1. Vectors

Section 1-1 : Basic Concepts

The general notation for a n-dimensional vector is,

$$\overrightarrow{v} = \langle a_1, a_2, \dots, a_n \rangle,$$

and each of the a_i's are called *components* of the vector.

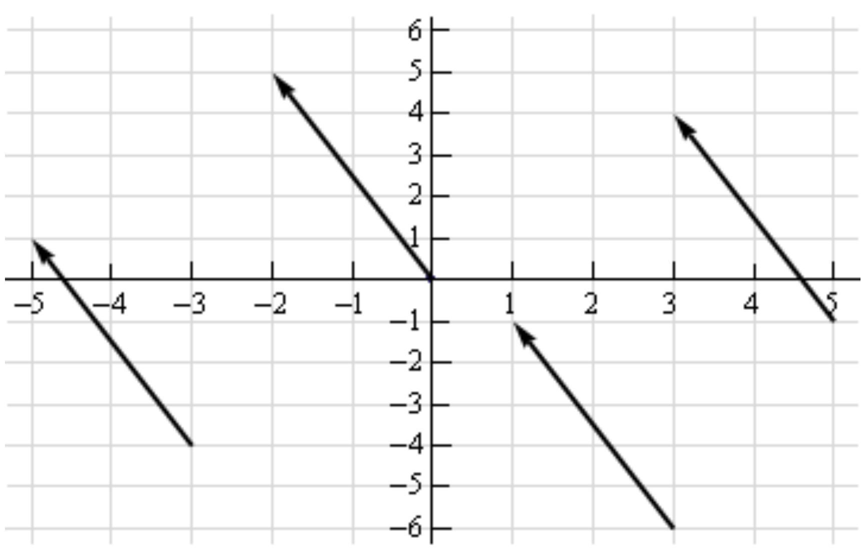
Because we will be working almost exclusively with *two* and *three* dimensional vectors in this course most of the formulas will be given for the two and/or three dimensional cases. However, most of the concepts/formulas will work with general vectors and the formulas are easily (and naturally) modified for general n-dimensional vectors. Also, because it is easier to visualize things in two dimensions most of the figures related to vectors will be two dimensional figures.

Given two vectors $\overrightarrow{v} = \langle a_1, a_2, \dots, a_n \rangle$, $\overrightarrow{w} = \langle b_1, b_2, \dots, b_n \rangle$, we have

$$\overrightarrow{v} = \overrightarrow{w} \quad \Longleftrightarrow \quad a_1 = b_1,\ a_2 = b_2,\ \dots,\ a_n = b_n\,.$$

Consider the following vector,

$$\overrightarrow{v} = \langle -2,5 \rangle$$



Each of the directed line segments in the sketch are called *representations* of the vector.

- vector notation, $\overrightarrow{v} = \langle -2,5 \rangle$
- coordinates of points, $P = (-2,5)$
- a representation of the vector $\overrightarrow{v} = \langle a_1, a_2 \rangle$ is any directed line segment, $\overrightarrow{AB}$, from the point $A = (x, y)$ to the point

$$B = (x + a_1, y + a_2).$$

- A representation of the vector $\overrightarrow{v} = \langle a_1, a_2, a_3 \rangle$ is any directed line segment, $\overrightarrow{AB}$, from the point $A = (x, y, z)$ to the point

$$B = (x + a_1, y + a_2, z + a_3).$$

- The representation of the vector $\overrightarrow{v} = \langle a_1, a_2, a_3 \rangle$ that starts at the point $A = (0,0,0)$ and ends at the point $B = (a_1, a_2, a_3)$ is called the *position vector* of the point (a_1, a_2, a_3).
- Given the two points

$$A = (a_1, a_2, a_3), \quad B = (b_1, b_2, b_3)$$

the vector with the representation $\overrightarrow{AB}$ is

$$\overrightarrow{AB} = \langle b_1 - a_1, b_2 - a_2, b_3 - a_3 \rangle.$$

- The vector above $\overrightarrow{AB}$ is the vector that starts at A and ends at B.
- The vector that starts at B and ends at A, with representation $\overrightarrow{BA}$ is

$$\overrightarrow{BA} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle.$$

Example 1 Give the vector for each of the following.

