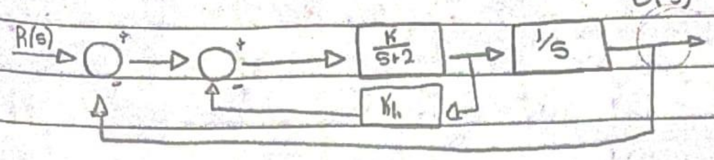
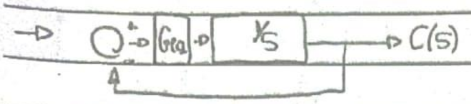
Lista 3

$\zeta = 0{,}7 \quad \omega_n = 4\,rad/s$

1)



$$G_{eq} = \frac{\frac{K}{s+2}}{1 + \frac{Kh}{s+2}} = \frac{K}{s+2+Kh} \qquad T(s) = \frac{\frac{K}{s(s+2+Kh)}}{1 + \frac{K}{s(s+2+Kh)}}$$



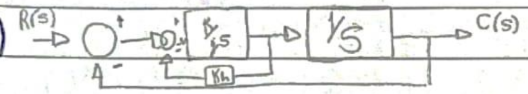
$$T(s) = \frac{K}{s^2 + s(2+Kh) + K} \qquad K = \omega_n^2 \quad K = 16$$

$2\zeta\omega_n = 2 + Kh$

$2 \cdot 0{,}7 \cdot 4 - 2 = 16h$

$h = 0{,}225$

2)



$K/J = 4 \quad \zeta = 0{,}6 \quad K_h = ?$

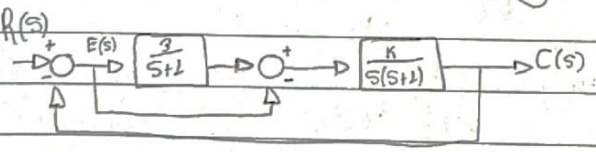
$$C_{eq} = \frac{K/Js}{1 + \frac{K}{Js}\cdot Kh} = \frac{K}{Js + K \cdot Kh} = \frac{4}{s+4h}$$

$$T(s) = \frac{\frac{1}{s}\cdot\frac{4}{s+4h}}{1 + \frac{4}{s+4h}\cdot\frac{1}{s}} = \frac{4}{s^2+4hs+4} \qquad \omega_n^2 = 4 \quad \omega_n = 2$$

$2\zeta\omega_n = 4h$

$h = \frac{\zeta\omega_n}{2} = 0{,}6$

3)



$k = ?$

$$G_{eq} = \frac{3}{s+1} - 1 = \frac{-s+2}{s+1} \qquad T(s) = \frac{\frac{K}{s(s+1)}\cdot\frac{(-s+2)}{(s+1)}}{1 + \frac{K(-s+2)}{s(s+1)(s+1)}} = \frac{K(-s+2)}{s^3+2s^2+s(1-K)+2K}$$

s^3	1	$1-K$	0
s^2	2	$2K$	0
s^1	b_1	0	
s^0	c_1	0	

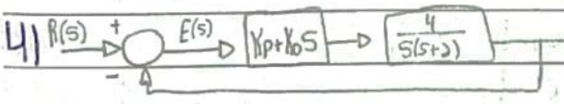
$$b_1 = -\frac{\begin{vmatrix} 1 & 1-K \\ 2 & 2K \end{vmatrix}}{2} = -\frac{[2K-2(1-K)]}{2} = -\frac{(2K-2+2K)}{2} = 1-2K$$

$$c_1 = -\frac{\begin{vmatrix} 2 & 2K \\ 1-2K & 0 \end{vmatrix}}{1-2K} = -\frac{(-2K(-2K+1))}{1-2K} = 2K$$

① $1-2K>0$

$2K<1$ $\quad K<\frac{1}{2}$

② $2K>0$ $\quad K>0$ $\quad$ logo, $\boxed{0<K<\frac{1}{2}}$

4) 

$$C_{eq} = (K_p+K_D s)\cdot\frac{4}{s(s+2)}$$

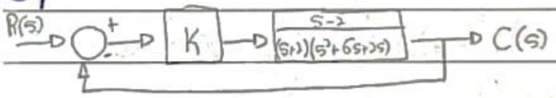
$$T(s) = \frac{4(K_p+K_D s)}{s(s+2)+4(K_p+K_D s)} = \frac{4(K_p+K_D s)}{s^2+s(2+4K_D)+4K_p}$$

s^2	1	$4K_p$	0
s^1	$2+4K_D$	0	
s^0	b_1	0	

$$b_1 = -\frac{\begin{vmatrix} 1 & 4K_p \\ 2+4K_D & 0 \end{vmatrix}}{2+4K_D} = \frac{4K_p\cancel{(2+4K_D)}}{\cancel{2+4K_D}} = 4K_p$$

