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Lista 2

1.a) $G(s) = \frac{5}{s+5}$ $\omega = 5$

$TR = \frac{2,2}{\omega} = \frac{2,2}{5} = 0,44$

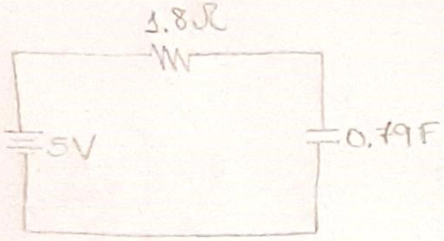
$TS = \frac{4}{\omega} = \frac{4}{5} = 0,8$

b) $G(s) = \frac{20}{s+20}$ $\omega = 20$

$TS = \frac{4}{20} = \frac{1}{2} = 0,2$

$TR = \frac{2,2}{20} = 0,11$

2.



$$Ri(t) + \frac{1}{C}\int_0^t i(\epsilon)d\epsilon = V(t)$$

$$R\frac{di(t)}{dt} \cdot \frac{1}{C}(t) = V(t) \quad \Big| \; i(t) = C.Vc(t)$$

$$RC\frac{dVc(t)}{dt} \cdot Vc(t) = V(t) \rightarrow \text{Aplicando Laplace}$$

$$RCsVc(s) + Vc(s) = V(s) \rightarrow Vc(s)(RCs+1) = V(s)$$

$$\frac{Vs}{Vc(s)} = RCs + 1 \rightarrow \frac{Vc(s)}{Vs} = \frac{1}{RCs+1} = \frac{Vc(s)}{Vs} = \frac{\frac{1}{C}}{Rs + \frac{1}{C}}$$

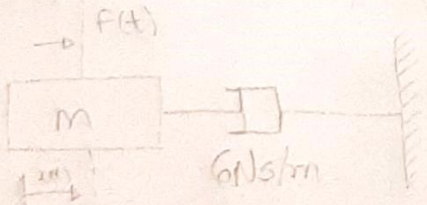
$$\boxed{\frac{Vc(s)}{5} = \frac{1,27}{1,8s+1,27}}$$

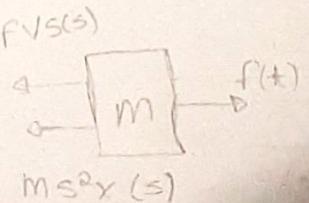
$\omega = 1,27$

$TS = \frac{4}{1,27} = 3,15$

$TR = \frac{2,2}{1,27} = 1,73$

3.





$$F(s) = 6s.X(s) + ms^2X(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{m} \cdot \left(\frac{1}{s^2 + \frac{6s}{m}}\right) = \frac{1}{m}\left[\frac{m}{6s} - \frac{m}{6(s-6/m)}\right] \cdot R(s) \cdot \frac{1}{s}$$

$$x(t) = -\frac{m}{36} + \frac{1}{6}t + e^{\frac{-6}{m}t}$$

$$x'(t) = \frac{1}{6} - \frac{6}{m}e^{\frac{-6}{m}t} \qquad x'(\infty) = \frac{1}{6}$$

$TR = X(0,9) - X(0,1)$

$$= -\frac{m}{6}\ln\left(\frac{m}{360}\right) \cdot \frac{m}{6}\ln\left(\frac{m}{40}\right) \Rightarrow TR = \frac{m}{6}\left(\ln\left(\frac{m}{40}\right) - \ln\left(\frac{m}{360}\right)\right)$$

$$TR = \frac{m}{6}\ln\left(\frac{m/40}{m/360}\right) \qquad \boxed{TR = \frac{\ln(9)m}{6}}$$

$$Xb = 0,9 \cdot \frac{1}{6} = 0,15 \qquad 0,15 = \frac{1}{6} - \frac{6}{m}e^{-6/m\,t}$$

$$\frac{m}{360} = e^{\frac{-6}{m}t} \Rightarrow \ln\left(\frac{m}{360}\right) = \frac{-6}{m}b \qquad b = \frac{-m}{6}\ln\left(\frac{m}{360}\right)$$

