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Problems 2 - Calculus II

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1. Determine the domain of the function $\vec{r}(t) = \langle \cos t, \ln(4-t), \sqrt{t+1} \rangle$.
2. Evaluate the following limit:

$$\lim_{t\to\infty} \left\langle \frac{1}{t^2}, \frac{2t^2}{1-t-t^2}, e^{-t} \right\rangle$$

3. Compute the derivative of the given vector function $\vec{r}(t) = \langle \ln(t^2+1), te^{-t}, 4 \rangle$.
4. Evaluate $\int_{-1}^{2} \vec{r}(t)dt$ where $\vec{r}(t) = \langle 6, 6t^2 - 4t, te^{2t} \rangle$.
5. Find the tangent line to the vector function $\vec{r}(t) = \langle \cos(4t), 3\sin(4t), t^3 \rangle$ and $t = \pi$.
6. Find the unit tangent, the unit normal and the binormal vectors for the vector function $\vec{r}(t) = \langle \cos(2t), \sin(2t), 3 \rangle$.
7. Determine the length of the vector function $\vec{r}(t) = \langle \frac{1}{3}t^3, 4t, \sqrt{2}\ t^2 \rangle$ on the interval $[0,2]$.
8. Find the curvature for $\vec{r}(t) = \langle \cos(2t), -\sin(2t), 4t \rangle$.
9. Evaluate the limit $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - y^6}{xy^3}$.
10. Evaluate the limit $\lim\limits_{(x,y)\to(2,1)} \dfrac{x^2 - 2xy}{x^2 - 4y^2}$.
11. Find all the first order partial derivatives of the function

$$f(x,y)=\frac{x^2}{y^2+1}-\frac{y^2}{x^2+y}.$$

12. Find $\dfrac{\partial z}{\partial x}$ for the function $x^2\sin(y^3)+xe^{3z}-\cos(z^2)=3y-6z+8.$

13. Find all second order derivatives for the function $f(s,t)=s^2t+\ln(t^2-s).$

14. Determine the gradient of the function $f(x,y,z)=x\cos(xy)+z^2y^4-7xz.$

15. Determine $D_{\vec{u}}f$ for the function $f(x,y,z)=x^2y^3-4xz$ in the direction of $\vec{v}=\langle -1,2,0\rangle.$

16. Find the equation of the tangent plane to $z=x\sqrt{x^2+y^2}+y^3$ at $(-4,3).$

17. Find the tangent plane and normal line to $x^2y=4ze^{x+y}-35$ at $(3,-3,2).$

18. Find $D_{\vec{u}}f$ for $f(x,y)=e^x\cos y$ in the direction 30 degrees from the positive x axis at the point $(1,\dfrac{\pi}{4})$.

19. Find the equation of the plane perpendicular to $\vec{r}(t)=\langle\cos t,\sin t,\cos(6t)\rangle$ when $t=\pi/4.$

20. Find the line of intersection of the plane given by $3x+6y-5z=-3$ and the plane given $-2x+7y-z=24.$