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## Faculty of Mathematics

# Problems 5 - Calculus II

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1. Evaluate $\int_1^2 \int_0^x \frac{1}{(x^2+y^2)^{3/2}}\, dy dx.$

2. Find the volume of the region above the $xy$-plane and below the graph of $z = 1 - x^2 - y^2$.

3. Evaluate $\iiint_E x\ dV$ where $E$ is enclosed by $z = 0,\ z = x + y + 5,\ x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

4. Find $\int_C x^2y\ dx - xy^2\ dy$ where $C$ is the circle $x^2 + y^2 = 4$ going counter-clockwise.

5. Evaluate $\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2 + y^2 + z^2)\ dz\ dx\ dy$. (Hint: Use the spherical coordinates).

6. Find all critical points of the function

$$f(x, y) = 2x^3 + 6xy^2 - 3y^3 - 150x,$$

and classify them using the Second Derivative Test.

7. Find a potential function for the given vector field

$$\overrightarrow{\mathbf{F}}(x, y) = (y \cos xy + 10x)\ \vec{\mathbf{i}} + (x \cos xy + 3y^2)\ \vec{\mathbf{j}}.$$

8. Find the volume between $x^2 + y^2 + z^2 = 2$ and $z = \sqrt{x^2 + y^2}$.

9. Compute $\oint_C x^2 z\,dx + 3x\,dy - y^3\,dz$ where $C$ is the unit circle $x^2 + y^2 = 1$ oriented counter-clockwise.

10. Let $F = \langle yz, xz, xy \rangle$. Find the work done by this force field on an object that moves from $(1,0,2)$ to $(1,2,3)$.