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Research paper

Estimation of longitudinal dispersion coefficient in rivers

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Abstract

The longitudinal dispersion coefficient is a crucial parameter for 1D water quality analyzing in natural rivers, and different types of empirical equations have been presented in the literature. To evaluate the precision of those commonly used equations, 116 sets of measured data for rivers in U.S. and UK have been collected for comparison. Firstly, the precisions of selected ten empirical equations under different aspect ratio (water surface width B/water depth H) have been compared, and calculation shows that most of the equations have underestimated the longitudinal dispersion when $20 < B/H < 100$, in which most of the natural rivers located. The regression analysis on the collected data sets proved that the product of water depth H and the cross-sectional averaged velocity U has a higher linear correlation with the longitudinal dispersion coefficient than the product of H and shear velocity $u*$, and then a new expression of longitudinal dispersion coefficient, which is a combination of the product of HU and other two nondimensional hydraulic and geometric parameters, was deduced and the exponents were determined by the regression analysis. The comparison between the measured data and the predicted results shows that the presented equation has the highest precision for the studied natural rivers. To further evaluate the precision of the empirical formulae to artificial open channels, comparison was made between laboratory measuring data and empirical equation prediction, and the results have shown that the newly presented model is effective at predicting longitudinal dispersion in trapezoidal artificial channels too.

Keywords: Longitudinal dispersion; Regression analysis; Empirical equation; Discrepancy ratio

1. Introduction

In a situation of contaminant and effluent discharged into natural rivers, there are three stages of mixing when they are transported downstream with the current (Jirka, 2004). In the first three dimensional entrainment and diffusion stage, jetted pollutant is fully mixed in the vertical direction very quickly. In the second stage, the pollutant diffuses transversely, and the lateral mixing is completed at this stage. In the last stage, the pollutant diffuses longitudinally and the dominant mechanism is the longitudinal dispersion, which is mainly caused by the lateral difference of longitudinal velocity. For the most commonly used 1D analytical model of water quality, the last stage is the focal one and its intensity can be quantified by the longitudinal dispersion coefficient D_x, which is a crucial parameter in predicting solute and/or sediment concentration distributions in water bodies (Kashefipour and Falconer, 2002), and has been extensively investigated (Rutherford, 1994; Marion and Zaramella, 2006; Kim, 2012).

Longitudinal dispersion coefficient can be affected by many hydrodynamic and geometrical parameters, and as a result the dispersion characteristic may vary from one river to another. Till now, three methods, namely the integral method, dye tracing measurements and empirical formulae have been widely used to estimate the longitudinal dispersion coefficient.

Fischer (1967) developed the triple integral expression of longitudinal dispersion coefficient,

$$D_x = -\frac{1}{A}\int_0^B hu' \int_0^y \frac{1}{\varepsilon_t h}\int_0^y hu' dydydy \tag{1}$$

In which $A=$ cross-sectional area; $B=$ river width; $h=$ local flow depth; $u' = u-U$, the deviation of local depth-average

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longitudinal velocity u from cross-sectional mean U; y= coordinate in the lateral direction; and ε_t = local transverse mixing coefficient.

Once detailed transverse profile of velocity and cross-sectional geometry have been provided, the Eq. (1) can give a rather accurate estimation. From then on, different assumptions for lateral profile of longitudinal velocity have been presented. For example, Sooky (1969) supposed that to be a combination of the logarithmic velocity profile and a linear function for a triangular section; Bogle (1997) suggested an empirical profile based on quartic function; Seo and Gadalrab (1999) proposed a combined form of a fourth-degree polynomial and exponential equations to represent both the skewed and flattened properties of the transverse velocity profiles; Deng et al. (2001) proposed a power-law function to determine the longitudinal dispersion coefficient. Seo and Baek (2004) reproduced the velocity profile with symmetrical and skewed distributions based on the probability density function (beta distribution). Since those supposed lateral profiles of longitudinal velocity were quiet different, the integrated results of Eq. (1) (i.e., the longitudinal dispersion coefficient) were different.