$$Xa = 0,1 \cdot \frac{1}{6} = 0,016 \Rightarrow 0,016 = \frac{1}{6} - \frac{6}{m}e^{\frac{-6}{m}a}$$

$$\frac{m}{40} = e^{\frac{-6}{m}a} \qquad \boxed{a = \frac{-m}{6}\ln\left(\frac{m}{40}\right)}$$

b) $X(Ts) = 0,98 \cdot \frac{1}{6} = \frac{49}{300}$; $\frac{49}{300} = \frac{1}{6} \cdot \frac{6}{m}e^{\frac{-5}{m}ts}$

$$\frac{-1}{300} = \frac{-6}{m}e^{\frac{-4}{m}ts} \rightarrow \frac{m}{1800} = e^{\frac{-6}{m}ts} \qquad ts = \frac{-m}{6}\ln\left(\frac{m}{1800}\right)$$

4. a) $T(s) = \frac{2}{s+2}$ polo -2 não tem ϕ $C(s) = \frac{1}{s}\frac{2}{(s+2)} = \frac{A}{s} \cdot \frac{B}{s+2}$

$= \frac{2}{s(s+2)}$ $As + 2A + Bs = 2$ $2A = 2$ $A = 1$

$A + B = 0$ $1 + B$ $B = -1$ $C(s) = \frac{1}{s} - \frac{1}{s+2}$

b) $T(s) = \frac{s}{(s+3)(s+6)}$ polos $= -3$ e -6 zeros

$C(s) = \frac{A}{s} + \frac{B}{(s+3)} + \frac{C}{(s+6)} = s \Rightarrow A(s+3)(s+6) + B(s)(s+6) + C(s)(s+3)$

$s = \phi$ $18A = 5$ $A = 5/18$ $C = 5/-9$ $C(s) = \frac{5}{18} + \frac{\frac{5}{18}}{s+3} + \frac{\frac{5}{9}}{s+6}$

$s = -3$ $C(-3)(-3+6) = 5$ $B = 5/18$

$s = -6$ $B(-6)(-6+3) = 5$

Superamortecida

c) $T(s) = \frac{10(s+7)}{(s+10)(s+20)}$ polos = -10 e -20 zeros = -7

$C(s) = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{s+20} = 10(s+7)$ → $s = 0$

$A(0+10)(0+20) = 10 \times 7$

$A = \frac{70}{200} = \frac{7}{20}$ $B = \frac{3}{10}$ $C = -\frac{13}{20}$ superamortecido

$C(s) = \frac{\frac{7}{20}}{s} + \frac{\frac{3}{10}}{s+10} = \frac{\frac{13}{20}}{s+20}$

d) $T(s) = \frac{20}{s^2+6s+144}$ polo $s = \frac{-6 \pm 6\sqrt{15}i}{2.1}$

$s = -3 \pm 3\sqrt{15}i$

e) $T(s) = \frac{s+2}{s^2+9}$ polos: ± 3 zeros: -2

$C(s) = \frac{(s-2)}{s(s+3)(s-3)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s-3}$

$As^2 + 9A - Bs^2 + 3Bs - Cs^2 - 3Cs = s+2$

$s=0$ $A.0 - 9A = 2$ $A = \frac{2}{9}$ $C = -\frac{1}{18}$ superamortecido

$s=3$ $9B + 9B = 5$ $B = \frac{5}{18}$

$s=-3$ $9C + 9C = -1$

$C(s) = \frac{\frac{2}{9}}{s} + \frac{\frac{5}{18}}{s+3} - \frac{\frac{1}{18}}{s+3} - \frac{\frac{1}{18}}{s-3}$

