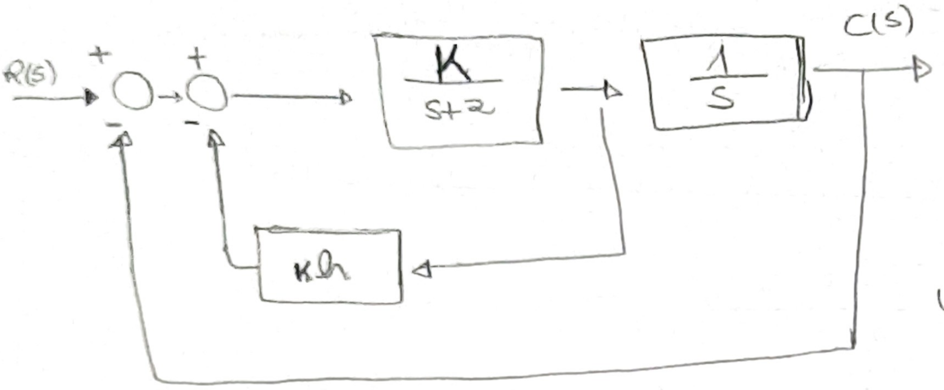
Sistemas de Controle 1 - Lista 3

1)



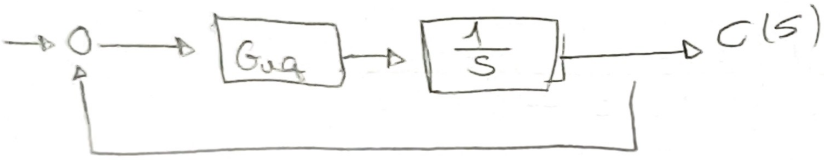
$K = ?$

$K_h = ?$

$\zeta = 0{,}7$

$\omega_n = 4 \text{ rad/s}$

$$G_{eq}(s) = \frac{\frac{K}{s+2}}{1 + \frac{K \cdot K_h}{s+2}} = \frac{K}{s+2+K \cdot K_h}$$



$$T(s) = \frac{\frac{K}{s(s+2+K \cdot K_h)}}{1 + 1 \cdot \frac{K}{s(s+2+K \cdot K_h)}}$$

$$T(s) = \frac{K}{s^2 + s(2+K K_h) + K}$$

$K = \omega_n^2$

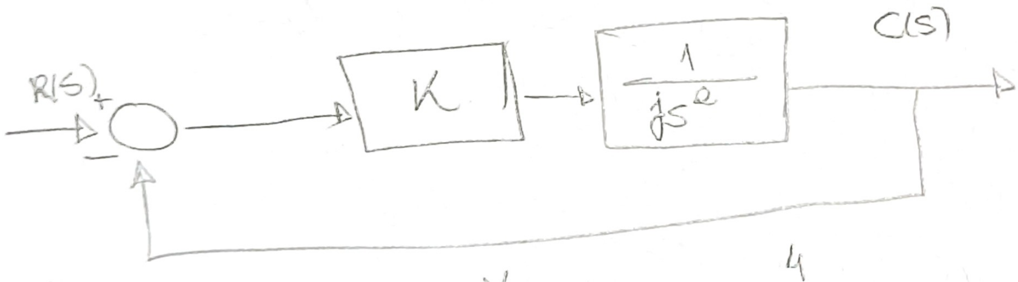
$K = 16$

$2 + K h = 2 \zeta \omega_n$

$16 h = 2 \cdot 0{,}7 \cdot 4 - 2$

$h = \frac{3{,}6}{16} = 0{,}225$

2)



$\frac{K}{J} = 4$

$\zeta = 0{,}6$

$K_h = ?$

$$G_{eq} = \frac{K/Js}{1 + \frac{K}{Js} \cdot K_h} = \frac{K}{Js + K K_h} = \frac{4}{s + 4h}$$

$$T(s) = \frac{\frac{1}{s} \cdot \frac{4}{s+4h}}{1 + \frac{4}{s+4h} \cdot \frac{1}{s}} = \frac{4}{s^2 + 4hs + 4} \rightarrow \omega_n^2 = 4$$

