SSY285 - Assignment 3

Group 35

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Question a)

From the last assignment, the following discrete system was given:

$$\underbrace{\begin{bmatrix} 0.4010 & 0.5964 & 0.0026 & 0.0004 & 0.0002 \\ 0.2076 & 0.7749 & 0.0175 & 0.0001 & 0.0009 \\ 0.0585 & 0.5686 & 0.3729 & 0.0000 & 0.0004 \\ -782.8328 & 773.7475 & 9.0853 & -0.0489 & 0.5964 \\ 342.5299 & -370.9589 & 28.4290 & 0.1491 & 0.7749 \end{bmatrix}}_{\text{Discrete A Matrix}} \underbrace{\begin{bmatrix} 0.0032 & 0.0003 \\ 0.0002 & 0.0032 \\ 0.0000 & 0.3168 \\ 4.4993 & 1.3088 \\ 0.5851 & 8.7634 \end{bmatrix}}_{\text{Discrete B Matrix}} \tag{1}$$

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Discrete C Matrix}} \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\text{Discrete D Matrix}} \tag{2}$$

In this task, we are supposed to define and calculate the Noise (N) matrix with its belonging covariance matrix Q_w for the disturbance vector w in the following equation considering some input noise to the system:

$$x(k+1) = Ax(k) + Bu(k) + Nw(k) \tag{3}$$

Since the noise is just affecting the inputs to the system a good choice of the N matrix would be to scale accordingly and as much as for the inputs (the B_d matrix). Meaning that B_d and N will be the same matrices.

$$\underbrace{\begin{bmatrix} 0.0032 & 0.0003 \\ 0.0002 & 0.0032 \\ 0.0000 & 0.3168 \\ 4.4993 & 1.3088 \\ 0.5851 & 8.7634 \end{bmatrix}}_{\text{Noise } N \text{ matrix}} \tag{4}$$

For the Q_w matrix the covariances can be calculated by the following approach. We were given the maximum and minimum disturbance values and to get the covariances would simply mean that we squared these disturbance values. In our case, we are affected by a confidence interval of 99.7 % of a normal distribution and therefore need to consider this. We know that the 99.7 % confidence interval corresponds to 3 sigmas of the normal distribution and can therefore simply use the following formula:

$$\mu \pm 3\sigma = \text{max or min(disturbance value)} \tag{5}$$

Which can be rewritten and σ can be expressed as:

$$\sigma = (\frac{\text{max or min(disturbance value)}}{3})^2 \tag{6}$$

This gives us the following covariance matrix Q_w:

$$\underbrace{\begin{bmatrix} 0.01 & 0 \\ 0 & 0.00111 \end{bmatrix}}_{\text{Covariance } Q_w \text{ matrix}} \tag{7}$$

Question b)

In this task we use the same approach as in subtask a) but instead of considering input disturbances we now consider measurement disturbances. By using the same approach but with the new given values for the disturbance interval, we get the following covariance Q_u matrix for the disturbance vector v:

$$\underbrace{\begin{bmatrix} 4.44 \cdot 10^{-5} & 0 \\ 0 & 1.11 \cdot 10^{-5} \end{bmatrix}}_{\text{Covariance } Q_u \text{ matrix}} \tag{8}$$

Question c)

To compute the Kalman filter gain, the command `kalman()` was used in Matlab. The estimator of the next state is given as $x(k+1) = Ax(k) + Bu(k) + Nw(k)$ and since the noise applied to the process is only due to the input noise, one can write it as $x(k+1) = Ax(k) + (B+N)u(k)$. From the questions before, the N matrix was said to be equal to the B-matrix ($N = B$). The input to the Kalman is therefore instead the discretized version of Bd twice:

$$B_d = \begin{bmatrix} 0.0032 & 0.0003 & 0.0032 & 0.0003 \\ 0.0002 & 0.0032 & 0.0002 & 0.0032 \\ 0.0000 & 0.3168 & 0.0000 & 0.3168 \\ 4.4993 & 1.3088 & 4.4993 & 1.3088 \\ 0.5851 & 8.7634 & 0.5851 & 8.7634 \end{bmatrix} \tag{9}$$

The Kalman or observer gain was given as:

$$K = \begin{bmatrix} 0.004571 & 0.010568 \\ 0.004554 & 0.010607 \\ 0.004664 & 0.010483 \\ -0.144021 & 0.101491 \\ -0.067985 & 0.090822 \end{bmatrix} \tag{10}$$

The covariance matrix was given as:

$$P = \begin{bmatrix} 0.000243 & 0.000262 & 0.000467 & 0.040741 & 0.041182 \\ 0.000262 & 0.000282 & 0.000505 & 0.043861 & 0.044342 \\ 0.000467 & 0.000505 & 0.000931 & 0.078825 & 0.079752 \\ 0.040741 & 0.043861 & 0.078825 & 6.839076 & 6.914311 \\ 0.041182 & 0.044342 & 0.079752 & 6.914311 & 6.990674 \end{bmatrix} \tag{11}$$

