Computer program for the 5th order Stokes standing waves

By: Danial Amini Baghbadorani, M.Sc.
Independent Researcher
Personal address: No 34, Golestan Dead End, Dehghan Alley, Bazarche St., Shahrestan Bridge, Moshtagh 2 St., Esfahan, Iran, Zip Code: 8158136647
Email: danial.amini@gmail.com
Phone: +989357815730

Submitted to: Engineering and Computational Mechanics journal, ICE publication
Date: 23 March 2018

Abstract

In this short paper, a previously published 5th order standing wave theory based on perturbation technique is reproved using an open source computer algebra system. The expansion, based on the small parameter kH which wave height H multiplied by wave number k, is derived for inviscid and incompressible standing waves.

1 Introduction

Standing waves are an important problem in waves and fluid mechanics because of their various applications. They can occur in lakes under the forcing of wind, in water tankers under the influence of external forcing and in fuel tankers of vehicles. It is important to determine maximum wave amplitude and the period of motion to be able to provide means of resistance against fluid forces.

Sobey (2009) gave a 5th order analytical solution for Stokes wave theory which is based on inviscid and irrotational potential function. The solution is performed by writing Fourier series for potential function ϕ as well as free surface position η, with respect to a dimensionless and small variable ε (wave height divided by wavelength) up to 5th order. The potential function already satisfies the Laplace equation for continuity of fluid, consequently, the coefficients of the Fourier series were obtained by substitution in the boundary conditions. Sobey (2009) provided coefficients in the appendix of their paper. Due to some typographical errors in that version, an errata paper was published to correct values of some of the coefficients in Sobey (2012). Due to tedious algebra involved in obtaining the coefficients, computer algebra system was used by Sobey (2009) to carry out the symbolic manipulations. The work is a continuation of work Tadjbakhsh and Keller (1960) who provided standing wave solutions up to 3rd order. Sobey (2012) extended the theory until 20th order approximation, although the coefficients were obtained by meshing the grid and solving a nonlinear optimization numerical problem.

In general, research papers are expected to provide novel results and methods to advance the state of science. However, it is also beneficial to reproduce the works of previous papers in order to obtain an independent verification of their results, and also to increase the trustworthiness of the paper in the eyes of the scientific community. In this short paper, we have tried to reproduce the analytical work of Sobey (2009). The technical details which are needed for implementation of the solution in the computer algebra system are included. The computer algebra software notebook file as well as the computer code for the standing wave calculations are provided as electronic supplementary files to facilitate the application of the method.

2 Governing equations and boundary conditions

The governing equation and boundary conditions, according to Sobey (2009) are:

$$\begin{aligned}
&\phi_{xx}+\phi_{zz}=0 \quad \text{(Continuity)},\\
&f_K=\eta_x+u\eta_x-w=0 \;\; @\, z=\eta \;\; \text{(Kinematic free surface boundary condition)},\\
&f_D=\phi_t+\left(u^2+w^2\right)/2+g\eta-\bar{B}=0 \;\; @\, z=\eta \;\; \text{(Dynamic free surface boundary condition)},\\
&u\big|_{x=0}=u\big|_{x=L/2}=w_{z=-h}=0 \;\; \text{(Impermeable bottom and side walls)},
\end{aligned} \tag{1}$$

$\phi|_{t=0} = 0$, $\eta|_{t=0} = \eta_0$ (Initial conditions),

$\phi(x,z,t) = \phi(x + 2\pi/k, z, t) = \phi(x, z, t + 2\pi/\omega)$ and

$\eta(x,t) = \eta(x + 2\pi/k, t) = \eta(x, t + 2\pi/\omega)$ (Periodic lateral boundary condition)

$\eta(0,0) - \eta(L/2, 0) - H = 0$ and $\eta(0,0) - \eta(0, T/2) - H = 0$ (Wave height constraint)

$\int_0^{L/2} \eta(x,t)\,dx = 0$ (Mean water level (MWL) is at z = 0)

Wherein ϕ = potential function, η = free surface position, x and z = horizontal and vertical coordinates, t = time, u and w = horizontal and vertical velocity components, g = acceleration of gravity, $\bar{B}$ = Bernoulli's constant, L = wavelength, h = mean water depth, η_0 = initial disturbance of water surface, k = $2\pi/L$ = wave number, $\omega = 2\pi/T$ = angular frequency, T = period, H = wave height, f_K and f_D = errors of kinematic and dynamic boundary conditions, and subscripts represent differentiation.