Tracer test, as a relative accurate way to determine the D_x, is commonly adopted to study the characteristic of longitudinal dispersion. Once the temporal concentration profile is obtained, statistical methods such as moment analysis (Guymer, 1998; Zhang et al., 2006), and Chatwin's analysis (Chatwin, 1980) can be used to estimate the dispersion coefficient. Since the process of tracer test is slow and laborious, during practical engineering application, it is preferable to use empirical equations with several hydraulic and geometric parameters which can be readily obtained from numerical models and field measuring to determine D_x. Those commonly used empirical equations for predicting D_x were the combined function of geometrical and hydraulic parameters, such as cross-sectional mean velocity U, water depth H, water surface width B, energy slope S, shearing velocity u_*, etc. Ten widely used empirical equations were listed in Table 1.

In this paper, predictions of D_x by equations listed in Table 1 were compared based on 116 sets of measured data obtained from above 50 rivers in U.S. and UK, and their accuracy within different range of B/H were analyzed. Then based on regression analysis, a new empirical formula with higher precision both for natural rivers and artificial trapezoidal flumes was presented.

Table 1
Commonly used empirical equations for predicting D_x.

No.	Empirical equations
EF(1)	$D_x = 0.011U^2B^2/(Hu_*)$ (Fischer, 1967)
EF(2)	$D_x = 5.93Hu_*$ (Elder, 1959)
EF(3)	$D_x = 0.18(U/u_*)^{0.5}(B/H)^2Hu_*$ (Liu, 1977)
EF(4)	$D_x = 0.6(B/H)^2Hu_*$ (Koussis and Rodriguez, 1998)
EF(5)	$D_x = 2.0(B/H)^{1.5}Hu_*$ (Iwasa and Aya, 1991)
EF(6)	$D_x = 0.55Bu_*/H^2$ (Li et al., 1998)
EF(7)	$D_x = 0.2(U/u_*)^{1.2}(B/H)^{1.3}Hu_*$ (Li et al., 1998)
EF(8)	$D_x = 5.92(U/u_*)^{1.43}(B/H)^{0.62}Hu_*$ (Seo and Cheong (1998)
EF(9)	$D_x = 10.612(U/u_*)HU$ (Kashefipour and Falconer, 2002)
EF(10)	$D_x = (7.428 + 1.775(B/H)^{0.62}(U/u_*)^{0.572})(U/u_*)HU$ (Kashefipour and Falconer, 2002)

2. Comparison between different empirical equations

To evaluate the precision of different empirical equations, totally 116 sets of measured longitudinal dispersion coefficient for above 50 rivers in U.S. and UK were collected for further analyzing (Table 2).

For natural rivers, the aspect ratio B/H is an important geometrical factor affecting longitudinal dispersion since the latter is caused by the transverse difference of longitudinal velocity. To investigate the effects of B/H to D_x, we divided the 116 sets of collected data into four groups with different span of B/H (i.e., $0 < B/H < 20$, $20 < B/H < 100$, $100 < B/H < 200$, $B/H > 200$), and the dispersion coefficient of each case was calculated by empirical equations listed in Table 1 separately.

The discrepancy ratio (DR), which is an error measure commonly used in the referred literature (Kashefipour and Falconer, 2002; Tayfur and Singh, 2005), was adopted to evaluate the prediction precision of different formulae,

$$\mathrm{DR} = \log\left(\frac{D_{\mathrm{xp}}}{D_{\mathrm{xm}}}\right) \tag{2}$$

where D_{xp} is the predicted longitudinal dispersion coefficient, and D_{xm} is the measured one. From Eq. (2), if DR = 0, then it is an exact prediction, otherwise it's either an overestimation (DR > 0, or $D_{\mathrm{xp}} > D_{\mathrm{xm}}$) or an underestimation (DR < 0, or $D_{\mathrm{xp}} < D_{\mathrm{xm}}$).

To evaluate the total effect, we define the total discrepancy ratio as:

$$\mathrm{DR_s} = \left(\sum_{i=1}^{N}\log\left(\frac{D_{\mathrm{xp}_i}}{D_{\mathrm{xm}_i}}\right)\right)\Big/N \tag{3}$$

where N is the number of the tested cases.