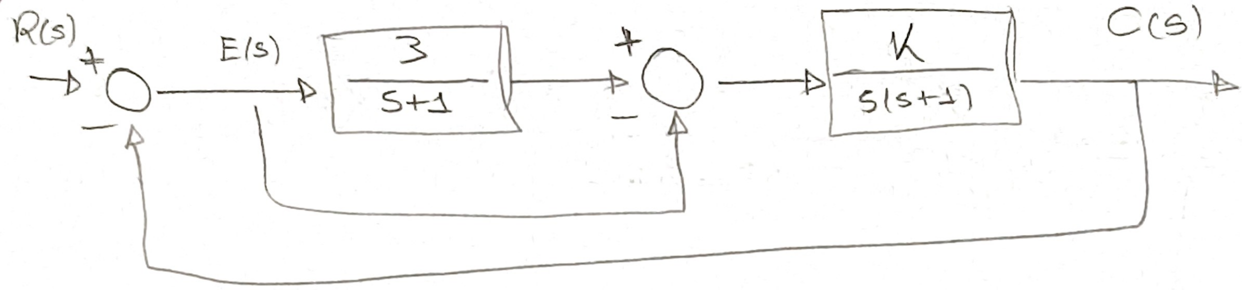
$\omega_n = 2$

$4h = 2 \zeta \omega_n$

$h = \frac{2 \cdot 0{,}6 \cdot 2}{4}$

$h = 0{,}6$

3)



$$G_{eq} = \frac{3}{s+1} - 1 = \frac{3-s-1}{s+1}$$

$$G_{eq}(s) = \frac{(-s+2)}{(s+1)} \qquad T(s) = \frac{\frac{K}{s(s+1)} \cdot \frac{(-s+2)}{(s+1)}}{1 + \frac{K(-s+2)}{s(s+1)(s+1)}} =$$

$$T(s) = \frac{K(-s+2)}{s(s+1)^2 \cdot K(-s+2)} = \frac{K(-s+2)}{s(s^2+2s+1) - Ks + 2K}$$

$$T(s) = \frac{K(-s+2)}{s^3+2s^2+s(1-K)+2K}$$

$$\begin{array}{c|cc} s^3 & 1 & 1-K \\ s^2 & 2 & 2K \\ s^1 & b_1 & b_2 \to 0 \\ s^0 & c_1 & c_2 \to 0 \end{array} \qquad b_1 = \frac{\begin{vmatrix} 1 & 1-K \\ 2 & 2K \end{vmatrix}}{2}$$

$$c_1 = -\frac{\begin{vmatrix} 2 & 2K \\ -2K-1 & 0 \end{vmatrix}}{-2K-1} = \frac{-(-2K(-2K-1))}{(-2K-1)} = 2K$$

I) $-2K+1>0$

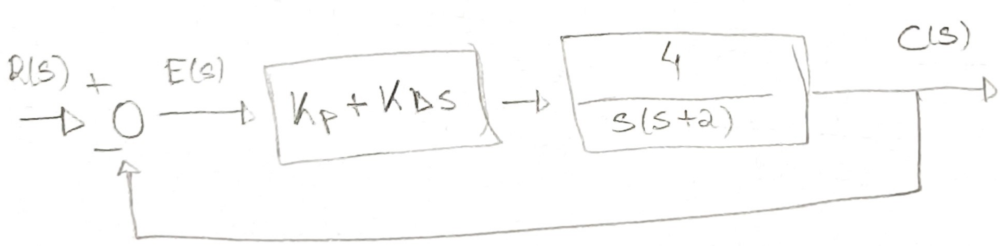
$2K<1 \quad K<\frac{1}{2}$

II) $2K>0$

$K>0$

$0<K<\frac{1}{2}$

4)



$$G_{eq}(s) = \frac{(K_p + K_D s)}{s(s+2)} \cdot 4 \qquad T(s) = \frac{4(K_p + K_D s)}{s(s+2)+4(K_p - K_D s)} = \frac{4(K_p+K_D s)}{s^2+s(2+4K_D)+4K_p}$$