3 Perturbation expansions for ϕ, η, ω, $\bar{B}$

The following Fourier series are written for the variables of interest as a perturbation expansion with respect to a small parameter, which is chosen to be $\varepsilon = kH/2$ in Sobey (2009). The expansions are given by:

$$\phi(x,z,t) = \left(\frac{g}{k^3}\right)^{1/2} \sum_{i=1}^{N} \varepsilon^i \sum_{j=0}^{i} \sum_{m=0}^{i} A_{ijm} \frac{\cosh jk(h+z)}{\cosh jkh} \cos jkx \sin m\omega t$$

$$\eta(x,t) = \frac{1}{k} \sum_{i=1}^{N} \varepsilon^i \sum_{j=0}^{i} \sum_{m=0}^{i} b_{ijm} \cos jkx \cos m\omega t \ , \quad \omega = \sqrt{gk} \sum_{i=1}^{N\omega} \varepsilon^{i-1} C_i \ , \quad \bar{B} = \frac{g}{k} \sum_{i=1}^{N} \varepsilon^i D_i \tag{2}$$

Here A_{ijm}, b_{ijm}, C_i and D_i = dimensionless coefficients for ϕ, η, ω and $\bar{B}$, respectively; N = summation bound for order of expression, $N\eta$ = order of calculating η, $N\omega$ = order of calculating ω; and i, j, and m are counters. Other quantities can be obtained by direct differentiation of ϕ and η. As a result (Sobey 2009) expressions for ϕ_t, u, w, η_x and η_t can be obtained as well.

For simplifying the ϕ calculated at $z = \eta$, a trigonometric expansion is applied on cosh function and then the Taylor series expansion with respect to $z = 0$ is applied based on Sobey (2009).

$$\frac{\cosh jk(h+\eta)}{\cosh jkh} = \cosh(jk\eta) + \tanh(jkh)\sinh(jk\eta) = \sum_{\tilde{n}=0}^{Nt} \frac{\left(1-(-1)^{\tilde{n}}\right)(jk\eta)^{\tilde{n}}}{2\tilde{n}!} + \tanh(jkh) \sum_{\tilde{n}=0}^{Nt} \frac{\left(1+(-1)^{\tilde{n}}\right)(jk\eta)^{\tilde{n}}}{2\tilde{n}!} \tag{3}$$

Multiple angle terms tanh jkh can be expressed in terms of single angle. By defining the dimensionless parameter q = tanh kh, the following formulas can be obtained:

$$\tanh 2kh = \frac{2q}{1+q^2}, \tanh 3kh = \frac{3q+q^3}{1+3q^2}, \tanh 4kh = \frac{4q\left(q^2+1\right)}{q^4+6q^2+1}, \tanh 5kh = \frac{q\left(q^4+10q^2+5\right)}{5q^4+10q^2+1} \tag{4}$$

The patterns show that tanh jkh is a rational function of q and j, denoted by $f(q,j) = \tanh jkh$.

4 Nondimensionalization

The following nondimensionalization is done in order to simplify equations:

$$\hat{x} = kx, \quad \hat{t} = \omega t, \quad \hat{\omega} = \frac{\omega}{\sqrt{gk}}, \quad \hat{\eta} = k\eta, \quad \hat{\eta}_x = \eta_x, \quad \hat{\eta}_t = \sqrt{\frac{k}{g}}\eta_t, \quad \hat{\phi} = \left(\frac{k^3}{g}\right)^{1/2}\phi, \quad \hat{\phi}_t = \frac{k}{g}\phi_t$$

$$, \quad \hat{u} = \sqrt{\frac{k}{g}}u, \quad \hat{w} = \sqrt{\frac{k}{g}}w, \quad \hat{\bar{B}} = \frac{k}{g}\bar{B}, \quad \hat{z} = kz, \quad \hat{h} = kh, \quad q = \tanh kh = \tanh \hat{h} \tag{5}$$

