PREDICTING LONGITUDINAL DISPERSION COEFFICIENT IN NATURAL STREAMS

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ABSTRACT: In this study, previous empirical equations used to compute the dispersion coefficient are analyzed in order to evaluate their behavior in predicting dispersion characteristics in natural streams. A comparative analysis of previous theoretical and empirical equations is reported. A new simplified method for predicting dispersion coefficients using hydraulic and geometric data that are easily obtained in natural streams is also developed. The one-step Huber method, which is one of the nonlinear multiregression methods, is applied to derive a new dispersion coefficient equation. This equation is proven to be superior in explaining dispersion characteristics of natural streams more precisely, as compared to existing equations.

INTRODUCTION

Contaminants and effluent, when discharged into a river, undergo stages of mixing as the flowing water transports them downstream. The effluent is dispersed longitudinally, transversely and vertically by advective and dispersive transport processes. Once the cross-sectional mixing is complete, the process of longitudinal dispersion is the most important mechanism, erasing all longitudinal concentration gradients (Fischer et al. 1979). For this case, the one-dimensional (1D) Fickian-type dispersion equation derived by Taylor (1954) has been widely used to obtain reasonable estimates of the rate of longitudinal dispersion. The 1D dispersion equation is

$$A\frac{\partial C}{\partial t} = -UA\frac{\partial C}{\partial x} + \frac{\partial}{\partial x}\left(KA\frac{\partial C}{\partial x}\right) \tag{1}$$

in which A = the cross-sectional area; C = the cross-sectional average concentration; U = the cross-sectional average velocity; K = the dispersion coefficient; t = time; and x = the direction of mean flow.

Analytical solutions of (1) are easily obtained with given initial and boundary conditions when the river flow is uniform and the dispersion coefficient is constant. However, use of the 1D dispersion equation is limited to locations far downstream from the source where the balance between advection and diffusion, assumed by Taylor, is achieved. Fischer et al. (1979) concluded that, during the early period of the transport process, the advective transport due to the velocity distribution is dominant. During this so-called "initial period," advection and diffusion are not in balance, and, as a result, Taylor's analysis cannot be applied. Fischer et al. (1979) also reasoned that, because of the dominant effect of the velocity distribution during the initial period, the longitudinal distribution of the cross-sectionally averaged concentration is highly skewed, with a steep gradient in the downstream direction and a long tail in the upstream direction. The variance of the longitudinal concentration distribution increases nonlinearly with time during the initial period; during the later, or so-called "Fickian (Taylor) period," the variance increases in a linear fashion for steady, uniform flow. Therefore, the 1D dispersion equation is applicable only after the Fickian period is reached.

When the 1D dispersion model is applied to predict concentration variations of pollutants in natural streams, the selection of a proper dispersion coefficient is the most important and also the most difficult task. It is a relatively simple task to use a measured dispersion coefficient, if it is known. However, for streams where mixing and dispersion characteristics are unknown, the dispersion coefficient can only be estimated using theoretical or empirical equations. A theoretical method to predict the longitudinal dispersion coefficient was first proposed by Taylor (1954) and expanded by Elder (1959), who derived an equation to compute the longitudinal coefficient for a uniform flow in an infinitely wide open channel, assuming a logarithmic velocity profile. Since then, a number of investigators have proposed empirical equations based on experimental and field data for predicting the dispersion coefficient. However, because most studies have been carried out based on specific assumptions and channel conditions, the behavior of the equations varies widely for the same flow condition and stream.

In this study, previous empirical equations used to compute the dispersion coefficient were analyzed to evaluate their behavior in predicting dispersion characteristics in natural streams. In addition, a new, simplified equation has been developed which predicts dispersion coefficients using hydraulic data that is more easily obtained for natural streams. Dimensional analysis was done to select physically meaningful parameters which relate to mechanisms of natural dispersion. A one-step Huber method, one of the nonlinear multiregression methods, was applied to derive a regression equation for the dispersion coefficient using 59 measurements collected in 26 streams in the United States.

EVALUATION OF PREVIOUS WORKS

Theoretical and Empirical Approaches

Taylor (1954) first introduced a concept for the longitudinal dispersion coefficient for longitudinal mixing in a straight circular tube in turbulent flow. Taylor derived his equation theoretically as follows:

in which r = tube radius; and U_* = shear velocity which is given as

$$U_* = \sqrt{gRS} \tag{3}$$

in which g = gravitational acceleration; R = hydraulic radius; and S = the slope of the energy grade line.

Elder (1959) extended Taylor's method for uniform flow in an open channel of infinite width. He derived a dispersion equation assuming a logarithmic velocity profile and assuming that the mixing coefficients for momentum transfer and mass transfer in the vertical direction are the same. Elder derived the equation as

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$$K = 5.93hU_* \tag{4}$$

in which h = the depth of flow.

Elder's equation has been widely used because it is simple and has a sound theoretical background. However, it has been suggested that his equation may not describe dispersion in natural streams (Fischer et al. 1979). Fischer (1966, 1968) showed that Elder's equation significantly underestimates the natural dispersion in real streams, because it does not consider the transverse variation of the velocity profile across the stream. He postulated that in most natural streams, the transverse profile of the velocity is far more important than the vertical profile in producing longitudinal dispersion.

