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# Mining Data Streams (Part 1)

Mining of Massive Datasets

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# New Topic: Infinite Data

## High dim. data

Locality  
sensitive  
hashing

Clustering

Dimensional  
ity  
reduction

## Graph data

PageRank,  
SimRank

Community  
Detection

Spam  
Detection

## Infinite data

Filtering  
data  
streams

Queries on  
streams

Web  
advertising

## Machine learning

SVM

Decision  
Trees

Perceptron,  
kNN

## Apps

Recommen  
der systems

Association  
Rules

Duplicate  
document  
detection

# Data Streams

- In many data mining situations, we do not know the entire data set in advance
- **Stream Management** is important when the input rate is controlled **externally**:
  - Google queries
  - Twitter or Facebook status updates
- We can think of the **data** as **infinite** and **non-stationary** (the distribution changes over time)

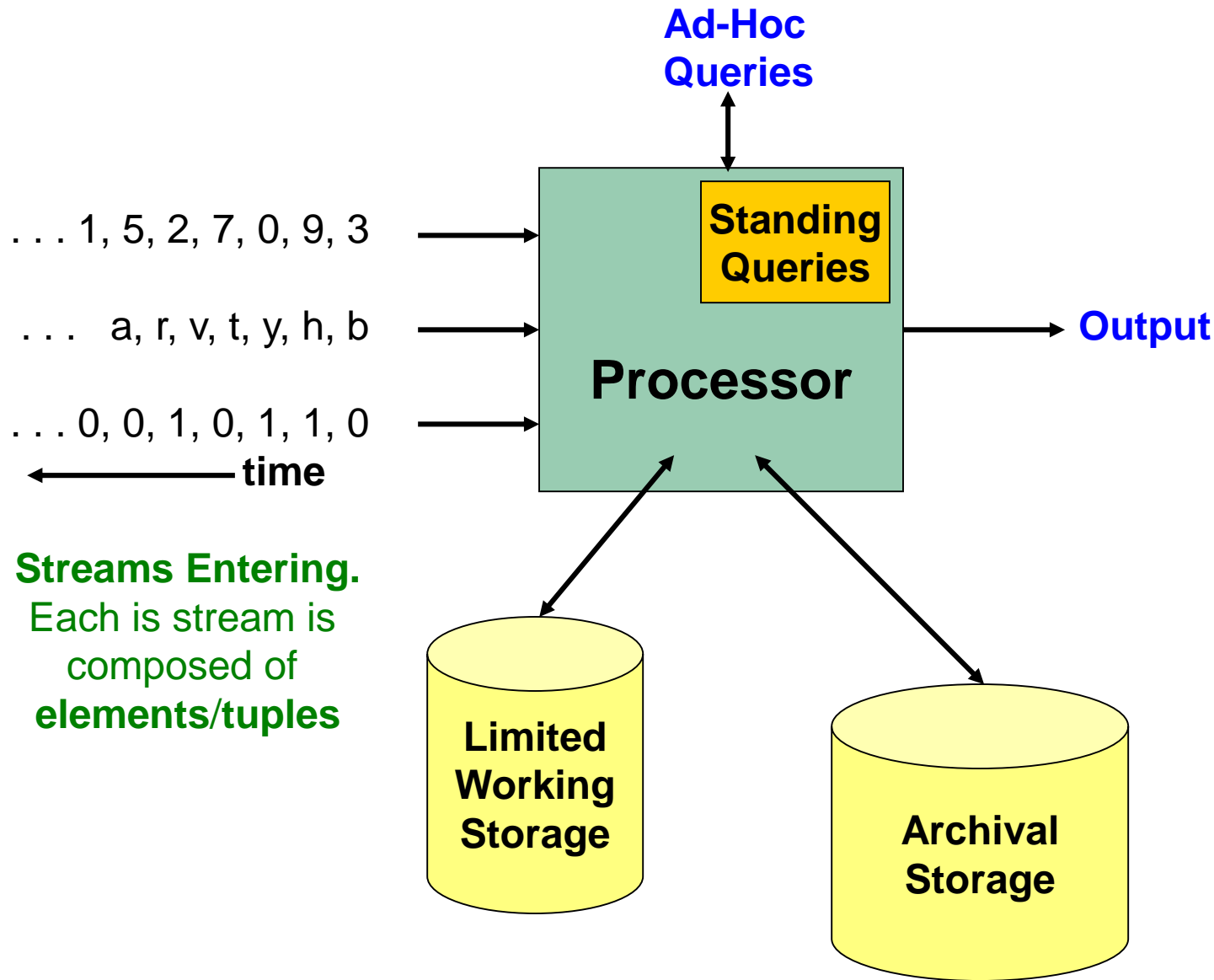
# The Stream Model

- Input **elements** enter at a rapid rate, at one or more input ports (i.e., **streams**)
  - **We call elements of the stream tuples**
- **The system cannot store the entire stream accessibly**
- **Q: How do you make critical calculations about the stream using a limited amount of (secondary) memory?**

# Side note: SGD is a Streaming Alg.

- **Stochastic Gradient Descent (SGD) is an example of a stream algorithm**
- **In Machine Learning we call this: Online Learning**
  - Allows for modeling problems where we have a continuous stream of data
  - We want an algorithm to learn from it and slowly adapt to the changes in data
- **Idea: Do slow updates to the model**
  - **SGD** (SVM, Perceptron) makes small updates
  - **So:** First train the classifier on training data.
  - **Then:** For every example from the stream, we slightly update the model (using small learning rate)

# General Stream Processing Model



# Problems on Data Streams

- **Types of queries one wants on answer on a data stream:** (we'll do these today)
  - **Sampling data from a stream**
    - Construct a random sample
  - **Queries over sliding windows**
    - Number of items of type  $x$  in the last  $k$  elements of the stream

# Problems on Data Streams

- **Types of queries one wants on answer on a data stream:** (we'll do these next time)
  - **Filtering a data stream**
    - Select elements with property  $x$  from the stream
  - **Counting distinct elements**
    - Number of distinct elements in the last  $k$  elements of the stream
  - **Estimating moments**
    - Estimate avg./std. dev. of last  $k$  elements