1. The vector from $A = (2, -7,0)$ to $B = (1, -3, -5)$.

$$\overrightarrow{AB} = \langle 1-2, -3-(-7), -5-0\rangle = \langle -1, 4, -5\rangle.$$

2. The vector from $B = (1, -3, -5)$ to $A = (2, -7, 0)$.

$$\overrightarrow{BA} = \langle 2-1, -7-(-3), 0-(-5)\rangle = \langle 1, -4, 5\rangle.$$

3. The position vector for $(-90, 4)$.

$$\langle -90, 4\rangle$$

Magnitude

The *magnitude*, or *length*, of the vector $\vec{v} = \langle a_1, a_2, a_3\rangle$ is given by,

$$\|\vec{v}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Example 2 Determine the magnitude of each of the following vectors.

1. $\vec{v} = \langle 3, -5, 10\rangle$

$$\|\vec{v}\| = \sqrt{9 + 25 + 100} = \sqrt{134}.$$

2. $\vec{v} = \left\langle \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}} \right\rangle$

$$\|\vec{v}\| = \sqrt{\frac{1}{5} + \frac{4}{5}} = \sqrt{1} = 1.$$

3. $\vec{v} = \langle 0, 0\rangle$

$$\|\overrightarrow{v}\| = \sqrt{0+0} = 0.$$

4. $\vec{i} = \langle 1,0,0 \rangle$

$$\|\vec{i}\| = \sqrt{1+0+0} = 1.$$

Zero Vector

The vector $\overrightarrow{0} = \langle 0,0,0 \rangle$ is called a *zero vector* since its components are all zero.
Note that:

$$\|\overrightarrow{v}\| = 0 \iff \overrightarrow{v} = \overrightarrow{0}\,.$$

Unit Vector

A vector $\overrightarrow{u}$ with magnitude of 1, i.e. $\|\overrightarrow{u}\| = 1$, is called a *unit vector*.

Standard Basis Vectors

In $\mathbb{R}^2$ there are two standard basis vectors,

$$\vec{i} = \langle 1,0 \rangle, \quad \vec{j} = \langle 0,1 \rangle\,.$$

In $\mathbb{R}^3$ there are three standard basis vectors,

$$\vec{i} = \langle 1,0,0 \rangle, \quad \vec{j} = \langle 0,1,0 \rangle, \quad \overrightarrow{k} = \langle 0,0,1 \rangle\,.$$

Note that standard basis vectors are also *unit vectors*.

Section 1-2 : Vector Arithmetic

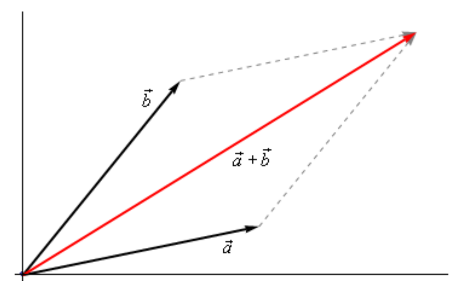
Addition. Given the vectors

$$\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle, \quad \overrightarrow{b} = \langle b_1, b_2, b_3 \rangle$$

the addition of the two vectors is given by the following formula:

$$\overrightarrow{a} + \overrightarrow{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

This is sometimes called the **parallelogram law** or **triangle law**.