The dimensionless quantities are demonstrated with hats. Consequently, the dimensionless expressions for ϕ and η along with the rest of variables (without showing the hat signs) are given by:

$$\eta = \sum_{i=1}^{N} \varepsilon^i \sum_{j=0}^{i} \sum_{m=0}^{i} b_{ijm} \cos jx \cos mt = \sum_{i=1}^{N} \eta_i \varepsilon^i \ , \quad \eta_x(x,t) = -\sum_{i=1}^{N} \varepsilon^i \sum_{j=0}^{i} \sum_{m=0}^{i} j\, b_{ijm} \sin jx \cos mt$$

$$\eta_t(x,t) = -\left(\sum_{i'=1}^{N\omega} \varepsilon^{i'-1} C_i\right)\left(\sum_{i=1}^{N} \varepsilon^i \sum_{j=0}^{i} \sum_{m=0}^{i} b_{ijm} m \cos jx \sin mt\right), \quad \omega = \sum_{i=1}^{N\omega} \varepsilon^{i-1} C_i, \quad \bar{B} = \sum_{i=1}^{N} \varepsilon^i D_i \tag{6}$$

$$\phi=\sum_{i=1}^{N}\left(\sum_{j=0}^{i}\sum_{m=0}^{i}\varepsilon^{i}A_{ijm}\cos jx\sin mt\left[\sum_{\tilde{n}=0}^{Nt}\left[\frac{\left(1+(-1)^{\tilde{n}}\right)}{2(\tilde{n}!)}+f(q,j)\frac{\left(1-(-1)^{\tilde{n}}\right)}{2(\tilde{n}!)}\right]\left(j\sum_{i''=1}^{N\eta}\eta_{i}\varepsilon^{i''}\right)^{\tilde{n}}\right]\right)$$

$$\phi_{t}=\left(\sum_{i'=1}^{N\omega}\varepsilon^{i'-1}C_{i}\right)\sum_{i=1}^{N}\left(\sum_{j=0}^{i}\sum_{m=0}^{i}\varepsilon^{i}mA_{ijm}\cos jx\cos mt\left[\sum_{\tilde{n}=0}^{Nt}\left[\frac{\left(1+(-1)^{\tilde{n}}\right)}{2(\tilde{n}!)}+f(q,j)\frac{\left(1-(-1)^{\tilde{n}}\right)}{2(\tilde{n}!)}\right]\left(j\sum_{i''=1}^{N\eta}\eta_{i}\varepsilon^{i''}\right)^{\tilde{n}}\right]\right)$$

$$u=-\sum_{i=1}^{N}\left(\sum_{j=0}^{i}\sum_{m=0}^{i}\varepsilon^{i}jA_{ijm}\sin jx\sin mt\left[\sum_{\tilde{n}=0}^{Nt}\left[\frac{\left(1+(-1)^{\tilde{n}}\right)}{2(\tilde{n}!)}+f(q,j)\frac{\left(1-(-1)^{\tilde{n}}\right)}{2(\tilde{n}!)}\right]\left(j\sum_{i''=1}^{N\eta}\eta_{i}\varepsilon^{i''}\right)^{\tilde{n}}\right]\right) \tag{7}$$

$$w=\sum_{i=1}^{N}\left(\sum_{j=0}^{i}\sum_{m=0}^{i}\varepsilon^{i}jA_{ijm}\cos jx\sin mt\left[\sum_{\tilde{n}=0}^{Nt}\left[\frac{\left(1-(-1)^{\tilde{n}}\right)}{2(\tilde{n}!)}+f(q,j)\frac{\left(1+(-1)^{\tilde{n}}\right)}{2(\tilde{n}!)}\right]\left(j\sum_{i''=1}^{N\eta}\eta_{i}\varepsilon^{i''}\right)^{\tilde{n}}\right]\right)$$

Here Nt = order of calculating the Taylor series, $\tilde{n}$ = counter for Nt, i' = counter for $N\omega$, and i'' = counter for $N\eta$. The boundary conditions in dimensionless form are:

$$\begin{aligned}
&\phi_N(x,z,t)=\phi_N(x+2\pi,z,t)=\phi_N(x,z,t+2\pi)\\
&\eta_N(x,t)=\eta_N(x+2\pi,t)=\eta_N(x,t+2\pi)\quad \text{(periodic boundary conditions)}\\
&\eta_N(0,0)-\eta_N(\pi,0)-2\varepsilon=0,\eta_N(0,0)-\eta_N(0,\pi)-2\varepsilon=0\quad \text{(wave height constraint)}\\
&f_k=\eta_{t;N}+u_N\eta_{x,N}-w_N=0\quad \text{(kinematic boundary condition)}\\
&f_D=\phi_{t;N}+(u_Nu_N+w_Nw_n)/2+\eta_N-\bar{B}_N=0\quad \text{(dynamic boundary condition)}
\end{aligned} \tag{8}$$

