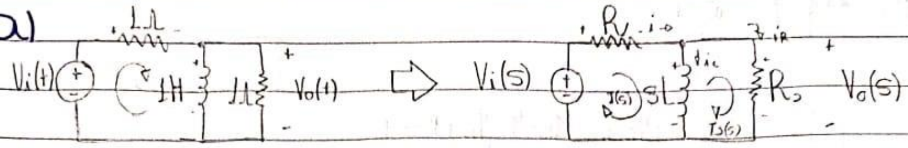
$Z_C = \frac{1}{sC}$ $\quad R \quad$ $Z_L = sL$

Lista 1)

$V_C(s) = \frac{1}{Cs} I(s)$ $\quad V_R(s) = R \cdot I(s)$ $\quad V_L(s) = Ls \cdot I(s)$

1) $G(s) = \frac{V_i(s)}{V_o(s)}$

a)



$-V_i + R_1 I_1(s) + sL I_1(s) - sL I_2(s) = 0$ $\qquad R_2 I_2(s) + sL I_2(s) - sL I_1(s) = 0$ $\qquad V_O = R_2 \cdot I_2(s)$

① $V_i(s) = I_1(s+1) - s I_2(s)$ $\qquad$ ② $I_2(s+1) - s I_1(s) = 0$ $\qquad$ $V_o = I_2(s)$

①+②

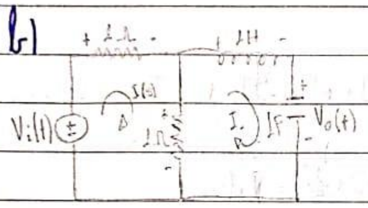
$$\begin{bmatrix} s+1 & -s \\ -s & s+1 \end{bmatrix} \begin{bmatrix} V_1 \\ 0 \end{bmatrix} \quad \Delta = s^2 + 2s + 1 - s^2 = 2s + 1$$

$V_O = I_2 = \frac{\Delta I_2}{\Delta} = \frac{s V_i(s)}{2s+1}$ $\qquad$ ② $\begin{vmatrix} 1+s & V_i \\ -s & 0 \end{vmatrix}$

$\Delta I_2 = s V_i$

$$\boxed{\frac{V_i(s)}{V_O(s)} = \frac{2s+1}{s}}$$

b)



① $-V_i(s) + I_1 + I_1 - I_2$ $\qquad$ ② $s I_2 + \frac{1}{s} I_2 + I_2 - I_1 = 0$

$2I_1 - I_2 = V_i(s)$ $\qquad$ $-I_1 + I_2\left(s + \frac{1}{s} + 1\right) = 0$

①+②

$$\begin{bmatrix} 2 & -1 \\ -1 & \frac{s^2+s+1}{s} \end{bmatrix} = \begin{bmatrix} V_i(s) \\ 0 \end{bmatrix} \quad \Delta = \frac{2s^2 + 2s + 2}{s} - 1 = \frac{2s^2+s+2}{s}$$

$V_O(s) = \frac{1}{s} I_2(s)$ $\qquad$ $I_2 = \frac{\Delta I_2}{\Delta}$

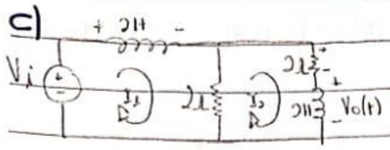
$I_2 = \frac{V_i(s) \cdot s}{2s^2+s+2}$

$$\begin{bmatrix} 2 & V_i \\ -1 & 0 \end{bmatrix} \quad \Delta I_2 = V_i(s)$$

$\frac{V_O(s)}{1} = \frac{V_i(s)}{2s^2+s+2}$

$$G(s) = \frac{V_i}{V_O} = \frac{2s^2+s+2}{1}$$

c)



① $-V_i + 2sI_1 + 2I_1 - 2I_2 = 0$

$I_1(2s+2) - 2I_2 = V_i$

② $2I_2 + 2sI_2 + 2I_2 - 2I_1 = 0$

$-2I_1 + I_2(4+2s) = 0$

$\boxed{V_0(s) = 2s\,I_2(s)}$

$$\begin{bmatrix} 2s+2 & -2 \\ -2 & 4+2s \end{bmatrix} = \begin{bmatrix} V_i \\ 0 \end{bmatrix} \quad \Delta = 8s + 4s^2 + 8 + 4s - 4 = \boxed{4s^2+12s+4}$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{2V_i}{4s^2+12s+4} = \frac{V_i}{2s^2+6s+2}$$