Parker (1961) adapted Taylor's turbulent flow equation to an open channel by substituting the hydraulic radius for the half pipe radius. The resulting equation is

$$K = 14.28R^{3/2}\sqrt{2gS} \tag{5}$$

Using the lateral velocity profile instead of the vertical velocity profile, Fischer (1966, 1968) obtained an integral relation for the dispersion coefficient in natural streams having large width-to-depth ratios. The result is given as

$$K = -\frac{1}{A}\int_0^W hu' \int_0^y \frac{1}{\varepsilon_t h}\int_0^y hu' \, dy \, dy \, dy \tag{6}$$

in which $h = h(y)$; u' = deviation of the velocity from the cross-sectional mean velocity; W = channel width; y = Cartesian coordinate in the lateral direction; and ε_t = transverse turbulent diffusion coefficient. He showed that the agreement of measurements and the prediction by (6) is within a factor of four in nonuniform streams, and within an error of 30% in uniform streams.

Eq. (6) is rather difficult to use because detailed transverse profiles of both velocity and cross-sectional geometry are required. As a result, Fischer (1975) developed a simpler equation by introducing a reasonable approximation of the triple integration, velocity deviation, and transverse turbulent diffusion coefficient. The result is

$$K = 0.011\frac{U^2W^2}{hU_*} \tag{7}$$

Eq. (7) has the advantage of simplicity in that it can predict dispersion coefficient by using only the data of cross-sectional mean parameters, which are easily obtained for a stream.

Sooky (1969) studied the effects of cross-sectional shape and velocity distribution on dispersion coefficient. Assuming a logarithmic velocity profile and power-function velocity profile, Sooky (1969) developed a dimensionless dispersion equation as the function of the width-to-depth ratio for a uniform flow in straight open channels for which cross section is triangular and circular. Through analysis of the field data of Godfrey and Frederick (1970), Sooky (1969) showed that the dimensionless dispersion coefficient increases as the width-to-hydraulic radius ratio increases. Sooky's work does not describe the natural dispersion in real streams adequately because his equation was derived assuming a uniform channel cross section. Bansal (1971) reviewed and summarized the empirical and theoretical equations to compute the dispersion coefficient. Using dispersion data obtained from the U.S. Geological Survey, he also demonstrated that the dimensionless dispersion coefficient increases as the width-to-hydraulic radius ratio increase.

McQuivey and Keefer (1974) developed a simple equation of dispersion coefficient using the similarity between the 1D solute dispersion equation and the 1D flow equation, especially when the Froude number is less than 0.5. They initially derived an equation which relates the longitudinal dispersion coefficient and the flow dispersion coefficient. Then, by the linear least-square regression of the field data, they derived an empirical equation for longitudinal dispersion coefficient as

$$K = 0.058\frac{hU}{S} \tag{8}$$

Even though the method suggested by McQuivey and Keefer (1974) is simple, Fischer (1975) suggested that their equation lacks an analytical basis since the mechanisms for dispersion of a flood wave and a dissolved contaminant would be expected to be quite different.

Abd El-Hadi and Daver (1976) attempted to relate the longitudinal dispersion coefficient to parameters such as bed roughness and the other hydrodynamic characteristics of channel flow. They performed experiments in a recirculating flume with different bed roughness simulations, and reported that the dimensionless dispersion coefficient is a function of both relative roughness height and the relative roughness spacing. They also showed that the relation between K and hU_* is clearly nonlinear beyond values of about 0.009 m^2/s for hU_*, and the degree of nonlinearity increases with increasing density of roughness.

Liu (1977) derived a dispersion coefficient equation using Fischer's equation [(6)] taking into account the role of lateral velocity gradients in dispersion in natural streams as

$$K = \beta\frac{U^2W^2}{hU_*} \tag{9}$$

in which β = a parameter which represents a function of the channel cross section shape and the velocity distribution across the stream. He suggested that the parameter β can be determined by considering sinuosity, sudden contractions and expansions, and dead zones in a natural stream. By least-square fitting to the field data obtained by Godfrey and Frederick (1970) and others, he deduced the following expression:

$$\beta = 0.18\left(\frac{U_*}{U}\right)^{1.5} \tag{10}$$

He postulated that the maximum deviation of the field data from the prediction by his equation is less than sixfold.

Chatwin and Sullivan (1982) investigated the effects of width-to-depth ratios on dispersion coefficient in channels for which the cross section is approximately rectangular. They determined analytically the dispersion coefficient for laminar flow, and expanded it for turbulent flow for a flat-bottomed channel of large width-to-depth ratio. However, in practice, it is difficult to use their method for predicting the dispersion coefficient, because detailed information on velocity profile and cross-sectional geometry are required to calculate the dispersion coefficient.

Magazine et al. (1988) experimentally studied the effect of large-scale bed and side roughness on dispersion. They derived an empirical predictive equation for the estimation of dimensionless dispersion coefficient using roughness parameters of the channel, such as the Reynolds number, details of boundary size, and spacing of roughness elements to account for blockage effects. Based on the experimental results of their study and an analysis of the available existing dispersion data, they developed the following expression:

$$\frac{K}{RU} = 75.86P^{-1.632} \tag{11}$$

in which P = a generalized roughness parameter incorporating the influence of the resistance and blockage effects, which are result of the roughness elements. For the prediction of dispersion coefficient in natural streams, Magazine et al. (1988) proposed the following equation:

$$P = 0.4 \frac{U}{U_*} \quad (12)$$

Asai and Fujisaki (1991) examined the dependence of the longitudinal dispersion coefficient on the width-to-depth ratio by using the k-ε model. They showed that the dispersion coefficient increases as the width-to-depth ratio increases up to 20; as the width-to-depth ratio increases further, the dispersion coefficient tends to decrease. Iwasa and Aya (1991), by analyzing their laboratory data and previous field data collected by Nordin and Sabol (1974) and others, derived an equation to predict the dispersion coefficient in natural streams and canals. The result is

$$\frac{K}{hU_*} = 2.0 \left(\frac{W}{h}\right)^{1.5} \quad (13)$$

Because natural streams are sinuous, and have sudden contractions, expansions and dead zones of water, the dispersion coefficient of the natural streams tends to increase compared to that of the simple, straight open channels. Thus, the effects of canal and flume data in the derivation of (13) cause the equation to underestimate the natural stream dispersion coefficient.

