Daniel Augusto Müller RA: 2039834

1) a) F

b) F

c) F

d) V

e) F

f) V

2)
$$\begin{cases} y_1(s)\left[s^2 m_1 + s b_1 + K_1 + K_{12}\right] - y_2(s)\left[K_{12}\right] = F(t) \\ y_2(s)\left[s^2 m_2 + K_{12}\right] - y_1(s)\left[K_{12}\right] = 0 \end{cases}$$

$$\begin{cases} \frac{d y_1(t)^2}{dt} m_1 + \frac{d y_1(t)}{dt} b_1 + y_1(t)(K_1 + K_{12}) - y_2(t) K_{12} = F(t) \\ \frac{d y_2(t)^2}{dt} m_2 + y_2(t) K_{12} - y_1(t) K_{12} = 0 \end{cases}$$

$$\frac{d y_1(t)}{dt} = V_1$$

$$\frac{d y_2(t)}{dt} = V_2$$

$$\begin{cases} V_1' m_1 + V_1 b_1 + y_1 . (K_1 + K_{12}) - y_2 . K_{12} = F(t) \\ V_2' m_2 + V_2 K_{12} - y_1 . K_{12} = 0 \end{cases}$$

$$\begin{bmatrix} y_1 \\ V_1 \\ y_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K_1 + K_{12}}{m_1} & \frac{b_1}{m_1} & -\frac{K_{12}}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_{12}}{m_2} & 0 & -\frac{K_{12}}{m_2} & 0 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ V_1 \\ y_2 \\ V_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \cdot a\,sen(w_0 t)$$

3) $T(S) = \dfrac{\dfrac{K}{S+9} \cdot \dfrac{S+6}{(S+2)(S+4)}}{1 + \dfrac{K}{S+9} \cdot \dfrac{S+6}{(S+2)(S+4)}} = \dfrac{K(S+6)}{S^3+15S^2+(K+62)S+(6K+72)}$

Tabela:

S^3	1	K+62	0
S^2	15	6K+72	0
S^1	b_1	0	
S^0	b_2	0	

$b_1 = \dfrac{-\begin{vmatrix} 1 & K+62 \\ 15 & 6K+72 \end{vmatrix}}{15} = \dfrac{9K+858}{15} = \dfrac{3K+286}{15}$

$b_2 = \dfrac{-\begin{vmatrix} 15 & 6K+72 \\ \frac{3K+286}{5} & 0 \end{vmatrix}}{\frac{3K+286}{5}} = 6K+72 = K+12$

a)

$\dfrac{3K+286}{15} > 0 \quad K_1 > -\dfrac{286}{3} \approx -95{,}3$

Se K for positivo, o sistema é estável

$K+12 > 0 \quad K_2 > -12$

b) $K_p = 20 \quad K_p = \lim_{S \to 0} \dfrac{K(S+6)}{(S+9)(S+2)(S+4)} = \dfrac{6K}{72}$

$\dfrac{6K}{72} = 20 \rightarrow \boxed{K=240}$

c) $e(\infty) = e_R(S)(\infty) + e_D(S)(\infty)$

$e_R(S)(\infty) = \dfrac{1}{20}$ $\quad e_D(S)(\infty) = \dfrac{-1}{\lim_{S \to 0} \frac{1}{\frac{S+6}{(S+2)(S+4)}} + \lim_{S \to 0} \frac{240}{S+9}} = \dfrac{-1}{\frac{8}{6}+\frac{240}{9}} = -\dfrac{1}{28}$

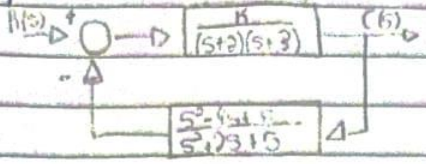
$e(\infty) = \dfrac{1}{20} - \dfrac{1}{28} = \dfrac{1}{70} = 1{,}428\%$

4) a) $G_M = 25dB$

b) $\theta_M = 67{,}5°$

c) Sim, o sistema de malha fechado é estável pois o sistema malha aberto é.

