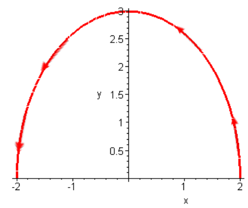
K. N. Toosi University of Technology

Faculty of Mathematics

Problems 3 - Calculus II

A. R. Moghaddamfar

1. Find the work done by the force $\vec{\mathbf{F}}(x, y) = (2x + e^{-y})\ \vec{\mathbf{i}} + (4y - xe^{-y})\ \vec{\mathbf{j}}$ along the indicated curve:



2. Find a potential function for the given vector field:
$\vec{\mathbf{F}}(x, y, z) = (2xy^2 + 3xz^2)\ \vec{\mathbf{i}} + (2x^2y + 2y)\ \vec{\mathbf{j}} + (3x^2z - 2z)\ \vec{\mathbf{k}}$.

3. Find the line integral $\int_C \vec{\mathbf{F}} \cdot d\vec{r}$ of the vector field

$$\vec{\mathbf{F}}(x, y, z) = 3x^2z\ \vec{\mathbf{i}} + z^2\ \vec{\mathbf{j}} + (x^3 + 2yz)\ \vec{\mathbf{k}},$$

along the curve C parametrized by $\vec{r}(t) = \left\langle \frac{\ln t}{\ln 2}, t^{\frac{3}{2}}, t\cos(\pi t) \right\rangle$, $1 \leqslant t \leqslant 4$.

4. Let $\vec{\mathbf{F}}(x, y, z) = -y\ \vec{\mathbf{i}} + x\ \vec{\mathbf{j}} + \vec{\mathbf{k}}$ and let C be the portion of the helix given by $\vec{r}(t) = \langle \cos t, \sin t, \frac{t}{2\pi} \rangle$ on $[0,2\pi]$. Evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{r}$.

5. Evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{r}$, where $\vec{\mathbf{F}}(x, y, z) = \langle xy, yz, zx \rangle$ and $\vec{r}(t) = \langle t, t^2, t^3 \rangle$, $t \in [0,1]$.

6. Compute $\operatorname{div} \vec{\mathbf{F}}$ for $\vec{\mathbf{F}}(x, y, z) = \langle x^2y, xyz, -x^2y^2 \rangle$.

7. Find the Laplacian of $f(x, y, z) = x^2y^2z + 2xz$.

8. If $f(x, y, z) = 2xz - y^2z$, find $\nabla \times \nabla f$.

9. Find the line of intersection of two planes $x + y + z = 1$ and $x + 2y + 2z = 1$.

10. Find the tangent line to the following curve:

$$C : \begin{cases} x^2 - 3xy + z^2 = 1, \\ 2x \tan^{-1}(xz) + 2y^2 - z = 1, \end{cases}$$

at point $(0,1,1)$.

11. Find an equation of the line perpendicular to two vectors $\overrightarrow{u} = \langle 1,1,4 \rangle$ and $\overrightarrow{v} = \langle 0, -1,2 \rangle$ passing through the point $(0,1,3)$.

12. Find the area of the triangle with vertices at $A = (1,0,2)$, $B = (3,1,0)$, and $C = (0,0,2)$.

13. Consider the function $f(x, y) = x^3 + 3xy + y^3$. Find the critical points of the function.

14. Find the arc length of the curve parametrized by $\vec{r}(t) = \langle e^t, e^{-t}, \sqrt{2}\ t \rangle$, on the range $0 \leqslant t \leqslant \ln 2$.

15. Find the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{2x^4 + y^2}$.

16. Find an equation of the plane tangent to the surface $xyz - \ln z = 0$ at point $(0,1,1)$.

17. Find and classify all critical points of the function $f(x, y) = 2x^2 + y^4 - 4xy$ on the entire plane.

18. Given two vectors $\overrightarrow{u} = \langle 1, -1,0 \rangle$ and $\overrightarrow{v} = \langle 1,0,1 \rangle$ find the angle between them. Compute the area of the parallelogram by the vectors.

19. Find and classify all critical points of the function $f(x, y) = x^2y - 2xy - 5x^2 + 10x$ and classify them using the Second Derivative Test.

20. Consider the function $f(x, y) = 3x^2 + 4y^2 - 2$. Find the directional derivative of $f(x, y)$ at the point $(1,1)$ in the direction of the vector $\overrightarrow{u} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$.