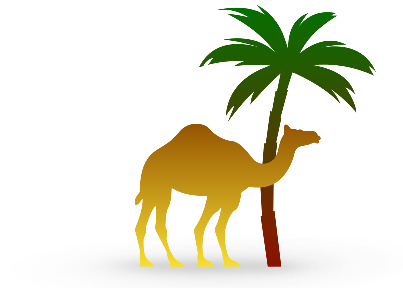
Pascal's Triangle



$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$$

$$(a+b)^0 = 1$$

$$(a+b)^1 = 1a+1b$$

$$(a+b)^2 = 1a^2+2ab+1b^2$$

$$(a+b)^3 = 1a^3+3a^2b+3ab^2+1b^3$$

$$(a+b)^4 = 1a^4+4a^3b+6a^2b^2+4ab^3+1b^4$$

$$(a+b)^5 = 1a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+1b^5$$

$$(a+b)^6 = 1a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+1b^6$$

$$(a+b)^7 = 1a^7+7a^6b+21a^5b^2+35a^4b^3+35a^3b^4+21a^2b^5+7ab^6+1b^7$$

$$\begin{array}{ccccccc}
1 & & & & & & \\
1 & 1 & & & & & \\
1 & 2 & 1 & & & & \\
1 & 3 & 3 & 1 & & & \\
1 & 4 & 6 & 4 & 1 & & \\
1 & 5 & 10 & 10 & 5 & 1 & \\
1 & 6 & 15 & 20 & 15 & 6 & 1
\end{array}$$

Khayyam - Pascal's Triangle

$1 = 2^0$	$\Longleftarrow$	1						
$2 = 2^1$	$\Longleftarrow$	1	1					
$4 = 2^2$	$\Longleftarrow$	1	2	1				
$8 = 2^3$	$\Longleftarrow$	1	3	3	1			
$16 = 2^4$	$\Longleftarrow$	1	4	6	4	1		
$32 = 2^5$	$\Longleftarrow$	1	5	10	10	5	1	
$64 = 2^6$	$\Longleftarrow$	1	6	15	20	15	6	1

1						
1	1					
1	2	1				
1	3	3	1			
1	4	6	4	1		
1	5	10	10	5	1	
1	6	15	20	15	6	1

1
1
2
3
5
8
13

Fibonacci Sequence: A00045 (***See also the On-Line Encyclopedia of Integer Sequences***)

$$\begin{cases} F_0 = 0, \quad F_1 = 1 \\ F_n = F_{n-1} + F_{n-2}, \quad n \geqslant 2 \end{cases}$$

$$L(7) = \begin{bmatrix} 1 & & & & & & \\ 1 & 1 & & & & & \\ 1 & 2 & 1 & & & & \\ 1 & 3 & 3 & 1 & & & \\ 1 & 4 & 6 & 4 & 1 & & \\ 1 & 5 & 10 & 10 & 5 & 1 & \\ 1 & 6 & 15 & 20 & 15 & 6 & 1 \end{bmatrix} \qquad L(n)$$

$$U(7) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ & 1 & 2 & 3 & 4 & 5 & 6 \\ & & 1 & 3 & 6 & 10 & 15 \\ & & & 1 & 4 & 10 & 20 \\ & & & & 1 & 5 & 15 \\ & & & & & 1 & 6 \\ & & & & & & 1 \end{bmatrix} \qquad U(n)$$

$$P(7) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 6 & 10 & 15 & 21 & 28 \\ 1 & 4 & 10 & 20 & 35 & 56 & 84 \\ 1 & 5 & 15 & 35 & 70 & 126 & 210 \\ 1 & 6 & 21 & 56 & 126 & 252 & 462 \\ 1 & 7 & 28 & 84 & 210 & 462 & 924 \end{bmatrix} \qquad P(n)$$

Pascal's Matrix

$$P(n) = L(n)U(n)$$

$$\det P(n) = \det L(n)U(n) = \det L(n) \det U(n) = 1 \times 1 = 1.$$

$$\begin{bmatrix} \boxed{1} & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 6 & 10 & 15 & 21 & 28 \\ 1 & 4 & 10 & 20 & 35 & 56 & 84 \\ 1 & 5 & 15 & 35 & 70 & 126 & 210 \\ 1 & 6 & 21 & 56 & 126 & 252 & 462 \\ 1 & 7 & 28 & 84 & 210 & 462 & 924 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 6 & 10 & 15 & 21 & 28 \\ 1 & 4 & 10 & 20 & 35 & 56 & 84 \\ 1 & 5 & 15 & 35 & 70 & 126 & 210 \\ 1 & 6 & 21 & 56 & 126 & 252 & 462 \\ 1 & 7 & 28 & 84 & 210 & 462 & 924 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 6 & 10 & 15 & 21 & 28 \\ 1 & 4 & 10 & 20 & 35 & 56 & 84 \\ 1 & 5 & 15 & 35 & 70 & 126 & 210 \\ 1 & 6 & 21 & 56 & 126 & 252 & 462 \\ 1 & 7 & 28 & 84 & 210 & 462 & 924 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 6 & 10 & 15 & 21 & 28 \\ 1 & 4 & 10 & 20 & 35 & 56 & 84 \\ 1 & 5 & 15 & 35 & 70 & 126 & 210 \\ 1 & 6 & 21 & 56 & 126 & 252 & 462 \\ 1 & 7 & 28 & 84 & 210 & 462 & 924 \end{bmatrix}$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