5 Extra conditions for coefficients *A* and *b*

From MWL condition, we can obtain that $b_{ijm}=0$ for $j=0$. Additional conditions can be used for speeding up calculations, which are: $A_{ijm}=b_{ijm}=0$ for $i+j$ = odd, or $i+m$ = odd and $j+m$ = odd , where the last condition was given by Sobey (2009). In addition, $C_i=0$ for i = even and $D_i=0$ for i = odd. Employing the additional conditions is not necessary for finding the solution.

6 Simplification of sine and cosine functions

The boundary conditions for order N, with N = even while eliminating all orders other than ε^N can be simplified to:

$$\begin{aligned}
&f_K=\sum_{\substack{2\le m\le N\\ m=\text{even}}}\sum_{\substack{2\le j\le N\\ j=\text{even}}}\left(\beta_1A_{i=N,j,m}+\beta_2b_{i=N,j,m}+\gamma\right)\cos jx\sin mt=0\\
&f_D=\left(\beta D_{i=N}+\gamma\right)_{j=0,m=0}+\sum_{\substack{2\le j\le N\\ j=even}}\left(\beta A_{i=N,j=0,m}+\gamma\right)\cos mt+\sum_{\substack{2\le m\le N\\ m=even}}\left(\beta b_{i=N,j,m=0}+\gamma\right)\cos jx\\
&+\sum_{\substack{2\le m\le N\\ m=\text{even}}}\sum_{\substack{2\le j\le N\\ j=\text{even}}}\left(\beta_1A_{i=N,j,m}+\beta_2b_{i=N,j,m}+\gamma\right)\cos jx\cos mt=0,\qquad \text{wave height constraint}=0
\end{aligned} \tag{9}$$

Here β and γ are coefficients which are functions of q, and γ is a function of previous order coefficients. The values of D_N, $A_{N,j=0,m}$ and $b_{N,j,m=0}$ are directly obtained from f_D. The remaining pairs of $A_{N,j,m}$ and $b_{N,j,m}$ are calculated by solving 2×2 systems from f_K and f_D. For N = even:

$$\begin{aligned}
&\text{wave height constraint}=\sum_{\substack{m=1\\ m=\text{odd}}}^{N}\sum_{\substack{j=1\\ j=\text{odd}}}^{N}b_{i=N,j,m}=0\rightarrow b_{N,1,1}=\sum_{\substack{m=1\\ m=\text{odd}}}^{N}\sum_{\substack{j=1\\ j=\text{odd}\\ j+m\ne2}}^{N}b_{i=N,j,m}\\
&f_k=\left(\beta_1A_{i=N,j=m=1}+\beta_2b_{i=N,j=m=1}+\beta_3C_{i=N}+\gamma\right)+\sum_{\substack{1\le m\le N\\ m=\text{odd}}}\sum_{\substack{1\le j\le N\\ j=\text{odd}\\ j+m\ne2}}\left(\beta_1A_{i=N,j,m}+\beta_2b_{i=N,j,m}+\gamma\right)\cos jx\sin mt=0\\
&f_D=\left(\beta_1A_{i=N,j=m=1}+\beta_2b_{i=N,j=m=1}+\beta_3C_{i=N}+\gamma\right)+\sum_{\substack{1\le m\le N\\ m=\text{odd}}}\sum_{\substack{1\le j\le N\\ j=\text{odd}\\ j+m\ne2}}\left(\beta_1A_{i=N,j,m}+\beta_2b_{i=N,j,m}+\gamma\right)\cos jx\cos mt=0
\end{aligned} \tag{10}$$