From Table 3, one can find that for most natural rivers considered here, their aspect ratios B/H were belong to [20,100] with a percentage of 83.6%, so the precision in this range can largely affect the prediction accuracy. When $20 < B/H < 100$, most of the equations underestimated the longitudinal dispersion coefficient. Comparatively speaking, EF(10) has the highest precision, for its $\mathrm{DR_s}$ is only −0.002. For Elder's (EF(2)) and Li's equation(EF(6)), the absolute values of $\mathrm{DR_s}$ were as large as 2.068 and 1.609, and the negative values of $\mathrm{DR_s}$ meant that the predicted values were much smaller than the measured ones. In other word, these two equations both largely underestimated D_x. By comparison one can notice that it seems as if the B/H term can help decreasing the underestimation effect, or B/H has a positive effect on the value of D_x.

To further compare the prediction precision of those empirical formulae with relatively high precision, EF(5), and EFs(8−10) were selected and corresponding calculated $\mathrm{DR_s}$ were plotted in Fig. 1. The results has shown that the EF(10) has a good precision for rivers within $20 < B/H < 200$, and EF(5) has a relative good precision for rivers within $0 < B/H < 100$.

Table 2
Experimentally measured data for longitudinal dispersion coefficient in natural streams.

Number	Stream	Width B (m)	Depth H (m)	Velocity U (m/s)	Shear velocity u_* (m/s)	Dispersion coefficient (m^2/s)
1[a]	Antietam Creek, Md.	12.8	0.3	0.42	0.057	17.5
2[a]		24.08	0.98	0.59	0.098	101.5
3[a]		11.89	0.66	0.43	0.085	20.9
4[a]		21.03	0.48	0.62	0.069	25.9
5[a]	Monocacy River, Md.	48.7	0.55	0.26	0.052	37.80
6[a]		92.96	0.71	0.16	0.046	41.40
7[a]		51.21	0.65	0.62	0.044	29.60
8[a]		97.54	1.15	0.32	0.058	119.80
9[a]		40.54	0.41	0.23	0.040	66.50
10[a]	Conococheague Creek, Md.	42.41	0.69	0.23	0.064	40.80
11[a]		49.68	0.41	0.15	0.081	29.30
12[a]		42.98	1.13	0.63	0.081	53.30
13[a]	Chattahoochee River, Ga.	75.59	1.95	0.74	0.138	88.90
14[a]		91.90	2.44	0.52	0.094	166.90
15[a]	Salt Creek, Nebr.	32.00	0.50	0.24	0.038	52.20
16[a]	Difficult Run, Va.	14.48	0.31	0.25	0.062	1.90
17[a]	Bear Creek, Colo.	13.72	0.85	1.29	0.553	2.90
