SSY285 - Assignment 1

Daniel Söderqvist Johannes Lundahl Marteinn Sigþórsson

December 1, 2023

Question a)

Using the system from previous assignment but with symbolic values of R and D_1 the controllability matrix and observability matrix was computed as following:

$$\mathcal{S}(A,B) = \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix} \tag{1}$$

$$\mathcal{O}(A,C) = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{2}$$

To confirm the rank computations, the function `rref()` was used to check for linear independence between the state equations. When checking if there were any set of values for which the different matrices lost rank, it was found that the relation $R = \frac{1}{2}$ and $D_1 = 10$ made the observability for case 1 lose one rank (making it unobservable), thereby not making it observable for all non zero, real positive values. For case 2 it was found that the system is always unobservable. The system was always controllable in both cases.

Question b)

If the system is controllable it is stabilizable by definition and it is detectable if it's observable. Therefore for both cases, it is always stabilizable since the controllability has full rank. In case 1 the observability matrix had full rank and is therefore detectable, but for the special case found in subtask a) there was a combination of the parameters R and D_1 ($R = \frac{1}{2}, D_1 = 10$) that gave a solution that didn't have full rank. Therefore, for this special case, one needs to investigate further to see if the case is detectable or not. By applying the PBH test for detectability:

$$\text{Rank of detectability} = rank([\lambda I - A; C]) \tag{3}$$

One can see that the detectability has full rank which in turn means it's detectable. In case 2 the observability matrix didn't have full rank and the same approach as for the special case needs to be considered. The results show that the rank of the detectability isn't full and therefore case 2 isn't fully detectable.

As a side note, the same test could be applied by checking for stabilizability with a small adjustment:

$$\text{Rank of stabilizability} = rank([\lambda I - A, B]) \tag{4}$$

After applying this one can confirm that system is fully stabilizable as by definition.

Question c)

By using the in-build functions in Matlab. The system becomes both uncontrollable and unobservable in both cases even if the system should be controllable for both cases and observable for case 1. This is due to a large conditional number of controllability and observability. By having a large conditional number the controllability and observability becomes sensitive and hard to control and could be prone to round-off error which happens with the in-build functions in this case.

conditional number for controllability:

$$\varkappa(S) = 1.019769175402598 * 10^{15}$$

conditional number for observability case 1:

$$\varkappa(O_1) = 3.064639270804582 * 10^{15}$$

conditional number for observability case 2:

$$\varkappa(O_2) = 1.367984332610595 * 10^{16}$$

Question d) and e)

From equations 4.3 to 4.7 in the course book, the discretized system is written as:

$$x(kT + T) = A_d x(kT) + B_d u(kT) \tag{5}$$

$$y(kT) = C_d x(kT) \tag{6}$$

where F and G are defined as:

$$A_d = e^{A_c T} \tag{7}$$

$$B_d = \int_{kT}^{(k+1)T} e^{(A_c(kT+T-s)} ds B_c \tag{8}$$

The final values of A_d and B_d was given as:

$$A_d = \begin{bmatrix} 0.4010 & 0.5964 & 0.0026 & 0.0004 & 0.0002 \\ 0.2076 & 0.7749 & 0.0175 & 0.0001 & 0.0009 \\ 0.0585 & 0.5686 & 0.3729 & 0.0000 & 0.0004 \\ -782.8328 & 773.7475 & 9.0853 & -0.0489 & 0.5964 \\ 342.5299 & -370.9589 & 28.4290 & 0.1491 & 0.7749 \end{bmatrix} \tag{9}$$

$$B_d = \begin{bmatrix} 0.0032 & 0.0003 \\ 0.0002 & 0.0032 \\ 0.0000 & 0.3168 \\ 4.4993 & 1.3088 \\ 0.5851 & 8.7634 \end{bmatrix} \tag{10}$$

Question f)

The minimal realization implies that the system is both controllable and observable. Looking at the observability and controllability matrices using Matlab built-in functions `rank()`, `ctrb()` and `obsv()`, one can observe that the rank is 5 (full rank) and thus the discrete system is controllable and observable.

Looking at the eigenvalues of the discrete A matrix, one can see that the eigenvalues/poles are within the unit circle which indicates stability for a discrete system. This can be seen both from the eigenvalues and figure 1 below. Although, there is one eigenvalue/pole that lies barely on the border of the unit circle which almost makes it a marginally stable system.

$$Eigenvalues = \begin{bmatrix} 0.061922159096908 + 0.673275975850638i \\ 0.061922159096908 - 0.673275975850638i \\ 0.999999999999999 + 0.000000000000000i \\ 0.762742585988058 + 0.000000000000000i \\ 0.388140238795048 + 0.000000000000000i \end{bmatrix} \tag{11}$$

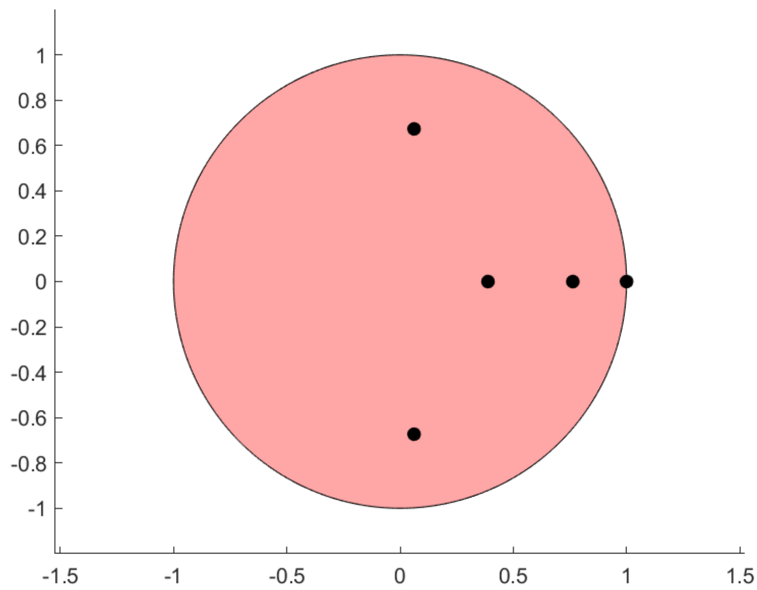

Figure 1: Poles for the discrete system.