SSY156 Homework 3 Group 19

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Exercise 2

1)

To be able to calculate the inertia matrix M, the Jacobian matrices for the manipulator were needed. The easiest way to obtain the Jacobians was in turn to calculate the Denavit Hartenberg parameters for the manipulator, see table 1 & 2. Since the manipulator is not a linear robot, which the DH parameters is based on, we had to make 2 DH tables.

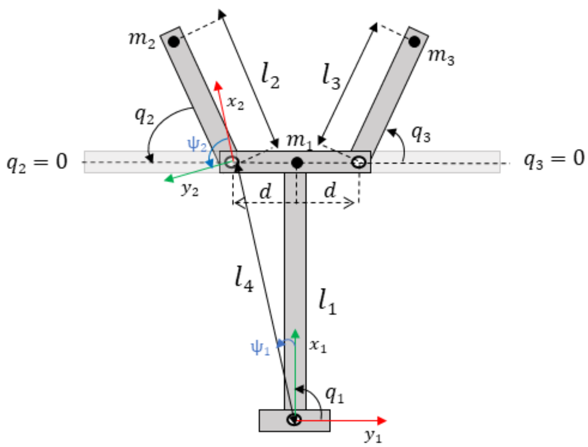

Figure 1: Caption

$$\psi_1 = \arctan \frac{d}{L_1} \tag{1}$$

$$\psi_2 = \frac{\pi}{2} - \psi_1 \tag{2}$$

$$L_4 = \sqrt{d^2 + L_1^2} \tag{3}$$

$link_i$	θ_i	d_i	a_i	α_i
1	$q_1 + \psi_1$	0	L_4	0
2	$-q_2 + \psi_2$	0	L_2	0

Table 1: Denavit Hartenberg parameters for the "arm" on the left.

$link_i$	θ_i	d_i	a_i	α_i
1	$q_1 - \psi_1$	0	L_4	0
2	$q_3 - \psi_2$	0	L_3	0

Table 2: Denavit Hartenberg parameters for the "arm" on the right.

The following transformation matrices were obtained from the DH parameters.

$$H_1^0 = \begin{bmatrix} \cos q1 & -\sin q1 & 0 & L_1 \cdot \cos q1 \\ \sin q1 & \cos q1 & 0 & L_1 \cdot \sin q1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$H_2^L = \begin{bmatrix} \cos q2 & -\sin q2 & 0 & -L_2 \cdot \cos q2 \\ \sin q2 & \cos q2 & 0 & L_2 \cdot \sin q2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$H_3^R = \begin{bmatrix} \cos q3 & -\sin q3 & 0 & L_3 \cdot \cos q3 \\ \sin q3 & \cos q3 & 0 & L_3 \cdot \sin q3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

To calculate the Jacobian matrices from the DH parameters/transformation matrices following equation was used:

$$\begin{bmatrix} J_{p_i} \\ J_{O_i} \end{bmatrix} = \underbrace{\begin{bmatrix} z_{i-1} \times (p_e - p_{i-1}) \\ z_{i-1} \end{bmatrix}}_{\text{Revolute joint}} \tag{7}$$

We decided to skip the second term of the inertia matrix regarding the angular velocity and the moment of inertia term. This is because in this case we have point masses and the inertia, $\mathcal{I}$, is calculated from the center of mass and thus the moment of inertia becomes 0.

$$M(q) = \sum_{i=1}^{n} (m_i \mathbf{J}_{vcm_i}^{0T} \mathbf{J}_{vcm_i}^{0} + \underbrace{\mathbf{J}_{\omega cm_i}^{0T} \mathbf{R}_{cm_i}^{0} \mathcal{I}_{cm_i} \mathbf{R}_{cm_i}^{0T} \mathbf{J}_{\omega cm_i}^{0})}_{=0} \tag{8}$$

Since the Jacobian matrices didn't have the same size and we needed to get a $3x3$ matrix to match the size of q, we had to pad the Jacobians with zeros as follows:

$$\mathbf{J}_{vcm_1}^{0} = [\mathbf{J}_{vcm_1}^{0} \;\; \mathbf{0} \;\; \mathbf{0}], \quad \mathbf{J}_{vcm_2}^{0} = [\mathbf{J}_{vcm_1}^{0} \;\; \mathbf{J}_{vcm_2}^{1} \;\; \mathbf{0}], \quad \mathbf{J}_{vcm_3}^{0} = [\mathbf{J}_{vcm_1}^{0} \;\; \mathbf{0} \;\; \mathbf{J}_{vcm_3}^{1}] \tag{9}$$

