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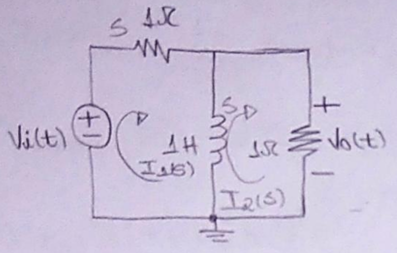
1) a)



$V_o(t) = 1 \cdot I_2(s)$

$\boxed{1}$ $-V_i(s) + I_1(s) + sI_1(s) - sI_2(s) = 0$

$\boxed{2}$ $sI_2(s) - sI_1(s) + I_2(s) = 0$

$\boxed{1}$ $I_1(s) + sI_1 - sI_2 = V_i(s)$

$I_1(1+s) - sI_2 = V_i(s)$

$$\begin{bmatrix} 1+s & -s \\ -s & 1+s \end{bmatrix} = \begin{bmatrix} V_i(s) \\ 0 \end{bmatrix}$$

$1 + 2s + s^2 - s^2 = 2s + 1 = \Delta$

$\boxed{2}$ $I_2 + sI_2 - sI_1(s) = 0$

$-sI_1 + I_2(1+s) = 0$

$I_2 = \dfrac{\Delta I_2}{\Delta} = \dfrac{V_i(s) \cdot s}{2s+1}$

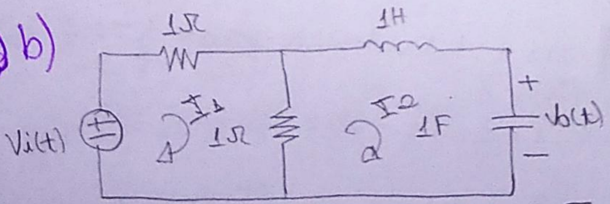
$$\begin{bmatrix} 1+s & V_i(s) \\ -s & 0 \end{bmatrix} =$$

$V_i(s) \cdot s = \Delta I_2$

$V_o(s) = 1 \cdot I_2$ $\quad V_o(s) = \dfrac{V_i(s) \cdot s}{2s+1}$

$\dfrac{V_i(s)}{V_o(s)} = \dfrac{2s+1}{s}$ $\qquad \dfrac{V_o(s)}{V_i(s)} = \dfrac{s}{2s+1}$

b)



$\boxed{1}$ $I_1 + I_1 - I_2 = V_i(s)$

$2I_1 - I_2 = V_i(s)$

$\boxed{2}$ $I_2 - I_1 + sI_2 + \frac{1}{s} I_2 = 0$

$-I_1 + I_2\left(s + \frac{1}{s} + 1\right) = 0$

$$\begin{bmatrix} 2 & -1 \\ -1 & s + \frac{1}{s} + 1 \end{bmatrix} = \begin{bmatrix} V_i(s) \\ 0 \end{bmatrix}$$

$\dfrac{2s^2 + 2 + 2s}{s} - 1 = \dfrac{2s^2 + 2 + 2s - s}{s} = \dfrac{2s^2 + s + 2}{s} = \Delta$

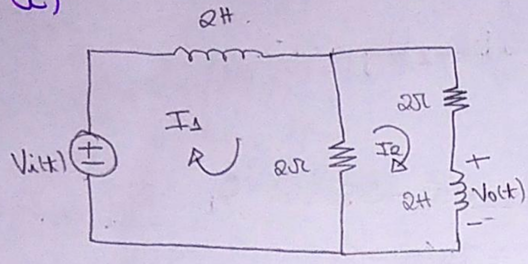
$I_2 = \dfrac{\Delta I_2}{\Delta} = \dfrac{V_i(s)}{\frac{2s^2+s+2}{s}} = \dfrac{V_i(s)}{2s^2+s+2} \cdot s$

$$\begin{bmatrix} 2 & V_i(s) \\ -1 & 0 \end{bmatrix} = -(-V_i(s)) = V_i(s) = \Delta I_2$$

$V_o(s) = \frac{1}{s} I_2 \rightarrow V_o(s) = \frac{1}{\cancel{s}} \cdot \left(\dfrac{V_i(s) \cdot \cancel{s}}{2s^2+s+2}\right)$