18[a]	Little Pincy Creek, Md.	15.85	0.22	0.39	0.053	7.10
19[a]	Bayou Anacoco, La.	17.53	0.45	0.32	0.024	5.80
20[a]	Comite River, La.	15.70	0.23	0.36	0.039	69.00
21[a]	Bayou Bartholomew, La	33.38	1.40	0.20	0.031	54.70
22[a]	Amite River, La.	21.34	0.52	0.54	0.027	501.40
23[a]	Tickfau River, La	14.94	0.59	0.27	0.080	10.30
24[a]	Tangipahoa, River, La.	31.39	0.81	0.48	0.072	45.10
25[a]		29.87	0.40	0.34	0.020	44.00
26[a]	Red River, La	253.59	1.62	0.61	0.032	143.80
27[a]		161.54	3.96	0.29	0.060	130.50
28[a]		152.40	3.66	0.45	0.057	227.60
29[a]		155.14	1.74	0.47	0.036	177.70
30[a]	Sabine River, La.	116.43	1.65	0.58	0.054	131.30
31[a]		160.32	2.32	1.06	0.054	308.90
32[a]	Sabine River, Tex.	14.17	0.50	0.13	0.037	12.80
33[a]		12.19	0.51	0.23	0.030	14.70
34[a]		21.34	0.93	0.36	0.035	24.20
35[a]	Mississippi River, La.	711.20	19.94	0.56	0.041	237.20
36[a]	Mississippi River, Mo.	533.40	4.94	1.05	0.069	457.70
37[a]		537.38	8.90	1.51	0.097	374.10
38[a]	Wind Bighorn River, Wyo	44.20	1.37	0.99	0.142	184.60
39[a]		85.34	2.38	1.74	0.153	464.60
40[a]	Copper Creep, Va.	16.66	0.49	0.20	0.080	16.84
41[a]	Clinch River, Va.	48.46	1.16	0.21	0.069	14.76
42[a]	Copper Creek, Va.	18.29	0.38	0.15	0.116	20.71
43[a]	Powell River, Tenn.	36.78	0.87	0.13	0.054	15.50
44[a]	Clinch River, Va.	28.65	0.61	0.35	0.069	10.70
45[a]	Copper River, Va.	19.61	0.84	0.49	0.101	20.82
46[a]	Clinch River, Va.	57.91	2.45	0.75	0.104	40.49
47[a]		53.24	2.41	0.66	0.107	36.93
48[a]	Copper Creek, Va.	16.76	0.47	0.24	0.080	24.62
49[a]	Missouri River, Iowa	180.59	3.28	1.62	0.078	1486.45
50[a]	Bayou Anacoco. La.	25.91	0.94	0.34	0.067	32.52
51[a]		36.58	0.91	0.40	0.067	39.48
52[a]	Nooksack River, Wash.	64.01	0.76	0.67	0.268	34.84
53[a]	Wind Bighorn River, Wyo.	59.44	1.10	0.88	0.119	41.81
54[a]		68.58	2.16	1.55	0.168	162.58
55[a]	John Day River, Oreg.	24.99	0.58	1.01	0.140	13.94
56[a]		34.14	2.47	0.82	0.180	65.03
57[a]	Yadkin River, N.C.	70.10	2.35	0.43	0.101	111.48
58[a]		71.63	3.84	0.76	0.128	260.13
59[a]	Minnesota River	80.00	2.74	0.034	0.0024	22.30
60[a]		80.00	2.74	0.14	0.0097	34.90
61[a]	Amite River	37.00	0.81	0.29	0.070	23.20
62[a]		42.00	0.80	0.42	0.069	30.20
63[a]	White River	67.00	0.55	0.35	0.044	30.20

