Lista 2

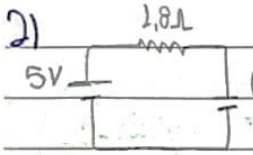
1) T_r e T_s

a) $G(s) = \frac{5}{s+5}$ $\quad a = 5$ $\quad T_r = \frac{2,2}{a} = 0,44$

$T_s = \frac{4}{a} = 0,8$

b) $G(s) = \frac{20}{s+20}$ $\quad a = 20$ $\quad T_r = 0,11$ $\quad T_s = 0,2$

2)



$R.i(t) + \frac{1}{C}\int_0^t i(c)\,dc = V(t)$

$V(t) = R\frac{di(t)}{dt} + \frac{1}{C}.t \Rightarrow i(t) = C.V_c(t)$

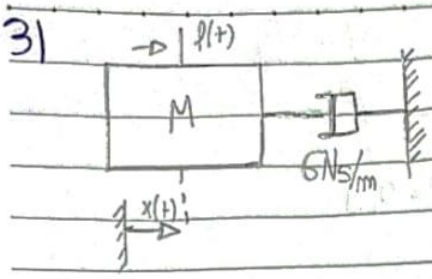
$RC\frac{dV_c(t)}{dt}.V_c(t) = V(t) \Rightarrow RCsV_c(s) + V_c(s) = V(s) \Rightarrow V_c(s)(RCs+1) = V(s)$

$\frac{V_c(s)}{V(s)} = \frac{1}{RCs+1} \Rightarrow \frac{V_c(s)}{V(s)} = \frac{1/C}{Rs+1/C}$ $\quad \frac{1}{C} = 1,27$

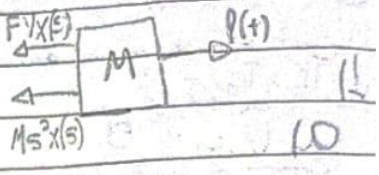
$\frac{V_c(s)}{5} = \frac{1,27}{1,8s+1,27}$ $\quad a = 1,27$ $\quad T_r = \frac{2,2}{a} = 1,73$

$T_s = \frac{4}{a} = 3,15$

3)



Simplificando



$$F(s) = 6sX(s) + Ms^2X(s) \rightarrow \frac{X(s)}{F(s)} = \frac{1}{M} \cdot \left(\frac{1}{s^2 + \frac{6s}{M}}\right)$$

$$\frac{X(s)}{F(s)} = \frac{1}{M}\left[\frac{M}{6s} - \frac{M}{6(s - \frac{6}{M})}\right]$$

$$x(t) = -\frac{M}{36} + \frac{1}{6}t + e^{-\frac{6}{M}t} \qquad x'(t) = \frac{1}{6} - \frac{6}{M}e^{-\frac{6}{M}t} \qquad x'(\infty) = \frac{1}{6}$$

$$Tr = x(0,9) + x(0,2) = -\frac{M}{6}\ln\left(\frac{M}{360}\right) \cdot \frac{M}{6}\ln\left(\frac{M}{40}\right) = \frac{M}{6}\left(\ln\left(\frac{M}{40}\right) - \ln\left(\frac{M}{360}\right)\right)$$

$$Tr = \frac{M}{6}\ln\left(\frac{M/40}{M/360}\right) = \frac{M}{6}\ln(9)$$

$$X_{lr} = 0,9 \cdot \frac{1}{6} = 0,15 \qquad x'(t) = \frac{1}{6} - \frac{6}{M}e^{-\frac{6}{M}t} = 0,15 \rightarrow \frac{0,15 - \frac{1}{6}}{-6/M} = e^{-\frac{6}{M}lr}$$

$$e^{-\frac{6}{M}lr} = \frac{M}{360} \qquad -\frac{6}{M}lr = \ln\left(\frac{M}{360}\right) \qquad lr = -\frac{M}{6}\ln\left(\frac{M}{360}\right)$$

$$X_a = 0,1 \cdot \frac{1}{6} = 0,016 \qquad 0,016 = \frac{1}{6} - \frac{6}{M}e^{-\frac{6}{M}a} \qquad \frac{0,016 - \frac{1}{6}}{-6/M} = e^{-\frac{6}{M}a}$$

$$a = -\frac{M}{6}\ln\left(\frac{M}{40}\right)$$

b) $x(T_s) = 0,98 \cdot \frac{1}{6} \cdot \frac{49}{300}$ $\quad \frac{49}{300} = \frac{1}{6} \cdot 6e^{-\frac{5}{M}t_s}$ (2)