Comparisons with Stream Data

In order to test the behavior of the existing dispersion coefficient equations, 59 data sets measured in 26 streams in the United States were collected from published reports of Godfrey and Frederick (1970), Yotsukura et al. (1970), McQuivey and Keefer (1974), and Nordin and Sabol (1974). These data sets contain hydraulic and geometric parameters including channel width, mean depth, mean velocity, slope, sinuosity, and dye test results.

To calculate the observed dispersion coefficient from the field data, both the moment method and the routing procedure developed by Fischer (1968) were considered. It was, however, very difficult to obtain physically meaningful dispersion coefficients by the moment method, because the longitudinal distribution of the concentration is highly skewed, and contains a steep gradient in the rising limb and a long tail in the falling limb. Thus, in this study, the measured dispersion coefficients were calculated using the routing procedure. The field data sets with measured dispersion coefficients are listed in Table 1.

Among the methods for predicting dispersion coefficient suggested by previous investigators, six simple theoretical and empirical equations were tested using these 59 field data sets. These included the dispersion equations proposed by Elder (1959), McQuivey and Keefer (1974), Fischer (1975), Liu (1977), Magazine et al. (1988), and Iwasa and Aya (1991). The dispersion coefficients that were calculated using the selected equations were compared with measured data and are shown in Fig. 1. In Fig. 1, K_p is the predicted dispersion coefficient, and K_m is the measured dispersion coefficient.

Fig. 1 shows that the use of Elder's equation significantly underestimates measured values. The equation of Magazine et al. also gives low values, whereas McQuivey and Keefer's equation and Iwasa and Aya's equation predict values which agree relatively well with measured values. Fischer's equation generally overestimates, whereas Liu's equation gives values with more accuracy compared to Fischer's equation. However, for large rivers having channel width larger than 200 m, both Liu's equation and Fischer's equation give high estimates as shown in Fig. 1. The most probable reason for overestimation is the fact that both equations include a term for the square of the channel width.

To evaluate the difference between measured and predicted values of the dispersion coefficient more quantitatively, discrepancy ratio is defined by White et al. (1973) [(14)] was used as an error measure.

$$\text{Discrepancy Ratio} = \log \frac{K_p}{K_m} \quad (14)$$

If the discrepancy ratio is 0, the predicted value of the dispersion coefficient is identical to the measured dispersion coefficient. If the discrepancy ratio is larger than 0, the predicted value of the dispersion coefficient overestimates, and if the discrepancy ratio is smaller than 0, it underestimates. Accuracy is defined as the proportion of numbers for which the discrepancy ratio is between -0.3 and 0.3 for the total number of data.

Discrepancy ratios for each equation for the 59 field data sets are shown in Fig. 2. Accuracy of each equation is listed in Table 2. Of the equations examined, Liu's equation shows the highest accuracy and Iwasa and Aya's equation ranks second.

DEVELOPMENT OF NEW EQUATION

Selection of Meaningful Parameters

Major factors which influence dispersion characteristics of pollutants in natural streams can be categorized into three groups: fluid properties, hydraulic characteristics of the stream, and geometric configurations. The fluid properties include fluid density, viscosity, and so on. The cross-sectional mean velocity, shear velocity, channel width, and depth of flow can be included in the category of bulk hydraulic characteristics. The bed forms and sinuosity can be regarded as the geometric configurations. The dispersion coefficient can be related to these parameters as

$$K = f_1(\rho, \mu, U, U_*, h, W, S_f, S_n) \quad (15)$$

in which ρ = fluid density; μ = fluid viscosity; S_f = bed shape factor; and S_n = sinuosity.

By using dimensional analysis, a new functional relationship between dimensionless terms was derived as

$$\frac{K}{hU_*} = f_2 \left(\rho \frac{Uh}{\mu}, \frac{U}{U_*}, \frac{W}{h}, S_f, S_n\right) \quad (16)$$

in which K/hU_* = dimensionless dispersion coefficient; $\rho hU/\mu$ = Reynolds number; W/h = width-to-depth ratio; U/U_* = friction term which can be defined as

$$U/U_* = (8/f)^{1/2} \quad (17)$$

in which f = Darcy-Weisbach's friction factor. Bed shape factor, S_f, and sinuosity, S_n, are vertical and transverse irregularities in natural stream, respectively. These vertical and transverse irregularities cause secondary currents and shear flow that affect the hydraulic mixing processes in streams. In this study, however, these two parameters were dropped because they represent parameters not easily collected for natural streams, and furthermore, the influences of these two parameters can be included in the friction team. For fully turbulent flow in rough open channels, such as natural streams, the effect of Reynolds number is negligible. Thus (16) reduces to

$$\frac{K}{hU_*} = f_3 \left(\frac{U}{U_*}, \frac{W}{h}\right) \quad (18)$$

This functional relationship indicates that, even though (18) is composed of dimensionless parameters, if it is converted