$\dfrac{V_i(s)}{V_o(s)} = 2s^2 + s + 2$ $\qquad \dfrac{V_o(s)}{V_i(s)} = \dfrac{1}{2s^2+s+2}$

c)



$\boxed{1}$ $2I_1 - 2I_2 + 2sI_1 = V_i(s)$

$I_1(2+2s) - 2I_2 = V_i(s)$

$\boxed{2}$ $2I_2 - 2I_1 + 2I_2 + 2sI_2 = 0$

$-2I_1 + I_2(4+2s) = 0$

$$\begin{bmatrix} (2+2s) & -2 \\ -2 & (4+2s) \end{bmatrix} = \begin{bmatrix} V_i(s) \\ 0 \end{bmatrix}$$

$8s + 4s^2 + 8 + 4s - 4 = \Delta$

$4s^2 + 12s + 4 = \Delta$

$$\begin{bmatrix} (2+2s) & V_i(s) \\ -2 & 0 \end{bmatrix} \Rightarrow -(-2V_i(s)) = \Delta I_2$$

$\Delta I_2 = 2V_i(s)$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{2V_i(s)}{4s^2+12s+4}$$

$$\frac{\cancel{2}V_i(s)}{\cancel{2}(2s^2+6s+2)} = I_2 \Rightarrow I_2 = \frac{V_i(s)}{2s^2+6s+2}$$

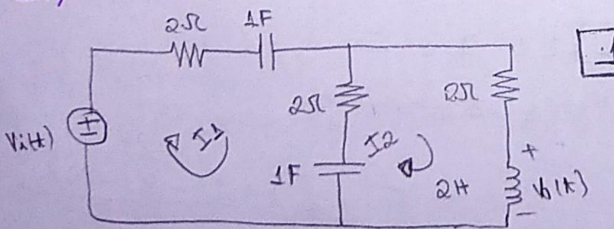
$$V_o(s) = 2sI_2 \Rightarrow V_o(s) = 2s\left(\frac{V_i(s)}{2s^2+6s+2}\right)$$

$$V_o(s) = \frac{2s \cdot V_i(s)}{2(s^2+3s+1)} = V_o(s) = \frac{sV_i(s)}{(s^2+3s+1)}$$

$(s^2+3s+1) \cdot V_o(s) = sV_i(s)$

$$\frac{V_i(s)}{V_o(s)} = \frac{s^2+3s+1}{s} \rightarrow \frac{V_o(s)}{V_i(s)} = \frac{s}{s^2+3s+1}$$

d)



$\boxed{1}$ $2I_1 + \frac{1}{s}I_1 + 2I_1 - 2I_2 + \frac{1}{s}I_1 - \frac{1}{s}I_2 = V_i(s)$

$4I_1 + \frac{2}{s}I_1 - 2I_2 - \frac{1}{s}I_2 = V_i(s)$

$I_1\left(4+\frac{2}{s}\right) \cdot I_2\left(-2-\frac{1}{s}\right) = V_i(s)$

$\boxed{2}$ $\frac{1}{s}I_2 - \frac{1}{s}I_1 + 4I_2 - 2I_1 + 2sI_2 = 0$

$I_1\left(-2-\frac{1}{s}\right) + I_2\left(4+\frac{1}{s}+2s\right) = 0$

$$\begin{bmatrix} 4+\frac{2}{s} & -2-\frac{1}{s} \\ -2-\frac{1}{s} & 4+\frac{1}{s}+2s \end{bmatrix} = \begin{bmatrix} V_i(s) \\ 0 \end{bmatrix}$$

$$\Delta = \frac{16s^2+8s+8s^3+1}{s^2}$$

$$\begin{bmatrix} 4+\frac{2}{s} & V_i(s) \\ -2-\frac{1}{s} & 0 \end{bmatrix} \Rightarrow \Delta I_2 = -V_i(s). \left(\frac{-2s}{s}-1\right)$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{-V_i(s) \cdot \left(\frac{-2s-1}{s}\right)}{8s^3 \cdot 16s^2+8s+1} = \frac{\cancel{s^2}V_i(s)\left(\frac{-2s-1}{\cancel{s}}\right)}{8s^3+16s^2+8s+1}$$

$$V_o(t) = \frac{V_i(s)\left(-4s^3-\frac{s^2}{2s^2}\right)}{8s^3+16s^2+8s+1} \qquad \frac{V_o(s)}{V_i(s)} = \frac{2s^2}{4s^2+6s+1}$$

$$\frac{V_i(s)}{V_o(s)} = \frac{8s^3+16s^2+8s+1}{4s^3+2s^2} = \frac{4s^2+6s+1}{2s^2}$$

