PROVA - SC25 CP

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① a) F

b) F

c) V

d) F

e) F

f) V

② CONSIDERANDO O SISTEMA, TEMOS

$$\begin{cases} y_1(s)\,[s^2 M_1 + s b_1 + K_1] - y_2(s)\,[K_{12}] = -a\,\text{sen}(w_0 t) \\ y_2(s)\,[s^2 M_2 + K_{02}] - y_1(s)\,[K_{12}] = 0 \end{cases}$$

REESCREVENDO A EDO DO SISTEMA, TEMOS:

$$\begin{cases} \dfrac{d y_1(t)^2}{dt^2}.M_1 + \dfrac{d y_1(t)}{dt}.b_1 + y_1(t).K_1 - y_2(t).K_{12} = f(t) \\ \dfrac{d y_2(t)^2}{dt^2}.M_2 + y_2(t).K_{12} - y_1(t).K_{12} = 0 \end{cases}$$

CONSIDERANDO $\dfrac{d\,y_1(t)}{dt} = y_1' = V_1$ e $\dfrac{dy_2(t)}{dt} = y_2' = V_2$

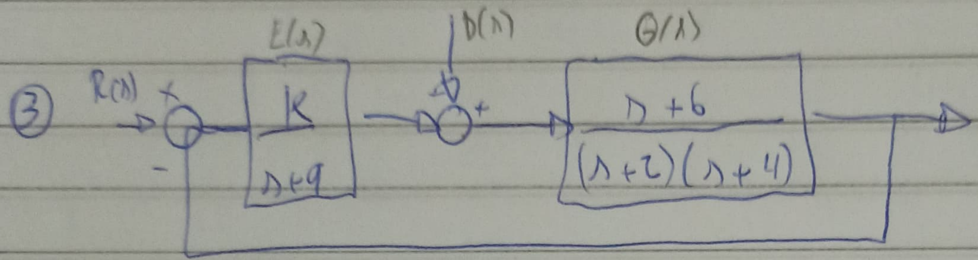
$$V_1' = -[V_1.b_1 + y_1.K_1 - y_2 K_{12} - f(t)]\,/M_1$$

$$V_2' = -[y_2.K_{12} - y_1.K_{12}]\,/M_2$$

$$\begin{cases} V_1'.M_1 + V_1.b_1 + y_1 K_1 - y_2.K_{12} = f(t) \\ V_2'.M_2 + y_2.K_{12} - y_1 K_{12} = 0 \end{cases}$$

$$\begin{bmatrix} \dot{y}_1 \\ v_1 \\ \dot{y}_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K_{11}+K_{12}/m_2 & b_1/m_1 & -K_{12}/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ K_{12}/m_2 & 0 & -K_{12}/m_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ v_1 \\ y_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 6 \end{bmatrix} a \cdot \text{sen}(w_0 t)$$

③



$$T(s) = E(s) \cdot G(s) \Rightarrow \frac{K(s+6)}{(s+9)(s+2)(s+4)}$$

$$G_{MF} = \frac{K(s+6)}{(s+9)(s+2)(s+4)+K(s+6)} \Rightarrow \frac{K(s+6)}{s^3+15s^2+s(K+62)+72+6K}$$

s^3	1	$K+62$
s^2	15	$72+6K$
s^1	$(9K+858)/15$	0
s^0	$72+6K$	

$a_1 = 15(K+62)-(72+6K)/15$

$a_1 = 15K+930-72-6K$

$a_1 = 9K+858/15$

$\frac{9K+858}{15} > 0 \Rightarrow K > -858/9$

$72+6K > 0 \Rightarrow K > -12$

a) $K > -12$ p/ ser estável

$$K_p = 20 = \lim_{s \to 0} \frac{K(s+6)}{(s+9)(s+2)(s+4)}$$

b) $20 = \frac{6K}{9 \cdot 2 \cdot 4} \Rightarrow K = 240$

c) $-e_D(\infty) = \dfrac{1}{\dfrac{1}{\lim\limits_{s\to 0} \dfrac{s+6}{(s+2)(s+4)}} + \lim\limits_{s\to 0} \dfrac{K}{s+9}}$ $\quad K = 240$

