

Leachate diffusion from two layer soil

compacted clay liner or CCL is at the top, with thickness h_1 , natural stratum with thickness h_2 is at the bottom.

Governing equations

$$R_{d1} \frac{\partial c_1}{\partial t} = D_1^* \frac{\partial^2 c_1}{\partial z^2} \quad 0 \leq z \leq h_1$$

$$R_{d2} \frac{\partial c_2}{\partial t} = D_2^* \frac{\partial^2 c_2}{\partial z^2} \quad h_1 \leq z \leq H$$

Variables

$D_1^* = 4 \times 10^{-10} \text{ m}^2/\text{s}$, $D_2^* = 1 \times 10^{-10} \text{ m}^2/\text{s}$ (diffusion coefficient)

$R_{d1} = 3.3$, $R_{d2} = 1.0$ (Retardation factor)

$n_1 = 0.444$, $n_2 = 0.375$ (porosity)

$h_1 = 0.9 \text{ m}$, $h_2 = 1.1 \text{ m}$ (layer thicknesses)

z = vertical direction, positive downwards (unit = m). $z=0$ corresponds to top of soil, and $z=H$ corresponds to bottom of the two-layer system

t = time (unit = s)

c = leachate concentration

$H = h_1 + h_2$

The boundary conditions: $c=c_0$ @ $z=0$

$c=0$ @ $z=H$

(there is an aquifer at the bottom that removes any leachate whenever it reaches the bottom of the second layer)

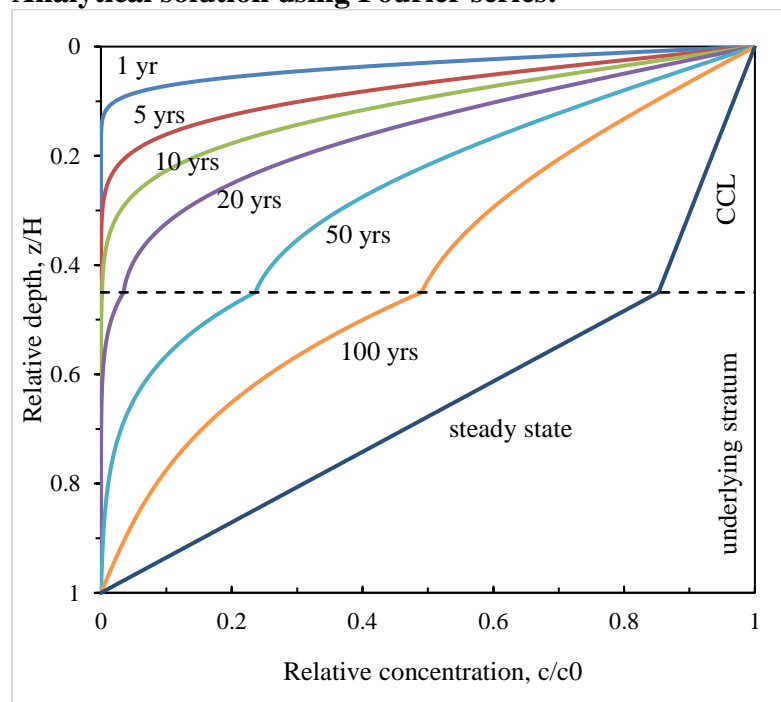
Initial condition: $c=0$ for $0 < z < H$

Interface boundary condition

$$c_1 = c_2 \quad @ \quad z = h_1 \quad n_1 D_1^* \frac{\partial c_1}{\partial z} = n_2 D_2^* \frac{\partial c_2}{\partial z} \quad @ \quad z = h_1$$

(The concentration and mass flux of concentration are continuous at the interface)

Analytical solution using Fourier series:



Reference

Li, Y. C., & Cleall, P. J. (2010). Analytical solutions for contaminant diffusion in double-layered porous media. *Journal of Geotechnical and Geoenvironmental Engineering*, 136(11), 1542-1554.

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