② a) massa $\boxed{1}$

$$x_1(s)\left[m_1 s^2 + (b_1+b_2)s + (K_1+K_2)\right] - x_2(s)\left[b_2 s + K_2\right] = u_1$$

$\ddot{x}_1 \quad \dot{x}_1 \quad x_1$

massa $\boxed{2}$

$$x_2(s)\left[m_2 s^2 + (b_2+b_3)s + (K_2+K_3)\right] - x_1(s)\left[b_2 s + K_2\right] - x_3(s)\left[b_3 s + K_3\right] = u_2$$

$\ddot{x}_2 \quad \dot{x}_2 \quad x_2$

massa $\boxed{3}$

$$x_3(s)\left[m_3 s^2 + b_3 s + K_3\right] - x_2(s)\left[b_3 s + K_3\right] = u_3$$

$\ddot{x}_3 \quad \dot{x} \quad x_3$

b)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-(K_1+K_2)}{m_1} & \frac{K_2}{m_1} & 0 & \frac{-(b_1+b_2)}{m_1} & \frac{b_2}{m_1} & 0 \\ \frac{K_2}{m_2} & \frac{-(K_2+K_3)}{m_2} & \frac{K_3}{m_2} & \frac{b_2}{m_2} & \frac{-(b_2+b_3)}{m_2} & \frac{b_3}{m_2} \\ 0 & \frac{K_3}{m_3} & \frac{-K_3}{m_3} & 0 & \frac{b_3}{m_3} & \frac{-b_3}{m_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{u_1}{m_1} \\ \frac{u_2}{m_2} \\ \frac{u_3}{m_3} \end{bmatrix}$$

③ massa $\boxed{1}$

$$y_1(s)\left[m_1 s^2 + (K_1 + K_{12}) + b_1 s\right] - y_2(s)\left[K_{12}\right] = F(t)$$

massa $\boxed{2}$

$$y_2\left[K_{12}\right] - y_1\left[K_{12}\right] = 0$$

④ a) Verifica-se que, VR está em paralelo a V(t), então:

$$VR = V(t)$$

Isolando na equação:

$$iR = 2e^{2VR}$$

Aplicando ln:

$$\ln\left(\frac{iR}{2}\right) = 2VR \rightarrow VR = \frac{\ln\left(\frac{iR}{2}\right)}{2} = V(t)$$

b) $f(V) \approx F(VS) + \left.\frac{df(V)}{dV}\right|_{VS} \delta V$

$$2e^{2(VS+\delta V)} \approx 2e^{2VS} + 4e^{2VS}\delta V \quad (1)$$

c) $\frac{C\,dV}{dt} \cdot iR - 4 = i(t)$

$C = \frac{1}{2}$ $\quad V = VS + \delta V \quad iR = 2e^{2VR} ; 2e^{2(VS-\delta V)}$

$$\frac{1}{2}\frac{d(VS+\delta V)}{dt} \cdot 2e^{2(VS+\delta V)} - 4 = i(t) \quad (2)$$

Substituindo

$$\frac{1}{2}\frac{d\,\delta V}{dt} \cdot 2e^{2}VS + 4e^{2VS}\delta V - 4 = i(t)$$