Khayyam - Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

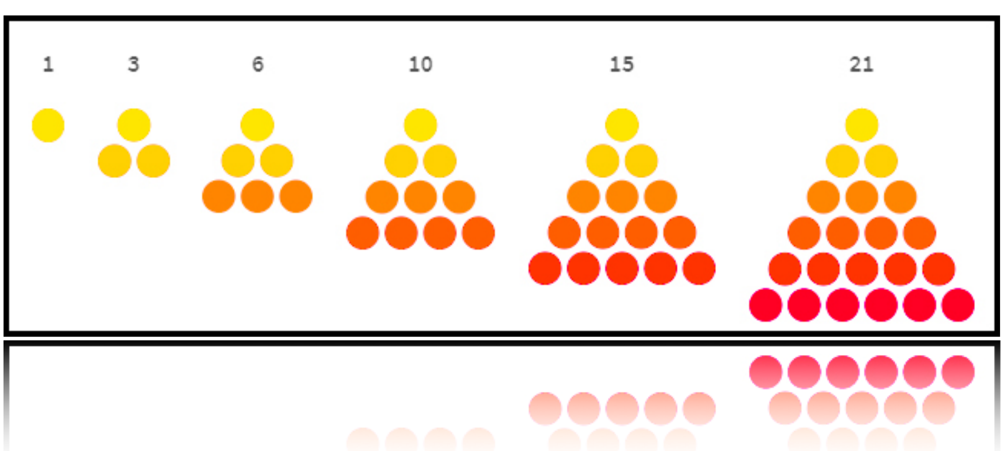
The sequence : 1,1,1,1,1,…

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

The natural numbers : 1,2,3,4,5,…

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

The triangular numbers: 1,3,6,10,15,…



1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

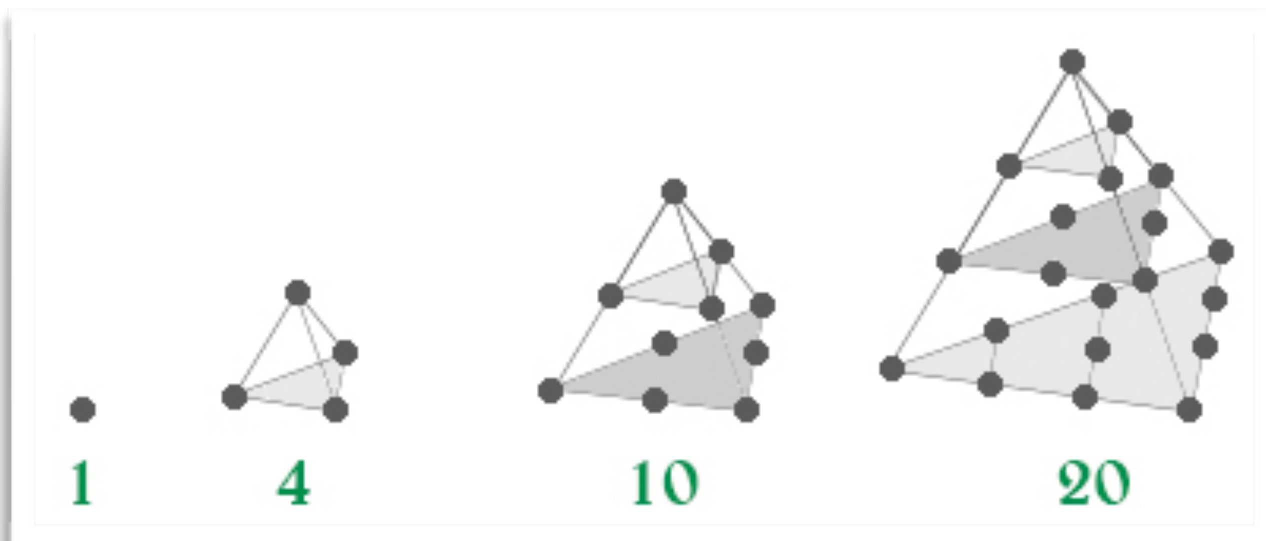
1 5 10 10 5 1

1 6 15 20 15 6 1

The tetrahedral numbers: 1,4,10,20,…

The first ten tetrahedral numbers are:

1,4,10,20,35,56,84,120,165,220,…



Strictly Come Counting Pascal's Triangle:

The *pentagonal numbers* are shown in the Figure below. Determine a formula for the nth pentagonal number.