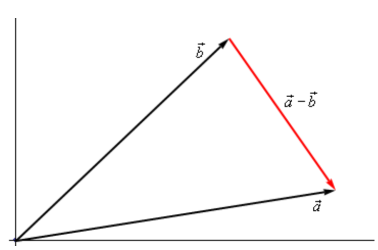


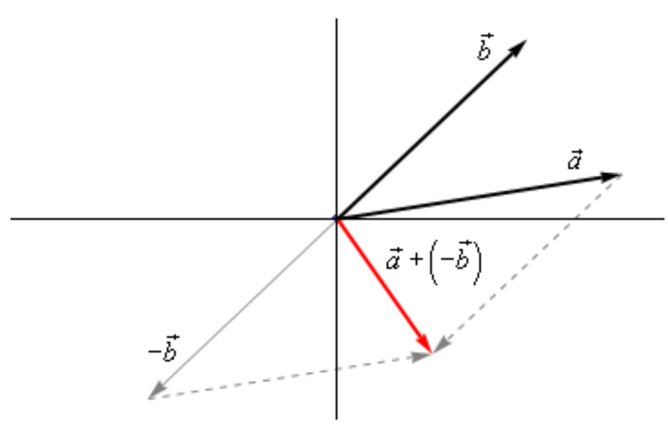
Subtraction. Given the vectors

$$\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle, \quad \overrightarrow{b} = \langle b_1, b_2, b_3 \rangle$$

the difference of the two vectors is given by,

$$\overrightarrow{a} - \overrightarrow{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$





Scalar multiplication. Given the vector $\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle$ and any number c the scalar multiplication is

$$c\overrightarrow{a} = \langle ca_1, ca_2, ca_3 \rangle$$

Actually, we multiply all the components by the constant c.

Several nice applications:

1. *Parallel vectors*. Given two vectors $\overrightarrow{u}, \overrightarrow{v}$ we have

$$\overrightarrow{u} \| \overrightarrow{v} \iff \exists c \quad \overrightarrow{v} = c\overrightarrow{u}.$$

2. *Unit vector in the same direction*. Given a non-zero vector $\overrightarrow{w}$,

$$\frac{1}{\|\overrightarrow{w}\|}\overrightarrow{w}$$

will be a unit vector that points in the same direction as $\overrightarrow{w}$.

3. *Standard Basis Vectors Revisited*. Given a vector $\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle$ we have

$$\langle a_1, a_2, a_3 \rangle = a_1\langle 1,0,0 \rangle + a_2\langle 0,1,0 \rangle + a_3\langle 0,0,1 \rangle .$$

So, we have

$$\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle = a_1\vec{i} + a_2\vec{j} + a_3\overrightarrow{k} .$$

Section 1-3 : Dot product

Dot Product (Inner product). Given the vectors

$$\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle, \quad \overrightarrow{b} = \langle b_1, b_2, b_3 \rangle$$

the dot product is,

$$\overrightarrow{a} \cdot \overrightarrow{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \,.$$

Example 3 Compute the dot product for each of the following.

1. $\overrightarrow{u} = 5\vec{i} - 8\vec{j}, \quad \overrightarrow{v} = \vec{i} + 2\vec{j}\,.$
2. $\overrightarrow{a} = \langle 0,3,-7 \rangle, \; \overrightarrow{b} = \langle 2,3,1 \rangle\,.$

$$\overrightarrow{u} \cdot \overrightarrow{v} = 5 - 16 = -11, \quad \overrightarrow{a} \cdot \overrightarrow{b} = 0 + 9 - 7 = 2.$$

Some properties of the dot product:

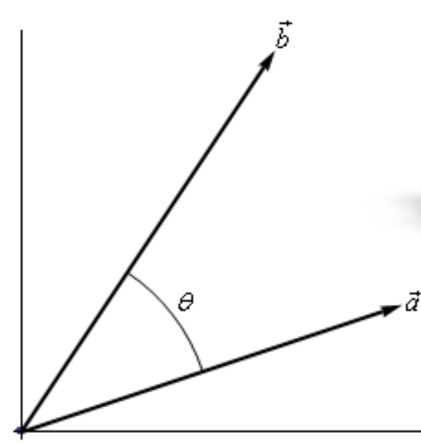
If $\overrightarrow{u}, \overrightarrow{v}, \overrightarrow{w}$ are vectors and c is a constant, then