Table 2 (*continued*)

Number	Stream	Width B (m)	Depth H (m)	Velocity U (m/s)	Shear velocity u_* (m/s)	Dispersion coefficient (m^2/s)
64[a]	Nooksack River	86.00	2.93	1.20	0.53	153.00
65[a]	Susquehanna River	203.00	1.35	0.39	0.065	92.90
66[a]	Bayou Anacoco	20.00	0.42	0.29	0.045	13.90
67[a]	Muddy River	13.00	0.81	0.37	0.081	13.90
68[a]		20.00	1.20	0.45	0.099	32.50
69[a]	Comite River	13.00	0.26	0.31	0.044	7.00
70[a]		16.00	0.43	0.37	0.056	13.90
71[a]	Missouri River	183.00	2.33	0.89	0.066	465.00
72[a]		201.00	3.56	1.28	0.084	837.00
73[a]	Missouri River	197.00	3.11	1.53	0.078	892.00
74[b]	Copper Creek, Va.	15.9	0.49	0.21	0.079	19.52
75[b]		18.3	0.84	0.52	0.100	21.40
76[b]		16.2	0.49	0.25	0.079	9.50
77[b]	Clinch river, Tn.	46.9	0.86	0.28	0.067	13.93
78[b]		59.4	2.13	0.86	0.104	53.88
79[b]		53.3	2.09	0.79	0.107	46.45
80[b]	Copper Creek, Va	18.6	0.39	0.14	0.116	9.85
81[b]	Power River, Tn.	33.8	0.85	0.16	0.055	9.50
82[b]	Clinch River, Va.	36.0	0.58	0.30	0.049	8.08
83[b]	Coachell Canal, Ca.	24.4	1.56	0.67	0.043	9.57
84[b]	Bayou Anacoco	19.8	0.41	0.29	0.044	13.94
85[b]	Nooksake River	86.0	2.94	1.20	0.514	153.29
86[b]	Antietam Creek	15.8	0.39	0.32	0.060	9.29
87[b]		19.8	0.52	0.43	0.069	16.26
88[b]		24.4	0.71	0.52	0.081	25.55
89[b]	Monocacy River	35.1	0.32	0.21	0.043	4.65
90[b]		36.6	0.45	0.32	0.051	13.94
91[b]		47.5	0.87	0.44	0.070	37.16
92[b]	Missouri River	182.9	2.23	0.93	0.065	464.52
93[b]		201.2	3.56	1.27	0.082	836.13
94[b]	Wind Bighorn Rivers	67.1	0.98	0.88	0.110	41.81
95[b]	Elkhorn River	32.6	0.30	0.43	0.046	9.29
96[b]		50.9	0.42	0.46	0.046	20.90
97[b]	Sabine River, Tex.	35.1	0.98	0.21	0.041	39.48
98[c]	Big blue river	75	1.6	0.22	0.990	17.00
99[c]	Embarrass river	30	1.1	0.38	0.025	35.90
100[c]	Illinois river, Henry	158	4.3	0.19	0.007	48.90
101[c]	Illinois river, Henry	232	3.4	0.24	0.043	52.00
102[c]	Illinois river, Kingston	202	4.6	0.18	0.036	49.10
103[c]	Illinois river, Kingston	194	6.3	0.22	0.039	537.70
104[c]	Illinois river, Marseilles	183	5.7	0.11	0.020	13.30
105[c]	Kanawha River	259	3.3	0.17	0.017	24.20
106[c]		259	3.4	0.17	0.018	22.10
107[c]	Missouri River	230	3.5	1.08	0.085	455.10
108[c]		176	3.4	1.61	0.082	966.20
109[c]		229	3.4	1.24	0.082	309.80
110[c]	New River	102	4.4	0.17	0.008	22.40
111[c]	Salt Creek	167	0.2	0.47	0.159	43.20
112[c]	Sangamon River	27	1.1	0.44	0.007	24.60
113[c]	Yampa River	78	1.2	1.42	0.026	325.60
114[c]		300	0.3	1.00	0.029	349.60
115[c]		300	0.4	0.97	0.032	227.70
116[c]		76	1.2	1.41	0.058	116.40

Note: Data sets with superscript indicate sources for experimental data.
[a] Deng et al. (2002).
[b] Kashefipour and Falconer (2002).
[c] Carr and Rehmann (2005).

3. Development of a new formula

By dimensional analysis, Kashefipour and Falconer (2002) concluded that combinations of U, H, B and bed shear velocity u_* can all have the dimension of D_x, and after regression analysis between U, H, B, S, u_* and D_x, they found that D_x is linearly related to the products of BU and HU with the corresponding correlation coefficients as 0.56 and 0.70 separately. Here regression analysis with the total 116 sets of data listed in Table 2 have shown that the linearly corresponding correlation

Table 3
Comparison between measured and predicted longitudinal dispersion coefficient.

Equation	DRs				
	$0 < B/H < 20$ ($N = 7$)	$20 < B/H < 100$ ($N = 97$)	$100 < B/H < 200$ ($N = 9$)	$B/H > 200$ ($N = 3$)	$0 < B/H < 1000$ ($N = 116$)
EF(1)	−0.265	0.182	0.907	2.237	0.264
EF(2)	−1.394	−2.068	−2.242	−3.223	−2.071
EF(3)	−0.126	0.123	0.853	1.706	0.205
EF(4)	0.039	0.221	0.953	1.647	0.304
EF(5)	−0.045	−0.077	0.428	0.704	−0.016
EF(6)	−1.472	−1.609	−0.923	−0.243	−1.512
EF(7)	−0.428	−0.387	0.024	0.513	−0.334
EF(8)	0.382	0.163	0.265	0.258	0.187
EF(9)	0.292	−0.118	−0.298	−0.643	−0.121
EF(10)	0.325	−0.002	−0.037	−0.141	0.011
Eq. (4)	0.225	0.000	0.139	0.104	0.027