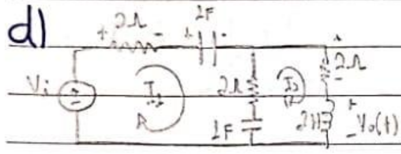
$$\begin{bmatrix} 2s+2 & V_i \\ -2 & 0 \end{bmatrix} \quad \Delta I_2 = 2V_i(s)$$

$$V_0(s) = 2s \cdot \frac{V_i}{2s^2+6s+2}$$

$$V_0(s) = \frac{s\,V_i(s)}{s^2+3s+1}$$

$$\boxed{G(s) = \frac{V_i(s)}{V_0(s)} = \frac{s^2+3s+1}{s}}$$

d)



① $-V_i + 2I_1 + \frac{1}{s}I_1 + 2I_1 - 2I_2 + \frac{1}{s}I_1 - \frac{1}{s}I_2 = 0$

$I_1\left(4 + \frac{2}{s}\right) - I_2\left(2 + \frac{1}{s}\right) = V_i(s)$

② $2I_2 + 2sI_2 + \frac{1}{s}I_2 - \frac{1}{s}I_1 + 2I_2 - 2I_1 = 0 \qquad -I_1\left(2+\frac{1}{s}\right) + I_2\left(4 + \frac{2s^2+1}{s}\right) = 0$

$$\begin{bmatrix} 4+\frac{2}{s} & -2-\frac{1}{s} \\ -2-\frac{1}{s} & 4+\frac{2s^2+1}{s} \end{bmatrix} = \begin{bmatrix} V_i \\ 0 \end{bmatrix} \quad \Delta = 16 + \frac{8s^2+4}{s} + \frac{8}{s} + \frac{4s^2+2}{s^2} - 4 - \frac{2}{s} - \frac{2}{s} - \frac{1}{s^2}$$

$$\Delta = \frac{8s^3+16s^2+8s+1}{s^2}$$

$\boxed{V_0(s) = 2s\,I_2(s)}$

$$I_2(s) = \frac{V_0}{2s}$$

$$\begin{bmatrix} 4+\frac{2}{s} & V_i \\ -2-\frac{1}{s} & 0 \end{bmatrix} \quad \Delta I_2 = V_i\left(\frac{2s-1}{s}\right)$$

$$I_2 = \frac{\Delta I_2}{\Delta} = V_i\left(\frac{2s-1}{s}\right)\frac{s^2}{8s^3+16s^2+8s+1}$$

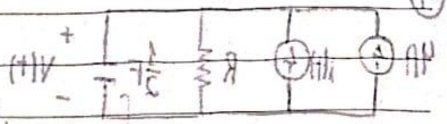
$$\frac{V_i}{V_0} = \frac{8s^3+16s^2+8s+1}{4s^3+2s^2} = \frac{4s^2+6s+1}{2s^2}$$

$$I_2 = V_i\left(\frac{-2s^2-s}{8s^3+16s^2+8s+1}\right)$$

(2)

a)

$\mathcal{IC} = \pi i$



Carro 1

$$u_1 = X_1(s)\left[m_1 s^2 + (b_1+b_2)s + (K_1+K_2)\right] - X_2(s)\left[b_2 s + K_2\right]$$

Carro 2

$$u_2 = -X_1(s)\left[b_2 s + K_2\right] + X_2(s)\left[m_2 s^2 + (b_2+b_3)s + (K_2+K_3)\right] - X_3(s)\left[b_3 s + K_3\right]$$

Carro 3

$$u_3 = -X_2(s)\left[b_3 S + K_3\right] + X_3(s)\left[m_3 s^2 + b_3 s + K_3\right]$$

b)