1. $\overrightarrow{u} \cdot (\overrightarrow{v} + \overrightarrow{w}) = \overrightarrow{u} \cdot \overrightarrow{v} + \overrightarrow{u} \cdot \overrightarrow{w}$
2. $(c\overrightarrow{v}) \cdot \overrightarrow{w} = \overrightarrow{v} \cdot (c\overrightarrow{w}) = c(\overrightarrow{v} \cdot \overrightarrow{w})$
3. $\overrightarrow{u} \cdot \overrightarrow{v} = \overrightarrow{v} \cdot \overrightarrow{u}$
4. $\overrightarrow{v} \cdot \overrightarrow{0} = 0$
5. $\overrightarrow{v} \cdot \overrightarrow{v} = \|\overrightarrow{v}\|^2$
6. $\overrightarrow{v} \cdot \overrightarrow{v} = 0 \implies \overrightarrow{v} = \overrightarrow{0}\,.$

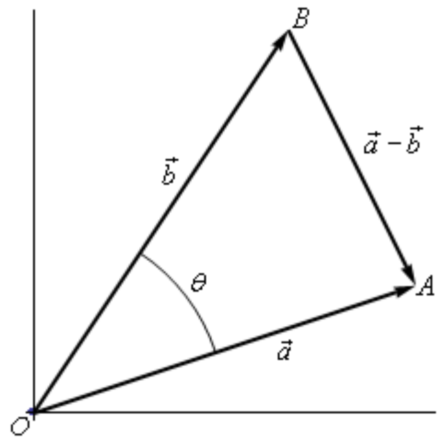
A nice geometric interpretation to the dot product.

First suppose that θ is the angle between $\overrightarrow{a}, \overrightarrow{b}$ such that $0 \leqslant \theta \leqslant \pi$ as shown in the image below. Then

$$\overrightarrow{a} \cdot \overrightarrow{b} = \|\overrightarrow{a}\| \; \|\overrightarrow{b}\| \; \cos\theta\,.$$



Proof. Considering



the *Law of Cosines* tells us that,

$$\|\vec{a}-\vec{b}\|^2 = \|\vec{a}\|^2 + \|\vec{b}\|^2 - 2\|\vec{a}\|\|\vec{b}\|\cos\theta.$$

Since

$$\|\vec{a}-\vec{b}\|^2 = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b}) = \|\vec{a}\|^2 - 2\vec{a}\cdot\vec{b} + \|\vec{b}\|^2,$$

we obtain

$$-2\vec{a}\cdot\vec{b} = -2\|\vec{a}\|\ \|\vec{b}\|\cos\theta,$$

and so

$$\vec{a}\cdot\vec{b} = \|\vec{a}\|\ \|\vec{b}\|\ \cos\theta$$

as desired.

Example 4 Determine the angle between $\vec{a} = \langle 3, -4, -1\rangle$ and $\vec{b} = \langle 0,5,2\rangle$.

We have $\vec{a}\cdot\vec{b} = -22$, $\|\vec{a}\| = \sqrt{26}$, $\|\vec{b}\| = \sqrt{29}$. The angle is then,

$$\cos\theta = \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{\|\overrightarrow{a}\|\ \|\overrightarrow{b}\|} = \frac{-22}{\sqrt{26}\sqrt{29}} = -\,0.8011927$$

and so $\theta = \cos^{-1}(-0.8011927)$.

Orthogonal vectors

If two vectors are orthogonal, then we know that the angle between them is 90 degrees. Hence, we have

$$\overrightarrow{a} \perp \overrightarrow{b} \quad \Longleftrightarrow \quad \overrightarrow{a} \cdot \overrightarrow{b} = 0.$$

Parallel vectors

if two vectors are parallel then the angle between them is either 0 degrees (pointing in the same direction) or 180 degrees (pointing in the opposite direction). This would mean that one of the following would have to be true.

$$\overrightarrow{a} \| \overrightarrow{b} \quad \Longleftrightarrow \quad \overrightarrow{a} \cdot \overrightarrow{b} = \|\overrightarrow{a}\|\ \|\overrightarrow{b}\| \text{ or } \overrightarrow{a} \cdot \overrightarrow{b} = -\|\overrightarrow{a}\|\ \|\overrightarrow{b}\|$$

Example 5 Determine if the following vectors are parallel, orthogonal, or neither.