$-\frac{1}{300} = -6e^{-\frac{5}{M}t_s}$

$\frac{M}{1800} = e^{-\frac{5}{M}t_s}$ $\quad t_s = -\frac{M}{6}\ln\left(\frac{M}{1800}\right)$

4)

a) $T(s) = \frac{2}{s+2}$ Polo -2, não tem zeros

$C(s) = \frac{1}{s} + \frac{2}{(s+2)} = \frac{A}{s} + \frac{B}{(s+2)}$ $\quad As + 2A + Bs = 2$ (1)

Se $s=0$ $\quad 2A = 2$ $\quad A = 1$, como $A+B=0$ $\quad B=-1$ (1b)

$$\boxed{C(s) = \frac{1}{s} - \frac{1}{s+2}}$$

b) $T(s) = \frac{s}{(s+3)(s+6)}$ polos -3 e -6, zeros $s=0$ (1)

$C(s) = \frac{A}{s} + \frac{B}{(s+3)} + \frac{C}{(s+6)} = s$ $\rightarrow A(s+3)(s+6) + B(s)(s+6) + C(s)(s+3) = s$ (1)

$s=0$ $\quad 18A = s$ $\quad A = \frac{s}{18}$ $\quad C(s) = \frac{s}{18} - \frac{s}{9} + \frac{s}{18}$

$s=-3$ $\quad B(-3)(-3+6) = s$ $\quad B = -\frac{s}{9}$ $\quad \frac{1}{(s+3)} \quad \frac{1}{(s+6)}$

$s=-6$ $\quad C(-6)(-6+3) = s$ $\quad C = \frac{s}{18}$ Supramortecido

c) $T(S) = \frac{10(S+7)}{(S+10)(S+20)}$ polos $-10, -20$ zeros -7

$C(S) = \frac{A}{S} + \frac{B}{(S+10)} + \frac{C}{(S+20)} = \frac{10(S+7)}{} \quad A(S+10)(S+20) + B(S)(S+20) + C(S)(S+10)$

Se $S=0 \quad 200A = 70 \quad A = \frac{7}{20}$

Se $S=-10 \quad B(-10)(-10+20) = -30 \quad B = \frac{3}{10}$

Se $S=-20 \quad C = \frac{-13}{20}$

$$C(S) = \frac{\frac{7}{20}}{S} + \frac{\frac{3}{10}}{S+10} - \frac{\frac{13}{20}}{S+20}$$

Supramortecido

d) $T(S) = \frac{20}{S^2+6S+144} \quad S^2+6S+144 = 0$

$S = \frac{-6 \pm \sqrt{36-4.1.144}}{2} = \frac{-6 \pm \sqrt{-540}}{2}$

$S = \frac{-6 \pm 6\sqrt{15}i}{2} = -3 \pm 3\sqrt{15}i$

e) $T(S) = \frac{S+2}{S^2+9}$ Polos $S^2+9=0 \quad S = \sqrt{-9} = \pm 3i$ zeros $S=-2$

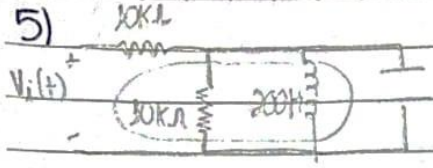
f) $T(S) = \frac{S+5}{(S+10)^2}$ Polo -10, zero -5

$C(S) = \frac{A}{S} + \frac{B}{S+10} + \frac{C}{(S+10)^2} = S+5 \rightarrow A(S+10)^2 + B(S)(S+10)^2 + C(S)(S+10) = S+5$