Pairs of $A_{N,j,m}$ and $B_{N,j,m}$ for j and m not both equal to zero are obtained from solving 2×2 systems from f_K and f_D. $B_{N,1,1}$ is subsequently found from wave height constraint, and the 2×2 equation for $A_{N,1,1}$

and C_N is solved in the end. Each term in the initial expression for f_K and d_D may be in terms of multiplication and powers of several angles (such as $\sin^2 x \sin 3x \cos^2 t$), which need to be converted to multiple angle expressions in Eqs. (9) and (10) either by simplification (as explained in Sobey (2009)), or by using orthogonality property of Fourier series. For example, the coefficient of $\cos jx \sin mt$ in f_D is equal to $\left(1/\pi^2\right)\int_0^{2\pi}\int_0^{2\pi} f_D \cos jx \sin mt \, dx\, dt$; The coefficient of $\cos mt$ is equal to $\left(1/2\pi^2\right)\int_0^{2\pi}\int_0^{2\pi} f_D \cos mt \, dx\, dt$ and the remaining constant coefficient is $\left(1/4\pi^2\right)\int_0^{2\pi}\int_0^{2\pi} f_D \, dx\, dt$.

7 Selection of N, $N\omega$, Nt and $N\eta$

When working with a given degree of accuracy of $\bar{N}$, the values of N, $N\omega$ and $N\eta$ (upper bounds in calculating ϕ, ω and η), as well as the Taylor series power N_t should be chosen as small as possible while still producing results of order $\bar{N}$ to save computation resources. For η, this means that $N = \bar{N}$. For η_t we have:

$$\eta_t \sim \left(\sum_{i'=1}^{N\omega} \varepsilon^{i'-1}\right)\left(\sum_{i=1}^{N} \varepsilon^{i}\right) \sim \sum_{i=1}^{N} \varepsilon^{i+N\omega-1} \sim O\left(\varepsilon^{\bar{N}}\right) \rightarrow N\omega = \bar{N} - i + 1, \quad N = \bar{N} \tag{11}$$

Where ~ means that the two terms are of the same order. For η_t, we must select $N = \bar{N}$ and $N\omega = N - i + 1$. This will ensure that the highest order term will be of order $O(\varepsilon^{\bar{N}})$ at most. For ϕ, u and w, the multinomial expansion involving the expansion for η and the powers of the Taylor series should be expanded, resulting in:

$$\begin{gathered}\phi \sim \sum_{i=1}^{N} \varepsilon^{i}\left[\sum_{\tilde{n}=0}^{Nt}\left(\sum_{i''=1}^{N\eta} \beta_{i''}\varepsilon^{i''}\right)^{\tilde{n}}\right] \sim O\left(\varepsilon^{\bar{N}}\right) \Rightarrow \sum_{\tilde{n}=0}^{Nt}\left(\sum_{i''=1}^{N\eta} \beta_{i''}\varepsilon^{i''}\right)^{\tilde{n}} = \sum_{\tilde{n}=0}^{Nt} \sum_{(i''1,i''2,\ldots,i''\tilde{n})=1}^{N\eta} \beta_{i''_1}\beta_{i''_2}\ldots\beta_{i''_{\tilde{n}}}\varepsilon^{i''1+i''2+\ldots+i''\tilde{n}} \\ \sim O\left(\varepsilon^{\bar{N}-i}\right) \Rightarrow i''_1 + i''_2 + \ldots + i''_{\tilde{n}} \leq \bar{N} - i \text{ for each } \tilde{n} \text{ for possible values of } i''_1, i''_2, \ldots, i''_{\tilde{n}};\ N_t = N\eta = \bar{N} - i\end{gathered} \tag{12}$$

For ϕ_t, as before we have $N\omega = \bar{N} - i + 1$. For the nonlinear terms $u\eta_x$, u^2 and w^2, the binomial expansion is written and decision is made to retain each term. For instance for $u\eta_x$ we can consider $(u_1\varepsilon + u_2\,\varepsilon^2 + u_3\,\varepsilon^3 + \ldots + u_{i1}\,\varepsilon^{i1} + \ldots + u_{N1}\,\varepsilon^{N1})\cdot(\eta_{x1}\,\varepsilon + \eta_{x2}\,\varepsilon^2 + \eta_{x3}\,\varepsilon^3 + \ldots + \eta_{xi1}\,\varepsilon^{i1} + \ldots + \eta_{xN2}\,\varepsilon^{N2}) = u_1\,\eta_{x1}\,\varepsilon^2 + (u_1\,\eta_{x2} + u_2\,\eta_{x1})\,\varepsilon^3 + \ldots$; and the expansion is continued until obtaining $\varepsilon^{\bar{N}}$.

