SSY156 Homework 2 Group 19

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Exercise 1

1.

We have defined our coordinate frames according to Figures (1 - 4) where Figures (1) and (2) define the coordinate frames of the links and Figures (3) and (4) the coordinate frames of each links center of mass. The world frame is not included in any of these figures, the first frame is the 0 frame which is located at the same place as the world frame but with a different orientation. The transformation between the world frame and the 0 frame is described by

$$H_0^w = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

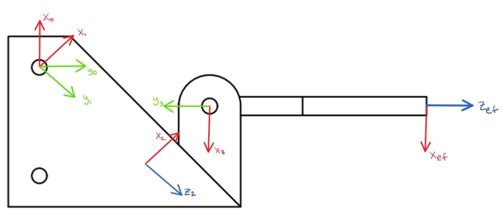

Figure 1: Side view of the robot with all designated frames for the links.

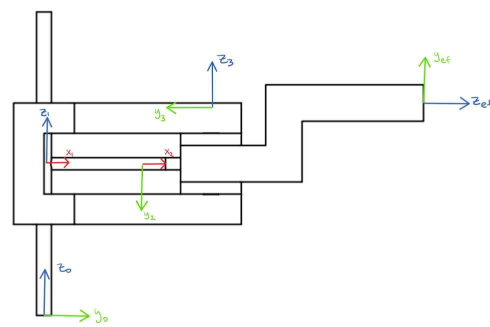

Figure 2: Top view of the robot with all designated frames for the links.

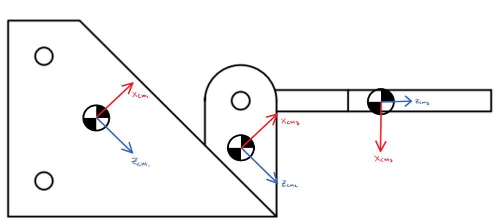

Figure 3: Side view of the robot with all designated frames for the center of masses of the links.

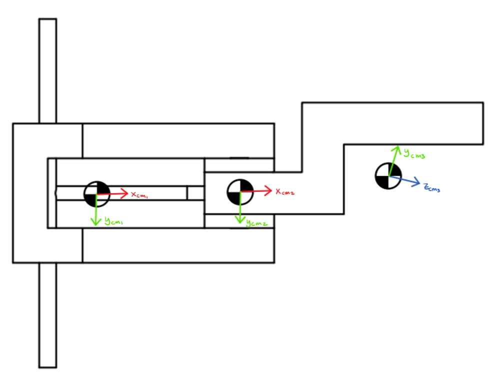

Figure 4: Top view of the robot with all designated frames for the center of masses of the links.

2.

The Denavit-Hartenberg (D-H) parameters were calculated using the proximal version, where the rotation and distance with respect to the previous x-axis are calculated before the parameters related to the z-axis, see table 1 and 2.

$$\alpha = \frac{\pi}{4} - \delta, \qquad \text{where} \quad \delta = \arctan(\frac{L_6 - L_2}{L_6}) \tag{2}$$

$$H = \sqrt{L_6^2 + (L_6 + L_2)^2} \tag{3}$$

$$d_1 = H \cdot \cos\alpha, \qquad d_2 = H \cdot \sin\alpha, \qquad d_4 = L_2 - L_{10} \tag{4}$$

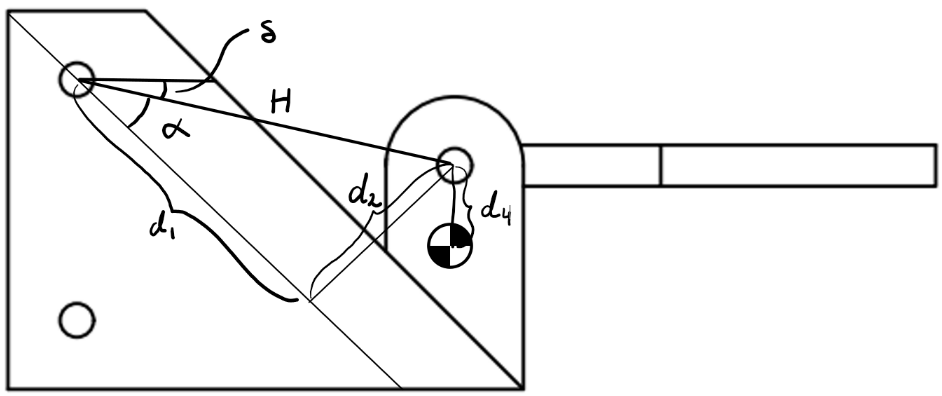

Figure 5: Distances and angles used to

$link_i$	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	$q_1 + L_8$	$\frac{\pi}{4}$
2	0	$-\frac{\pi}{2}$	$q_2 + d_1$	0
3	d_2	$\frac{\pi}{2}$	L_5	$q_3 - \frac{3\pi}{4}$
ef	0	$\frac{\pi}{2}$	$L_3 + L_4$	0

Table 1: Denavit Hartenberg proximal parameters for the joints.

$link_i$	a_{i-1}	α_{i-1}	d_i	θ_i
cm_1	0	$-\frac{\pi}{2}$	$\sqrt{2} \cdot L_9$	0
cm_2	$d_2 - d_4 \cdot \cos(\frac{\pi}{4})$	0	$(L_2 - L_{10}) \cdot \sin(\frac{\pi}{4})$	0
cm_3	0	$\frac{\pi}{2} + \arctan(\frac{L_5}{2(L_3+L_{11})})$	$\sqrt{(\frac{L_5}{2})^2 + (L_3 + L_{11})^2}$	0

Table 2: Denavit Hartenberg proximal parameters for the center of mass.

