K. N. Toosi University of Technology

Faculty of Mathematics

Problems 1 - Calculus II

A. R. Moghaddamfar

1. Find the intersection of the lines $\langle 2,1,0\rangle + t\langle -1,-1,-1\rangle$ and $\langle 3,0,5\rangle + t\langle 2,0,6\rangle$.

Answer: $(1,0,-1)$.

2. Find the equation of the plane that contains the point $(1,3,0)$ and the line given by $x = 3 + 2t, y = -4t, z = 7 - t$.

Answer: $31x + 12y + 2z = 67$.

3. Find the equation of the plane through the points $A = (1,3,2)$, $B = (5,2,0)$ and $C = (3,-1,6)$.

Answer: $6x + 10y + 7z - 50 = 0$.

4. Find the intersection, if any, of the line $x = 2 + 3t, y = -4t, z = 5 + t$ and the plane $4x + 5y - 2z = 18$.

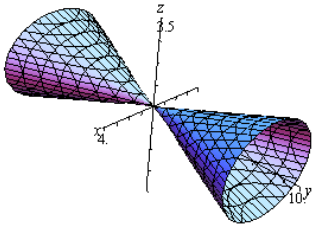
Answer: $(-4,8,3)$.

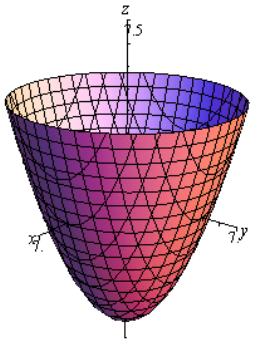
5. Given two planes $\pi_1 : x + y + z = 1$ and $\pi_2 : x - 2y + 3z = 1$, find the line of intersection.

Answer: $x = 1 + 5t,\ y = -2t,\ z = -3t$.

6. Sketch each of the quadric surface $y^2 = 4x^2 + 16z^2$.

7. Sketch the quadric surface: $z = \dfrac{x^2}{4} + \dfrac{y^2}{4} - 6$.





8. Compute the volume of the parallelepiped formed by t
$\overrightarrow{u} = \langle 1,2,4\rangle$, $\overrightarrow{v} = \langle -5,3,-7\rangle$, $\overrightarrow{w} = \langle -1,4,2\rangle$.

Answer: 0.

9. Convert the coordinates $(-2,2,3)$ from Cartesian to cylindrical.

Answer: $(2\sqrt{2}, \frac{3\pi}{4}, 3)$.

10. Convert the coordinates $(2\sqrt{3}, 6, -4)$ from Cartesian to spherical.

Answer: $(8, \frac{\pi}{3}, \frac{2\pi}{3})$.

11. Convert the coordinates $(1, \frac{\pi}{2}, 1)$ from cylindrical to spherical.

Answer: $(\sqrt{2}, \frac{\pi}{2}, \frac{\pi}{4})$.

12. Change the equation $x^2 + y^2 - z^2 = 1$ to spherical coordinates.

Answer: $\rho^2 = -\sec(2\phi)$.

13. If A, B and C are three points, find $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$.

Answer: $\overrightarrow{0}$.

14. Find the angle between the vectors $\overrightarrow{u} = \langle 3,3,0\rangle$ and $\overrightarrow{v} = \langle 1,0,0\rangle$.

Answer: $\frac{\pi}{4}$.

15. Let $\overrightarrow{u} = \langle 1,1,0\rangle$ and $\overrightarrow{v} = \langle 2,4,2\rangle$. Find a unit vector that is perpendicular to both $\overrightarrow{u}$ and $\overrightarrow{v}$.

Answer: $\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$.

16. Find the area of the parallelogram with vertices $(0,0), (1,2), (3,7)$ and $(2,5)$.

Answer: 1.

17. Find the distance from $(2,-1,-1)$ to the plane $2x-3y+z=2$.

Answer: $\dfrac{4}{\sqrt{14}}$.

18. Find the distance from $(1,0,1)$ to the line $\langle 3,2,1\rangle + t\langle 2,-1,-2\rangle$.

Answer: $\dfrac{\sqrt{68}}{3}$.

19. Find an equation for the sphere with radius 1 and center at $(0,1,0)$ in spherical coordinates.

Answer: $\rho = 2\sin\theta\sin\phi$.

20. Find the cosine of the angle between the planes $x+y+z=2$ and $x+2y+3z=8$.

Answer: $\dfrac{\sqrt{42}}{7}$.