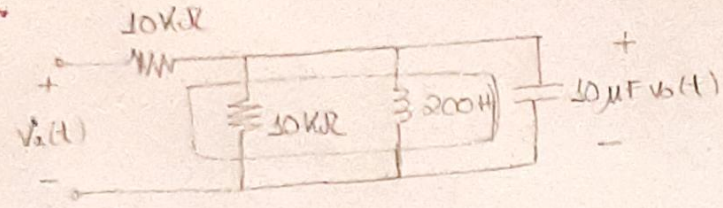
f) $T(s) = \frac{s+5}{(s+10)^2}$ $T(s) = \frac{s+5}{(s+10)^2}$ zeros = -5 polos = -10

$C(s) = \frac{s+5}{s(s+10)^2} \Rightarrow \frac{A}{s} + \frac{B}{s+10} + \frac{C}{(s+10)^2} = s+5$

$s=0$ $A(0+10)(0+10) = 5$ $A = \frac{1}{20}$ $B = \frac{1}{180}$ $C = -\frac{1}{180}$

$C(s) = \frac{\frac{1}{20}}{s} + \frac{\frac{1}{180}}{s+10} - \frac{\frac{1}{180}}{(s+10)^2}$

5.



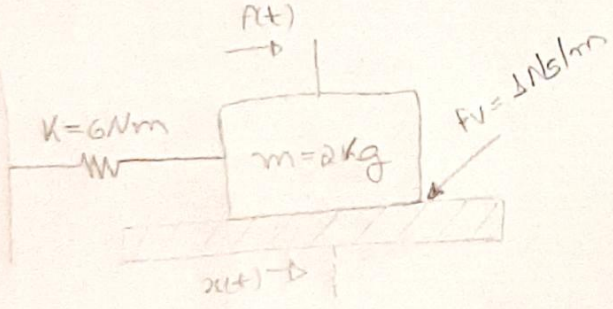
$$A = \frac{10K \cdot 200S}{10K + 200S} \Rightarrow \frac{2\times10^6}{10K + 200S}$$

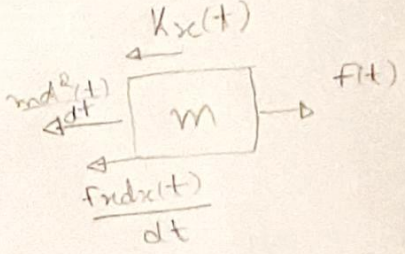
$$V_o = V_i(S) \cdot \frac{\frac{2\cdot10^6}{10K+200S}}{10K + \frac{2\cdot10^6}{10K\cdot200S}} \qquad \frac{V_o}{V_i(S)} = \frac{\frac{2\times10^6}{10K+200S}}{10K + \frac{2\times10^6 S}{10K+200S}}$$

$$\frac{V_o}{V_i(S)} = \frac{2\cdot10^6}{(10K+200S)} \cdot \frac{(10K+200S)}{10K(10K+200S) + 2\times10^6 S} = \frac{2\times10^6}{4\times10^6 S + 100} \quad (\div S)$$

$$\frac{V_o}{V_i(S)} = \frac{2\times10^6}{\frac{100\times10^6}{S}} \cdot 4\times10^6 \Rightarrow \frac{V_o}{V_i(S)} = \frac{\frac{2}{100}}{\frac{1}{S} + \frac{4}{100}}$$

6.





$$K\,x(t) + m\frac{d^2x(t)}{dt} + Fv\frac{dx(t)}{dt} = F(t)$$

$$K\,x(S) + mS^2x(S) + FvS\,x(t) = F(S)$$

$$x(S)\left[K + mS^2 + FvS\right] = F(S) \rightarrow \frac{x(S)}{F(S)} = \frac{1}{mS^2 + F.V.S + K} \quad (\div m)$$

$$\frac{x(S)}{F(S)} = \frac{\frac{1}{m}}{S^2 + \frac{Fv}{m}S + \frac{K}{m}} = \frac{0,5}{S^2 + 0,5S + 0,36}$$