8 Solution of the system

For the first order solution, we can use $\bar{N} = 1$. The nonlinear terms formed from multiplication need not be included. The wave height constraint becomes $2\varepsilon(b_{1,1,0}+b_{1,1,1}-1)=0$; the kinematic boundary condition becomes $f_K = \varepsilon \cos x \sin t\,(b_{1,1,1}C_1+A_{1,1,1})=0$, and the dynamic boundary condition becomes $f_D = \varepsilon(-D_1+b_{1,1,0}\cos x + A_{1,0,1}\,C_1 \cos t + (A_{1,1,1}\,C_1 + B_{1,1,1})\cos t \cos x) = 0$. These nonlinear equations have two solutions, but since $\omega > 0$, therefore $D_1 = b_{1,1,0} = A_{1,0,1} = 0$, $C_1 = \sqrt{q}$, $A_{1,1,1} = -1/\sqrt{q}$ and $b_{1,1,1} = 1$. For higher orders, the resulting equations are linear, as pointed out by Sobey (2009).

9 Iteration for calculating wavelength, L

The method employed by Sobey (2009) starts with a given value of water depth h, wave height H and wavelength L, then the wave period T is explicitly obtained without the need for iteration. If H, h and T are specified, L is found from iteration as outlined below:

1- Input g, h, H and T

2- Guess first value of L. By using the first order approximation for ω, we have $\omega = \sqrt{gk \tanh kh}$. If the deep water approximation is used, then $\omega = \sqrt{gk}$, leading to $L = gT^2/2\pi$. If shallow water approximation is used, then $\omega = \sqrt{ghk^2}$, leading to $L = T\sqrt{gh}$. The average of these two values can be used as the first guess for L.

3- Calculate $\omega = 2\pi/T$, $k = 2\pi/L$, $\varepsilon = kH/2$ and $q = \tanh kh$

4- Calculate ω_{calc} from the perturbation series solution, and obtain $T_{calc} = 2\pi / \omega_{calc}$

5- Update L using the formula $L_{new} = L_{old} + (T - T_{calc}) \times 10$

6- Repeat steps 3-5 about 20 times until $T - T_{calc}$ becomes small enough.

10 Checking the order of the method

In order to numerically check the order of the method, Sobey (2009) suggests calculating residual of kinematic and dynamic free surface boundary conditions f_k and f_D for different wave heights by using sample values of h = 100 m, T = 10 s, H = 0.5 m, 1 m, 5 m and 10 m. The residual of f_K is computed on a 50×50 grid via:

$$f_{k,value} = \sum_{j=0}^{50} \sum_{i=0}^{50} \left| f_k\left(x = iL/100, t = jT/100\right) \right| / 51^2 \tag{13}$$

The plot of f_K and f_D vs. ε is shown in Figure 1, confirming that the expressions of Sobey (2009) are $O(\varepsilon^5)$.