① $4K_p>0$ $\quad \boxed{K_p>0}$

② $2+4K_D>0$ $\quad \boxed{K_D>-\frac{1}{2}}$

5) 

$$C_{eq}(s) = \frac{K(s-2)}{s^3+7s^2+31s+25}$$

$$T(s) = \frac{K(s-2)}{s^3+7s^2+31s+25+K(s-2)} = \frac{K(s-2)}{s^3+7s^2+s(31+K)+(25-2K)}$$

s^3	1	$31+K$	0
s^2	7	$25-2K$	0
s^1	b_1	0	
s^0	c_1	0	

$$b_1 = -\frac{\begin{vmatrix} 1 & 31+K \\ 7 & 25-2K \end{vmatrix}}{7} = \frac{-25+2K+7K+217}{7} = \frac{9K+192}{7}$$

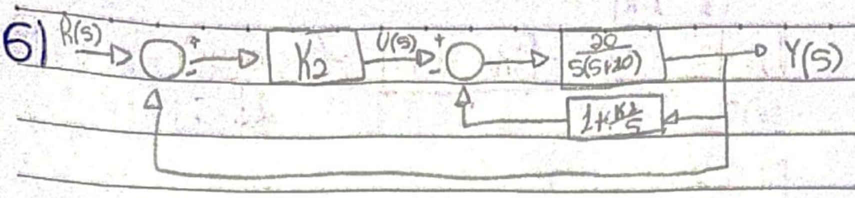
$$c_1 = -\frac{\begin{vmatrix} 7 & 25-2K \\ \frac{9K+192}{7} & 0 \end{vmatrix}}{\frac{9K+192}{7}} = 25-2K$$

① $25-2K>0$ $\quad K<\frac{25}{2}$

② $\frac{9K+192}{7}>0$ $\quad K>-\frac{192}{9}$

$$\boxed{-\frac{64}{3}<K<\frac{25}{2}}$$

6)



a)

$$T(s) = \frac{\frac{20}{s(s+10)}}{1+\left(1+\frac{K_1}{s}\right)\left(\frac{20}{(s+10)s}\right)} = \frac{20}{s(s+10)} \cdot \frac{s^2(s+10)}{s^2(s+10)+20s+20K_1} = \frac{20s}{s^3+10s^2+20s+20K_1}$$

s^3	1	20	0
s^2	10	$20K_1$	0
s^1	b_1	0	
s^0	c_1	0	

$$b_1 = -\frac{\begin{vmatrix} 1 & 20 \\ 10 & 20K_1 \end{vmatrix}}{10} = \frac{-20K_1+200}{10} = -2K_1+20$$

$$c_1 = -\frac{\begin{vmatrix} 10 & 20K_1 \\ -2K_1+20 & 0 \end{vmatrix}}{-2K_1+20} = 20K_1$$

① $20K_1>0$, $K_1>0$

② $-2K_1+20>0$, $K_1<10$

$$\boxed{0< K_1 < 10}$$

b) K= 5

c) $T(s)= \frac{20s \cdot K_2}{s^3+10s^2+s(20+20K_2)+100}$

s^3	1	$20+20K_2$	0
s^2	1	10	0
s^1	b_1	0	
s^0	c_1	0	

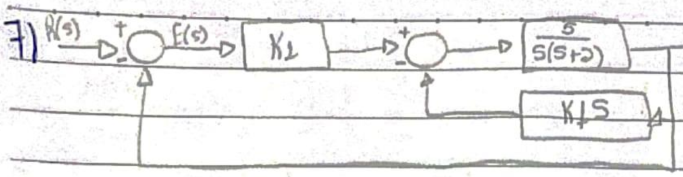
$$b_1 = -\frac{\begin{vmatrix} 1 & 20+20K_2 \\ 1 & 10 \end{vmatrix}}{1} = -\cancel{10}+\cancel{20}+20K_2 = 20K_2+10$$

$c_1 = 10$

$20K_2+10>0$

$$\boxed{K_2 > -\frac{1}{2}}$$

7)



$K_v = 20$

$\xi = 0{,}7$ 1,0

$$G(s) = \frac{\frac{5}{s(s+2)}}{1 + \frac{5K_f s}{(s+2)s}} = \frac{5}{s(s+2) + 5K_f s} = \frac{5}{s^2 + s(2+5K_f) + 0} = K_v = \lim_{s \to 0} s \cdot \frac{5}{s(s+2+5K_f)}$$

$$K_v = \frac{5}{2+5K_f} = 20 \quad 100K_f + 40 = 5 \quad \boxed{K_f = -\frac{35}{100}}$$