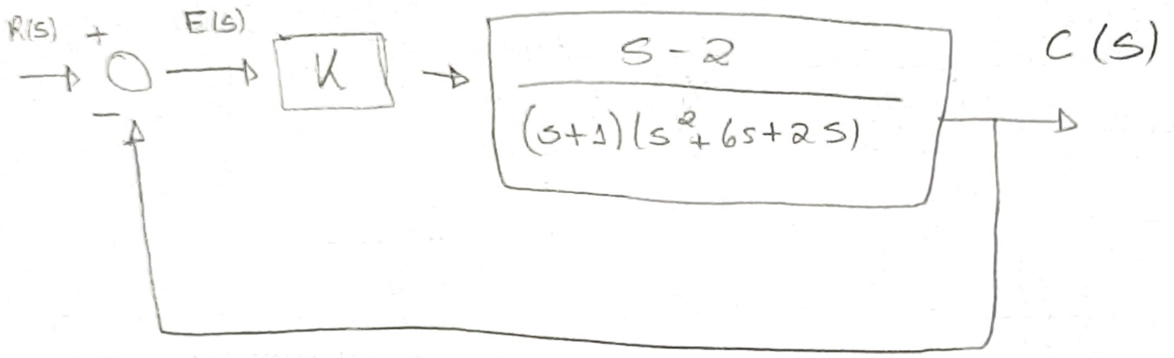
$$\begin{array}{c|cc} s^2 & 1 & 4K_p \\ s^1 & 2+4K_D & 0 \\ s^0 & b_1 & b_2 = 0 \end{array}$$

$$b_1 = -\frac{\begin{vmatrix} 1 & 4K_p \\ 2+4K_D & 0 \end{vmatrix}}{2+4K_D} = \frac{4K_p(2+4K_D)}{(2+4K_D)} = 4K_p$$

I) $2+4K_D>0 \quad K_D > -\frac{1}{2}$

II) $4K_p>0 \quad \boxed{K_p>0}$

5)



$$T(s) = \frac{\dfrac{K(s-2)}{(s+1)(s^2+6s+25)}}{1 + \dfrac{K(s-2)}{(s+1)(s^2+6s+25)}} = \frac{K(s-2)}{s^3+7s^2+s(31+K)+(25-2K)}$$

$$\begin{array}{c|cc} s^3 & 1 & 31+K \\ s^2 & 7 & 25-2K \\ s^1 & b_1 & \cancel{b_2}\ 0 \\ s^0 & c_1 & \cancel{c_2}\ 0 \end{array}$$

$$b_1 = \frac{-\begin{vmatrix} 1 & 31+K \\ 7 & 25-2K \end{vmatrix}}{7} = \frac{-(25-2K-217-7K)}{7}$$

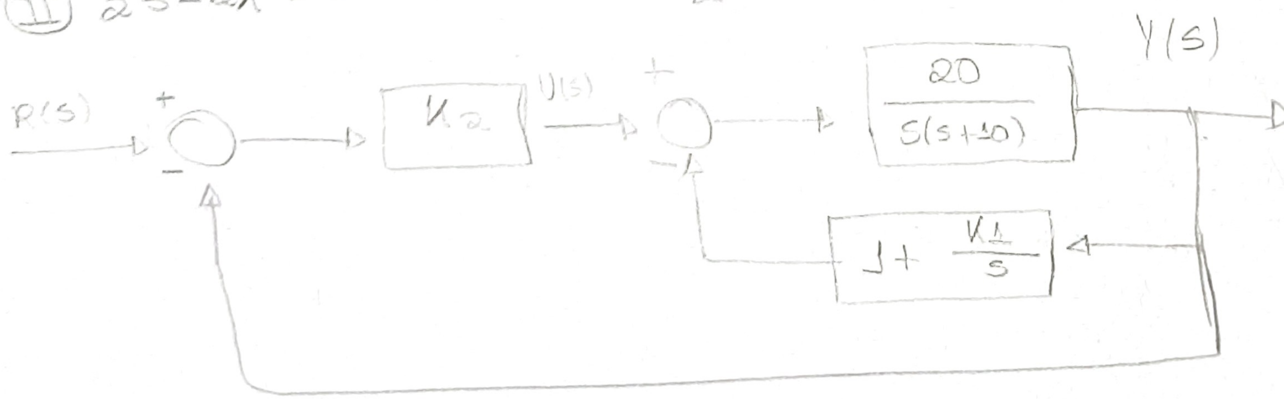
$$b_1 = \frac{9K+192}{7}$$

$$c_1 = \frac{-\begin{vmatrix} 7 & 25-2K \\ \frac{9K+120}{2} & 0 \end{vmatrix}}{\frac{9K+190}{7}} = \frac{-\left(-(25-2K)\left(\frac{9K+192}{7}\right)\right)}{\left(\frac{9K+192}{7}\right)} = 25-2K$$

I $\dfrac{9K+192}{7} > 0$ $\qquad K > \dfrac{64}{3}$ $\qquad \dfrac{64}{3} < K < \dfrac{25}{2}$

