Modelling and Control of Mechatronics Systems Assignment 1 Group 21

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1 Introduction

Given a 2DOF PR Robot with a prismatic joint q1 and a revolute joint q2 with motor force f1 and torque τ :

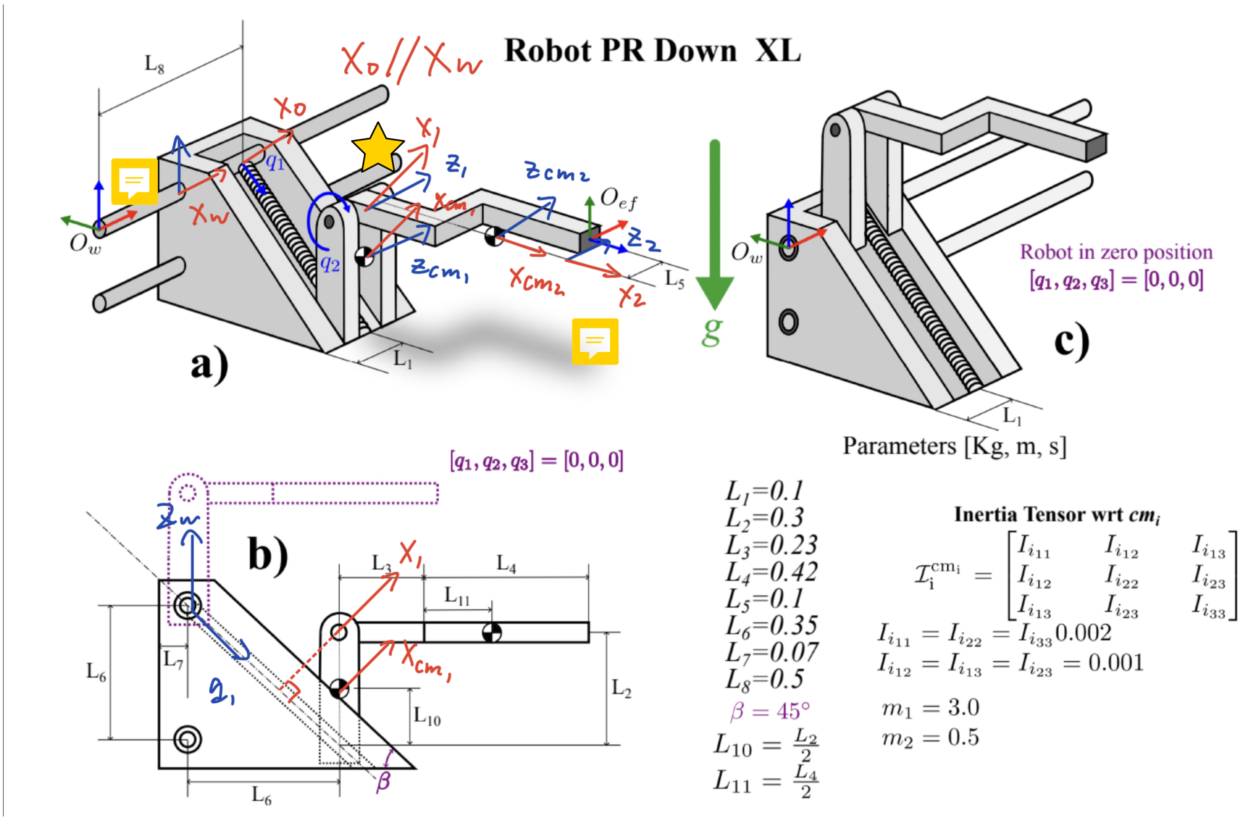

Figure 1: Coordinate frame setting

2 Q1: The dynamical model:-

i	θ	d	a	α
1	$\frac{\pi}{2}$	$q_1 - L_2 sin\beta$	$L_2 cos\beta$	$\frac{\pi}{2}$
1	$q_2 + \frac{\pi}{2} - \beta$	0	$L_3 + L_4$	0

Table 1: D-H parameters for the joints

i	θ	d	a	α
cm1	$\frac{\pi}{2}$	$q_1 - L_{10} sin\beta$	$L_{10} cos\beta$	$\frac{\pi}{2}$
cm2	$q_2 + \frac{\pi}{2} - \beta$	0	$L_3 + L_{11}$	0

Table 2: D-H parameters for the center of mass

Homogeneous Transformation Matrices:
Based on the DH tables we can compute the homogeneous transformation matrices between the different joints (H_{i-1}^{i}) and between the different centers of masses ($H_{i-1,cm}^{i}$).