The eigenvalues of the observer was given as:

$$\text{Eigenvalues}(\lambda) = \begin{bmatrix} 0.995016881577413 + 0.000000000000000i \\ -0.026520637421506 + 0.000000000000000i \\ -0.010713245534839 + 0.015932182447477i \\ -0.010713245534839 - 0.015932182447477i \\ -0.000000000334191 + 0.000000000000000i \end{bmatrix} \tag{12}$$

Question d)

When designing the Linear Quadratic Gaussian (LQG-) controller, the command `dlqr` was used in Matlab. To add integral action to the controller an integral state measuring the integral of the error was added to the state space model. Therefore a new augmented A and B matrix was obtained as follows:

$$\frac{d}{dt}\begin{bmatrix} x \\ x_I \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & 0 \\ -C & \mathbf{1} \end{bmatrix}}_{A_{d\text{aug}}} \begin{bmatrix} x \\ x_I \end{bmatrix} + \underbrace{\begin{bmatrix} B_d \\ 0 \end{bmatrix}}_{B_{d\text{aug}}} u + \begin{bmatrix} 0 \\ I \end{bmatrix} r \tag{13}$$

The optimal controller gain was found by changing the Q_u and Q_x matrices. Since it was given that the control input was supposed to be kept low, the values of Q_u were chosen to be high (penalizing large input values due to cost). The values of Q_x were obtained by using trial & error and fine-tuning. The values were finally given as follows:

$$Q_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11 \end{bmatrix} \tag{14}$$

$$Q_u = \begin{bmatrix} 100000 & 0 \\ 0 & 100000 \end{bmatrix} \tag{15}$$

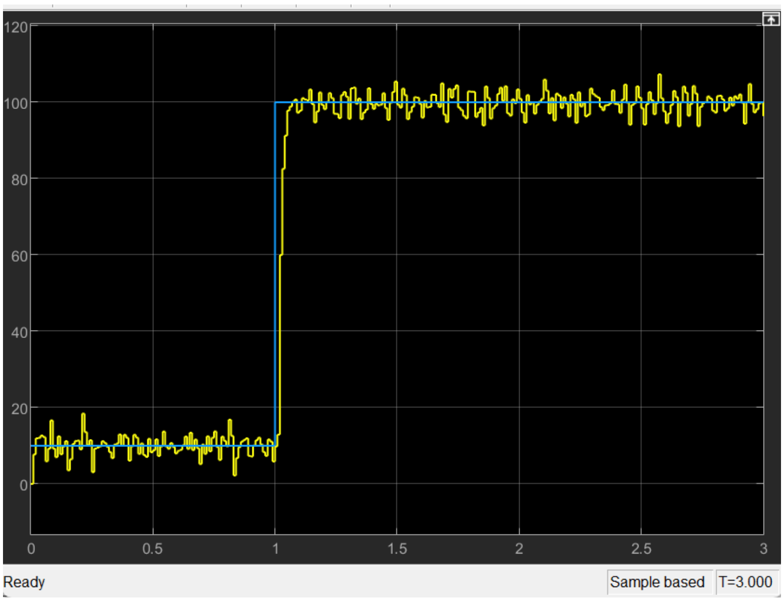

Figure 1: Simulation results, the angular velocity of the second wheel ω_2 (yellow line) vs the reference (blue line).

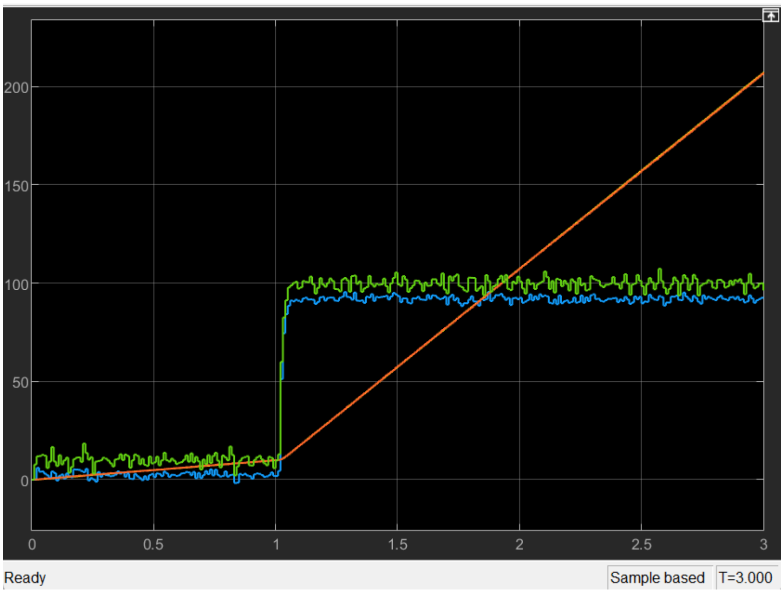

Figure 2: Simulation results, actual angular velocity ω_2 (green line), estimated angular velocity estimated ω_2 (blue line) and the angle of the second wheel ϕ (orange line).

We can from the result see that the angular velocity still is a bit noisy but it follows the reference well, has a relatively short rising time, and is without overshot. However, with some more fine-tuning it might be possible to reduce the noise and get a smother control.