The final inertia matrix became:

$$M(q) = \begin{bmatrix} L_1^2(m_1 + m_2 + m_3) & L_1 L_2 m_2 \sin q_2 & L_1 L_3 m_3 \sin q_3 \\ L_1 L_2 m_2 \sin q_2 & L_2^2 m_2 & 0 \\ L_1 L_3 m_3 \sin q_3 & 0 & L_3^2 m_3 \end{bmatrix} \tag{10}$$

When examining the inertia matrix it is clear that the first link effects both the second and third link but the second and third link does not effect each other which can be seen with the cross terms is the inertia matrix.

2)

To find the joint varibles for which the coupling inertial forces equal to zero, we used the off diagonal elements in the inertia matrix calculated above. Further we substituted L_1, L_2, L_3 & d to 1 and set them equal to zero and solved for q_2 and q_3 as follows:

$$m_2 \cdot \sin q_2 = 0, \quad m_3 \cdot \sin q_3 = 0 \tag{11}$$

With following constraints:

$$-\pi < q_i \leq \pi, \quad i = 1, 2, 3 \tag{12}$$

We obtained the coupling inertial forces to be zero when:

$$q_2 = 0 \;\; \& \;\; q_3 = 0 \tag{13}$$

$$q_2 = \pi \;\; \& \;\; q_3 = 0 \tag{14}$$

$$q_2 = 0 \;\; \& \;\; q_3 = \pi \tag{15}$$

$$q_2 = \pi \;\; \& \;\; q_3 = \pi \tag{16}$$

Q1

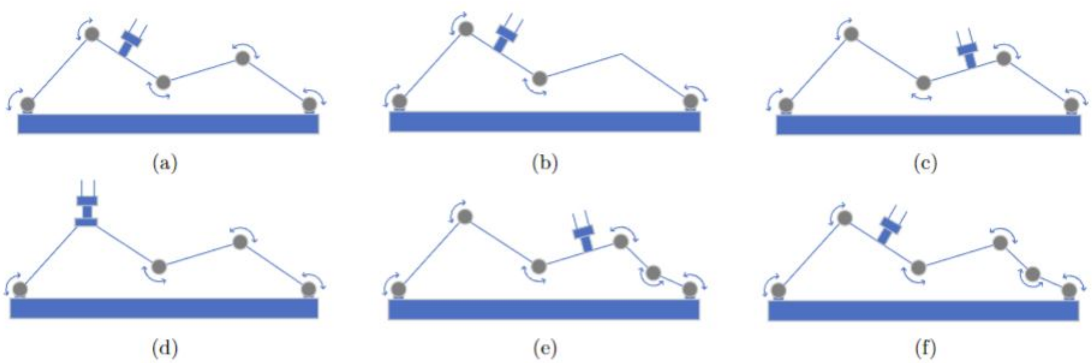

Figure 2: Given models from task.

By using the equation of Gruebler's for a planar constrained robotic mechanisms with revolute joints and deciding the amount of links and joints by counting them in each model figure 2 the degrees of freedom could be calculated. Gruebler's equation is shown down below (17):

$$M = \text{ degrees of freedom } = \underbrace{3(n-1)}_{DOFs} - \underbrace{(2j_{\mathrm{p}} + j_{\mathrm{h}})}_{Constraints} \tag{17}$$

Where n is the amount of links, j_p the amount of joints and j_b can be disregarded in all our cases due to that we are just considering revolute joints. By first considering the hole system and counting the ground as 1 link the following DoF's could be calculated:

DoF's for the hole system of each case:

Case a):
Amount of links, $n = 5$.
Amount of joints, $j_b = 5$.
DoF's for the hole system: $DoF's = 3(5-1) - 2 \cdot 5 = \underline{\mathbf{2}}$

Case b):
Amount of links, $n = 4$.
Amount of joints, $j_b = 4$.
DoF's for the hole system: $DoF's = 3(4-1) - 2 \cdot 4 = \underline{\mathbf{1}}$

Case c):
Amount of links, $n = 5$.
Amount of joints, $j_b = 5$.
DoF's for the hole system: $DoF's = 3(5-1) - 2 \cdot 5 = \underline{\mathbf{2}}$

Case d):
Amount of links, $n = 4$.
Amount of joints, $j_b = 4$.
DoF's for the hole system: $DoF's = 3(4-1) - 2 \cdot 4 = \underline{\mathbf{1}}$