$-e_D(\infty) = \dfrac{1}{\dfrac{1}{\frac{6}{8}} + \dfrac{240}{9}} \Rightarrow 0,035 \Rightarrow 3,5\% = e_D(\infty)$

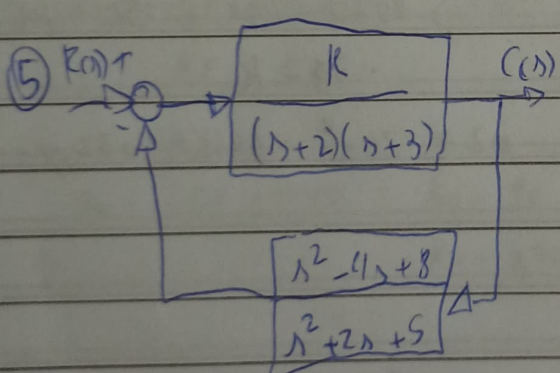
4)

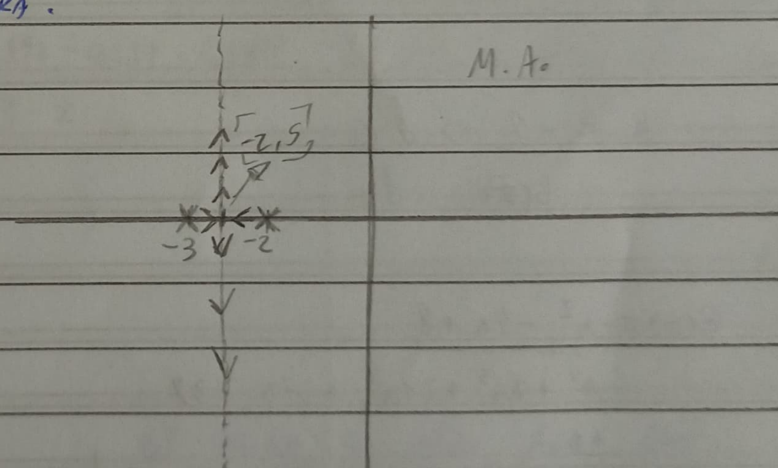
a) $G_M = 25\ dB$

b) $\Phi_M = 112,5^{\circ}$

c) SIM, POIS SE O SISTEMA EM MALHA ABERTA É ESTÁVEL, EM MALHA FECHADA TAMBÉM SERÁ.

5)





$\sigma_a = \dfrac{-5+0}{2} = -2,5$

$\theta_a = \dfrac{(2k+1)180}{2} \Rightarrow k_0 = 90$

$k_1 = 270$

$MF = \dfrac{K(s^2-4s+8)}{s^4 + 7s^3 + (21+K)s^2 + (37-4K)s + 38+K}$

s^4	1	$21+k$	$38+k$
s^3	7	$37-4k$	0
s^2	$(110+5k)/7$	$38+k$	
s^1	$-20k^2-256k+4032$	0	
s^0	$38+k$		

$$a1 = \frac{7(21+k)-(37-4k)}{7}$$

$$a1 = \frac{110+5k}{7}$$

$$a2 = \frac{\left(\frac{110+5k}{7}\right)\cdot(37-4k) - 7.(38+k)}{\frac{110+5k}{7}}$$

$$a2 = 37-4k - \frac{\cancel{7}(38+k)}{\frac{110+5k}{\cancel{7}}}$$

$$a2 = 37-4k - \frac{38+k}{110+5k}(110+5k)$$

$$a2 = (37-4k).(110+5k) - 38+k$$

$$\Rightarrow 4070-440k+185k-20k^2$$
$$-38-k$$

$$a2 = -20k^2-256k+4032$$

$$-20k^2-256k+4032 > 0$$

$r_1 = -21,974$

$r_2 = 9,174$

$$[-21,974 < k < 9,174]$$

c)

$$k = \frac{-1}{G(\sigma)} \Rightarrow \frac{dk}{d\sigma}$$

$$G(\sigma) = \frac{s^2-4s+8}{s^4+7s^3+21s^2+37s+38}$$

$$\Rightarrow \frac{dk}{d\sigma} = \left(\frac{\sigma^4+7\sigma^3+21\sigma^2+37\sigma+38}{\sigma^2-4\sigma+8}\right)' \Rightarrow \frac{4\sigma^3+21\sigma^2+42\sigma+37}{2\sigma-4}$$