  - **Finding frequent elements**



# Applications (1)

- **Mining query streams**

- Google wants to know what queries are more frequent today than yesterday

- **Mining click streams**

- Yahoo wants to know which of its pages are getting an unusual number of hits in the past hour

- **Mining social network news feeds**

- E.g., look for trending topics on Twitter, Facebook

# Applications (2)

- **Sensor Networks**

- Many sensors feeding into a central controller

- **Telephone call records**

- Data feeds into customer bills as well as settlements between telephone companies

- **IP packets monitored at a switch**

- Gather information for optimal routing
- Detect denial-of-service attacks

# Sampling from a Data Stream: Sampling a fixed proportion

As the stream grows the sample  
also gets bigger

# Sampling from a Data Stream

- Since **we can not store the entire stream**, one obvious approach is to store a **sample**
- **Two different problems:**
  - (1) Sample a **fixed proportion** of elements in the stream (say 1 in 10)
  - (2) Maintain a **random sample of fixed size** over a potentially infinite stream
    - At any “time”  $k$  we would like a random sample of  $s$  elements
      - **What is the property of the sample we want to maintain?**  
For all time steps  $k$ , each of  $k$  elements seen so far has equal prob. of being sampled

# Sampling a Fixed Proportion

- **Problem 1: Sampling fixed proportion**
- **Scenario:** Search engine query stream
  - **Stream of tuples:** (user, query, time)
  - **Answer questions such as:** How often did a user run the same query in a single days
  - Have space to store  $1/10^{\text{th}}$  of query stream
- **Naïve solution:**
  - Generate a random integer in **[0..9]** for each query
  - Store the query if the integer is **0**, otherwise discard

# Problem with Naïve Approach

- **Simple question:** What fraction of queries by an average search engine user are duplicates?
  - Suppose each user issues  $x$  queries once and  $d$  queries twice (total of  $x+2d$  queries)
    - **Correct answer:**  $d/(x+d)$
  - **Proposed solution:** We keep 10% of the queries
    - Sample will contain  $x/10$  of the singleton queries and  $2d/10$  of the duplicate queries at least once
    - But only  $d/100$  pairs of duplicates
      - $d/100 = 1/10 \cdot 1/10 \cdot d$
    - Of  $d$  “duplicates”  $18d/100$  appear exactly once
      - $18d/100 = ((1/10 \cdot 9/10) + (9/10 \cdot 1/10)) \cdot d$
  - **So the sample-based answer is** 
$$\frac{\frac{x}{10} + \frac{d}{100} + \frac{18d}{100}}{\frac{x}{10} + \frac{d}{100} + \frac{18d}{100}} = \frac{d}{10x + 19d}$$

