Lösungsvorschlag Blatt 13

1) a) $\int_{-1}^{1} x e^{x^2} dx = \frac{1}{2}\int_{-1}^{1} 2x e^{x^2} dx = \frac{1}{2}\left[e^{x^2}\right]_{-1}^{1} = 0$

(ii) $\int_{-2}^{0} \frac{2x^3-7x^2+7x-2}{x-1} dx$

Polynomdivision: $(2x^3-7x^2+7x-2):x-1 = 2x^2-5x+2$

$$\begin{array}{l} -(2x^3-2x^2) \\ \quad -5x^2+7x \\ \quad -(-5x^2+5x) \\ \qquad 2x-2 \\ \qquad -(2x-2) \\ \qquad\quad 0 \end{array}$$

$\rightarrow$ kein Rest $\Rightarrow$ keine PBZ nötig

$\Rightarrow \int_{-2}^{0} \frac{2x^3-7x^2+7x-2}{x-1} dx = \int_{-2}^{0} 2x^2-5x+2\, dx = \left[\frac{2x^3}{3}-\frac{5x^2}{2}+2x\right]_{-2}^{0} = 0-\left(-\frac{58}{3}\right) = \frac{58}{3}$

(iii) $\int_0^1 \frac{\ln(x+\sqrt{1+x^2})}{\sqrt{1+x^2}} dx$ $\left(\begin{array}{l} \text{Es ist } \frac{d}{dx}\ln(x+\sqrt{1+x^2}) = \frac{1}{\sqrt{1+x^2}} \\ \text{also } \int_0^1 f(x)\cdot f'(x) = \frac{1}{2}\left[f^2(x)\right]_0^1 \end{array}\right)$

$= \left[\frac{1}{2}\left(\ln\left(x+\sqrt{1+x^2}\right)\right)^2\right]_0^1 = \frac{1}{2}\left(\ln\left(1+\sqrt{2}\right)\right)^2$

2) a) $\int_2^\infty \frac{1}{\sqrt{x}} dx = \lim_{b\to\infty} \int_2^b \frac{1}{\sqrt{x}} dx = \lim_{b\to\infty} \left[2\sqrt{x}\right]_2^b = \lim_{b\to\infty} 2\sqrt{b}-2\sqrt{2} = \infty$

b) $\int_0^1 \frac{\arccos(x)}{\sqrt{1-x^2}} dx = \lim_{a\to 0} \int_a^1 \frac{\arccos(x)}{\sqrt{1-x^2}} = \lim_{a\to 0} \left[-\frac{1}{2}\arccos^2(x)\right]_a^1 = 0 + \frac{\left(\frac{\pi}{2}\right)^2}{2} = \frac{\pi^2}{8}$

c) $\int_0^\infty \frac{k}{\mu}\cdot\left(\frac{x}{\mu}\right)^{k-1} e^{-\left(\frac{x}{\mu}\right)^k} dx = \lim_{b\to\infty} \left[-e^{-\left(\frac{x}{\mu}\right)^k}\right]_0^b = 0-(-1) = 1$

d) $\int_0^1 \ln(x) dx = \lim_{a\to 0} \int_a^1 \ln(x) = \lim_{a\to 0} \left[x\ln(x)-x\right]_a^1 = \lim_{a\to 0}\left(-1-a\ln(a)+a\right) = -1$

denn $\lim_{a\to 0} a\ln(a) = \lim_{a\to 0} \frac{\ln(a)}{\frac{1}{a}} \overset{\frac{\infty}{\infty}}{=} \ln\left(\frac{\frac{1}{a}}{-\frac{1}{a^2}}\right) = 0$

e) $\int_0^\infty \frac{x}{1+x^2} dx = \lim_{b\to\infty} \int_0^b \frac{x}{1+x^2} dx \overset{*}{=} \lim_{b\to\infty} \left[\frac{1}{2}\ln(1+x^2)\right] = \lim_{b\to\infty} \frac{1}{2}\ln(1+b^2) = \infty$

* wegen $\int \frac{f'}{f} = \ln$

f) $\int_0^\infty e^{-\mu x} dx = \lim_{b\to\infty} \int_0^b e^{-\mu x} = \lim_{b\to\infty} \left[-\frac{1}{\mu}e^{-\mu x}\right]_0^b = \lim_{b\to\infty} -\frac{1}{\mu}e^{-\mu b}+\frac{1}{\mu}$

$= \begin{cases} \frac{1}{\mu}, & \mu>0 \\ \infty, & \mu<0 \end{cases}$

3) $f: [-1, 1] \to \mathbb{R}$, $f(x) = -3(x^2-1)$ Volumen Rotationskörper:

$$V = \pi \int_{-1}^{1} (f(x))^2 \, dx = 9\pi \int_{-1}^{1} (x^2-1)^2 \, dx = 9\pi \int_{-1}^{1} x^4 - 2x^2 + 1 \, dx$$

$$= 9\pi \left[\frac{1}{5}x^5 - \frac{2}{3}x^3 + x\right]_{-1}^{1} = \frac{48}{5}\pi$$

4)

a) $x\,\phi(x)$ ist punktsymmetrisch $\Rightarrow \int_{-\infty}^{\infty} x\phi(x)\,dx = 0$. Wir rechnen nach:

$$\int_{-\infty}^{\infty} x\,\phi(x)\,dx = \lim_{c\to\infty} \int_{-c}^{c} x\,\phi(x)\,dx = \lim_{c\to\infty} \int_{-c}^{c} x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx$$

$$= \lim_{c\to\infty} \left[-\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\right]_{-c}^{c} = 0$$

$$\int_{-\infty}^{\infty} x^2 \phi(x)\,dx = 2 \cdot \int_{0}^{\infty} x^2 \phi(x)\,dx = 2 \lim_{c\to\infty} \frac{1}{\sqrt{2\pi}} \int \underset{g}{x} \cdot \underset{f'}{x e^{-\frac{x^2}{2}}}$$

$$\overset{PI}{=} \frac{2}{\sqrt{2\pi}} \lim_{c\to\infty} \left(\left[-x e^{-\frac{x^2}{2}}\right]_0^c + \int_0^c e^{-\frac{x^2}{2}}\,dx\right) = \frac{2}{\sqrt{2\pi}} \lim_{c\to\infty} \Big(\underbrace{-c e^{-\frac{c^2}{2}}}_{0}\Big) + \int_0^{\infty}$$

$$= 1$$

(Wir haben benutzt, dass $\int_0^{\infty} \phi(x)\,dx = \frac{1}{2}$, da $\phi(x) = \phi(-x)$)

b) $z: \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$$\Gamma\left(\tfrac{1}{2}\right) = \int_0^{\infty} x^{-\frac{1}{2}} e^{-x}\,dx = \lim_{b\to\infty} \int_0^{b} x^{-\frac{1}{2}} e^{-x}\,dx$$

$$\underset{\frac{dx}{du} = u}{\overset{x = \frac{u^2}{2}}{=}} \lim_{b\to\infty} \int_0^{\sqrt{2b}} \frac{\sqrt{2}}{u} e^{-\frac{u^2}{2}} u\,du = \sqrt{2} \int_0^{\infty} e^{-\frac{u^2}{2}}\,du = \sqrt{2} \cdot \frac{\sqrt{2\pi}}{2} = \sqrt{\pi}$$

(gleiche Begründung wie oben)