Se $S=0 \quad A(0+10)(0+20)^2 = 0+5 \quad A = \frac{1}{200} \quad B = \frac{1}{100} \quad C = \frac{-1}{180}$

$$C(S) = \frac{\frac{1}{200}}{S} + \frac{\frac{1}{180}}{S+10} - \frac{\frac{1}{180}}{(S+10)^2}$$

5)



$A = \frac{10K \cdot 200S}{10K + 200S} = \frac{2.10^6 S}{10K + 200S}$

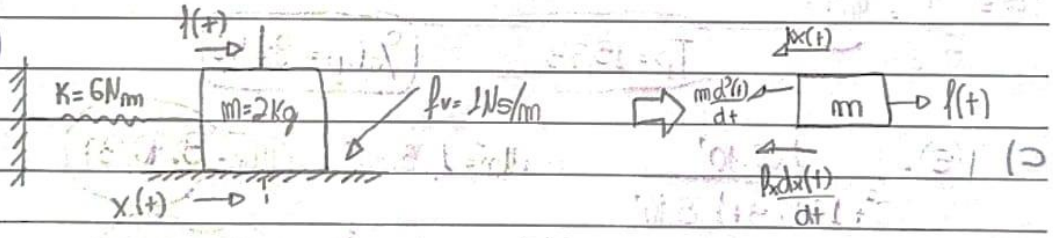
$V_o = V_i(S) \cdot \frac{2.10^6 S}{10K + 200S}$ $\quad \frac{V_o}{V_i(S)} = \frac{2.10^6 S \cdot (10K + 200S)}{(10K + 200S)\ 10K(10K + 200S) + 2.10^6 S}$

$\frac{10K + 200S}{10K + 2.10^6 S}$

$\frac{V_o}{V_i(S)} = \frac{2.10^6 S}{9.10^5 S + 100K^2}$

$\frac{V_o}{V_i(S)} = \frac{\frac{1}{50}}{\frac{1}{S} + \frac{1}{5S}}$

6)



$Kx(t) + m\frac{d^2x(t)}{dt} + fx\frac{dx(t)}{dt} = F(t)$ $\quad Kx(S) + mS^2x(S) + fvSx(S) = F(S)$

$X(S)\left[K + mS^2 + fvS\right] = F(S)$ $\quad \frac{X(S)}{F(S)} = \frac{1}{K + mS^2 + fvS} \div m$

$\frac{X(S)}{F(S)} = \frac{\frac{1}{m}}{S^2 + \frac{fv}{m}S + \frac{K}{m}}$ substituindo os valores $\quad \frac{X(S)}{F(S)} = \frac{0,5}{S^2 + 0,5S + 3}$

7)

a) $T(s) = \frac{16}{s^2+3s+16}$ $\quad \omega_m^2 = 16$ $\quad \omega_m = 4$

$2\zeta\omega_m = 3$ $\quad \zeta = 3/8$

$T_s = \frac{4}{\zeta.\omega_m} = \frac{4}{\frac{3}{8}.4}$ $\quad T_s = 8/3$ $\quad T_p = \frac{\pi}{\omega_m\sqrt{1-\zeta^2}}$ $\quad T_p = 0,85$

$\%U_p = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$ $\quad \%U_p = 28,05\%$

b) $T(s) = \frac{0,04}{s^2+0,02s+0,04}$ $\quad \omega_m^2 = 0,04$ $\quad \omega_m = 0,2$

$\zeta = \frac{0,02}{2.\omega_m} = 0,05$

$T_s = \frac{4}{\zeta.\omega_m} = 400$ $\quad T_p = 15,73$ $\quad \%U_p = 85,45$

c) $T(s) = \frac{1,05.10^7}{s^2+1,6.10^3 s+1,05.10^7}$ $\quad \omega_m^2 = 1,05.10^7$ $\quad \omega_m = 3240,37$

$2\zeta\omega_m = 1,6.10^3$ $\quad \zeta = 0,25$

$T_s = \frac{4}{\zeta.\omega_m} = 0,005$ $\quad T_p = 0,001$ $\quad \%U_p = 44,91$

8) a) $\%U_p = 10\%$, $T_s = 0,6D$ $\quad \zeta = \frac{\ln(\%U_p/100)}{\sqrt{\pi^2+\ln^2(\%U_p/100)}} = 0,56$