5)



$$\theta a = \frac{-5+0}{2} = -2{,}5$$

$$\frac{(2K+1)180}{2} \quad K_0 = 90° \quad K_1 = 270°$$

$$M_F = \frac{\frac{K}{(s+2)(s+3)} \cdot \frac{s^2-4s+8}{s^2+2s+5}}{1+\frac{K}{(s+2)(s+3)} \cdot \frac{s^2-4s+8}{s^2+2s+5}} = \frac{K(s^2-4s+8)}{s^4+7s^3+(K+21)s^2+(37-4K)s+(8K+30)}$$

s^4	1	$K+21$	$8K+30$
s^3	7	$-4K+37$	0
s^2	a_1	a_2	0
s^1	b_1	0	
s^0	c_1	0	

$$a_1 = \frac{(7K+147)-(-4K+37)}{7} = \frac{11K-110}{7}$$

$$a_2 = \frac{7(8K+30)}{7} = 8K+30$$

$$b_1 = \frac{a_1 \cdot (37-4K) - 7a_2}{a_1} = \frac{-44K^2-455K+5540}{11(K-10)}$$

$$c_1 = a_2 = 8K+30$$

$$\frac{-44K^2-455K+5540}{11(K-10)} > 0 \quad \begin{array}{l} K_1 < -17{,}52 \\ K_2 < 7{,}18 \\ K_3 < 10 \end{array}$$

c) $K = \frac{-1}{G(\sigma)} \rightarrow \frac{dK}{d\sigma}$ $\quad G(\sigma) = \frac{s^2-4s+8}{s^4+7s^3+21s^2+37s+30}$

$$\frac{dK}{d\sigma} = \frac{\sigma^4+7\sigma^3+21\sigma^2+37\sigma+30}{\sigma^2+4\sigma+8} =$$

$$\boxed{\begin{array}{ll} s_1 = -3 & s_2 = -2 \\ s_{2,3} = \pm 2i - 1 & \end{array}}$$

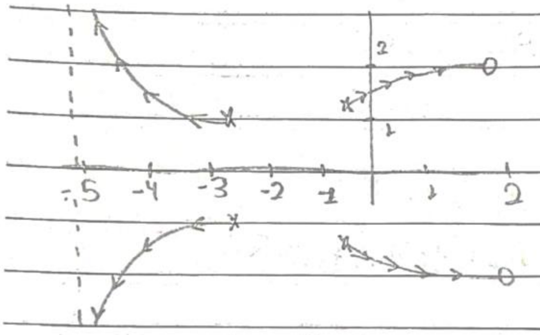
FORONI

Para a malha fechada:

$G(s) = \dfrac{s^2-4s+8}{s^4+7s^3+22s^2+33s+38}$ $\quad$ P/ K=1 $\quad$ Polos $= -2{,}892 \pm 1{,}835\,i$

$-0{,}607 \pm 1{,}694\,i$

Zeros $= 2 \pm 2i$



$\theta_a = \dfrac{\sum p - \sum z}{\#p - \#z} = 5{,}499$

b) $-22{,}974 < K < 9{,}174$ cortaju

c) $\sigma = -2{,}56$

Saída do Eixo Real

d) Polo $= -2{,}892 \pm 1{,}833\,i$ $\quad \theta = 71{,}9°$

Polo $= -0{,}607 \pm 1{,}694\,i$ $\quad \theta = 61{,}52°$

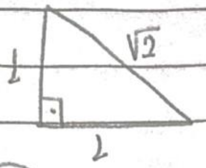
e) $K = 250$ p/ $\xi = 0{,}3$

Polos Dominantes $= -5{,}22 \pm 16{,}5\,i$

6) $\theta_3 = ?$

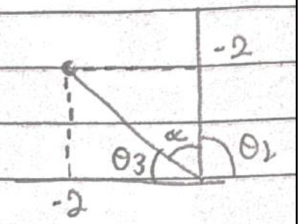
polos $= 0, -2$

zeros $= -z$



$\theta = 90^\circ$

θ_1:



$\theta_3 = 180 - \theta_1 - 45^\circ$

$\theta_1 = 90^\circ + \alpha$

$tg\,\alpha = |2/2| = 1$

$arctg\,\alpha = 45^\circ$

$\theta_1 = 135^\circ$

b) $Tg\theta = \dfrac{m_2}{|z-2|}$

$tg\,45 = \dfrac{2}{|z-2|}$

1

$|z-2| = 2$

$|z| = 4$

$-z = -4$

c) $K = \dfrac{1}{|G(s) \cdot H(s)|} = \dfrac{1}{\left|(s+z) \cdot \dfrac{1}{s(s+2)}\right|_{s=-2+2j}}$

$K_1 = \left|\dfrac{s(s+2)}{s+z}\right|_{s=-2+2j} = \dfrac{2+2j}{-4-4j} = \dfrac{1+j}{-2-2j} \cdot \dfrac{-2+2j}{-2+2j} = \dfrac{-4}{8}$

$K_2 = \dfrac{1}{|G(s)|_{s=-2+2j}} = \dfrac{m_3}{m_1 m_2} = \dfrac{2\sqrt{2}}{2(2\sqrt{2})} = 2$