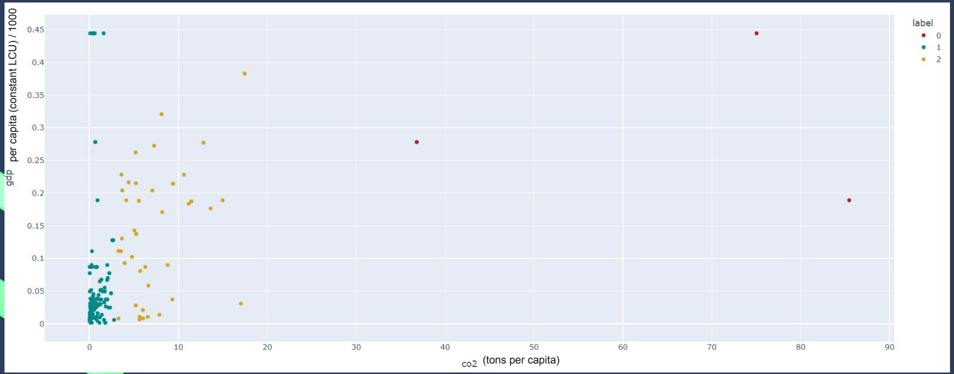


264 Countries

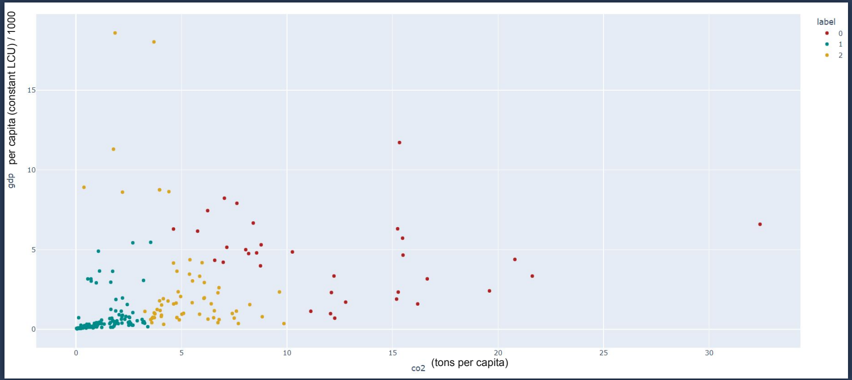


Graphs

Year 1965



Year 2018



DEMO