$\sigma_1 = -2,56$

$\sigma_{2,3} = -1,34 \pm 1,34j$

AGORA PARA MALHA FECHADA TEMOS

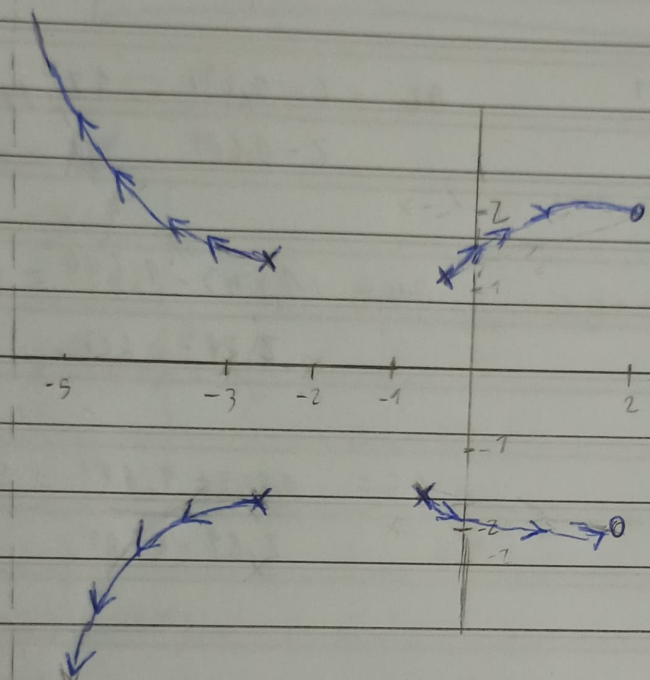
$$G(s) = \frac{(s^2 - 4s + 8)}{s^4 + 7s^3 + 22s^2 + 33s + 38}$$

P/ K = 1

Polos = $-2{,}892 \pm 1{,}833$

e = $-0{,}607 \pm 1{,}694$

Zeros = $2 \pm 2i$



$$\theta_a = \frac{\Sigma p - \Sigma z}{\#p - \#z} = \frac{-2{,}892 - 2{,}892 - 0{,}607 - 0{,}607 - (2+2)}{4 - 2} = 5{,}499$$