TABLE 1. Summary of Hydraulic and Dispersion Data Measured at 26 Streams in the United States

Stream (1)	Width, W (m) (2)	Depth, h (m) (3)	Velocity, U (m/s) (4)	Slope, S (5)	Shear velocity, U_* (m/s) (6)	Dispersion coefficient, K (m^2/s) (7)	Reference (8)
Antietam Creek, MD	12.80	0.30	0.42	0.00095	0.057	17.50	Nordin and Sabol (1974)
	24.08	0.98	0.59	0.00135	0.098	101.50	
—[a]	11.89	0.66	0.43	0.00095	0.085	20.90	
—[a]	21.03	0.48	0.62	0.00100	0.069	25.90	
Monocacy River, MD	48.70	0.55	0.26	0.00050	0.052	37.80	
—[a]	92.96	0.71	0.16	0.00045	0.046	41.40	
—[a]	51.21	0.65	0.62	0.00040	0.044	29.60	
—[a]	97.54	1.15	0.32	0.00045	0.058	119.80	
	40.54	0.41	0.23	0.00045	0.040	66.50	
Conococheague Creek, MD	42.21	0.69	0.23	0.00060	0.064	40.80	
—[a]	49.68	0.41	0.15	0.00060	0.081	29.30	
—[a]	42.98	1.13	0.63	0.00060	0.081	53.30	
Chattahoochee River, GA[a]	75.59	1.95	0.74	0.00072	0.138	88.90	
	91.90	2.44	0.52	0.00037	0.094	166.90	
Salt Creek, NE	32.00	0.50	0.24	0.00033	0.038	52.20	
Difficult Run, VA[a]	14.48	0.31	0.25	0.00127	0.062	1.90	
Bear Creek, CO[a]	13.72	0.85	1.29	0.02720	0.553	2.90	
Little Pincy Creek, MD[a]	15.85	0.22	0.39	0.00130	0.053	7.10	
Bayou Anacoco, LA[a]	17.53	0.45	0.32	0.00054	0.024	5.80	
Comite River, LA	15.70	0.23	0.36	0.00058	0.039	69.00	
Bayou Bartholomew, LA[a]	33.38	1.40	0.20	0.00007	0.031	54.70	
Amite River, LA[a]	21.34	0.52	0.54	0.00048	0.027	501.40	
Tickfau River, LA	14.94	0.59	0.27	0.00117	0.080	10.30	
Tangipahoa River, LA[a]	31.39	0.81	0.48	0.00061	0.072	45.10	
	29.87	0.40	0.34	0.00069	0.020	44.00	
Red River, LA[a]	253.59	1.62	0.61	0.00007	0.032	143.80	
—[a]	161.54	3.96	0.29	0.00009	0.060	130.50	
—[a]	152.40	3.66	0.45	0.00009	0.057	227.60	
	155.14	1.74	0.47	0.00008	0.036	177.70	
Sabine River, LA	116.43	1.65	0.58	0.00014	0.054	131.30	
—[a]	160.32	2.32	1.06	0.00013	0.054	308.90	
Sabine River, TX[a]	14.17	0.50	0.13	0.00029	0.037	12.80	
	12.19	0.51	0.23	0.00018	0.030	14.70	
—[a]	21.34	0.93	0.36	0.00013	0.035	24.20	
Mississippi River, LA[a]	711.20	19.94	0.56	0.00001	0.041	237.20	
Mississippi River, MO[a]	533.40	4.94	1.05	0.00012	0.069	457.70	
	537.38	8.90	1.51	0.00012	0.097	374.10	
Wind/Bighorn River, WY	44.20	1.37	0.99	0.00150	0.142	184.60	
—[a]	85.34	2.38	1.74	0.00100	0.153	464.60	
Copper Creek, VA	16.66	0.49	0.20	0.00135	0.080	16.84	Godfrey and Frederick (1970)
Clinch River, VA	48.46	1.16	0.21	0.00085	0.069	14.76	
Copper Creek, VA[a]	18.29	0.38	0.15	0.00332	0.116	20.71	
Powell River, TN	36.78	0.87	0.13	0.00032	0.054	15.50	
Clinch River, VA[a]	28.65	0.61	0.35	0.00039	0.069	10.70	
Copper Creek, VA	19.61	0.84	0.49	0.00132	0.101	20.82	
Clinch River, VA[a]	57.91	2.45	0.75	0.00041	0.104	40.49	
Coachella Canal, CA[a]	24.69	1.58	0.66	0.00010	0.041	5.92	
Clinch River, VA[a]	53.24	2.41	0.66	0.00043	0.107	36.93	
Copper Creek, VA[a]	16.76	0.47	0.24	0.00135	0.080	24.62	
Missouri River, IA	180.59	3.28	1.62	0.00020	0.078	1486.45	Yotsukura et al. (1970)
Bayou Anacoco, LA[a]	25.91	0.94	0.34	0.00049	0.067	32.52	McQuivey and Keefer (1974)
	36.58	0.91	0.40	0.00050	0.067	39.48	
Nooksack River, WA	64.01	0.76	0.67	0.00963	0.268	34.84	
Wind/Bighorn River, WY[a]	59.44	1.10	0.88	0.00131	0.119	41.81	
—[a]	68.58	2.16	1.55	0.00133	0.168	162.58	
John Day River, OR[a]	24.99	0.58	1.01	0.00346	0.140	13.94	
	34.14	2.47	0.82	0.00134	0.180	65.03	
Yadkin River, NC[a]	70.10	2.35	0.43	0.00044	0.101	111.48	
	71.63	3.84	0.76	0.00044	0.128	260.13	

[a]Data sets used for derivation of new equation.

into a dimensionless form, only hydraulic and geometric parameters readily obtained in natural streams are included in the equation. These parameters are channel width (W), depth (h), mean velocity (U), and shear velocity (U_*). These are the same parameters that were used in Fischer's and Liu's equations.