$T_s = \frac{4}{\omega_m\zeta}$ $\quad \omega_m = \frac{4}{T_s\zeta}$ $\quad \omega_m = 11,9$

$2.\omega_m.\zeta =$ $\quad T(s) = \frac{141,61}{s^2+13,33s+141,61}$

Polo de $T(s)$ $\quad s^2 + 13{,}33s + 141{,}61 = 0$

$$s = \frac{-13{,}33 \pm \sqrt{13{,}33^2 - 4.1.141{,}61}}{2.1} \qquad s_1 = -6{,}67 + 11{,}9\sqrt{0{,}59}\,i$$

$$s_2 = -6{,}67 - 11{,}9\sqrt{0{,}59}\,i$$

b) $\%Up = 10\%$ e $Tp = 5s$

$$\zeta = \frac{-\ln\left(\frac{10}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{10}{100}\right)}} = 0{,}59 \qquad Tp = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \qquad \omega_n = \frac{\pi}{Tp\sqrt{1-\zeta^2}} \qquad \boxed{\omega_n = 0{,}78}$$

$2.5.\omega_n = 0{,}92$ $\qquad \boxed{T(s) = \frac{0{,}61}{s^2 + 0{,}92s + 0{,}61}}$

$s^2 + 0{,}92s + 0{,}61 = 0$

$s_1 = -0{,}46 + 0{,}78\sqrt{0{,}65}\,i \qquad s_2 = -0{,}46 - 0{,}78\sqrt{0{,}65}\,i$

c) $Ts = 7seg$ e $Tp = 3seg$

$$Ts = \frac{4}{\zeta.\omega_n} = \frac{4}{\zeta\left(\frac{\pi}{Tp\sqrt{1-\zeta^2}}\right)} \qquad \frac{4}{Ts} = \frac{\zeta\pi}{Tp\sqrt{1-\zeta^2}} \qquad 4Tp\sqrt{1-\zeta^2} = \zeta\pi Ts$$

$16Tp^2(1-\zeta^2) = \pi^2 Ts^2 \zeta^2 \rightarrow 16Tp^2 - 16Tp^2\zeta^2 = \pi^2 Ts^2 \zeta^2$

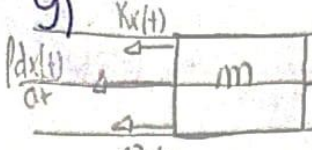
$$\zeta^2 = \frac{16Tp^2}{\pi^2 Ts^2 + 16Tp^2} \qquad \zeta = 0{,}48 \qquad \omega_n = \frac{4}{Ts\zeta} = 1{,}19$$

$$\boxed{T(s) = \frac{1{,}42}{s^2 + 1{,}14s + 1{,}42}} \qquad s^2 + 1{,}14s + 1{,}42 = 0$$

$$s = \frac{-1{,}14 \pm \sqrt{1{,}3 - 4.1.1{,}42}}{2} \qquad s_1 = 0{,}57 + \frac{\sqrt{4{,}38}}{2}\,i$$

$$s_2 = -0{,}57 - \frac{\sqrt{4{,}38}}{2}\,i$$

9)



$m\frac{d^2x(t)}{dt} + f\frac{dx(t)}{dt} + Kx(t) = F(t)$

$ms^2X(s) + FcsX(s) + KX(s) = F(s)$

(1)

$\frac{X(s)}{F(s)} = \frac{1}{ms^2+Fcs+K} = \frac{1/m}{s^2+\frac{Fc}{m}s+\frac{K}{m}} = \frac{0,2}{s^2+0,4s+4}$

$w_n^2 = 4$ | $w_n = 2$ | $2\zeta w_n = 0,4$ | $\zeta = 0,1$

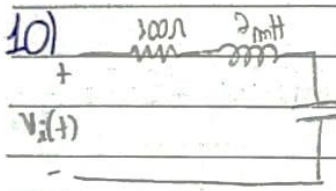
(1)

$\%U_p = e^{\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)} \cdot 100 = 72,9\%$ | $T_p = \frac{\pi}{w_n\sqrt{1-\zeta^2}} = 1,58$