$$H_0^w : \begin{pmatrix} 1 & 0 & 0 & L_1 \\ 0 & -\cos(\beta) & -\sin(\beta) & 0 \\ 0 & \sin(\beta) & -\cos(\beta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_1^0\colon \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & L_2\,\cos\left(\beta\right) \\ 0 & 1 & 0 & q_1 - L_2\,\sin\left(\beta\right) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_2^1\colon \begin{pmatrix} \cos\left(q_2-\beta+1.5708\right) & -\sin\left(q_2-\beta+1.5708\right) & 0 & \cos\left(q_2-\beta+1.5708\right)\,\left(L_3+L_4\right) \\ \sin\left(q_2-\beta+1.5708\right) & \cos\left(q_2-\beta+1.5708\right) & 0 & \sin\left(q_2-\beta+1.5708\right)\,\left(L_3+L_4\right) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_2^0\colon \begin{pmatrix} 0 & 0 & 1 & 0 \\ \cos\left(q_2-\beta+1.5708\right) & -\sin\left(q_2-\beta+1.5708\right) & 0 & L_2\,\cos\left(\beta\right)+\cos\left(q_2-\beta+1.5708\right)\,\left(L_3+L_4\right) \\ \sin\left(q_2-\beta+1.5708\right) & \cos\left(q_2-\beta+1.5708\right) & 0 & q_1-L_2\,\sin\left(\beta\right)+\sin\left(q_2-\beta+1.5708\right)\,\left(L_3+L_4\right) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Hcm1\colon \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & L_{10}\,\cos\left(\beta\right) \\ 0 & 1 & 0 & q_1 - L_{10}\,\sin\left(\beta\right) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Hcm2\colon \begin{pmatrix} \cos\left(q_2-\beta+1.5708\right) & -\sin\left(q_2-\beta+1.5708\right) & 0 & \cos\left(q_2-\beta+1.5708\right)\,\left(L_3+L_{11}\right) \\ \sin\left(q_2-\beta+1.5708\right) & \cos\left(q_2-\beta+1.5708\right) & 0 & \sin\left(q_2-\beta+1.5708\right)\,\left(L_3+L_{11}\right) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Hcm_2^0\colon \begin{pmatrix} 0 & 0 & 1 & 0 \\ \sin\left(\beta-q_2\right) & -\cos\left(\beta-q_2\right) & 0 & L_2\,\cos\left(\beta\right)+L_3\,\sin\left(\beta-q_2\right)+L_{11}\,\sin\left(\beta-q_2\right) \\ \cos\left(\beta-q_2\right) & \sin\left(\beta-q_2\right) & 0 & q_1-L_2\,\sin\left(\beta\right)+L_3\,\cos\left(\beta-q_2\right)+L_{11}\,\cos\left(\beta-q_2\right) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Jacobian matrices of two masses can be compute as below:

$$J_{ef}^0 = \begin{bmatrix} Z_0^0 & Z_1^0 & Z_2^0 x(t_{ef}^0 - t_2^0) \\ 0 & 0 & Z_2^0 \end{bmatrix} \tag{1}$$

$$J0cm1\text{:} \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$J0cm2\text{:} \begin{pmatrix} 0 & 0 \\ 0 & -\cos\left(\beta - q_2\right)\left(L_3 + L_{11}\right) \\ 1 & \sin\left(\beta - q_2\right)\left(L_3 + L_{11}\right) \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Dynamic Model Matrices

So we have dynamic model matrices constructed as:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{2}$$

where as:

$$M(q) = \begin{bmatrix} \frac{7}{2} & -\frac{11 sin(q2 - \frac{\pi}{4})}{50} \\ -\frac{11 sin(q2 - \frac{\pi}{4})}{50} & \frac{1497}{2500} \end{bmatrix} \tag{3}$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & -\frac{11 \dot{q}2 cos(q2 - \frac{\pi}{4})}{50} \\ 0 & 0 \end{bmatrix} \tag{4}$$

$$G(q) = \begin{bmatrix} \frac{7\sqrt{2}g}{4} \\ -\frac{11 g sin(q2 - \frac{\pi}{2})}{50} \end{bmatrix} \tag{5}$$

3 Q2: Design a controller in the Joint Space

We are now building a simple PD controller in joint space, in order to compensate its internal robot's forces.Once we cancel out the effects of gravity and inertia then we can almost pretend that the system behaves linearly. This means that we can also treat control of each of the joints independently, since their movements no longer affect one another. So in our control system we're actually going to have a PD controller for each joint.

Rewriting the system dynamics, in terms of acceleration gives:

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{q})(\mathbf{u} - \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{g}(\mathbf{q})) \tag{6}$$

Ideally, the control signal would be constructed as:

$$\mathbf{u} = (\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}_{\text{des}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q})) \tag{7}$$

where $\ddot{\boldsymbol{q}}_{\text{des}}$is the desired acceleration of the system. When the Coriolis centrifugal effects are not considered, the PD control signal is:

$$\mathbf{u} = (\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}_{\text{des}} + \mathbf{g}(\mathbf{q})) \tag{8}$$

So we use a standard PD control formula to generate the desired acceleration:

$$\ddot{\mathbf{q}}_{des} = k_p\left(\mathbf{q}_{\text{des}} - \mathbf{q}\right) + k_v\left(\dot{\mathbf{q}}_{\text{des}} - \dot{\mathbf{q}}\right) \tag{9}$$

where k_p and k_v are our gain values, and the control signal has been fully defined:

$$\mathbf{u} = \left(\mathbf{M}(\mathbf{q})\left(k_p\left(\mathbf{q}_{\text{des}} - \mathbf{q}\right) + k_v\left(\dot{\mathbf{q}}_{\text{des}} - \dot{\mathbf{q}}\right)\right) + \mathbf{g}(\mathbf{q})\right) \tag{10}$$