To test the correlation between the dimensionless dispersion coefficient and dimensionless parameters included in (18), plots of measured dispersion coefficient versus measured hydraulic and geometric parameters were constructed using a log-log scale. The plot of dimensionless dispersion coefficient versus Reynolds number is shown in Fig. 3. This figure shows that, for the data collected in natural streams, the Reynolds number has an insignificant effect on the dimensionless dispersion coefficient. This confirms the assumption that, for turbulent flow in rough natural stream, the effect of Reynolds number is probably negligible.

The plots of K/hU_* versus W/h and K/hU_* versus U/U_* are shown in Figs. 4 and 5. These figures demonstrate that the dimensionless dispersion coefficient appears to have some dependency on these two dimensionless parameters, even though the data are somewhat scattered. It increases as the friction term and the width-to-depth ratio increase.

Regression Method

A standard nonlinear multiple model in which dependent variable $\mathbf{Y}$ is related to p unknown independent variables $\mathbf{X}$ can be given as

$$\mathbf{Y} = \alpha \mathbf{X}_1^{\beta} \mathbf{X}_2^{\gamma} \mathbf{X}_3^{\delta} \cdots \mathbf{X}_p^{\lambda} \varepsilon \tag{19}$$

in which $\mathbf{X}$ = independent variables which represent the hydraulic and geometric parameters; α, β, γ, . . . , λ = unknown regression coefficients; and ε = independent random residuals,

J. Hydraul. Eng. 1998.124:25-32.

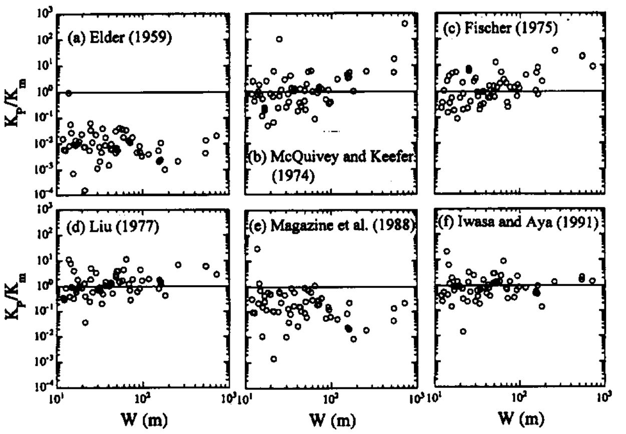

FIG. 1. Comparison of Estimated Dispersion Coefficients with Measured Data

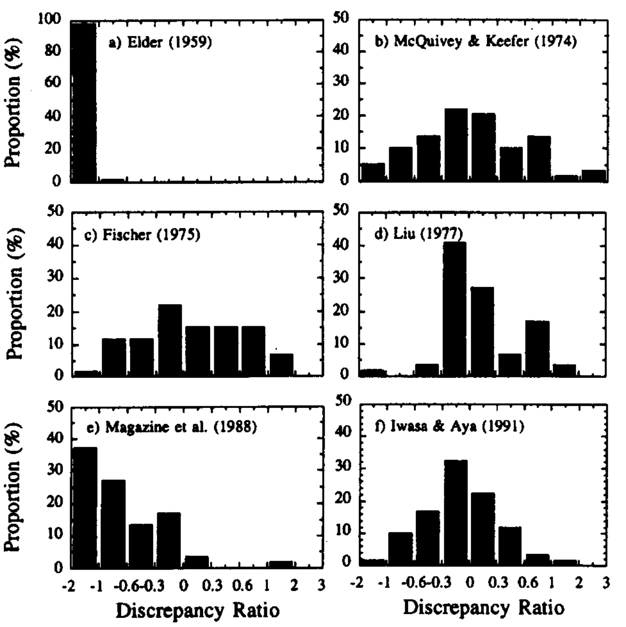

FIG. 2. Comparison of Discrepancy Ratios of Selected Equations

TABLE 2. Accuracy of Selected Dispersion Coefficient Equations

Dispersion coefficient equation (1)	Accuracy (%) (2)
Elder (1959)	0.0
McQuivey and Keefer (1974)	42.4
Fischer (1975)	37.3
Liu (1977)	67.8
Magazine et al. (1988)	20.3
Iwasa and Aya (1991)	54.5

which usually follow arbitrary distributions. In (19), vectors and matrices are depicted in bold letters. Taking logarithms of (19), a linear multiple form can be derived as follows:

$$\ln \mathbf{Y} = \ln \alpha + \beta \ln \mathbf{X}_1 + \gamma \ln \mathbf{X}_2 \cdots + \lambda \ln \mathbf{X}_p + \ln \varepsilon \quad (20)$$

The solution of the transformed model [(20)] is usually obtained by a least-squares method in which a sum of the squares of the residuals is minimized. However, for cases where residuals do not follow the normal distribution, especially when none of the residuals is not exceptionally large compared to the estimated standard deviation of the observations, the estimates of regression coefficients reflect leverage points that have an overriding influence on the fit.