# Solution: Sample Users

## Solution:

- Pick  $1/10^{\text{th}}$  of **users** and take all their searches in the sample
- Use a hash function that hashes the user name or user id uniformly into 10 buckets

# Generalized Solution

- **Stream of tuples with keys:**
  - Key is some subset of each tuple's components
    - e.g., tuple is (user, search, time); key is **user**
  - Choice of key depends on application
- **To get a sample of  $a/b$  fraction of the stream:**
  - Hash each tuple's key uniformly into  **$b$**  buckets
  - Pick the tuple if its hash value is at most  **$a$**



Hash table with  **$b$**  buckets, pick the tuple if its hash value is at most  **$a$** .

**How to generate a 30% sample?**

Hash into  $b=10$  buckets, take the tuple if it hashes to one of the first 3 buckets



# Sampling from a Data Stream: Sampling a fixed-size sample

As the stream grows, the sample is of  
fixed size



# Maintaining a fixed-size sample

- **Problem 2: Fixed-size sample**
- **Suppose we need to maintain a random sample  $S$  of size exactly  $s$  tuples**
  - E.g., main memory size constraint
- **Why?** Don't know length of stream in advance
- **Suppose at time  $n$  we have seen  $n$  items**
  - **Each item is in the sample  $S$  with equal prob.  $s/n$**

**How to think about the problem: say  $s = 2$**

**Stream:** a x c y z k q d e g...

At  $n=5$ , each of the first 5 tuples is included in the sample  $S$  with equal prob.

At  $n=7$ , each of the first 7 tuples is included in the sample  $S$  with equal prob.

**Impractical solution would be to store all the  $n$  tuples seen so far and out of them pick  $s$  at random**

# Solution: Fixed Size Sample

## ■ Algorithm (a.k.a. Reservoir Sampling)

- Store all the first  $s$  elements of the stream to  $S$
- Suppose we have seen  $n-1$  elements, and now the  $n^{th}$  element arrives ( $n > s$ )
  - With probability  $s/n$ , keep the  $n^{th}$  element, else discard it
  - If we picked the  $n^{th}$  element, then it replaces one of the  $s$  elements in the sample  $S$ , picked uniformly at random

## ■ Claim: This algorithm maintains a sample $S$ with the desired property:

- After  $n$  elements, the sample contains each element seen so far with probability  $s/n$

# Proof: By Induction

- **We prove this by induction:**
  - Assume that after  $n$  elements, the sample contains each element seen so far with probability  $s/n$
  - We need to show that after seeing element  $n+1$  the sample maintains the property
    - Sample contains each element seen so far with probability  $s/(n+1)$
- **Base case:**
  - After we see  $n=s$  elements the sample  $S$  has the desired property
    - Each out of  $n=s$  elements is in the sample with probability  $s/s = 1$

# Proof: By Induction

- **Inductive hypothesis:** After  $n$  elements, the sample  $S$  contains each element seen so far with prob.  $s/n$

- **Now element  $n+1$  arrives**

- **Inductive step:** For elements already in  $S$ , probability that the algorithm keeps it in  $S$  is:

$$\left(1 - \frac{s}{n+1}\right) + \left(\frac{s}{n+1}\right)\left(\frac{s-1}{s}\right) = \frac{n}{n+1}$$

Element  $n+1$  discarded

Element  $n+1$   
not discarded

Element in the  
sample not picked

- So, at time  $n$ , tuples in  $S$  were there with prob.  $s/n$
- Time  $n \rightarrow n+1$ , tuple stayed in  $S$  with prob.  $n/(n+1)$
- So prob. tuple is in  $S$  at time  $n+1 = \frac{s}{n} \cdot \frac{n}{n+1} = \frac{s}{n+1}$

# Queries over a (long) Sliding Window

# Sliding Windows

- A useful model of stream processing is that queries are about a **window** of length  $N$  – the  $N$  most recent elements received
- **Interesting case:**  $N$  is so large that the data cannot be stored in memory, or even on disk
  - Or, there are so many streams that windows for all cannot be stored
- **Amazon example:**
  - For every product  $X$  we keep 0/1 stream of whether that product was sold in the  $n$ -th transaction
  - We want answer queries, how many times have we sold  $X$  in the last  $k$  sales