II $25-2K > 0$ $\qquad K < \dfrac{25}{2}$

6)



(a) $$G_{eq}(s) = \frac{\dfrac{20}{s(s+10)}}{1 \cdot \left[\dfrac{20}{s(s+10)} \cdot \left(1+\dfrac{K_1}{s}\right)\right]} = \frac{20}{s(s+10)} \cdot \frac{s^2(s+10)}{s^2(s+10)+20s+20K_1} *$$

$$1 \cdot \left[\frac{20}{s(s+10)} + \frac{20K_1}{s^2(s+10)}\right] \qquad * \; G_{eq} = \frac{20s}{s^3+10s^2+20s+20K_1}$$

$$\begin{array}{lll} s^3 & 1 & 20 \\ s^2 & 10 & 20K_1 \\ s^1 & b_1 & \cancel{b_2} \to 0 \\ s^0 & c_1 & c_2 \end{array}$$

$$b_1 = \frac{-\begin{vmatrix} 1 & 20 \\ 10 & 20K_1 \end{vmatrix}}{10} = \frac{-(2\cancel{0}K_1 - 20\cancel{0})}{1\cancel{0}} = -2K_1+20$$

$$c_1 = \frac{-\begin{vmatrix} 10 & 20K_1 \\ -2K+20 & 0 \end{vmatrix}}{-2K+20} = \frac{-(-20K_1(-2K_1+20))}{(2K_1+20)} = c_1 = 20K_1$$

(I) $-2K_1+20>0$

$K_1<10$ $\qquad 0<K_1<10$

(II) $20K_1>0$

$K_1>0$

b) $0<K<10 \rightarrow \boxed{K=5}$

c) $$T(s) = \frac{20s\;K_2}{s^3+10s^2+s(20+20K_2)+100}$$

$$c_1 = \frac{-(-(20K_2+10) \cdot 100)}{20K_2+10} = 100$$

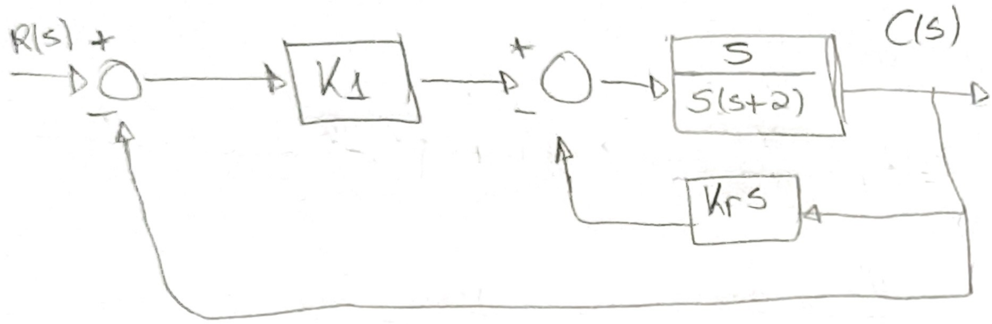
$$b_1 = \frac{-(\cancel{100} - 20\cancel{0} - 20\cancel{0}K_2)}{1\cancel{0}} = 20K_2+10$$

(I) $20K_2>10$

$K_2>\frac{1}{2}$

$$\begin{array}{l|ll} s^3 & 1 & 20+20K_2 \\ s^2 & 10 & 100 \\ s^1 & b_1 & \cancel{b_2} \to 0 \\ s^0 & c_1 & \cancel{c_2} \to 0 \end{array}$$

7)



$K_V = 20$

$\zeta = 0{,}7$

$$G(s) = \frac{\frac{5}{s(s+2)}}{1 + \frac{5K_r}{s(s+2)}} = \frac{5}{s^2+2s+5K_r} = K_V = \lim_{s\to 0} \cancel{s} \cdot \frac{5}{\cancel{s}\left(s+2+\frac{5K_F}{s}\right)} + \infty$$

$$\boxed{T(s) = \frac{5K_2}{s^2+2s+5K_r+5K_2}}$$

8)

$$G(s) = \frac{K(s+\alpha)}{(s+\beta)^2} \qquad E(s) = \frac{R(s)}{1+G(s)} = \frac{1}{s}\left(1+\frac{1}{\frac{K(s+\alpha)}{(s+\beta)^2}}\right)$$

$$= \frac{1}{s} \cdot \frac{s^2+2\beta s+\beta^2}{s^2+s(2\beta+K)+\beta^2+K\alpha} \qquad K_p = \lim_{s\to 0} G(s)$$