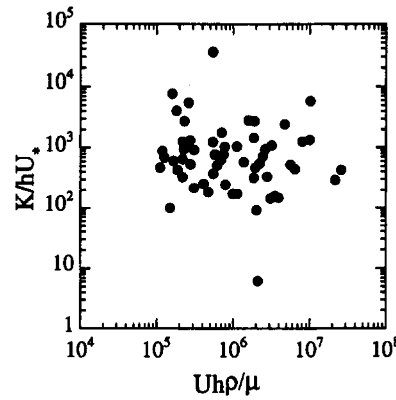

FIG. 3. Plots of Nondimensional Dispersion Coefficient (K/hU_*) versus Reynolds Number ($Uh\rho/\mu$)

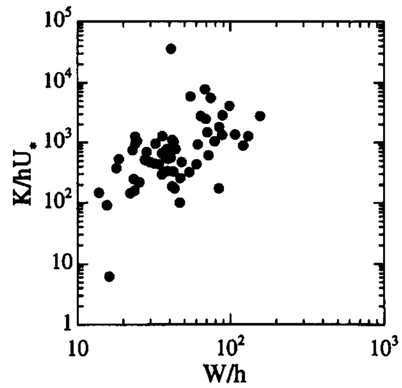

FIG. 4. Plots of Nondimensional Dispersion Coefficient (K/hU_*) versus Width-to-Depth Ratio (W/h)

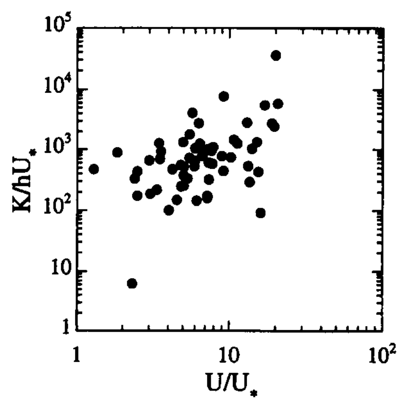

FIG. 5. Plots of Nondimensional Dispersion Coefficient (K/hU_*) versus Friction Term (U/U_*)

In this study, results of the application of the least-squares method indicates that the distribution of residuals does not follow the normal distribution. Several leverage points have been identified in the data set of the 35 measured dispersion coefficients that are used for derivation of the regression equation. Physically, it is probable that these leverage points came from the data measured in streams which are so irregular in nature that the 1D dispersion model cannot adequately explain natural dispersion characteristics of the streams. In these streams, concentration-time curves are significantly more skewed than the concentration distribution predicted by solution of the 1D dispersion equation.

The robust estimation minimizes a sum of less rapidly increasing functions of residuals rather than minimizing a sum of squares. The one-step method developed by Huber (1981), is one of the robust regression methods, gives reasonably good estimates even in the presence of moderately bad leverage points. The one-step Huber method in which the sum of the robust loss function of the residuals is minimized, can be written as

$$\sum_{i=1}^{p} \xi \left(\frac{y_i - \sum x_{ij}T_j}{s} \right) = \min, \quad j = 1, 2, 3, \ldots, n \tag{21}$$

in which y_i = measured value of the dependent variable; T_j = computed value of the robust estimator; s = a known or previously estimated scale parameter; ξ = robust loss function. If we let $\psi = \xi'$, which is the first derivative of ξ, then a necessary condition for a minimum is that the robust estimator, $\hat{T}_j$ satisfies the following equation.

$$\sum_{i=1}^{p} \psi \left(\frac{y_i - \sum x_{ij}T_j}{s} \right) x_{ij} = 0 \tag{22}$$

in which ψ = weight function.

In general, (22) is a set of nonlinear equations; thus an iterative method is required to solve this equation. If a starting value, $\hat{\mathbf{T}}_0$ is given, then there are various iteration schemes for obtaining a solution to (22). Among various iteration schemes, in this study, Newton's rule was used, and the result obtained is

$$\hat{\mathbf{T}}_{0+1} = \hat{\mathbf{T}}_0 + s(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\psi \left(\frac{\mathbf{Y} - \mathbf{X}\hat{\mathbf{T}}_0}{s} \right) \tag{23}$$

To minimize the least absolute residuals estimator, $\hat{\mathbf{T}}_{0+1}$, the following equation was used in this study.

$$r_i = r_i(T) = y_i - \hat{y}_i = y_i - \sum x_{ij}T_j \tag{24}$$

in which $\hat{y}_i$ = predicted value of dependent variable; and r_i = residual. It is also necessary to estimate the value of the scale parameter to solve (23). In this study, the scale parameter was estimated by the following function:

$$s = 1.48 \left[\operatorname*{med}_{i} \left| (y_i - x_{ij}T_j) - \operatorname*{med}_{j}(y_i - x_{ij}T_j) \right| \right] \tag{25}$$

in which the factor 1.48 makes the scale parameter an approximately unbiased estimate of scale when the error model is Gaussian (Holland and Welsch 1977). In this study, the weight function proposed by Huber (1981) was used. That is

$$\psi(r) = r, \quad |r| \leq a \tag{26a}$$

$$\psi(r) = a \sin(r), \quad |r| > a \tag{26b}$$

in which a = constant which Huber proposes to be 1.5.