# Sliding Window: 1 Stream

## ■ Sliding window on a single stream:

**N = 6**

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past

Future →



# Counting Bits (1)

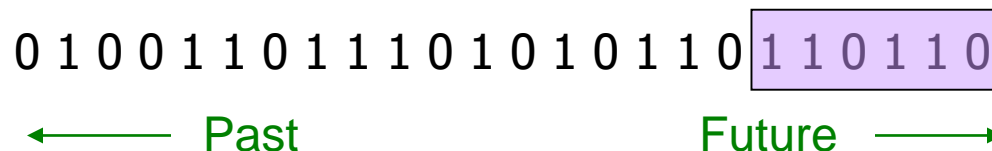
## ■ Problem:

- Given a stream of **0s** and **1s**
- Be prepared to answer queries of the form  
**How many 1s are in the last  $k$  bits?** where  $k \leq N$

## ■ Obvious solution:

Store the most recent  $N$  bits

- When new bit comes in, discard the  $N+1^{\text{st}}$  bit



Suppose  $N=6$

# Counting Bits (2)

- You can not get an exact answer without storing the entire window

- **Real Problem:**

**What if we cannot afford to store  $N$  bits?**

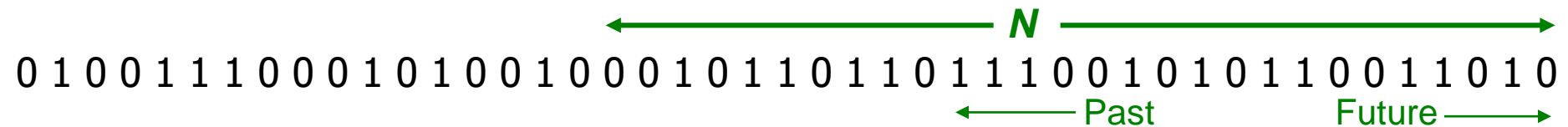
- E.g., we're processing 1 billion streams and  
 **$N = 1$  billion**



- **But we are happy with an approximate answer**

# An attempt: Simple solution

- **Q: How many 1s are in the last  $N$  bits?**
- A simple solution that does not really solve our problem: **Uniformity assumption**



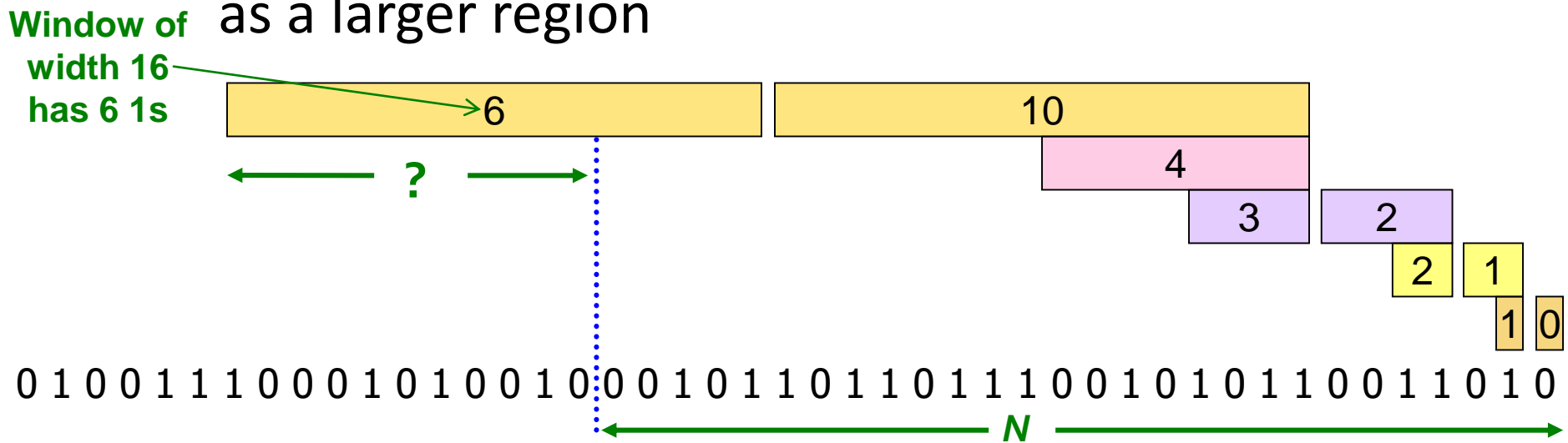
- **Maintain 2 counters:**
  - $S$ : number of 1s from the beginning of the stream
  - $Z$ : number of 0s from the beginning of the stream
- **How many 1s are in the last  $N$  bits?**  $N \cdot \frac{S}{S+Z}$
- **But, what if stream is non-uniform?**
  - What if distribution changes over time?