$e_{\infty} = 0{,}1$

$\zeta = 0{,}5$

$\omega_n = \sqrt{10}$

$\alpha = ?$

$\beta = ?$

$K = ?$

$$= \lim_{s\to 0} \frac{K(s+\alpha)}{(s+\beta)^2} = \frac{K\alpha}{\beta^2} \qquad e = (\infty) = \frac{1}{1+K_p}$$

$$0{,}1 = \frac{1}{1+\frac{K\alpha}{\beta^2}} = 0{,}1 + 0{,}1\frac{K\alpha}{\beta^2} = 1$$

$$0{,}1\frac{K\alpha}{\beta} = 0{,}9$$

$$K\alpha = 9\beta^2$$

$$T(s) = \frac{K(s+\alpha)}{s^2+s(2\beta+K)+\underbrace{\beta^2+K\alpha}_{\omega_n^2 = \beta^2+K\alpha}}$$

$10 = \beta^2 + 9\beta^2$

$\beta = \pm 1$

$2\beta + K = 2\zeta\omega_n$

$2 \cdot 1 + K = 3{,}16$

$K = 1{,}16$

$2(-1) + K = 3{,}16$

$K = 5{,}16$

$\beta = \pm 1$

$p/K = 1{,}16 \quad \alpha = 7{,}76$

$p/K = 5{,}16 \quad \alpha = 1{,}74$

9) $W_n = 10$

a) entrada rampa, tipo 1

b) $e(\infty) = \frac{1}{K_V}$ $\quad G(s) = \frac{K}{s(s+a)} \rightarrow K_V = \lim_{s \to 0} \cancel{s} \cdot \frac{K}{\cancel{s}(s+a)} = \frac{K}{a}$

$e(\infty) = \frac{1}{0{,}01}$ $\quad \frac{K}{a} = 100$ $\quad \frac{100}{a} = 100$

$\frac{K}{a} = 100$ $\quad \boxed{a = 1}$

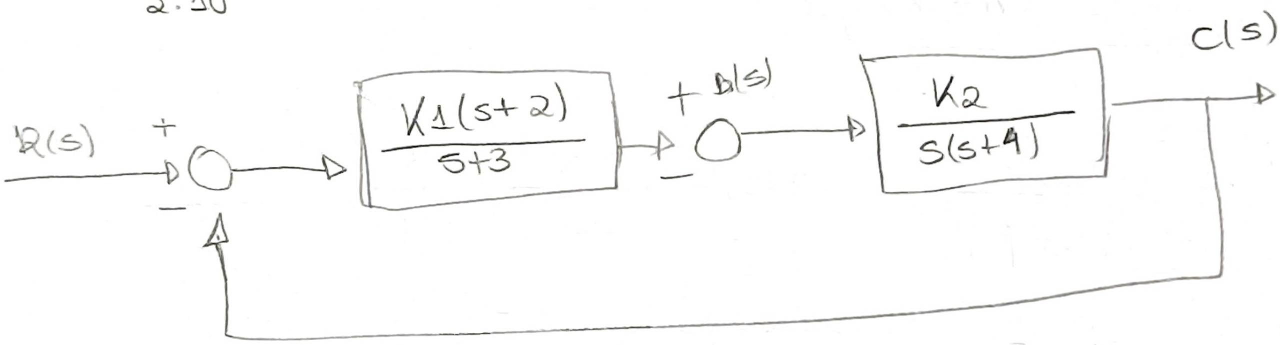
$T(s) = \frac{K}{s^2 + sa + K}$ $\quad W_n^2 = K$

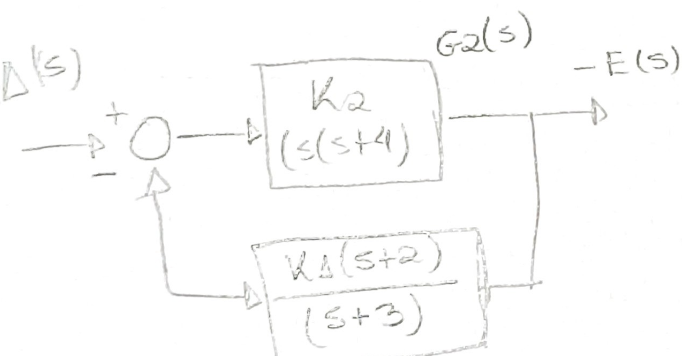
$K = 100$

c) $2 \zeta W_n = 1$

$\zeta = \frac{1}{2 \cdot 10}$ $\quad \zeta = 0{,}05$

10)