A. $\overrightarrow{a} = \langle 6, -2, -1 \rangle$, $\overrightarrow{b} = \langle 2,5,2 \rangle$,

B. $\overrightarrow{u} = 2\overrightarrow{i} - \overrightarrow{j}$, $\overrightarrow{v} = -\frac{1}{2}\overrightarrow{i} + \frac{1}{4}\overrightarrow{j}$.

- We have

$$\overrightarrow{a} \cdot \overrightarrow{b} = 12 - 10 - 2 = 0,$$

  and hence two vectors are orthogonal.

- Since

$$\overrightarrow{u} \cdot \overrightarrow{v} = -1 - \frac{1}{4} = -\frac{1}{5},$$

So, they aren not orthogonal. Let us get the magnitudes and see if they are parallel. We have

$$\|\vec{u}\| = \sqrt{4+1} = \sqrt{5}, \quad \|\vec{v}\| = \sqrt{\frac{1}{4}+\frac{1}{16}} = \sqrt{\frac{5}{16}} = \frac{\sqrt{5}}{4}$$

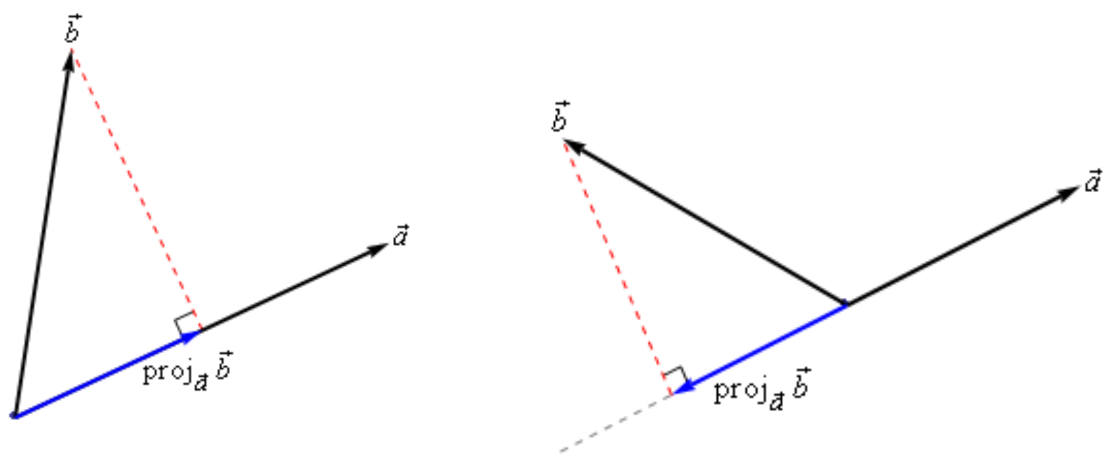
Now, notice that,

$$\vec{u}\cdot\vec{v} = -\frac{1}{5} = -\sqrt{5}\left(\frac{\sqrt{5}}{4}\right) = -\|\vec{u}\|\ \|\vec{v}\|,$$

So, the two vectors are parallel.

Projections

Given two vectors $\vec{a}$ and $\vec{b}$ we want to determine the projection of $\vec{b}$ onto $\vec{a}$:



- There is a nice formula for finding the projection of $\vec{b}$ onto $\vec{a}$:

$$\mathrm{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|^2}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|}\,\frac{\vec{a}}{\|\vec{a}\|}$$

- The projection of $\vec{a}$ onto $\vec{b}$:

$$\mathrm{proj}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|^2}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|}\,\frac{\vec{b}}{\|\vec{b}\|}$$

Example 6 Determine the projections $\text{proj}_{\vec{a}} \vec{b}$ and $\text{proj}_{\vec{b}} \vec{a}$ for vectors $\vec{a} = \langle 1,0,-2 \rangle$ and $\vec{b} = \langle 2,1,-1 \rangle$.