# DGIM Method

- DGIM solution that does not assume uniformity
- We store  $O(\log^2 N)$  bits per stream
- Solution gives approximate answer, never off by more than 50%
  - Error factor can be reduced to any fraction  $> 0$ , with more complicated algorithm and proportionally more stored bits

# Idea: Exponential Windows

- **Solution that doesn't (quite) work:**
  - Summarize **exponentially increasing** regions of the stream, looking backward
  - Drop small regions if they begin at the same point as a larger region



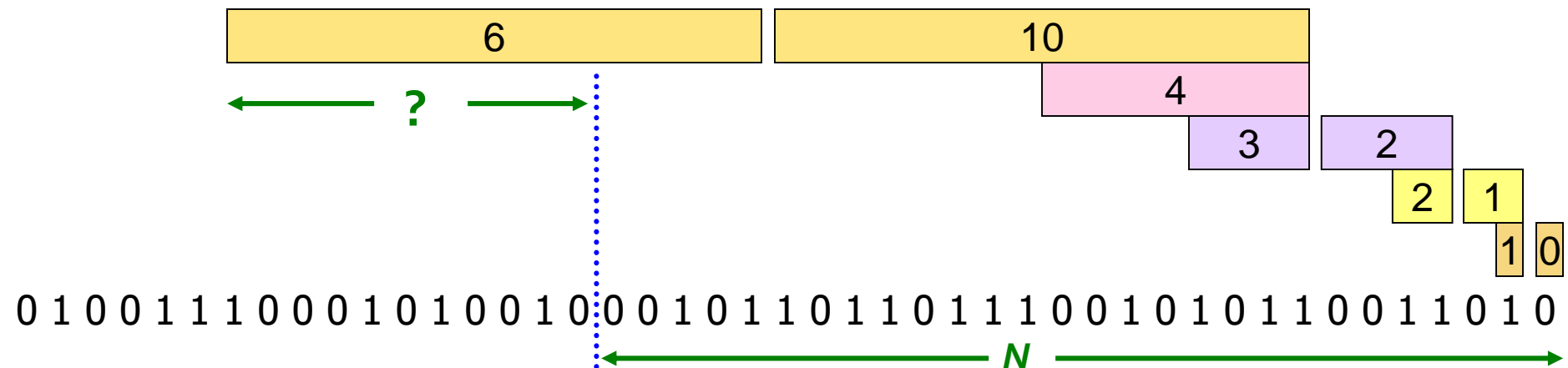
We can reconstruct the count of the last  $N$  bits, except we are not sure how many of the last **6 1s** are included in the  $N$

# What's Good?

- Stores only  $O(\log^2 N)$  bits
  - $O(\log N)$  counts of  $\log_2 N$  bits each
- Easy update as more bits enter
- Error in count no greater than the number of **1s** in the “**unknown**” area

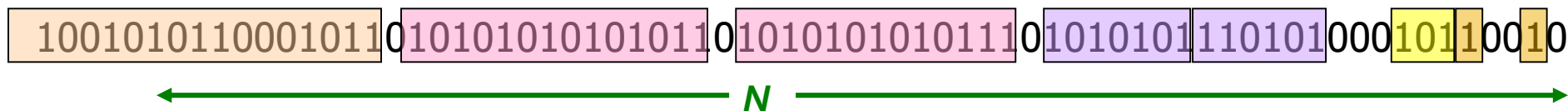
# What's Not So Good?

