# K. N. Toosi University of Technology

## Faculty of Mathematics

## Problems 5 - Calculus II

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1. Evaluate $\displaystyle\int_1^2 \int_0^x \frac{1}{(x^2+y^2)^{3/2}} \, dy dx$.

   Answer: $\dfrac{\sqrt{2}}{4}$.

2. Find the volume of the region above the $xy$-plane and below the graph of $z = 1 - x^2 - y^2$.

   Answer: $\dfrac{\pi}{2}$.

3. Evaluate $\displaystyle\iiint_E x \, dV$ where $E$ is enclosed by $z = 0$, $z = x + y + 5$, $x^2 + y^2 = 4$

   and $x^2 + y^2 = 9$.

   Answer: $\dfrac{65\pi}{4}$

4. Find $\displaystyle\int_C x^2 y \, dx - xy^2 \, dy$ where $C$ is the circle $x^2 + y^2 = 4$ going counter-
   clockwise.

   Answer: $-8\pi$.

5. Evaluate $\displaystyle\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2 + y^2 + z^2) \, dz \, dx \, dy$. (Hint: Use spherical
   coordinates).

   Answer: $\dfrac{486}{5}\pi$.

6. Find and classify all critical points of the function

$$f(x, y) = 2x^3 + 6xy^2 - 3y^3 - 150x$$

and classify them using the Second Derivative Test.

Answer: $(5,0)$ minimum, $(-5,0)$ maximum, $(3,4), (-3, -4)$ saddle points.

7. Find a potential function for the given vector field

$$\overrightarrow{\mathbf{F}}(x, y) = (y \cos xy + 10x) \, \vec{\mathbf{i}} + (x \cos xy + 3y^2) \, \vec{\mathbf{j}}.$$

Answer: $f(x, y) = \sin xy + 5x^2 + y^3$.

8. Find the volume between $x^2 + y^2 + z^2 = 2$ and $z = \sqrt{x^2 + y^2}$.

Answer: $\dfrac{4\pi}{3}(\sqrt{2} - 1)$.

9. Compute $\displaystyle\oint_C x^2 z\,dx + 3x\,dy - y^3\,dz$ where $C$ is the unit circle $x^2 + y^2 = 1$ oriented counter-clockwise.

Answer: $3\pi$.

10. Let $F = \langle yz, xz, xy \rangle$. Find the work done by this force field on an object that moves from $(1,0,2)$ to $(1,2,3)$.

Answer: $6$.