We have

$$\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\| \vec{a} \|^2} \vec{a} = \frac{4}{5} \langle 1,0,-2 \rangle = \langle \frac{4}{5}, 0, -\frac{8}{5} \rangle,$$

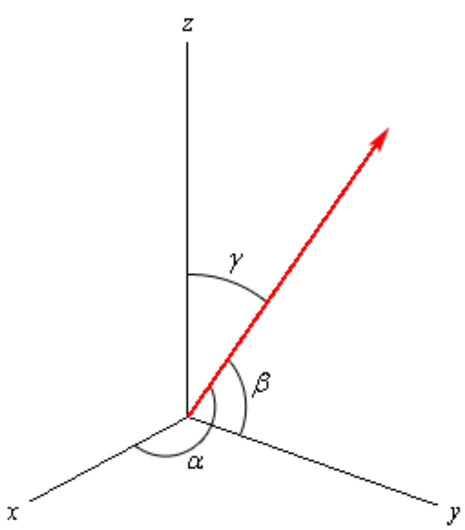
and

$$\text{proj}_{\vec{b}} \vec{a} = \frac{\vec{a} \cdot \vec{b}}{\| \vec{b} \|^2} \vec{b} = \frac{4}{6} \langle 2,1,-1 \rangle = \langle \frac{4}{3}, \frac{2}{3}, -\frac{2}{3} \rangle .$$

Direction Cosines

This application of the dot product requires that we be in $\mathbb{R}^3$. We start with a vector, $\vec{a} = \langle a_1, a_2, a_3 \rangle$ in $\mathbb{R}^3$. This vector will form *angles* with the x-axis (α), the y-axis (β), and the z-axis (γ). These angles α, β, γ are called *direction angles* and the cosines of these angles $\cos \alpha, \cos \beta, \cos \gamma$ are called *direction cosines.*

The formulas for the direction cosines are,



$$\cos \alpha = \frac{\vec{a} \cdot \vec{i}}{\| \vec{a} \|} = \frac{a_1}{\| \vec{a} \|} ,$$

$$\cos \beta = \frac{\vec{a} \cdot \vec{j}}{\| \vec{a} \|} = \frac{a_2}{\| \vec{a} \|},$$

$$\cos \gamma = \frac{\vec{a} \cdot \vec{k}}{\| \vec{a} \|} = \frac{a_3}{\| \vec{a} \|}$$

Some facts about the direction cosines:

- $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.
- The vector $\overrightarrow{u} = \langle \cos\alpha, \cos\beta, \cos\gamma \rangle$ is a unit vector.
- $\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle = \|\overrightarrow{a}\| \langle \cos\alpha, \cos\beta, \cos\gamma \rangle$.

Example 7 Determine the direction cosines for $\overrightarrow{a} = \langle 2,1,-4 \rangle$.

$$\cos\alpha = \frac{2}{\sqrt{21}}, \quad \cos\beta = \frac{1}{\sqrt{21}}, \quad \cos\gamma = \frac{-4}{\sqrt{21}}.$$

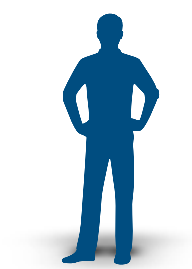
Section 1-4 : Cross Product

Cross Product. Given the vectors

$$\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle, \quad \overrightarrow{b} = \langle b_1, b_2, b_3 \rangle$$

the cross product is given by the formula

$$\overrightarrow{a} \times \overrightarrow{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}.$$



or equivalently

$$\overrightarrow{a} \times \overrightarrow{b} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \overrightarrow{k}$$

$$= \langle a_2b_3 - b_2a_3, b_1a_3 - a_1b_3, a_1b_2 - b_1a_2 \rangle.$$

Example 8 If $\overrightarrow{a} = \langle 2,1,-1 \rangle$ and $\overrightarrow{b} = \langle -3,4,1 \rangle$ compute each of the cross products $\overrightarrow{a} \times \overrightarrow{b}$ and $\overrightarrow{b} \times \overrightarrow{a}$.