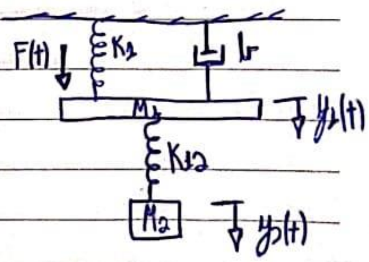
$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-(K_1+K_2)}{m_1} & \frac{K_2}{m_1} & 0 & \frac{-(b_1+b_2)}{m_1} & \frac{b_2}{m_1} & 0 \\ \frac{K_2}{m_2} & \frac{-(K_2+K_3)}{m_2} & \frac{K_3}{m_2} & \frac{b_2}{m_2} & \frac{-(b_2+b_3)}{m_2} & \frac{b_3}{m_2} \\ 0 & \frac{K_3}{m_3} & \frac{-K_3}{m_3} & 0 & \frac{b_3}{m_3} & \frac{-b_3}{m_3} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ u_1/m_1 \\ u_2/m_2 \\ u_3/m_3 \end{bmatrix}$$

(3) M_1

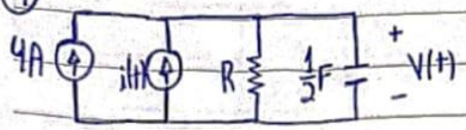
$$F(t) = y_1(s)\left[m_1 s^2 + (K_1+K_2) + bS\right] - y_2(s)\left[K_{12}\right]$$

M_2

$$0 = y_2\left[K_{12}\right] - y_1\left[K_{12}\right]$$



4



$i_R = 2e^{2V_r}$

a) $V_R = V(t)$ $\qquad i_R = 2e^{2V_r}$

$\frac{i_R}{2} = e^{2V_r} \Rightarrow \ln\left(\frac{i_R}{2}\right) = 2V_R$

$$\boxed{V_R = \frac{\ln\left(\frac{i_R}{2}\right)}{2}} = V(t)$$

b) $F(V) \approx F(V_S) + \left.\frac{df(V)}{dV}\right|_{V_S} \delta V$

$\frac{df(V)}{dV} = \frac{d}{dV} 2e^{2V_S} = 4e^{2V_S}$

$$\boxed{2e^{2(V_S+\delta V)} \approx 2e^{2V_S} + 4e^{2V_S}\delta V}$$

c) $G(s) = \frac{V(s)}{I(s)}$ $\qquad$ Do circuito temos: $CV' + i_R - 4 = i(t)$

$\frac{1}{2}\frac{d(V_S+\delta V)}{dt} + 2e^{2(V_S+\delta V)} - 4 = i(t) \Rightarrow \frac{1}{2}\frac{d\delta V}{dt} + 2e^{2V_S} + 4e^{2V_S}\delta V - 4 = i(t)$

$i(t) = 0 \quad V_S = V_R \quad i_R = 4$

então de a)

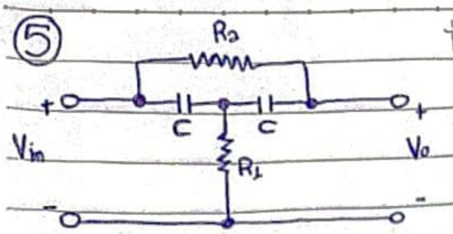
$V_R = \frac{1}{2}\ln 2$ logo, $e^{2V_S} = 2$

$\frac{1}{2}\frac{d\delta V}{dt} + 8\delta V = i(t)$

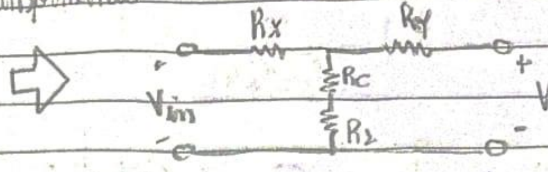
$\delta V(S)\left(\frac{S}{2} + 8\right) = I(S)$

$$\boxed{\frac{V(S)}{I(S)} = \frac{2}{S+16}}$$

⑤



transformando



$R_X = \frac{C_R \cdot R_2}{2C_R + R_2} = \frac{R_2}{2+sCR_2}$ $\quad R_Y = \frac{C_R \cdot R_2}{2C_R + R_2} = R_X$ $\quad R_C = \frac{C_R^2}{2C_R + R_2} = \frac{1}{sC(2+sCR_2)}$

$R_C + R_1 = \frac{1 + sC(2+sCR_2) \cdot R_1}{sC(2+sCR_2)}$ $\quad R_X + R_C + R_1 = \frac{1 + sC(2+sCR_2)R_1 + sCR_2}{sC(2+sCR_2)}$