- As long as the **1s** are fairly evenly distributed, the error due to the unknown region is small – **no more than 50%**
- But it could be that all the **1s** are in the unknown area at the end
- In that case, **the error is unbounded!**



# Fixup: DGIM method

- **Idea:** Instead of summarizing fixed-length blocks, summarize blocks with specific number of **1s**:
  - Let the block *sizes* (number of **1s**) increase exponentially
- When there are few 1s in the window, block sizes stay small, so errors are small



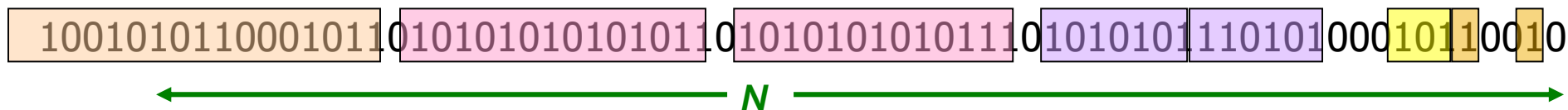


# DGIM: Timestamps

- Each bit in the stream has a *timestamp*, starting **1, 2, ...**
- Record timestamps modulo  **$N$**  (**the window size**), so we can represent any **relevant** timestamp in  **$O(\log_2 N)$**  bits

# DGIM: Buckets

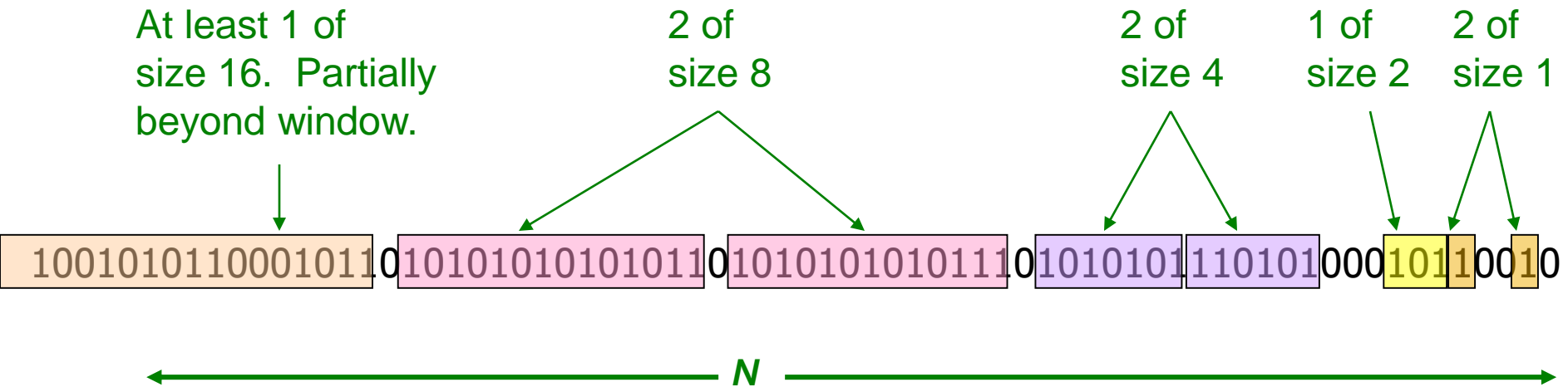
- A **bucket** in the DGIM method is a record consisting of:
  - (A) The timestamp of its end [ $O(\log N)$  bits]
  - (B) The number of 1s between its beginning and end [ $O(\log \log N)$  bits]
- **Constraint on buckets:**  
Number of **1s** must be a power of 2
  - That explains the  $O(\log \log N)$  in (B) above



# Representing a Stream by Buckets

- Either **one** or **two** buckets with the same **power-of-2 number of 1s**
- **Buckets do not overlap in timestamps**
- **Buckets are sorted by size**
  - Earlier buckets are not smaller than later buckets
- Buckets disappear when their end-time is  $> N$  time units in the past

# Example: Bucketized Stream



## Three properties of buckets that are maintained:

- Either **one** or **two** buckets with the same **power-of-2** number of **1s**
- Buckets do not overlap in timestamps
- Buckets are sorted by size

# Updating Buckets (1)

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to  $N$  time units before the current time
- **2 cases:** Current bit is **0** or **1**
- **If the current bit is 0:**  
**no other changes are needed**

# Updating Buckets (2)

- **If the current bit is 1:**
  - (1) Create a new bucket of size **1**, for just this bit
    - End timestamp = current time
  - (2) If there are now **three buckets of size 1**,  
**combine the oldest two into a bucket of size 2**
  - (3) If there are now **three buckets of size 2**,  
**combine the oldest two into a bucket of size 4**
  - (4) And so on ...