We have

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -1 \\ -3 & 4 & 1 \end{vmatrix} = 5\vec{i} + \vec{j} + 11\vec{k},$$

and similarly,

$$\vec{b} \times \vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 4 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -5\vec{i} - \vec{j} - 11\vec{k},$$

Example 9 A plane is defined by any three points that are in the plane. If a plane contains the points

$$P = (1,0,0), \quad Q = (1,1,1), \quad R = (2,-1,3)$$

find a vector that is orthogonal to the plane.

Since all three points lie in the plane any vector between them must also be in the plane. We will use the following two

$$\overrightarrow{PQ} = \langle 1-1, 1-0, 1-0 \rangle = \langle 0,1,1 \rangle$$
$$\overrightarrow{PR} = \langle 2-1, -1-0, 3-0 \rangle = \langle 1,-1,3 \rangle$$

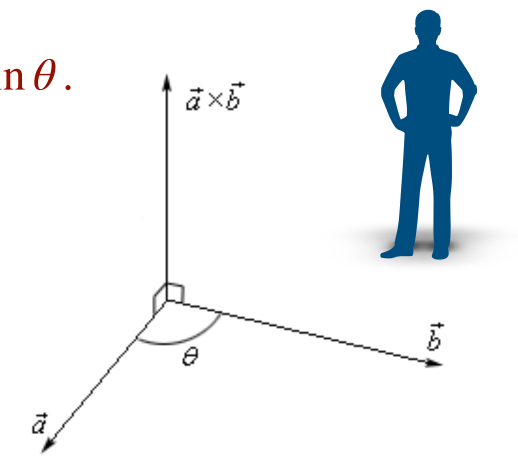
The cross product of these two vectors will be orthogonal to the plane. So, we find the cross product:

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 1 \\ 1 & -1 & 3 \end{vmatrix} = 4\vec{i} + \vec{j} - \vec{k}.$$

A nice geometric interpretation to the cross product.

First suppose that θ is the angle between $\overrightarrow{a}, \overrightarrow{b}$ such that $0 \leqslant \theta \leqslant \pi$ as shown in the image below. Then

$$\|\overrightarrow{a} \times \overrightarrow{b}\| = \|\overrightarrow{a}\| \ \|\overrightarrow{b}\| \ \sin\theta .$$



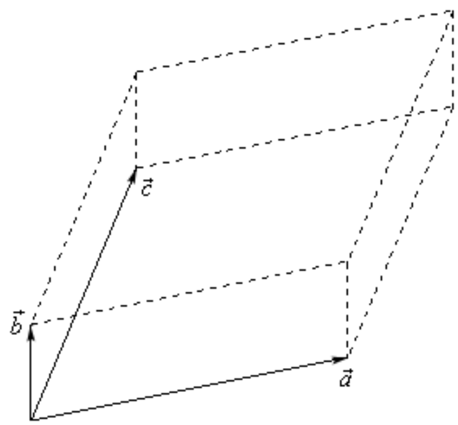
Some properties of the dot product:

If $\overrightarrow{u}, \overrightarrow{v}, \overrightarrow{w}$ are vectors and c is a constant, then

1. $\overrightarrow{u} \times (\overrightarrow{v} + \overrightarrow{w}) = \overrightarrow{u} \times \overrightarrow{v} + \overrightarrow{u} \times \overrightarrow{w}$
2. $(c\overrightarrow{v}) \times \overrightarrow{w} = \overrightarrow{v} \times (c\overrightarrow{w}) = c(\overrightarrow{v} \times \overrightarrow{w})$
3. $\overrightarrow{u} \times \overrightarrow{v} = -\overrightarrow{v} \times \overrightarrow{u}$
4. $\overrightarrow{v} \times \overrightarrow{0} = \overrightarrow{0}$
5. $\overrightarrow{v} \times \overrightarrow{v} = \overrightarrow{0}$
6. $\overrightarrow{u} \cdot (\overrightarrow{v} \times \overrightarrow{w}) = (\overrightarrow{u} \times \overrightarrow{v}) \cdot \overrightarrow{w}$
7. $\overrightarrow{u} \| \overrightarrow{v} \iff \overrightarrow{u} \times \overrightarrow{v} = \overrightarrow{0}$
8. If $\overrightarrow{u} \times \overrightarrow{v} \neq \overrightarrow{0}$, then $\overrightarrow{u} \times \overrightarrow{v} \perp \overrightarrow{u}$ and $\overrightarrow{u} \times \overrightarrow{v} \perp \overrightarrow{v}$.

