SSY156 Homework 1 Group 19

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Problem 6

a)

In this question the transformation matrices to the frames $\{C\}, \{Ob\}, \{LA\}, \{LH\}, \{RA\}$ and $\{RH\}$ from the origin $\{O\}$ are calculated. The first transformation that was calculated was the transformation from the origin to the object frame i.e. H_{Ob}^{O} which only required simple translation in x, y and z to place the frame in the correct location. For the other frames which were all connected to the robot itself we created an intermediate frame located between the shoulders inline with the "spine" of the robot which we called the torso frame (T) and was used to take care of the robots ability to rotate around its spine. All frames on the robot was then calculated relative to this torso frame which rotates with the robot making them simpler to compute. The total transformation from the origin to the part on the robot was then calculated as the product of the transformation to the torso and then the transformation from the torso to the part which we will clarify in the calculations below.

$$H_T^O = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & -L_3 \\ \sin\theta & \cos\theta & 0 & L_2 \\ 0 & 0 & 1 & L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1}$$

$$H_{Ob}^O = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & L_2 \\ 0 & 0 & 1 & \frac{z_{ob}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$H_C^O = \begin{bmatrix} -\sin\theta & 0 & \cos\theta & L_4\cos\theta - L_3 \\ \cos\theta & 0 & \sin\theta & L_2 + L_4\sin\theta \\ 0 & 1 & 0 & L_6 + L_7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$H_{LA}^O = H_T^O \cdot H_{LA}^T = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & -L_3 - L_5 \cdot \frac{\sin\theta}{2} \\ \sin\theta & 0 & \cos\theta & L_2 + L_5 \cdot \frac{\cos\theta}{2} \\ 0 & -1 & 0 & L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$H_{RA}^O = H_T^O \cdot H_{RA}^T = \begin{bmatrix} \cos\theta & 0 & \sin\theta & L_5 \cdot \frac{\sin\theta}{2} - L_3 \\ \sin\theta & 0 & -\cos\theta & L_2 - L_5 \cdot \frac{\cos\theta}{2} \\ 0 & 1 & 0 & L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$H_{RH}^O = H_T^O \cdot H_{RA}^T \cdot H_{RH}^{RA} \tag{6}$$

$$H_{LH}^O = H_T^O \cdot H_{LA}^T \cdot H_{LH}^{LA} \tag{7}$$

b)

We decided to call the given point $P_O = [x_{ob} \quad y_{ob} \quad z_{ob}]^T$. What we are asked to calculated in this question is the location of this point in the reference frame $\{RA\}$. What we first did was set up the relation seen in equation (8) which uses the transformation matrix to translate points from the perspective of $\{RA\}$ to $\{O\}$ which is the opposite of what we want in this case.

$$P_O = H_{RA}^O \cdot P_{RA} \tag{8}$$

What we instead want is the transformation matrix from $\{RA\}$ to $\{O\}$ which is the inverse of H_{RA}^O. The inverse of the transformation matrix was done according to equation (9).

$$H_{RA}^O = (H_O^{RA})^{-1} = \begin{bmatrix} (R_O^{RA})^T & -(R_O^{RA})^T p_O^{RA} \\ 0_{1x3} & 1 \end{bmatrix} \tag{9}$$

The transformation matrix from RA to O will thus become:

$$H_O^{RA} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & L_3 \cdot \cos\theta - L_2 \cdot \sin\theta \\ 0 & 0 & 1 & -L_6 \\ \sin\theta & -\cos\theta & 0 & L_2 \cdot \cos\theta - \frac{L_5}{2} + L_3 \cdot \sin\theta \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

Given H_O^{RA} the relationship presented in equation (11) can be set up and P_{RA} can be calculated from the given point P_O

$$P_{RA} = H_O^{RA} \cdot P_O \tag{11}$$

$$P_{RA} = \begin{bmatrix} L_3 \cos\theta - L_2 \sin\theta + x_{ob} \cos\theta + y_{ob} \sin\theta \\ z_{ob} - L_6 \\ L_2 \cos\theta - \frac{L_5}{2} + L_3 \sin\theta - y_{ob} \cos\theta + x_{ob} \sin\theta \end{bmatrix} \tag{12}$$

c)

To begin the transformation matrix H_{LA}^{Ob} which can be computed using a combination of the previously presented matrices according to equation (13).

$$H_{LA}^{Ob} = (H_O^{Ob})^{-1} H_{LA}^{O} \tag{13}$$

$$H_{LA}^{Ob} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & -L_1 - L_3 - \frac{L_5 \sin\theta}{2} \\ \sin\theta & 0 & \cos\theta & \frac{L_5 \cos\theta}{2} \\ 0 & -1 & 0 & L_6 - z_{ob} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

The rotation matrix between the two frames can be extracted from H_{LA}^{Ob} as it's the top left 3 by 3 matrix.

$$R = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\theta & 0 & \cos\theta \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{15}$$

Unit Quaternion

The unit quaternion are defined as $\mathcal{Q} = \{\eta, \epsilon\}$, where η is the scalar part of the quaternion and ϵ is the vector part which in turn has 3 components, $\epsilon = [\epsilon_x \epsilon_y \epsilon_z]^T$. Given the rotation matrix R, η is calculated as:

$$\eta = \frac{1}{2}\sqrt{r_{11} + r_{22} + r_{33} + 1} = \frac{\sqrt{2}}{2} \tag{16}$$

Given $\eta \neq 0$ we can calculate ϵ using Chiaverini-Sciliano's method

$$\begin{aligned} \epsilon_x &= \text{sgn}(r_{32} - r23)\sqrt{r_{22} + r_{33} + r_{11}} = -\tfrac{\sqrt{2}}{2} \\ \epsilon_y &= \text{sgn}(r_{13} - r31)\sqrt{r_{11} + r_{22} + r_{33}} = 0 \\ \epsilon_z &= \text{sgn}(r_{21} - r12)\sqrt{r_{33} + r_{11} + r_{22}} = 0 \end{aligned} \tag{17}$$

Euler angles

A set of Euler angles are incomplete if not also presented with the order of which the rotations should be carried out. In this case the rotations will be carried out in the ZYZ order.

$$R(\phi) = R(\varphi) R_{z'}(\vartheta) R_{y''}(\psi) \tag{18}$$

For the ZYZ order the angles φ, ϑ and ψ are calculated using equation (19) with the values from (15).

$$\begin{aligned} \varphi &= \text{Atan2}(r_{23}, r_{13}) \\ \vartheta &= \text{Atan2}(\sqrt{r_{13}^2 + r_{23}^2}, r_{33}) \\ \psi &= \text{Atan2}(r_{32}, -r_{31}) \end{aligned} \tag{19}$$

Performing the calculations gives the following result

$$\begin{aligned} \varphi &= \tfrac{\pi}{2} \\ \vartheta &= \tfrac{\pi}{2} \\ \psi &= -\tfrac{\pi}{2} \end{aligned} \tag{20}$$

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