# Example: Updating Buckets

Current state of the stream:

1001010110001011010101010101011010101010101110101010111010101011101010100010110010

Bit of value 1 arrives

00101011000101101010101010101101010101010111010101011101010111010101000101100101

Two orange buckets get merged into a yellow bucket

00101011000101101010101010101101010101010111010101011101010111010101000101100101

Next bit 1 arrives, new orange bucket is created, then 0 comes, then 1:

010110001011010101010101010110101010101110101010111010101000101100101101

Buckets get merged...

010110001011010101010101010110101010101110101010111010101000101100101101

State of the buckets after merging

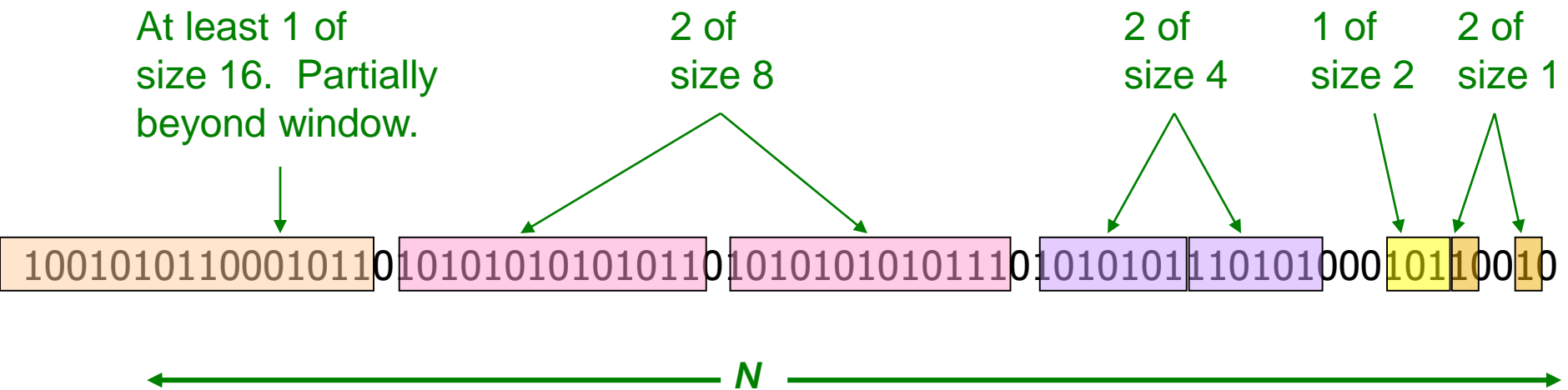
010110001011010101010101010110101010101110101010111010101000101100101101

# How to Query?

- **To estimate the number of 1s in the most recent  $N$  bits:**
  1. **Sum the sizes of all buckets but the last**  
(note “size” means the number of 1s in the bucket)
  2. **Add half the size of the last bucket**
- **Remember:** We do not know how many **1s** of the last bucket are still within the wanted window

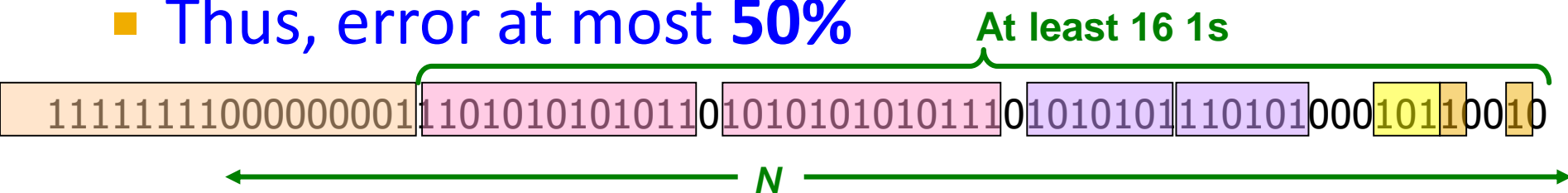


# Example: Bucketized Stream



# Error Bound: Proof

- **Why is error 50%? Let's prove it!**
- Suppose the last bucket has size  $2^r$
- Then by assuming  $2^{r-1}$  (i.e., half) of its **1s** are still within the window, we make an error of at most  $2^{r-1}$
- Since there is at least one bucket of each of the sizes less than  $2^r$ , the true sum is at least  $1 + 2 + 4 + \dots + 2^{r-1} = 2^r - 1$
- Thus, error at most 50%



