Question: Show that

$$\binom{n}{n}+\binom{n+1}{n}+\binom{n+2}{n}+\binom{n+3}{n}+\cdots+\binom{n+m}{n}=\binom{n+m+1}{n+1}$$

Solution: We have

$$\binom{n}{n}+\binom{n+1}{n}+\binom{n+2}{n}+\binom{n+3}{n}+\cdots+\binom{n+m}{n}=$$

$$\binom{n+1}{n+1}+\binom{n+1}{n}+\binom{n+2}{n}+\binom{n+3}{n}+\cdots+\binom{n+m}{n}=$$

$$\binom{n+2}{n+1}+\binom{n+2}{n}+\binom{n+3}{n}+\cdots+\binom{n+m}{n}=$$

$$\binom{n+3}{n+1}+\binom{n+3}{n}+\cdots+\binom{n+m}{n}=$$

$$\vdots$$

$$\binom{n+m}{n+1}+\binom{n+m}{n}=\binom{n+m+1}{n+1}. \quad \square$$