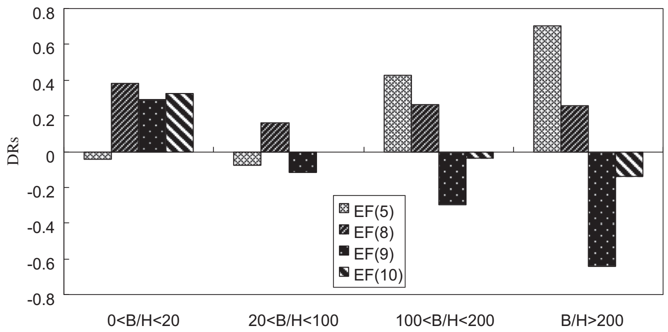

Fig. 1. Comparison of discrepancy ratio for empirical equations with different B/H.

coefficient of UH to D_x is 0.37, while that of u_*H to D_x is only 0.05. Considering the B/H term has the tendency to eliminate the underestimation, we adopt the expression $D_x = k(B/H)^{\alpha}(U/u_*)^{\beta}HU$ to deduce a new model for predicting the longitudinal dispersion coefficient.

To obtain the exponents of α and β, the variations of the dimensionless parameters $D_x/(HU)$ with B/H and U/u_* of cases listed in Table 2 were plotted in Figs. 2 and 3 respectively, with those plots demonstrating relatively pronounced power relationships between $D_x/(HU)$ and B/H and U/u_*. The correlations have been analyzed and $\alpha = 0.7$ and $\beta = 0.13$ were determined.

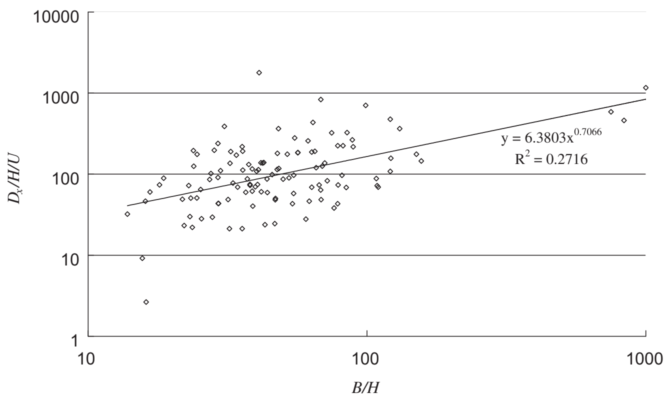

Fig. 2. Relationship between $D_x/(HU)$ and B/H.

Then a new equation for predicting the longitudinal dispersion coefficient was obtained as following:

$$D_x = 5.4(B/H)^{0.7}(U/u_*)^{0.13}HU \tag{4}$$

To compare the prediction precision, the discrepancy ratios of Eq. (4) with different interval of B/H were listed in Table 3. It has shown that Eq. (4) can predict the longitudinal dispersion coefficient well especially for rivers within $20 < B/H < 100$.

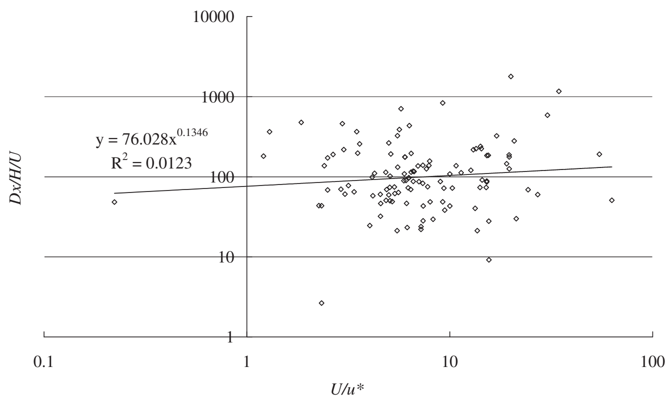

Fig. 3. Relationship between $D_x/(HU)$ and U/u_*.