7. a) $T(S) = \frac{16}{S^2+3S+16}$ $\quad T(S) = \frac{16}{S^2+3S+16}$ $\quad \omega_n^2 = 16$ $\quad \omega_n = 4$ $\quad 2\zeta\omega_n = 3$ $\quad \zeta = \frac{3}{8}$

$$T_e = \frac{4}{\zeta\omega_n} = \frac{4}{\frac{3}{2}} = \frac{8}{3} \qquad T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0,85$$

$$\%UP = e^{\frac{-\pi}{\sqrt{1-0,5^2}}} \cdot 100 = 28,1\%$$

b) $T(s) = \dfrac{0{,}04}{s^2 + 0{,}02s + 0{,}04}$ $\quad \omega_n^2 = 0{,}04$ $\quad 2\zeta\omega_n = 0{,}2$

$\omega_n = 0{,}2$ $\quad \zeta = \frac{1}{2}$

$T_s = \dfrac{4}{\zeta\omega_n} = 10$ $\quad T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 18{,}14$ $\quad \%UP = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100 = 16{,}3\%$

c) $T(s) = \dfrac{1{,}05 \times 10^{-7}}{s^2 + 1{,}6 \times 10^{-3}s + 1{,}05 \times 10^{-7}}$ $\quad \omega_n^2 = 1{,}05 \times 10^{-7}$ $\quad 2\zeta\omega_n = 1{,}6 \times 10^{-3}$

$\omega_n = 3{,}24 \times 10^{-4}$ $\quad \zeta = 0{,}247$

$T_s = \dfrac{4}{\zeta\omega_n} = 5 \times 10^{3}$ $\quad T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1 \times 10^{-3}$ $\quad \%UP = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100 = 44{,}9\%$

8. a) $\%UP = 12\%$ e $T_s = 0{,}6$ segundos

$\zeta = \dfrac{-\ln(\%UP/100)}{\sqrt{\pi^2 - \ln^2(\%UP/100)}} = 0{,}56$ $\quad T_s = \dfrac{4}{\zeta\omega_n}$ $\quad \omega_n = 11{,}9$

$T(s) = \dfrac{141{,}61}{s^2 + 13{,}33s + 141{,}61}$

pelo $s = -6{,}67 \pm 11{,}9\sqrt{(0{,}56)^2 \cdot 1} =$

$s_1 = -6{,}67 + 11{,}9\sqrt{0{,}69}\,i$

$s_2 = -6{,}67 - 11{,}9\sqrt{0{,}69}\,i$

b) $\%UP = 10\%$ e $T_p = 5$ segundos

$\zeta = \dfrac{-\ln(10/100)}{\sqrt{\pi^2 - \ln^2(10/100)}} = 0{,}59$ $\quad T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 5$

$\omega_n = 0{,}78$

$T(s) = \dfrac{0{,}6084}{s^2 + 0{,}9204s + 0{,}6084}$ $\quad \Delta = 0$

$s = -0{,}4602 \pm 0{,}78\sqrt{0{,}3484 - 1}$

$s_{1,2} = -0{,}4602 \pm 0{,}78\sqrt{0{,}65}\,i$

c) $T_s = 7$ segundos e $T_p = 3$ segundos.

$T_s = \dfrac{4}{\zeta\omega_n}$ $\quad \omega_n = 0{,}68$

$\dfrac{4}{\zeta\left(\frac{\pi}{T_p\sqrt{1-\zeta^2}}\right)} = \zeta\left(\dfrac{\pi}{T_p\sqrt{1-\zeta^2}}\right) = \dfrac{4}{T_s}$

$\zeta T_s \cdot \pi = 4\pi\sqrt{1-\zeta^2}$

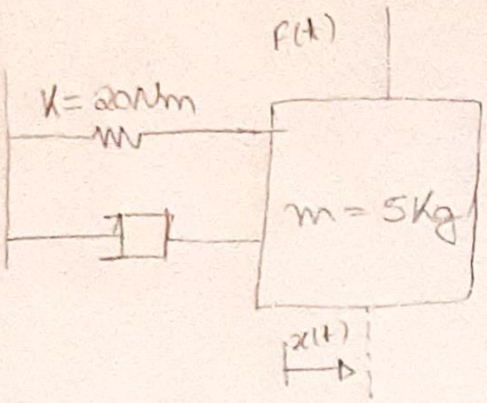
$\zeta^2 T_s^2 \pi^2 = 16\pi^2(1-\zeta^2)$