# Further Reducing the Error

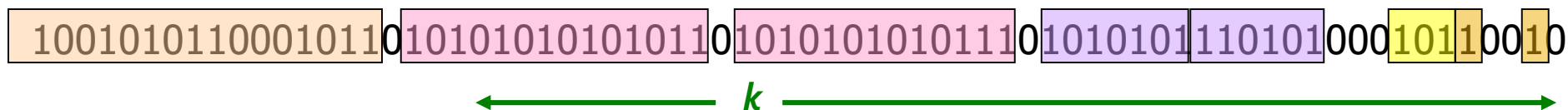
- Instead of maintaining **1** or **2** of each size bucket, we allow either  **$r-1$**  or  **$r$**  buckets ( **$r > 2$** )
  - Except for the largest size buckets; we can have any number between **1** and  **$r$**  of those
- **Error is at most  $O(1/r)$**
- By picking  **$r$**  appropriately, we can tradeoff between number of bits we store and the error

# Extensions

- Can we use the same trick to answer queries

**How many 1's in the last  $k$ ?** where  $k < N$ ?

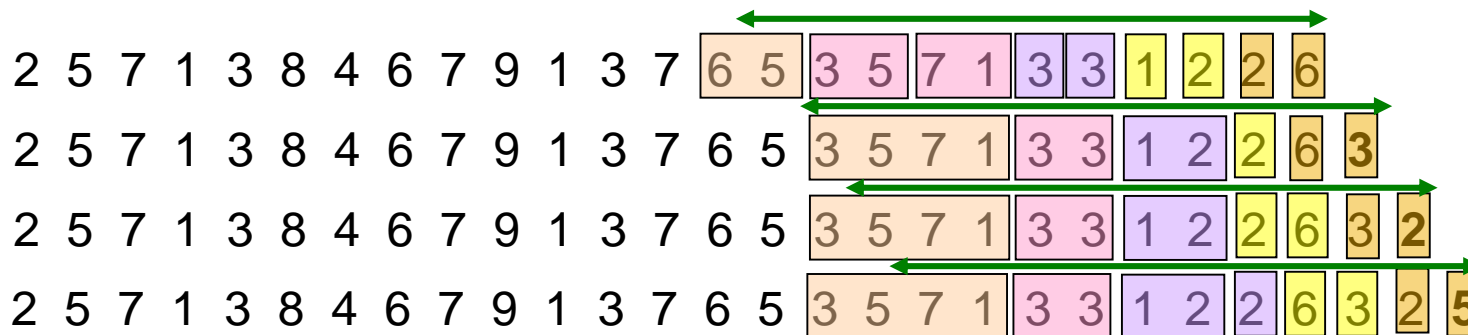
- **A:** Find earliest bucket **B** that at overlaps with  $k$ .  
Number of 1s is the **sum of sizes of more recent buckets +  $\frac{1}{2}$  size of B**



- Can we handle the case where the stream is not bits, but integers, and we want the sum of the last  $k$  elements?

# Extensions

- **Stream of positive integers**
- **We want the sum of the last  $k$  elements**
  - Amazon: Avg. price of last  $k$  sales
- **Solution:**
  - **(1) If you know all have at most  $m$  bits**
    - Treat  $m$  bits of each integer as a separate stream
    - Use DGIM to count **1s** in each integer  $c_i$  ...estimated count for  **$i$ -th bit**
    - The sum is  $= \sum_{i=0}^{m-1} c_i 2^i$
  - **(2) Use buckets to keep partial sums**
    - **Sum of elements in size  $b$  bucket is at most  $2^b$**



**Idea:** Sum in each bucket is at most  $2^b$  (unless bucket has only 1 integer)  
**Bucket sizes:**

16 8 4 2 1

# Summary

- **Sampling a fixed proportion of a stream**
  - Sample size grows as the stream grows
- **Sampling a fixed-size sample**
  - Reservoir sampling
- **Counting the number of 1s in the last  $N$  elements**
  - Exponentially increasing windows
  - Extensions:
    - Number of 1s in any last  $k$  ( $k < N$ ) elements
    - Sums of integers in the last  $N$  elements