$-E(s) = \frac{G_2(s)}{1 + G_2(s) G_1(s)} \cdot D(s)$

$-e_D(\infty) = \frac{1}{\lim_{s \to 0} \frac{1}{G_2(s)} + \lim_{s \to 0} G_1(s)} \rightarrow -e_1(\infty) = \frac{1}{\lim_{s \to 0} \frac{1}{\frac{K_2}{s(s+1)}} + \lim_{s \to 0} G_1(s)}$

$\boxed{K_1 = 1{,}5 \times 10^5}$

$e(\infty) = \frac{1}{K_V}$

$K_V = \lim_{s \to 0} s G(s)$ $\quad K_V = \lim_{s \to 0} s \cdot \left(\frac{G_1(s)}{(1 + G_1 G_2)} \right)$

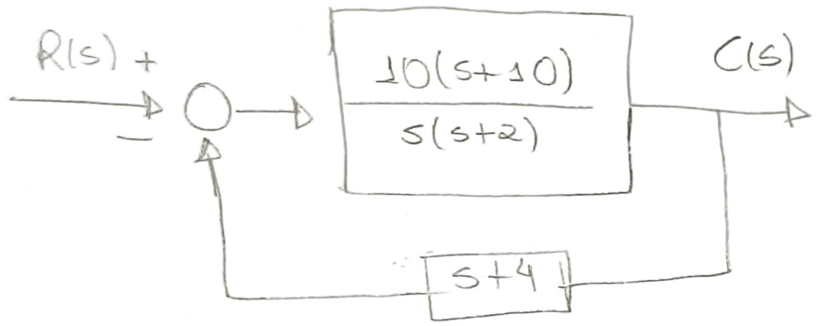
$K_V = \lim_{s \to 0} s \cdot \left(\frac{\frac{K_1(s+2)}{s+3}}{1 + \frac{K_1(s+2)}{s+3} \cdot \frac{K_2}{(s(s+4))}} \right) \Rightarrow$

$$\Rightarrow K_V = \lim_{s\to 0} s \cdot \left[\frac{K_1(s+2)}{(s+3)} \cdot \left(\frac{(s+3)(s^2+4s)}{(s+3)(s^2+4s)} + \frac{K_1(s+2)(s^2+4s)}{(s+3)(s^2+4s)} + \frac{K_2(s+3)}{(s+3)(s^2+4s)} \right) \right]$$

$$K_V = \lim_{s\to 0} \frac{\cancel{2} K_2 K_1}{4\cancel{s}} \qquad K_V = \frac{K_2 K_1}{2} \qquad \boxed{K_2 = 0{,}0067}$$

$$0{,}002 = \frac{2}{K_2 K_1}$$

11)



$$G_{eq}(s) = \frac{\dfrac{10(s+10)}{s(s+2)}}{1 + \dfrac{10(s+10)(s+4)}{s(s+2)} - \dfrac{10(s+10)}{s(s+2)}}$$

$$G_{eq} = \frac{10(s+10)}{s(s+2)+10(s+10)(s+4)-10(s+10)} = \frac{10(s+10)}{s^2+2s+10(s^2+14s+40)-10s-100}$$

$$= \frac{10(s+10)}{11s^2+132s+300} \qquad K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{1\cancel{00}}{3\cancel{00}} = \frac{1}{3}$$

a) tipo 0 $\qquad$ b) $K_p = \frac{1}{3}$ $\qquad K_v = K_a = 0$

c) entrada em degrau

d) $e(\infty) = \dfrac{1}{1+K_p} \qquad \dfrac{1}{1+\frac{1}{3}} = \dfrac{1}{\frac{4}{3}} = \boxed{\dfrac{3}{4}}$

e) $$e(\infty) = \lim_{s\to 0} \frac{\cancel{s}\left(\dfrac{1}{\cancel{s}}\right)}{1 + \dfrac{10(s+10)(s+4)}{s(s+2)}} = \frac{1}{\dfrac{s(s+2)+10(s+20)(s+4)}{s(s+2)}} = 0$$