$$\theta_a = \frac{(2K+1)180}{2} = K_0 = 90 \qquad K_1 = 270$$

b) $-27{,}974 < K < 9{,}774$

CORTA JW

c) SAÍDA DO EIXO REAL EM

$\sigma = -2{,}56$ //

d) POLO: $-2{,}892 \pm 1{,}833 \Rightarrow$

ANGULO DE SAÍDA $\theta = 71{,}4^0$ //

POLO: $-0{,}607 \pm 1{,}694$

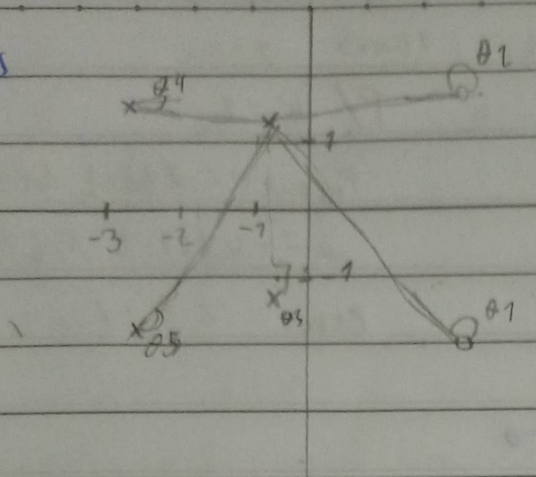
ANGULO DE SAÍDA $\theta = 61{,}52^0$ //

e) $K = 250$

PARA $\zeta = 0{,}3$ e

POLOS DOMINANTES = $-5{,}22 \pm 16{,}5i$ //

ANGULOS DA SAÍDA



$\theta_1 = \dfrac{2-0,607}{2+1,694} = 20°$

$\theta_1 = 160°$

$\theta_2 = \dfrac{2-1,694}{2-0,607} = 12°$

$\theta_4 = \dfrac{1,833-1,694}{2,89-0,607} = 3,48°$

$\theta_5 = \dfrac{1,833+1,694}{2,89-0,607} = 57°$

$tg = \dfrac{CO}{CA}$

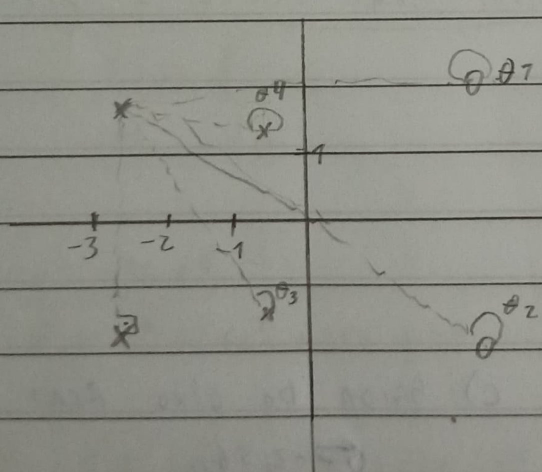
Polo $-0,607 \pm 1,694$

$\Sigma_{\theta z} - \Sigma_{\theta p} = 180$

$32° - 3,48+57+90+\theta = 180$

$\theta = 180° + 118,43°$

SAÍDA $\theta = 61,52°$



$\theta_1 = \dfrac{2-1,83}{2+2,89} = 2°$

$\theta_2 = \dfrac{2+1,83}{2+2,89} = 38° - 180$

$\theta_2 = 142°$

$\theta_3 = \dfrac{1,69+1,89}{2,89-0,607} = 122,5°$

$\theta_4 = \dfrac{2,89-0,607}{1,83-1,694} = 3,4°$

POLO $-2,89 \pm 1,83$

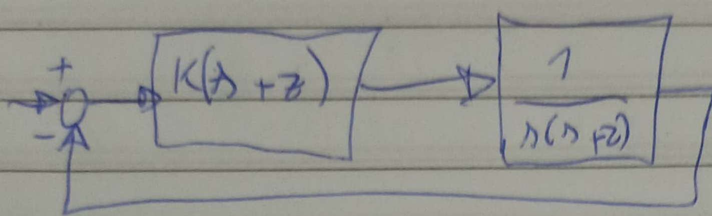
$\Sigma_{\theta z} - \Sigma_{\theta p} = 180$

$182° + 142° - (122,5° + 3,4° + 90° + \theta) = 180$

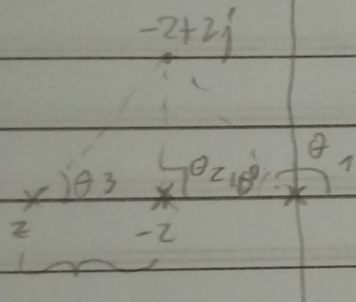
$\theta = 180 - 108,1$

SAÍDA $\theta = 71,9°$

⑥



a) $\theta_3 = 45^\circ$



$\theta_1 = \frac{2}{2} \Rightarrow \theta_1 = 135^\circ$

$\theta_1 = 90$

$\angle \left. \frac{1}{s(s+2)} \right|_{s=-2+2j}$

$\Rightarrow 135^\circ$

$\Phi = 180^\circ - 135^\circ = 45^\circ$

$\theta_3 = 45^\circ$

$x = \frac{2}{tg(\Phi)} = 2$ Logo $z = 2+2$

$z = -4$

$K = \frac{1}{\left| \frac{s+z}{} \cdot \frac{1}{s(s+2)} \right|_{s=-2+2j}} \Rightarrow K = 2$

b) $z = -4$

c) $K = 2$