Also, if $\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle$, $\overrightarrow{b} = \langle b_1, b_2, b_3 \rangle$ and $\overrightarrow{c} = \langle c_1, c_2, c_3 \rangle$ then

$$\overrightarrow{a} \cdot (\overrightarrow{b} \times \overrightarrow{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}.$$



The area of the parallelogram (two dimensional front of this object) is given by,

$$\text{Area} = \| \overrightarrow{a} \times \overrightarrow{b} \|$$

and the volume of the parallelepiped (the whole three dimensional object) is given by,

$$\text{Valume} = | \overrightarrow{a} \cdot (\overrightarrow{b} \times \overrightarrow{c}) |$$

Example 10 Determine if the vectors $\overrightarrow{a} = \langle 1,4,7 \rangle$, $\overrightarrow{b} = \langle 2, -1,4 \rangle$ and $\overrightarrow{c} = \langle 0, -9,18 \rangle$ lie in the same plane or not.

All we need to do is compute the volume of the parallelepiped formed by these three vectors. If the volume is zero they lie in the same plane. We have

$$\overrightarrow{a} \cdot (\overrightarrow{b} \times \overrightarrow{c}) = \begin{vmatrix} 1 & 4 & -7 \\ 2 & -1 & 4 \\ 0 & -9 & 18 \end{vmatrix} = 1(-18+36) - 4(36-0) - 7(-18-0) = 0,$$

So, the volume is zero and so they lie in the same plane.

Practical Problems

For problems 1- 4 give the vector for the set of points. Find its magnitude and determine if the vector is a unit vector.

1. The line segment from $(-9,2)$ to $(4,-1)$.
2. The line segment from $(4,5,6)$ to $(4,6,6)$.
3. The position vector for $(-3,2,10)$.
4. The position vector for $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.
5. The vector $\overrightarrow{v} = \langle 6,-4,0\rangle$ starts at the point $P = (-2,5,-1)$. At what point does the vector end?
6. Determine if $\overrightarrow{a} = \langle 3,-5,1\rangle$ and $\overrightarrow{b} = \langle 6,-2,2\rangle$ are parallel vectors.
7. Determine if $\overrightarrow{v} = 9\overrightarrow{i} - 6\overrightarrow{j} - 24\overrightarrow{k}$ and $\overrightarrow{w} = \langle -15,10,40\rangle$ are parallel vectors.
8. Determine if the two vectors are parallel, orthogonal or neither.
$$\overrightarrow{v} = -3\overrightarrow{i} - 12\overrightarrow{j} + 6\overrightarrow{k}, \quad \overrightarrow{w} = \overrightarrow{i} + 4\overrightarrow{j} - 2\overrightarrow{k}.$$
9. Given $\overrightarrow{u} = 7\overrightarrow{i} - \overrightarrow{j} + \overrightarrow{k}$ and $\overrightarrow{w} = -2\overrightarrow{i} + 5\overrightarrow{j} - 6\overrightarrow{k}$ compute $\mathrm{proj}_{\overrightarrow{w}}\overrightarrow{u}$.
10. Are the vectors $\overrightarrow{u} = \langle 1,2,-4\rangle$, $\overrightarrow{v} = \langle -5,3,-7\rangle$ and $\overrightarrow{w} = \langle -1,4,2\rangle$ are in the same plane?