$i(t) = 0 \quad VS = VR \quad iR = 4$

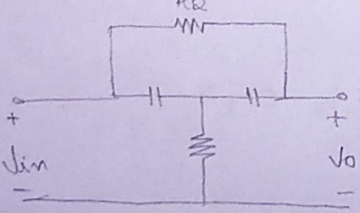
$VS = \frac{1}{2}\ln 2 \quad \therefore e^{2VS} = 2$

$$\rightarrow \frac{1}{2}\frac{d\,\delta V}{dt} + 8\delta V = i(t)$$

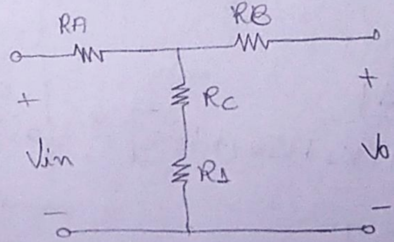
$$\delta V(s)\left(\frac{s}{2} + 8\right) = I(s)$$

$$\frac{V(s)}{I(s)} = \frac{2}{s+16}$$

(5)



$\rightarrow$



$$RA = \frac{CRR2}{2CR+R2} = \frac{R2}{2+SCR2}$$

$$Rc + R1 = \frac{1 + SC(2+SCR2).R1}{SC(2+SCR2)}$$

$$RB = \frac{CRR2}{2CR+R2}$$

$$RA + Rc + R1 = \frac{1 + SC(2+SCR2)R1 + SCR2}{SC(2+SCR2)}$$

$$Rc = \frac{CR^2}{2CR+R2} = \frac{1}{SC(2+SCR2)}$$

$$Vo(s) = \frac{\frac{1 + SC(2+SCR2)R1}{SC(2+SCR2)}}{\frac{1 + SC(2+SCR2)R1 + SCR2}{SC(2+SCR2)}} \cdot Vin$$

$$\frac{Vo(s)}{Vin(s)} = \frac{s^2C^2R1R2 + 2SCR1 + 1}{s^2C^2R1R2 + 2SCR1 + SCR2 + 1}$$

⑥ (a)

$$\dot{x} = \overset{A}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & 0 & 1 \end{bmatrix}} x + \overset{B}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} u \qquad y = \overset{C}{\begin{bmatrix} 2 & 1 & 1 \end{bmatrix}} x$$

$$(SI-A)^{-1} = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & 0 & -1 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 4 & 0 & s+1 \end{bmatrix}$$

$$\begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 4 & 0 & s+1 \end{vmatrix} \begin{matrix} s & -1 \\ 0 & s \\ 4 & 0 \end{matrix} = s^3 + s^2 + 4$$

$$\operatorname{adj}(SI-A) = \begin{pmatrix} \begin{vmatrix} s & -1 \\ 0 & s+1 \end{vmatrix} & \begin{vmatrix} 0 & -1 \\ 4 & s+1 \end{vmatrix} & \begin{vmatrix} 0 & s \\ 4 & 0 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ 0 & s+1 \end{vmatrix} & \begin{vmatrix} s & 0 \\ 4 & s+1 \end{vmatrix} & \begin{vmatrix} s & -1 \\ 4 & 0 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ s & -1 \end{vmatrix} & \begin{vmatrix} s & 0 \\ 0 & -1 \end{vmatrix} & \begin{vmatrix} s & -1 \\ 0 & s \end{vmatrix} \end{pmatrix}$$

$$(SI-A)^{-1} = \frac{\operatorname{adj}(SI-A)}{\det(SI-A)}$$

$$\operatorname{adj}(SI-A) = \begin{vmatrix} s^2+s & -4 & -4s \\ s+1 & s^2+s & -4 \\ 1 & s & s^2 \end{vmatrix}$$

$$C.(SI-A)^{-1} \qquad C = \begin{bmatrix} 2 & 1 & 1 \end{bmatrix}$$

$$C.(SI-A)^{-1} = \begin{bmatrix} 2(s^2+s)+(s+1)+1 & -8+s^2+s+s & -8s-4s^2 \end{bmatrix}$$

$$(SI-A)^{-1} = \frac{\begin{vmatrix} s^2+s & -4 & -4s \\ s+1 & s^2+s & -4 \\ 1 & s & s^2 \end{vmatrix}}{s^3+s^2+4}$$

$$C.(SI-A)^{-1} = \frac{\begin{bmatrix} 2s^2+3s+2 & +s^2+2s-8 & +s^2-8s-4 \end{bmatrix}}{s^3+s^2+4}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C.(SI-A)^{-1}.B = \frac{s^2-8s-4}{s^3+s^2+4}$$