K. N. Toosi University of Technology

Faculty of Mathematics

Problems 4 - Calculus II

A. R. Moghaddamfar

1. Evaluate $\int_{\pi/2}^{\pi}\int_{0}^{x^2} \frac{1}{x^2}\cos\frac{y}{x}\,dydx.$

Answer:$1.$

2. Evaluate $\int_{1}^{4}\int_{0}^{\sqrt{y}} e^{x/\sqrt{y}}\,dxdy.$

Answer: $\frac{14}{3}(e-1).$

3. Evaluate $\iint_D e^{-(x^2+y^2)}\,dxdy$ where D is the region between the two circles

$x^2+y^2=1$ and $x^2+y^2=4.$

Answer: $\pi(e^{-1}-e^{-4})$

4. Evaluate $\iint_D (x+y)^2\,dxdy$ where D is the parallelogram bounded by the lines

$x+y=0,\ x+y=1,\ 2x-y=0$ and $2x-y=3.$

Answer: $\frac{1}{3}.$

5. Let D be the region in the first quadrant bounded by the hyperbolas $xy=1,\ xy=9$ and the lines $y=x,\ y=4x$. Evaluate

$$\iint_D \left(\sqrt{\frac{y}{x}} + \sqrt{xy}\right)\, dxdy$$

Answer: $8 + \frac{52}{3}\ln 2.$

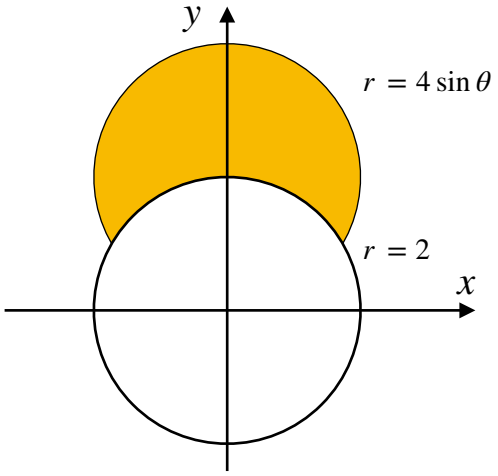
6. Evaluate $\int_0^{\frac{\sqrt{\pi}}{2}} \int_{2y}^{\sqrt{\pi}} \sin(x^2)\, dxdy.$

Answer: $\frac{1}{2}.$

7. Find the volume under $z = \sqrt{4 - r^2}$ above the region enclosed by the curve $r = 2\cos\theta,\ -\pi/2 \leqslant \theta \leqslant \pi/2.$

Answer: $\frac{8}{3}\pi - \frac{32}{9}.$

8. Find the area outside the circle $r = 2$ and inside $r = 4\sin\theta.$



Answer: $\frac{4}{3}\pi + 2\sqrt{3}.$

9. Let D be the region bounded by the lines $y = x,\ y = x - 1,\ x + 2y = 0$ and $x + 2y = 1.$ Evaluate

$$\iint_D \frac{x+2y}{\cos(x-y)}\,dxdy$$

Answer: $\frac{1}{6}\ln\left|\tan\frac{\pi+2}{4}\right|$.

10. Evaluate $\int_0^2\int_{y/2}^1 e^{x^2}\,dxdy$.

Answer: $e-1$.

11. Evaluate the triple integral $\iiint_D 12xy^2z^3dV$ over the rectangular box D given by $D=\{(x,y,z)\in\mathbb{R}^3 \mid -1\leqslant x\leqslant 2,\ 0\leqslant y\leqslant 3,\ 0\leqslant z\leqslant 2\}$.

Answer: 648.

12. Find the volume of the solid within the cylinder $x^2+y^2=9$ and between the planes $z=1$ and $x+z=5$.

Answer: 36π.

13. Evaluate $\int_C xydx+x^2dy$, where C is the arc of the parabola $y=x^2$ from $(0,0)$ to $(2,4)$, followed by a straight line from $(2,4)$ back to $(0,0)$.

Answer: $\frac{4}{3}$.

14. Evaluate the line integral of the function $f(x,y,z)=xy+y+z$ along the curve $r(t)=\langle 2t,t,2-2t\rangle$ in the interval $t\in[0,1]$.

Answer: $\frac{13}{2}$.