3.

To compute the relative homogeneous transformation matrices the following relationship between the (D-H) parameters and the homogeneous transformation matrix was used.

$$H_i^{i-1}(\alpha, a, \theta, d) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i \cos\alpha_{i-1} & \cos\theta_i \cos\alpha_{i-1} & -\sin\alpha_{i-1} & -d_i \sin\alpha_{i-1} \\ \sin\theta_i \sin\alpha_{i-1} & \cos\theta_i \sin\alpha_{i-1} & \cos\alpha_{i-1} & d_i \cos\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{5}$$

$$H_0^w = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$H_1^0 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & q_1 + \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$H_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & q_2 + \frac{\sqrt{2}}{5} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

$$H_3^2 = \begin{bmatrix} \cos\left(q_3 + \frac{3\cdot\pi}{4}\right) & -\sin\left(q_3 + \frac{3\cdot\pi}{4}\right) & 0 & \frac{3\cdot\sqrt{2}}{20} \\ 0 & 0 & -1 & -\frac{1}{10} \\ \sin\left(q_3 + \frac{3\cdot\pi}{4}\right) & \cos\left(q_3 + \frac{3\cdot\pi}{4}\right) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

$$H_{ef}^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -\frac{13}{20} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

$$H_{cm_1}^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.2475 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$H_{cm_2}^2 = \begin{bmatrix} 1 & 0 & 0 & 0.1061 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.1061 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$H_{cm_3}^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.1129 & -0.9936 & -0.44 \\ 0 & 0.9936 & -0.1129 & -0.05 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

4.

To calculate the absolute Homogeneous Transformations from each joint and center of mass with respect to the world frame, table 1 and 2 together with transformation matrix 5 is used. How the transformation is done is described in question 3.

$$H_0^w = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

$$H_1^w = H_0^w \cdot H_1^0 = \begin{bmatrix} 0 & 0 & 1 & q_1 + \frac{1}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$H_2^w = H_1^w \cdot H_2^1 = \begin{bmatrix} 0 & -1 & 0 & q_1 + \frac{1}{2} \\ -\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}\cdot q_2}{2} - \frac{1}{5} \\ \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}\cdot q_2}{2} - \frac{1}{5} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

$$H_3^w = H_2^w \cdot H_3^2 = \begin{bmatrix} 0 & 0 & 1 & q_1 + \frac{3}{5} \\ \sin(q_3) & \cos(q_3) & 0 & \frac{\sqrt{2}\cdot q_2}{2} - \frac{7}{20} \\ -\cos(q_3) & \sin(q_3) & 0 & \frac{\sqrt{2}\cdot q_2}{2} - \frac{1}{20} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

$$H_{ef}^w = H_3^w \cdot H_{ef}^3 = \begin{bmatrix} 0 & 1 & 0 & q_1 + \frac{3}{5} \\ \sin(q_3) & 0 & -\cos(q_3) & -\frac{13\cdot\cos(q_3)}{20} - \frac{\sqrt{2}\cdot q_2}{2} - \frac{7}{20} \\ -\cos(q_3) & \sin(q_3) & 0 & -\frac{13\cdot\sin(q_3)}{20} - \frac{\sqrt{2}\cdot q_2}{2} - \frac{1}{20} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

$$H_{cm_1}^w = \begin{bmatrix} 0 & -1 & 0 & q_1 + \frac{1}{2} \\ -\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & \frac{-7}{40} \\ \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & \frac{-7}{40} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

$$H_{cm_2}^w = \begin{bmatrix} 0 & -1 & 0 & q_1 + \frac{1}{2} \\ \frac{-\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}\cdot q_2}{2} - \frac{7}{20} \\ \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}\cdot q_2}{2} - \frac{1}{5} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

$$H_{cm_3}^w = \begin{bmatrix} 0 & 0.9936 & -0.1129 & q_1 + \frac{11}{20} \\ \sin(q_3) & -0.1129\cdot\cos(q_3) & -0.9936\cdot\cos(q_3) & -\frac{11}{25}\cdot\cos(q_3) - \frac{\sqrt{2}}{2}\cdot q_2 - \frac{7}{20} \\ -\cos(q_3) & -0.1129\cdot\sin(q_3) & -0.9936\cdot\sin(q_3) & -\frac{11}{25}\cdot\sin(q_3) - \frac{\sqrt{2}}{2}\cdot q_2 - \frac{1}{20} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

5.

The jacobians for the different coordinate frames w.r.t. the 0 frame in question were calculated using equation (22) where P_e is the location of the frame that the jacobian is computed for.

$$\begin{bmatrix} J_{p_i} \\ J_{O_i} \end{bmatrix} = \underbrace{\begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}}_{\text{Prismatic joint}} = \underbrace{\begin{bmatrix} z_{i-1} \times (p_e - p_{i-1}) \\ z_{i-1} \end{bmatrix}}_{\text{Revolute joint}} \tag{22}$$

$$J_{ef}^0 = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & -\frac{13\cdot\cos(q_3)}{20} \\ 0 & \frac{\sqrt{2}}{2} & -\frac{13\cdot\sin(q_3)}{20} \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{23}$$

$$J_{cm_1}^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{24}$$

$$J_{cm_2}^0 = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} \\ 0 & -\frac{\sqrt{2}}{2} \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{25}$$

$$J_{cm_3}^0 = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & -\frac{11\cdot\cos(q_3)}{25} \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{11\cdot\sin(q_3)}{25} \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{26}$$