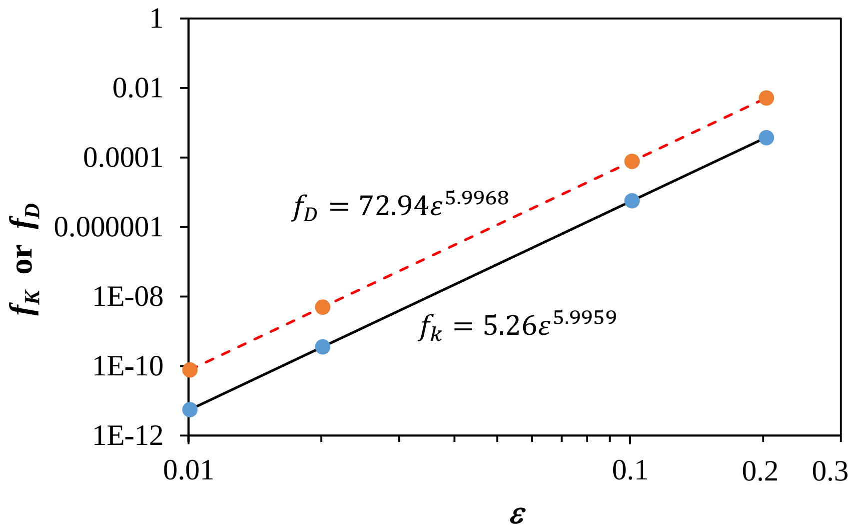

Figure 1. Plot of f_K and f_D vs. ε

Using the proposed formula, the wave profiles for a specific case with $\varepsilon \approx 0.2$ were plotted in Figure 2, similar to graphs of Sobey (2009). It is observed that the location of zero height of the wave does not exactly fall at $x = L/4$ point, but always towards the peak side, because in nonlinear and steep waves, it is well known that the peaks are steep and the troughs are flat. The length of locus of $z = 0$ is equal to 0.027 L for the particular example. Two pathlines are also drawn in Figure 2 which show the path of the particles in standing waves. The path clearly shows that any particle on the free surface stays on the free surface, and only moves along the free surface but cannot detach itself from it. Another notable point is that unlike traveling waves, the pathlines are not closed loops, but are curved lines. If the plots are drawn from $T/2$ until T, it will be seen that the particle will traverse back along the exact same pathline to its initial position. Also the particle which resides at $z = \eta$ and $x = 0$ moves along a vertical pathline, always attached to both the sidewall and the free surface. The theory of Sobey (2009) is only valid for the first mode of standing waves, meaning that the dimension of the container is half the wavelength.

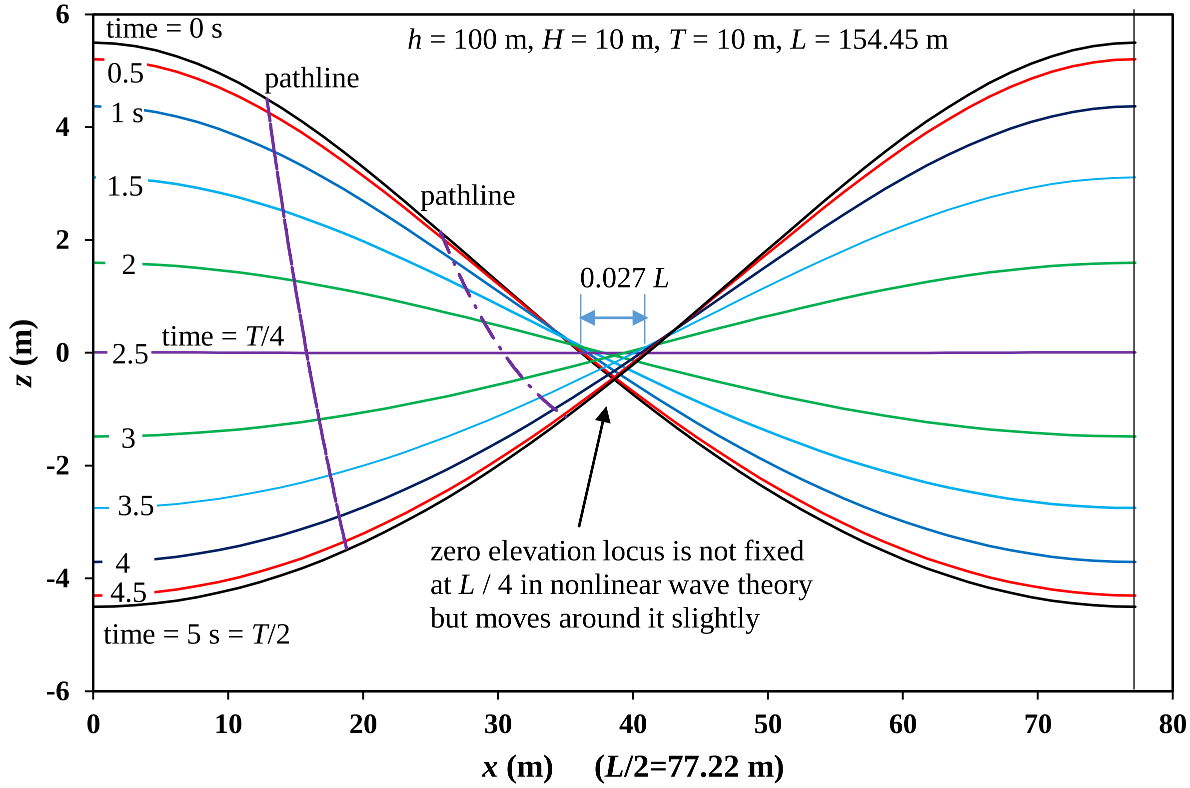

Figure 2. Standing wave profile for different instances for perturbation series up to order N=5