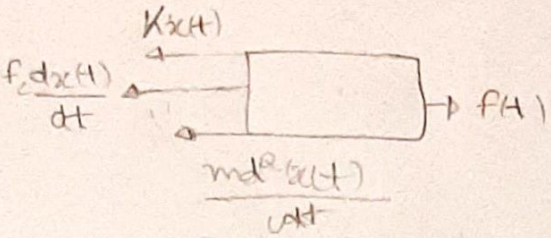
$\zeta^2 = \dfrac{16\pi^2}{T_s^2\pi^2 + 16T_p^2} = 0{,}84$

$T(s) = \dfrac{0{,}46}{s^2 + 1{,}145 + 046}$

pelo $s_{1,2} = -0{,}57 \pm 0{,}68\sqrt{0{,}294}\,i$

9.





$$\frac{md^2x(t)}{dt^2} + f_c\frac{dx(t)}{dt} + Kx(t) = f(t)$$

Laplace: $ms^2x(s) + Fcs\,x(s) + Kx(s) = F(s)$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2+f_cs+K} = \frac{\frac{1}{m}}{s^2+\frac{f_c}{m}\cdot s+\frac{K}{m}} = \frac{0,2}{s^2+0,4s+4}$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{0,2}{s^2+0,4s+4}}$$

$\omega_n^2 = 4$

$\boxed{\omega_n = 2}$

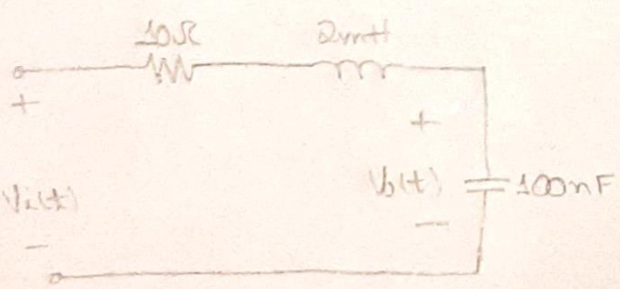
$2\zeta\omega_n = 0,4$

$\zeta = \frac{0,4}{4} = \boxed{0,1}$

$$\%UP = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100 = e^{-0,314} \times 100 = \boxed{72,9\%}$$

$$TP = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \boxed{1,58} \qquad TS = \frac{4}{\zeta\omega_n} = \boxed{20}$$

10.



a) $GS = \frac{V_o(s)}{V_i(s)} \rightarrow V_o = V_i \frac{100n/s}{\left(100 + 2\times10^{-3}s + \frac{100n}{s}\right)}$

$$\frac{V_o}{V_i} = \frac{100\cdot10^{-9}/s}{\frac{s(100+2\times10^{-3}s)+100\times10^{-9}}{s}}$$

$$= \frac{100\times10^{-9}}{(2\times10^{-3}s^2+100s+100\times10^{-9})} \quad (\div 2\times10^{-3})$$

$$\frac{V_o(s)}{V_i(s)} = \frac{5\times10^{-5}}{s^2+50\times10^3\cdot s+5\times10^{-5}}$$

11. % UP = 20% $\zeta = \frac{-\ln(20/100)}{\sqrt{\pi^2 + \ln^2(20/100)}} = 0,46$

$TP = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \rightarrow \omega_n = \frac{\pi}{TP\sqrt{1-\zeta^2}}$

$TP = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \rightarrow \omega_n = \frac{\pi}{TP\sqrt{1-\zeta^2}}$