$V_o(s) = \frac{R_C + R_1}{R_X + R_C + R_1} \cdot V_{in}(s)$ $\quad \boxed{\frac{V_o(s)}{V_{in}(s)} = \frac{s^2C^2R_1R_2 + 2sCR_1 + 1}{s^2C^2R_1R_2 + 2sCR_1 + sCR_2 + 1}}$

⑥ a)

$$\dot{X} = \overset{A}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & 0 & -1 \end{bmatrix}} X + \overset{B}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} r \qquad y = \overset{C}{\begin{bmatrix} 2 & 1 & 1 \end{bmatrix}} X \qquad (sI-A)^{-1} = \frac{\mathrm{adj}(sI-A)}{\det(sI-A)}$$

$$\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & 0 & -1 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 4 & 0 & s+1 \end{bmatrix} \Rightarrow \left|\begin{matrix} s & -1 & 0 \\ 0 & s & -1 \\ 4 & 0 & s+1 \end{matrix}\right|\begin{matrix} s & -1 \\ 0 & s \\ 4 & 0 \end{matrix} = \boxed{s^3 + s^2 + 4}$$

$$\mathrm{adj}(sI-A) = \begin{pmatrix} \begin{vmatrix} s & -1 \\ 0 & s+1 \end{vmatrix} & \begin{vmatrix} 0 & -1 \\ 4 & s+1 \end{vmatrix} & \begin{vmatrix} 0 & s \\ 4 & 0 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ 0 & s+1 \end{vmatrix} & \begin{vmatrix} s & 0 \\ 4 & s+1 \end{vmatrix} & \begin{vmatrix} s & -1 \\ 4 & 0 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ s & -1 \end{vmatrix} & \begin{vmatrix} s & 0 \\ 0 & -1 \end{vmatrix} & \begin{vmatrix} s & -1 \\ 0 & s \end{vmatrix} \end{pmatrix} = \begin{vmatrix} s^2+s & -4 & -4s \\ s+1 & s^2+s & -4 \\ 1 & s & s^2 \end{vmatrix}$$

$$(SI-A)^{-1} = \frac{\begin{bmatrix} S^2+S & -4 & -4S \\ S+1 & S^2+S & -4 \\ 1 & S & S^2 \end{bmatrix}}{S^3+S^2+4}$$

$$C = \begin{bmatrix} 2 & 1 & 1 \end{bmatrix} \qquad C.(SI-A)^{-1} = \frac{\begin{bmatrix} 2S^2+2S+S+1+1 & -8+S^2+S+S & -8S-4+S^2 \end{bmatrix}}{S^3+S^2+4}$$

$$C.(SI-A)^{-1} = \frac{\begin{bmatrix} 2S^2+3S+2 & S^2+2S-8 & S^2-8S-4 \end{bmatrix}}{S^3+S^2+4}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad C.(SI-A)^{-1}.B = \boxed{\frac{S^2-8S-4}{S^3-S^2+4}}$$

b)

$$\dot{x} = \overset{A}{\begin{bmatrix} 4 & -2 \\ 1 & 0 \end{bmatrix}} x + \overset{B}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}} r \qquad y = \overset{C}{\begin{bmatrix} 3 & 1 \end{bmatrix}} x$$

$$\begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} S-4 & 2 \\ -1 & S \end{bmatrix} \Rightarrow \begin{vmatrix} S-4 & 2 \\ -1 & S \end{vmatrix} = \boxed{S^2-4S+2}$$

$$adj(SI-A) = \begin{bmatrix} S & -2 \\ +1 & S-4 \end{bmatrix} \qquad (SI-A)^{-1} = \frac{adj(SI-A)}{det(SI-A)} = \frac{\begin{bmatrix} S & -2 \\ 1 & S-4 \end{bmatrix}}{S^2-4S+2}$$

$$C = \begin{bmatrix} 3 & 1 \end{bmatrix} \qquad C.(SI-A)^{-1} = \frac{\begin{bmatrix} 3S+1 & -6+S-4 \end{bmatrix}}{S^2-4S+2} = \frac{\begin{bmatrix} 3S+1 & S-10 \end{bmatrix}_{1x2}}{S^2-4S+2}$$

$$B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2x1} \qquad C.(SI-A)^{-1}.B = \frac{\begin{bmatrix} 3S+1 & S-10 \\ 6S+2 & 2S-20 \end{bmatrix}}{S^2-4S+2}$$