For the solution of the one-step Huber method, preliminary estimates of $\hat{\mathbf{T}}_0$ are assumed with a value computed by the least-squares method and values of scale parameter are then computed by (25). Eq. (22) is then solved approximately for robust estimates $\hat{\mathbf{T}}_{0+1}$ by applying Newton's rule. The iteration method is used in which estimates $\hat{\mathbf{T}}_{0+1}$ are computed with a guessed value, $\hat{\mathbf{T}}_0$, until the sum of the residual of computed value reaches a minimum. The flow chart explaining this procedure is shown in Fig. 6.

New Dispersion Coefficient Equation

In this study, a nonlinear multiregression equation for predicting the dimensionless dispersion coefficient as a function of the friction factor and width-to-depth ratio is derived by using the one-step Huber method. The data sets used in the development of the new dispersion coefficient equation are the same as those used in the comparison of the previous dispersion coefficient equations. Among 59 data sets, 35 measured data sets (see Table 1) were selected to derive the dispersion coefficient, and 24 measured data sets were used to verify the new dispersion coefficient equation. Data sets were separated into two groups having similar statistical characteristics. The statistical characteristics of dimensionless values of the measured dispersion coefficient and hydraulic and geometric parameters for both derivation and verification data sets are summarized in Table 3.

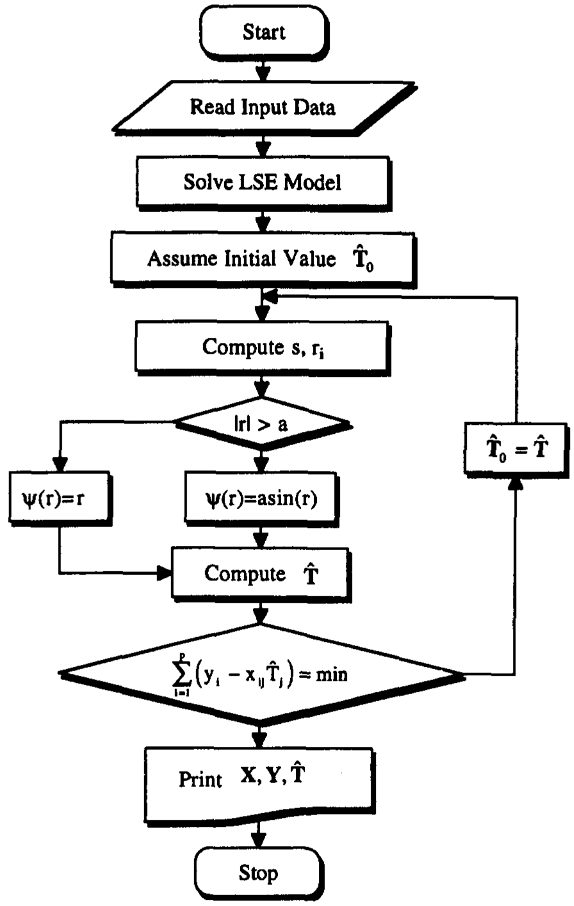

FIG. 6. Flowchart of One-Step Huber Method

TABLE 3. Statistical Characteristics of Dimensionless Hydraulic and Dispersion Parameters

Parameter (1)	Derivation Data Set: Range (2)	Median (3)	Quartile (4)	Verification Data Set: Range (5)	Median (6)	Quartile (7)
K/hU_*	6.16–35,712	582	305	146–7,692	936	330
W/h	15.6–157	39.0	28.0	13.82–100	42.5	25.3
$U/U*$	1.29–20.0	7.21	4.55	2.41–20.8	5.95	3.59

The new regression equation derived by using the one-step Huber method is given as

$$\frac{K}{hU_*} = 5.915 \left(\frac{W}{h} \right)^{0.620} \left(\frac{U}{U_*} \right)^{1.428} \tag{27}$$

In deriving (27), the correlation coefficient is 0.75. In this study, regression values were also computed by using the least-squares method, and those values were used as an initial condition for the solution of the one-step Huber method. The regression equation derived by the least-squares method is given as

J. Hydraul. Eng. 1998.124:25-32.

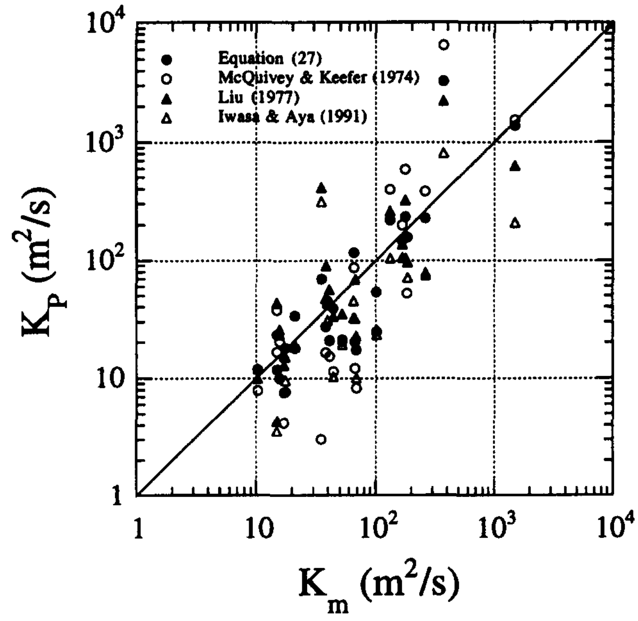

FIG. 7. Comparison of Estimated Dispersion Coefficients with 24 Measured Data Used in Verification