11 Supplementary materials

The computer algebra system used for symbolic computations was SageMath (2018), which is an open source and free software, and it can be downloaded from http://www.sagemath.org/. Sage notebooks have *.ipynb file extension. A simple program was made for calculating wave profiles using Visual Studio Express, a free software provided by Microsoft Coorporation (2018), downloadable from https://www.visualstudio.com/vs/visual-studio-express/. The files are available from the journal website and also from the Github repository of Amini Baghbadorani (2018).

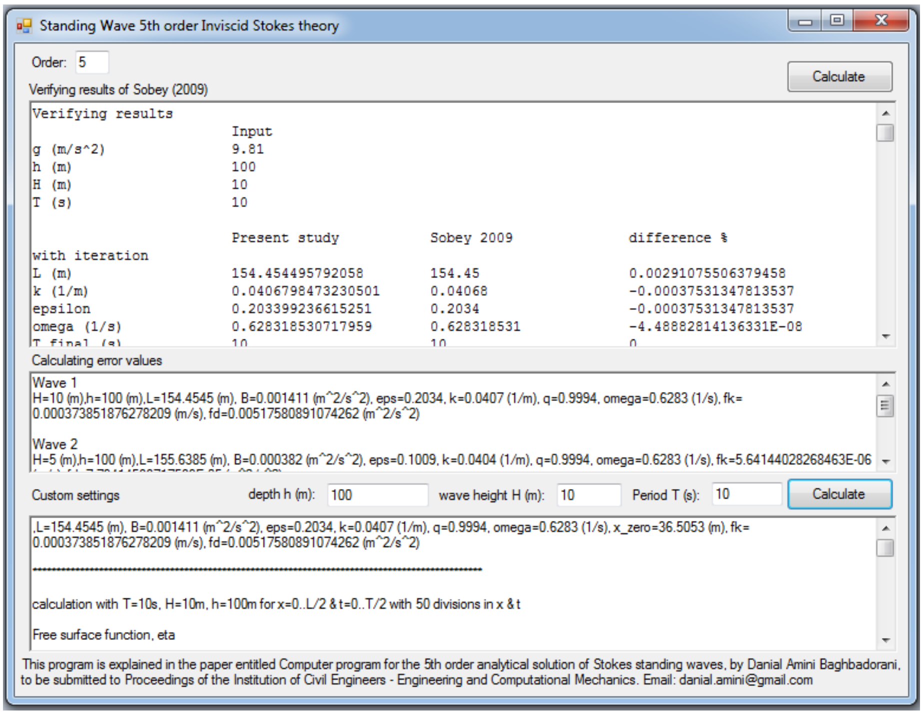

Figure 3. Wave calculator software

References

Amini Baghbadorani D (2018) Standing-waves-5th-order, https://github.com/DanialAmini/Standing-waves-5th-order. GitHub,

Microsoft Coorporation (2018) Visual Studio Express.

SageMath (2018) the Sage Mathematics Software System (Version 8.1). The Sage Developers

Sobey R (2012) Errata: Analytical solutions for steep standing waves. Proceedings of the Institution of Civil Engineers-Engineering and Computational Mechanics 165:75-75 DOI https://doi.org/10.1680/eacm.2012.165.1.75

Sobey RJ (2009) Analytical solutions for steep standing waves. Proceedings of the Institution of Civil Engineers - Engineering and Computational Mechanics 162:185-197 DOI https://doi.org/10.1680/eacm.2009.162.4.185

Sobey RJ (2012) Stokes Approximation Solutions for Steep Standing Waves. Coastal Engineering Journal 54:1250014 DOI https://doi.org/10.1142/S0578563412500143

Tadjbakhsh I, Keller JB (1960) Standing surface waves of finite amplitude. Journal of Fluid Mechanics 8:442-451 DOI https://doi.org/10.1017/S0022112060000724