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Blatt 3

Auf. 1

(a) $\Omega = [6]^2 \quad \mathcal{F} = \mathcal{P}(\Omega) \quad P(A) = \frac{|A|}{|\Omega|}$

(b) $C = [6]$

(c) $p_k(x_k) = \frac{2x_k - 1}{36}$

Auf. 2

(a) $X(w) = \begin{cases} 0 & w < \frac{1}{2} \\ 1 & \text{sonst} \end{cases} \qquad \forall B \in \mathbb{B}_{\mathbb{R}} \quad X^{-1}(B) = \begin{cases} \emptyset & 0 \in B \\ [0, \frac{1}{2}) & 0 \notin B \end{cases} \cup \begin{cases} \emptyset & 1 \in B \\ [\frac{1}{2}, 1] & 1 \notin B \end{cases} \quad \begin{array}{l} \Rightarrow X^{-1}(B) \in \mathcal{F} \\ \Rightarrow X \text{ ist ZV} \end{array}$

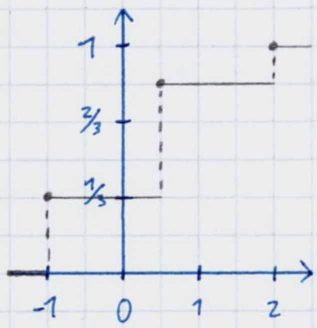
(b) $X(w) = \begin{cases} 0 & w < \frac{1}{3} \\ 1 & \frac{1}{3} \leq w < \frac{2}{3} \\ 2 & \text{sonst} \end{cases} \qquad \forall B \in \mathbb{B}_{\mathbb{R}} \quad X^{-1}(B) = \begin{cases} \emptyset & 0 \in B \\ [0, \frac{1}{3}) & 0 \notin B \end{cases} \cup \begin{cases} \emptyset & 1 \in B \\ [\frac{1}{3}, \frac{2}{3}) & 1 \notin B \end{cases} \cup \begin{cases} \emptyset & 2 \in B \\ [\frac{2}{3}, 1] & 2 \notin B \end{cases}$

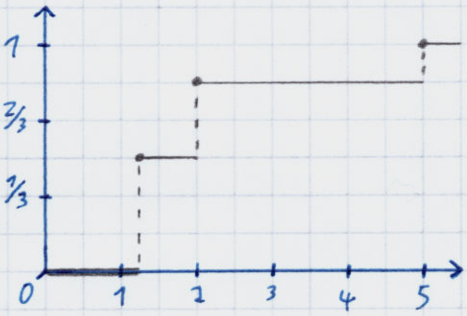
$\Rightarrow X^{-1}(B) \in \mathcal{F} \quad \Rightarrow X \text{ ist ZV}$

Auf. 3

(a) $F_X(x) = \begin{cases} 0 & x < -1 \\ \frac{2}{6} & -1 \leq x < \frac{1}{2} \\ \frac{5}{6} & \frac{1}{2} \leq x < 2 \\ 1 & 2 \leq x \end{cases}$

(b) $F_{X^2+1}(x) = \begin{cases} 0 & x < 1{,}25 \\ \frac{3}{6} & 1{,}25 \leq x < 2 \\ \frac{5}{6} & 2 \leq x < 5 \\ 1 & 5 \leq x \end{cases}$





Auf. 4

(a) $p_k(k) = \begin{cases} \left(\frac{2}{3}\right)^{0{,}5k} \cdot (1-p)^{0{,}5k-1} \cdot p & k \text{ gerade} \\ \left(\frac{2}{3}\right)^{0{,}5(k-1)} \cdot (1-p)^{0{,}5(k-1)} \cdot \frac{1}{3} & k \text{ ungerade} \end{cases}$

(b) $p_k(k) = \begin{cases} \sum_{n=1}^{\infty} P(X = 2n) & k = 5 \\ \sum_{n=1}^{\infty} P(X = 2n-1) & k = 2 \end{cases}$

(c) $P(Y=2) = P(Y=5) \Leftrightarrow \sum_{n=1}^{\infty} P(X = 2n) = \sum_{n=1}^{\infty} P(2n-1) \Leftrightarrow$

$\Leftrightarrow \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n (1-p)^{n-1} p = \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^{n-1} (1-p)^{n-1} \frac{1}{3}$

$\Leftrightarrow \frac{2}{3} p = \frac{1}{3}$

$\Leftrightarrow p = \frac{1}{2}$

Auf. 5

(a) $F_X(x) = \sum_{k=1}^{x} P(X=k) = \sum_{k=1}^{x} (1-p)^{k-1} p = \frac{1-(1-p)^x}{1-(1-p)} p = 1-(1-p)^x$

(b) $P(X=n+k \mid X>n) = \frac{P(X=n+k \cap X>n)}{P(X>n)} = \frac{P(X=n+k)}{1-P(X\leq k)} = \frac{(1-p)^{n+k-1}}{1-(1-(1-p)^k)} p = (1-p)^{n-1} p = P(X=n)$

Auf. 6

(a) 150 172 179 179 182 198 205 206 230 247

(b) (i) 194,8 (iv) 190

(ii) 192,9348 (v) 179

(iii) 191,1036 (vi) 206