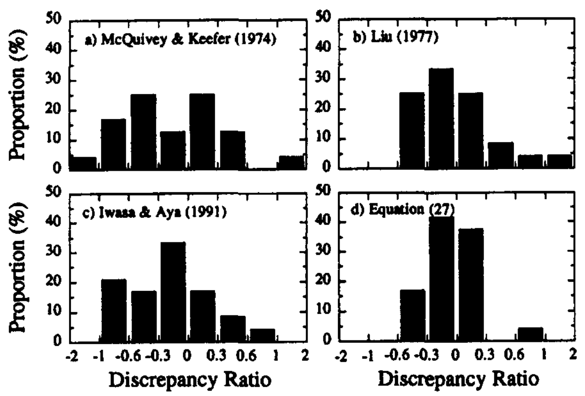

FIG. 8. Comparison of Discrepancy Ratios of Existing and New Equations for 24 Measured Data Used in Verification

TABLE 4. Accuracy of Existing Dispersion Coefficient Equations and Proposed Equation

Dispersion coefficient equation (1)	Accuracy (%) (2)
McQuivey and Keefer (1974)	50.0
Liu (1977)	66.7
Iwasa and Aya (1991)	58.3
Eq. (27)	79.2

$$\frac{K}{hU_*} = 0.64\left(\frac{W}{h}\right)^{1.23}\left(\frac{U}{U_*}\right)^{1.25} \tag{28}$$

In deriving (28), the correlation coefficient is 0.66.

VERIFICATION

Twenty-four measured data sets that were not used in the derivation of the regression equation were used to verify the proposed equation [(27)] for predicting dispersion coefficient. The dispersion coefficients predicted by the proposed equation and the existing equations were compared with measured dispersion coefficients. Three existing dispersion equations that were proven to be relatively better than other equations in predicting dispersion coefficient in natural streams were selected; they are the equations by McQuivey and Keefer (1974), Liu (1977), and Iwasa and Aya (1991).

The comparisons of estimated dispersion equations with measured data are shown in Fig. 7. This figure shows that the proposed equation [(27)] predicts quite well, whereas McQuivey and Keefer's equation and Iwasa and Aya's equation underestimate in many cases. Predictions by Liu's equation are generally in good agreement with the measured data.

Discrepancy ratios of each equation for 24 field data sets are shown in Fig. 8. The accuracy of each equation is listed in Table 4. The proposed equation predicts better than the existing equations, and the discrepancy ratio of the new dispersion coefficient equation ranges from −0.6 to 1. The accuracy of the proposed equation is 79%, which is the highest of all. These results demonstrate that the new dispersion coefficient equation developed in this study is superior to the existing equations in predicting dispersion coefficient more precisely in natural streams.

CONCLUSIONS

The results of this study show that, among the existing dispersion coefficient equations, Elder's equation is not amenable to estimating the dispersion coefficient of the 1D dispersion model because it underestimates significantly. McQuivey and Keefer's equation and Iwasa and Aya's equation predict relatively well, whereas the equation of Magazine et al. underestimates in most cases. Fischer's equation generally overestimates, whereas Liu's equation predicts relatively well. However, for large rivers with channel widths larger than 200 m, both Liu's equation and Fischer's equation overestimate significantly.

In addition to the comparative analysis of previous theoretical and empirical equations, a new, simple method for predicting dispersion coefficients by using hydraulic and geometric data that are easily obtained for natural streams has been developed. Dimensional analysis was implemented to select physically meaningful parameters that are required for the new equation in order to predict longitudinal dispersion in natural streams. The one-step Huber method was applied to derive a new dispersion coefficient equation. The proposed equation allows superior prediction as compared to the existing equations, and the discrepancy ratio of the new dispersion coefficient equation ranges from −0.6 to 1. The accuracy of the proposed equation is 79%. The dispersion coefficient estimated by the proposed equation can be used when the 1D dispersion model is applied to streams where mixing and dispersion data has not been collected, and thus the measured dispersion coefficient is not available.

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APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = flow cross-sectional area, m^2;
C = cross-sectional average concentration, mg/L;
f = Darcy-Weisbach's friction factor;
g = gravitational acceleration, m/s^2;
h = depth in flow, m;
K = longitudinal dispersion coefficient, m^2/s;
P = generalized roughness parameter;
R = hydraulic radius, m;
r = residual;
S = slope of energy gradient;
S_f = bed shape factor;
S_n = sinuosity of the stream;
s = scale parameter;
$\hat{\mathbf{T}}_{0+1}$ = robust estimate;
t = time coordinate, s;
U = cross-sectional average velocity, m/s;
U_* = shear velocity, m/s;
u' = deviation of point velocity from mean velocity, m/s;
W = channel width, m;
$\mathbf{X}$ = vector of independent variable;
x = longitudinal coordinate;
$\mathbf{Y}$ = vector of dependent variable;
y = lateral coordinate;
β = parameter predicted from frictional factor;
ε = random residual;
ε_t = transverse turbulent mixing coefficient, m^2/s;
μ = fluid viscosity, m^2/s;
ξ = robust loss function;
ρ = fluid density, mg/L;
σ_t^2 = variance of concentration-time distribution, s^2;
σ_x^2 = variance of concentration-distance distribution, m^2; and
ψ = weight function.

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