#implementation of middle boundary conditional

#Hickson, R. I., Barry, S. I., Mercer, G. N., & Sidhu, H. S. (2011).
#Finite difference schemes for multilayer diffusion.
#Mathematical and Computer Modelling,
#Vol 54, No(1-2), Pages 210 — 220.

```
restart;
D1 := 1.121212e-10 :
D2 := 1.0e-10 :
K1 := 4.736 :
K2 := 1.0 :
```

$$Eq1 := U(j-1) = UjLeft - dx \cdot UjLeftPrime + \frac{dx^2}{2} \cdot UjLeftPrimePrime$$

$$U(j-1) = UjLeft - dx\ UjLeftPrime + \frac{1}{2}\ dx^2\ UjLeftPrimePrime \quad (1)$$

$$Eq2 := U(j-2) = UjLeft - 2 \cdot dx \cdot UjLeftPrime + 2 \cdot dx^2 \cdot UjLeftPrimePrime$$

$$U(j-2) = 2\ dx^2\ UjLeftPrimePrime - 2\ dx\ UjLeftPrime + UjLeft \quad (2)$$

$$Eq3 := U(j+1) = UjRight + dx \cdot UjRightPrime + \frac{dx^2}{2} \cdot UjRightPrimePrime$$

$$U(j+1) = UjRight + dx\ UjRightPrime + \frac{1}{2}\ dx^2\ UjRightPrimePrime \quad (3)$$

$$Eq4 := U(j+2) = UjRight + 2 \cdot dx \cdot UjRightPrime + 2 \cdot dx^2 \cdot UjRightPrimePrime$$

$$U(j+2) = 2\ dx^2\ UjRightPrimePrime + 2\ dx\ UjRightPrime + UjRight \quad (4)$$

$$Eq5 := UjLeft = UjRight$$

$$UjLeft = UjRight \quad (5)$$

$$Eq6 := K1 \cdot UjLeftPrime = K2 \cdot UjRightPrime$$

$$4.736\ UjLeftPrime = 1.0\ UjRightPrime \quad (6)$$

```
soll := solve([Eq1, Eq2, Eq3, Eq4, Eq5, Eq6], [UjRight, UjLeft, UjLeftPrime, UjLeftPrimePrime,
    UjRightPrime, UjRightPrimePrime]) :
sol := soll[1] :
sol
```

$$\left[ UjRight = -0.2752208275\ U(j-2) - 0.05811250581\ U(j+2) + 0.2324500232\ U(j+1) \right. \quad (7)$$

$$+ 1.100883310\ U(j-1),\ UjLeft = -0.2752208275\ U(j-2) - 0.05811250581\ U(j+2)$$

$$+ 0.2324500232\ U(j+1) + 1.100883310\ U(j-1),\ UjLeftPrime$$

$$= \frac{0.08716875872\ (U(j-2) - 1.\ U(j+2) + 4.\ U(j+1) - 4.\ U(j-1))}{dx},$$

$$\left. UjLeftPrimePrime \right.$$

$$= \frac{1}{dx^2}\,(0.0004649000465\,(1559.\,U(j-2) - 125.\,U(j+2) + 500.\,U(j+1)$$

$$- 1934.\,U(j-1))),\ UjRightPrime$$

$$= \frac{0.4128312413\,(U(j-2) - 1.\,U(j+2) + 4.\,U(j+1) - 4.\,U(j-1))}{dx},$$

$$UjRightPrimePrime =$$

$$-\frac{1}{dx^2}\,(0.0009298000930\,(296.\,U(j-2) - 1013.\,U(j+2) + 1901.\,U(j+1)$$

$$- 1184.\,U(j-1)))\Big]$$

$lhs(sol[4])$

$$UjLeftPrimePrime \tag{8}$$

$lhs(sol[6])$

$$UjRightPrimePrime \tag{9}$$

$$simplify\left(\frac{\left(\frac{rhs(sol[4])\cdot D1}{2.0} + \frac{rhs(sol[6])\cdot D2}{2.0}\right)}{1.0e{-}10}\right)\cdot 1e{-}10$$

$$\frac{1}{dx^2}\,(1.\,10^{-10}\,(0.2687051389\,U(j-2) + 0.4383655277\,U(j+2) - 0.7534621107\,U(j+1) \tag{10}$$

$$+ 0.0463914442\,U(j-1)))$$

1

$$1 \tag{11}$$