Case e):
Amount of links, $n = 6$.
Amount of joints, $j_b = 6$.
DoF's for the hole system: $DoF's = 3(6-1) - 2 \cdot 6 = \underline{\mathbf{3}}$

Case f):
Amount of links, $n = 6$.
Amount of joints, $j_b = 6$.
DoF's for the hole system: $DoF's = 3(6-1) - 2 \cdot 6 = \underline{\mathbf{3}}$

DoF's for the end defector of each case:

And when calculating the DoF's for the end defector we consider the shortest way (least amount of links and joints) from 1 fixed joint to the end defector and just consider everything in between those 2. To note is that the DoF's for the end defector cannot to exceed the DoF's for the hole structure:

Case a):
Amount of links, $n = 3$.
Amount of joints, $j_b = 2$.
DoF's for the end defector: $DoF's = 3(3-1) - 2 \cdot 2 = \underline{\mathbf{2}}$

Case b):
Amount of links, $n = 3$.
Amount of joints, $j_b = 2$.
DoF's for the end defector: $DoF's = 3(3-1) - 2 \cdot 2 = 2$
Exceeds the number of DoF's for the strucutre and is therefore set to, $DoF's = \underline{\mathbf{1}}$

Case c):
Amount of links, $n = 3$.
Amount of joints, $j_b = 2$.
DoF's for the end defector: $DoF's = 3(3-1) - 2 \cdot 2 = \underline{\mathbf{2}}$

Case d):
Amount of links, $n = 2$.
Amount of joints, $j_b = 1$.
DoF's for the end defector: $DoF's = 3(2-1) - 2 \cdot 1 = \underline{\mathbf{1}}$

Case e):
Amount of links, $n = 4$.
Amount of joints, $j_b = 3$.
DoF's for the end defector: $DoF's = 3(4-1) - 2 \cdot 3 = \underline{\mathbf{3}}$

Case f):
Amount of links, $n = 3$.
Amount of joints, $j_b = 2$.
DoF's for the end defector: $DoF's = 3(3-1) - 2 \cdot 2 = \underline{\mathbf{2}}$

Q2

Given model and equations from task is shown down below in figure 3 and equations (18) and (19):

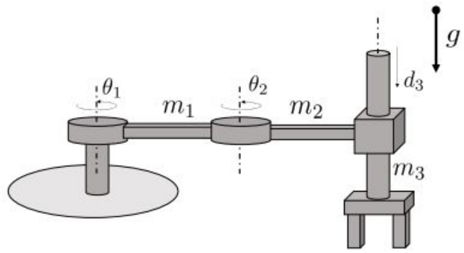

Figure 3: Given model from task.

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = 0 \tag{18}$$

$$\boldsymbol{g} = \begin{bmatrix} g_1 & g_2 & g_3 \end{bmatrix}^T \text{ and } \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} \tag{19}$$

a)

By adding extra mass to the second link in the form of a camera will not change how the gravity effects the dynamics of the system. This can be seen in (20) as $g_2 = 0$ this is because link 2 is only capable of rotating around an axis co-linear with the axis of which the gravitational force is acting i.e. gravity will not have any effect on the second links dynamics. The added weight on the second link would change the dynamics of the system as the inertia of the second link would increase but this would only change the M matrix not $\boldsymbol{g}$. Since the third link is a prismatic joint which is aligned with the axis of gravity the g_3 in (20) will show that gravity has an effect of the dynamics of the third link.

$$\boldsymbol{g} = [0 \quad 0 \quad m_3 \cdot g]^T \tag{20}$$

b)

In this case the gravitational effects of the third link is increased as it is now heavier which is shown with the masses m_0 and m_3 summed together.

$$\boldsymbol{g} = [0 \quad 0 \quad (m_o + m_3) \cdot g]^T \tag{21}$$

c)

In this case the camera is moved from the second link to the gripper and the gravitational effects of the third link is increased even more with the mass m_c summed together with m_0 and m_3.

$$\boldsymbol{g} = [0 \quad 0 \quad (m_c + m_o + m_3) \cdot g]^T \tag{22}$$

d)

Since the first prismatic joint is connected to the kinematic chain in the beginning and the axis of motion is aligned with the gravitational force this links dynamics will be affected by gravity. Due to the fact that this link sits at the beginning of the chain it needs to hold up all other links which results in (23)

$$g_1 = (m_1 + m_2 + m_3 + m_c + m_o) \cdot g. \tag{23}$$