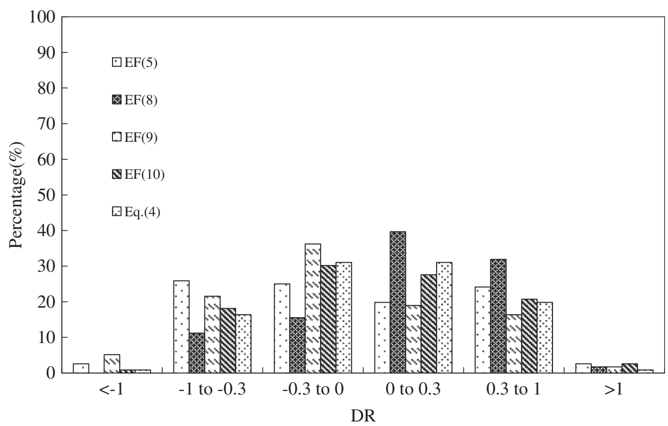

Fig. 4. Comparison of discrepancy ratio for different models.

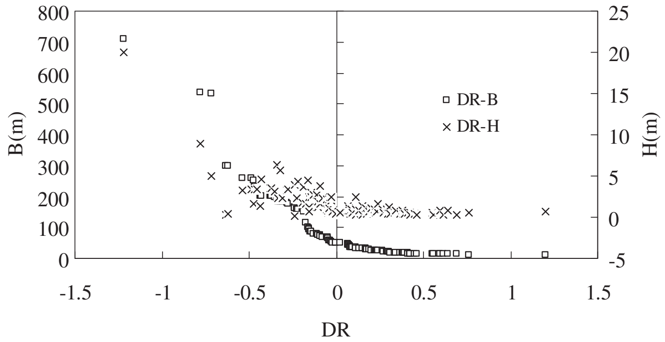

Fig. 5. Variation of DR of Eq. (4) with different B and H.

To further analyze the distribution of error, the DR of the selected models, EF(5), EFs(8–10) together with the newly presented Eq. (4) were statistically analyzed and the results were shown in Fig. 4. For EF.(10) and Eq. (4), their discrepancy ratios were almost symmetrically distributed, and large number of discrepancy ratios were located in the range of [−0.3,0.3]. For Eq. (4), the percentage of DR located in the range of [-0.3,0.3] is 62.1%, while that of EF(10) is 57.7%, 55.2% for EFs(8–9), and 44.8% for EF(5). So the conclusion can be made that Eq. (4) has the highest precision for the 116 sets of measured data.

To further evaluate the precision of Eq. (4) under different river width and water depth, the DR varied with different B and H were plotted in Fig. (5) separately. Most of the DR were located in the range of [-0.5, 0.5]. Comparably speaking, low precisions of Eq. (4) were obtained in rivers with relatively large or small B (e.g., $B > 259$ m or $B < 15$ m).

4. Comparison with laboratory measurement

Eq. (4) was proved to have a higher precision for predicting the longitudinal dispersion coefficients of the collected 116 natural rivers in U.S. and UK, but its application to geometrically uniform artificial channels still need to be evaluated. Here two series of tests, one was conducted by Fischer (1967) in a trapezoidal flume with roughened side slope (cases 1 to 6 in Table 4), the other was conducted by Wang and Lin (2003) in a rectangular flume with smooth bank wall (cases 7 to 8 with comprehensive roughness $n = 0.01$, cases 9 to 10 with roughened bed and $n = 0.012$, and cases 11 to 12 with roughened bed and $n = 0.02$), were adopted for comparison (details in Table 4). From Table 4, the conclusion can be made that the increase of cross-sectional velocity U, B/H leads to the increase of D_x; for rectangular flumes, increase the bed roughness can increase the D_x; the D_x in a trapezoidal channel is much greater than that in a rectangular one, which is identical with Guymer's (1998) observation that for natural cross-sectional channel geometries, the value of the longitudinal dispersion coefficient can be as much as over 150% greater than the corresponding values obtained for regular channel cross-sections.