P/preto $Tp = 0,2 \cdot \frac{2}{3} = 0,133$ $\omega_n = 26,6$

P/azul $Tp = 0,2$ $\omega_n = 17,69$

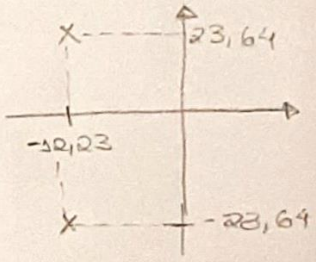
azul tracejado $\omega_n = 11,8$

vermelho $Tp = 0,4$ $\omega_n = 8,85$

vermelho tracejado $TP = 0,5$ $\omega_n = 7,1$

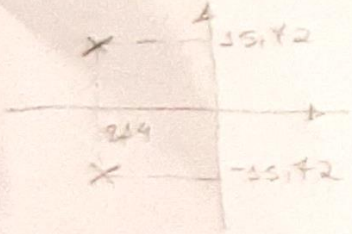
preto $\frac{707,6}{s^2 + 24,56 + 707,6} \rightarrow s_{1,2} = -12,23 \pm 26,6\sqrt{0,73}\,i$

$= -12,23 \pm 23,64\,i$



azul $\frac{3109,9}{s^2 + 16,35 + 3109,9} \rightarrow s_{1,2} = -8,14 \pm 17,69\sqrt{0,79}\,i$

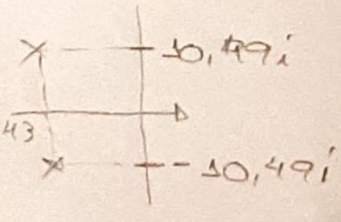
$= -8,14 \pm 15,72\,i$



azul tracejado: $\frac{139,24}{s^2 + 10,95 + 139,24}$

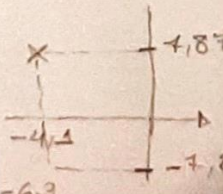
$s_{1,2} = -5,47 \pm 11,8\sqrt{0,79}\,i$

$= -5,43 \pm 10,49\,i$



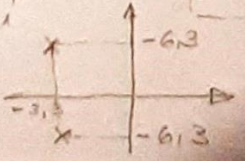
vermelho: $\frac{78,32}{s^2 + 8,84s + 78,32}$ $s_{1,2} = -4,1 \pm 8,85\sqrt{0,49}\,i$

$= -4,1 \pm 7,87\,i$

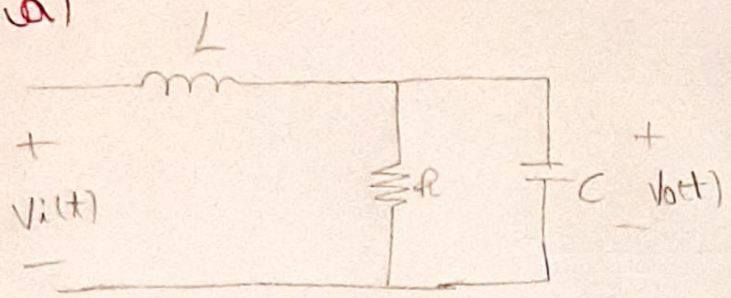


vermelho tracejado: $\frac{50,4}{s^2 + 6,3s + 50,4}$ $s_{1,2} = -3,2 \pm 7,1\sqrt{0,49}\,i$

$= -3,2 \pm 6,3\,i$



12. a)



$$V_i(t) = V_L(t) + V_o(t)$$

$$V_i(t) = LC\frac{d^2V_o(t)}{dt}$$

Laplace

$$V(s) = LCs^2V_o(s) = \frac{L}{R}sV_o(s) + V_o(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{LCs^2 + \frac{Ls}{R} + 1}$$

$$i = \frac{V_o(t)}{R}$$

$$V_L(t) = L\left(C\frac{d^2V_o(t)}{dt}\cdot\frac{1}{R}\frac{dV_o(t)}{dt}\right.$$

b) $\%UP = 36\%$

$$\zeta = \frac{-\ln(36/100)}{\sqrt{\pi^2 + \ln^2(36/100)}} = 0,33$$

$$T_P = 0,3\times10^{-4}$$

$$0,3\times10^{-4} = \frac{\pi}{\omega_n\sqrt{1-0,33^2}} = \omega_n = 99196,6$$

$$T_S = \frac{4}{\zeta\omega_n} = 1,33\times10^{-4}$$