8) $G(s) = \dfrac{K(s+\alpha)}{(s+\beta)^2}$, $e_{rp} = 0{,}1$, $\xi = 0{,}5$, $\omega_n = \sqrt{10}$

$$E(s) = \frac{1/s}{1 + \frac{K(s+\alpha)}{(s+\beta)^2}} = \frac{1}{s} \cdot \frac{(s+\beta)^2}{(s+\beta)^2 + K(s+\alpha)} = \frac{1}{s} \frac{s^2+2s\beta+\beta^2}{s^2+2s\beta+\beta^2+Ks+K\alpha} = \frac{1}{s} \cdot \frac{s^2+2s\beta+\beta^2}{s^2+s(2\beta+K)+\beta^2+K\alpha}$$

$$K_p = \lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{K(\cancel{s}+\alpha)}{(\cancel{s}+\beta)^2} = \frac{K\alpha}{\beta^2} \qquad e = \frac{1}{1+K_p} = \frac{1}{1+\frac{K\alpha}{\beta^2}} = \frac{\beta^2}{\beta^2+K\alpha} = 0{,}1$$

$0{,}9\beta^2 = 0{,}1K\alpha \quad K\alpha = 9\beta^2$ $\qquad 2\xi\omega_n = 2\beta + K$

$K = 2\xi\omega_n - 2\beta$

$\boxed{K_1 = 1{,}16}$ $\boxed{K_2 = 5{,}16}$

$\omega_n^2 = \beta^2 + K\alpha$

$10 = \beta^2 + 9\beta^2$

$\beta^2 = 1$ $\boxed{\beta = \pm 1}$

$$\boxed{\begin{array}{lll} \beta = 1, & K = 1{,}16 , & \alpha = 7{,}76 \\ \beta = -1, & K = 5{,}16 , & \alpha = 1{,}74 \end{array}}$$

$\alpha = \dfrac{9\beta^2}{K} = \dfrac{9}{K}$

9) $\omega_n = 10$ $K_v = 0{,}01$

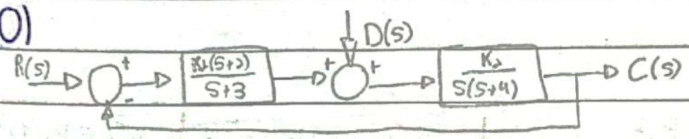
a) entrada rampa, tipo 1

b) $e(\infty) = \frac{1}{K_v}$ $G(s) = \frac{K}{s(s+a)} \to K_v = \lim_{s\to 0} s \cdot \frac{K}{s(s+a)} = \frac{K}{a}$

$e(\infty) = \frac{1}{0{,}01}$ $\frac{K}{a} = 100$ $\boxed{a=1}$ $T(s) = \frac{K}{s^2+sa+K}$ $\to \omega_n = 10$ $\omega_n^2 = K$ $\boxed{K=100}$

c) $2\zeta\omega_n = 1$ $\zeta = \frac{1}{2\omega_n} = 0{,}05$

10)



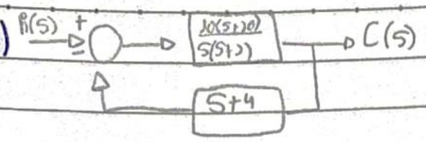
$-E(s) = \frac{G_2(s)}{1+G_2\cdot G_1} \cdot D(s)$ $-e_1(\infty) = \frac{1}{\lim_{s\to 0}\frac{1}{G_2(s)} + \lim_{s\to 0} G_1(s)}$

$-e_1(\infty) = \frac{1}{\lim_{s\to 0}\frac{1}{\frac{K_2}{s(s+4)}} + \lim_{s\to 0} G_1(s)}$ $\boxed{K_1 = 1{,}5 \cdot 10^5}$

$e(\infty) = \frac{1}{K_v}$ $K_v = \lim_{s\to 0} s \cdot G(s) = \lim_{s\to 0} s\left[\frac{G_1(s)}{1+G_1\cdot G_2}\right] = \lim_{s\to 0} s\left[\frac{\frac{K_1(s+2)}{s+3}}{1+\frac{K_1(s+2)}{s+3}\cdot\frac{K_2}{s(s+4)}}\right]$

$K_v = \lim_{s\to 0} \frac{2K_2K_1}{4}$ $\boxed{K_v = \frac{K_2K_1}{2}}$ $K_2 = \frac{2K_v}{K_1}$ $\boxed{K_2 = 0{,}0067}$

$K_v = 0{,}002$

11) 

$$G_{eq} = \frac{\frac{10(S+10)}{S(S+2)}}{1 + \frac{10(S+10)}{S(S+2)} \cdot (S+4) - \frac{10(S+10)}{S(S+2)}} = \frac{10(S+10)}{11S^2+132S+300} \qquad K_p = \lim_{S \to 0} G(S) = \frac{100}{300} = \frac{1}{3}$$

a) tipo 0

b) $K_p = 1/3 \quad K_v = K_a = 0$

c) Entrada em degrau

d) $e(\infty) = \frac{1}{1+K_p} = \frac{1}{1+\frac{1}{3}} = \frac{3}{4}$

e) $e(\infty) = \lim_{S \to 0} S \cdot \frac{1/S}{1 + \frac{10(S+10)(S+4)}{S(S+2)}} = 0$