

# Linear control system design SSY285

## Assignment M3: Linear state estimation and control of DC motor with flywheel

### Problem

Consider the DC motor with flywheel modeled and analyzed in the previous assignments. The starting point of this assignment is the discrete time state space representation obtained in sub-problems d) and e) of assignment **M2**. The angle,  $\phi_2$ , and angular velocity,  $\omega_2$ , of the flywheel are both assumed to be measured in this assignment. In the following questions, assume that disturbance bounds correspond to a confidence interval of 99.7% of a normal distribution.

### Questions

- Suppose that (discrete time) white noise is added to both the external torque  $T_e$  and the applied motor voltage  $v_a$ , where the noise sequences are zero mean and uncorrelated. The voltage disturbances are mostly due to variations in the power supply unit, and are bounded by  $\pm 0.3V$ . The torque disturbance is estimated to be less than 10% of the maximum applied external torque value, which is assumed to be  $T_e^{\max} = 1Nm$ . Based on this information, propose a covariance matrix  $Q_w$  for the disturbance vector  $w$ . Which  $N$  matrix should be used in  $x(k+1) = Ax(k) + Bu(k) + Nw(k)$ ?
- Suppose also that measurement disturbances  $v_1, v_2$  are added to the output. The disturbances are upper bounded by  $0.02rad$  and  $0.01rad/s$ , respectively. Like above, they are assumed to be discrete time, zero mean uncorrelated white noises. Propose a covariance matrix  $Q_v$  for the measurement disturbance vector  $v$ .
- Provided the cross spectrum between  $w$  and  $v$  is zero, compute a (discrete time) Kalman filter to estimate the “current” state  $\hat{x}(k|k)$  of the system  $x(k+1) = Ax(k) + Bu(k) + Nw(k)$ ,  $y(k) = Cx(k) + v(k)$ . Find the observer gain matrix,  $L$ . What is the covariance matrix,  $P$ , of the state estimation error? What are the observer eigenvalues in this case?
- Design a (discrete time) Linear Quadratic Gaussian controller and simulate the closed-loop answer to a step  $r_{\omega_2}$ , jumping from an initial value of  $10rad/s$  to  $100rad/s$ , for the discrete time and noise corrupted system above. Implement reference tracking by integral action. Use the previously computed Kalman filter gain, answered in c) to reconstruct system states. For the LQ controller, choose the two appropriate weighting matrices  $Q_u$  and  $Q_x$ , in order to keep the control inputs low. Add an input disturbance of  $1V$ , and see how the results change.

Pre-approval of solution is mandatory before submission (by TA in tutorial session)