$T_s = \frac{4}{\zeta w_n} = 20$

(1)

10)



$G(s) = \frac{V_o(s)}{V_i(s)}$

$V_o = V_i \frac{100.10^{-9}/s}{\left(100 + 2.10^{-3}s + \frac{100.10^{-9}}{s}\right)}$

$\frac{V_o}{V_i} = \frac{100.10^{-9}}{2.10^{-3}s^2 + 100s + 100.10^{-3}} \div 2.10^{-3}$

$\frac{V_i}{V_o} = \frac{s^2 + 5.10^4 s + 50}{5.10^{-5}} = G(s)$

$w_n = 223,61$ | $\zeta = 0,11$

$T_s = 0,16$ | $T_p = 0,014$ | $\%O_p = 70,28\%$

11) %Mp = 20% $\zeta = \frac{-\ln(\frac{20}{100})}{\sqrt{\pi^2 + \ln^2(\frac{20}{100})}} = 0,46$

$2.\zeta.\omega_n$

$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \rightarrow \omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}}$ ω_n^2

Preto: $T_p = 0,2.\frac{2}{3} = 0,133\,seg$ $\omega_n = 26,54$ $T(s) = \frac{704,4}{s^2 + 24,45 + 704,4}$

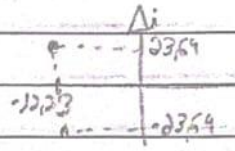
Azul: $T_p = 0,2\,seg$ $\omega_n = 17,65$ $T(s) = \frac{311,5}{s^2 + 16,26 + 311,5}$

Azul 2: $T_p = 0,3\,seg$ $\omega_n = 11,77$ $T(s) = \frac{138,5}{s^2 + 10,85 + 138,5}$

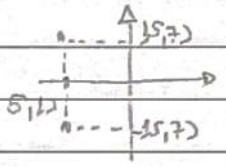
Vermelho: $T_p = 0,4\,seg$ $\omega_n = 8,82$ $T(s) = \frac{77,8}{s^2 + 8,15 + 77,8}$

Vermelho 2: $T_p = 0,5\,seg$ $\omega_n = 7,06$ $T(s) = \frac{49,8}{s^2 + 6,55 + 49,8}$

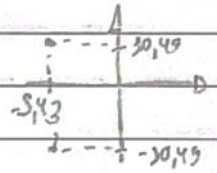
Preto: $S_{1,2} = -12,23 \pm 25,64\sqrt{0,73}\,i$



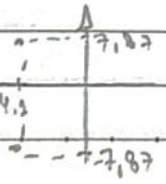
Azul: $S_{1,2} = -8,14 \pm 15,72\,i$



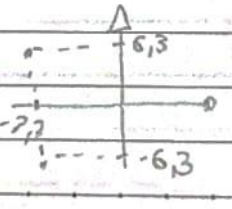
Azul 2: $S_{1,2} = -5,43 \pm 10,49\,i$



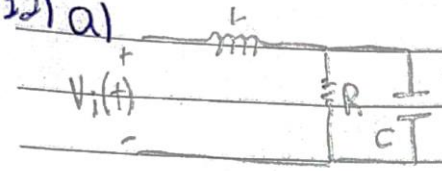
Vermelho: $S_{1,2} = -4,1 \pm 7,87\,i$



Vermelho 2: $S_{1,2} = -3,3 \pm 6,3\,i$



12) a)



$i = \frac{V_o(t)}{R}$ $\quad V_i(t) = V_L(t) + V_o(t)$

$V(s) = LCs^2 V_o(s) = \frac{Ls}{R} V_o(s) + V_o(s)$

$$\frac{V_o}{V_i} = \frac{1}{LCs^2 + \frac{Ls}{R} + 1}$$

b) $\%U_p = 36\%$ $\quad \zeta = \frac{-\ln(\frac{36}{100})}{\sqrt{\pi^2 + \ln(\frac{36}{100})^2}} = 0,32$ $\quad T_p = 0,3.10^{-4}$

$\omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}} = 99196,6$ $\quad T_s = \frac{4}{\zeta\omega_n} = 1,32.10^{-4}$