The dispersion coefficient calculated by EF(5), EFs(8–10) and Eq. (4) were compared with the measured D_x, and the discrepancy ratios were listed in Table 4. For EFs(8–10), the values of DR_s were relatively high, especially for EF(10), the DR_s is 2.027, which means EF(10) has largely overestimated the D_x. Eq. (4) was good at predicting cases conducted in trapezoidal flumes, however it overestimated the longitudinal dispersion in rectangular flumes. This can be explained by that for open channel with the same cross-sectional area, the increase of u_* with the increase of U in a rectangular cross-section is much smaller than that in a trapezoidal one, since the latter has a longer wetted perimeter. In other words, the HU term in Eq. (4) can cause an overestimate in a rectangular channel. Comparatively speaking, EF(5) with a Hu^* term has a best prediction for longitudinal dispersion in a laboratory flume especially in a rectangular one, which is in accordance with the former conclusion that EF(5) is suitable for small scale rivers.

5. Conclusions

In this paper, the precisions of 10 selected empirical equations commonly used to predict the longitudinal dispersion coefficient in natural rivers have been compared with the

Table 4
Comparison between measured and predicted longitudinal dispersion coefficient for laboratory flumes.

Run	B/m	H/m	U/ms^{-1}	u_*/ms^{-1}	D_x/m^{2}s^{-1}	DR				
						EF(5)	EF(8)	EF(9)	EF(10)	Eq. (4)
1	0.4	0.035	0.25	0.0202	0.123	−0.352	0.750	0.970	1.561	0.467
2	0.43	0.047	0.45	0.0359	0.253	−0.433	0.763	1.046	1.591	0.470
3	0.4	0.035	0.45	0.0351	0.415	−0.641	0.484	0.713	1.311	0.196
4	0.34	0.035	0.44	0.0348	0.25	−0.530	0.648	0.917	1.476	0.356
5	0.33	0.021	0.45	0.0328	0.4	−0.668	0.376	0.537	1.220	0.091
6	0.19	0.021	0.46	0.0388	0.22	−0.696	0.470	0.742	1.273	0.184
DR_s						−0.553	0.582	0.821	1.405	0.294
7	0.5	0.05	0.05	0.0027	0.00304	0.448	1.852	2.208	2.852	1.512
8	0.5	0.05	0.1	0.0053	0.00787	0.328	1.744	2.105	2.752	1.401
9	0.5	0.05	0.05	0.0032	0.00291	0.541	1.840	2.154	2.762	1.522
10	0.5	0.05	0.1	0.0064	0.00925	0.340	1.638	1.952	2.561	1.320
11	0.5	0.05	0.05	0.0053	0.00501	0.524	1.510	1.699	2.206	1.257
12	0.5	0.05	0.1	0.0106	0.01129	0.473	1.458	1.647	2.154	1.205
DR_s						0.443	1.674	1.961	2.548	1.370

measured data of above 50 natural rivers in U.S. and UK. The result has shown that most of the equations have underestimated the D_x, and EF.(10) has the best precision compared with other existing empirical equations especially in the range of $20 < B/H < 100$, while EF(2) and EF(6) both underestimate the longitudinal dispersion coefficient.

A new empirical equation has been established to predict the longitudinal dispersion coefficient by relating this process to the main hydraulic and geometrical parameters of river depth, width, cross-sectional-averaged velocity, and bed shear velocity. Comparison of this new equation with the selected published empirical equations has shown that the accuracy of the newly presented Eq. (4) was comparatively favorable except for those rivers with very small B or large B.

Further comparison between the selected empirical equations, including the newly presented one to the laboratory measured data has shown that the EF(10) was not suitable for the laboratory cases listed here, while EF(5) is more suitable for the small scale laboratory cases especially for those conducted in rectangular open channel. Eq. (4) was good at predicting the dispersion coefficient for trapezoidal flume, however it overestimated the longitudinal dispersion in rectangular flume. Since natural rivers are generally trapezoidal shaped, one can make the conclusion that the newly presented empirical equation may have a good performance in predicting the longitudinal dispersion coefficients in natural rivers.

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