Chapter 1

Derivative Formulæ

1.1 General Functions

For any two functions u and v:

$$(u+v)' = u' + v'$$

When there is a constant, c:

$$(c \cdot u)' = c \cdot u'$$

1.2 Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x$$
$$\frac{d}{dx}\cos x = -\sin x$$

1.3 Product Rule

$$(u \cdot v)' = v \cdot u' + u \cdot v' \tag{1.1}$$

Example 1.1. To differentiate $f(x) = x^3 \sin x$, we let $u = x^3$ and $v = \sin x$:

$$\begin{aligned} \therefore \quad u' &= 3x^2 \\ v' &= \cos x \end{aligned}$$

From Equation 1.1, we know:

$$\begin{aligned} f'(x) &= vu' + uv' \\ &= 3x^2 \sin x + x^3 \cos x \end{aligned}$$

1.4 Quotient Rule

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

1.5 Chain Rule

The *chain rule* (in Leibniz's notation) can be written in the following way:

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

Example 1.2. To differentiate $y = \sin^{10} t$, we let $x = \sin t$:

$$\begin{aligned} y &= x^{10} \\ \therefore \quad \frac{dy}{dx} &= 10x^9 \end{aligned}$$

From the chain rule, we get:

$$\frac{dy}{dt} = 10x^9 \cos t \quad \because \quad \frac{dx}{dt} = \cos t$$

Finally, we need to substitute $x = \sin t$:

$$\frac{dy}{dt} = 10 \sin^9 t \cos t$$

Example 1.3. To differentiate $\sin(10t)$, we let $x = 10t$ and $y = \sin x$: We now have:

$$\begin{aligned} \frac{dx}{dt} &= 10 \\ \frac{dy}{dx} &= \cos x \\ \frac{d}{dt}\sin(10t) &= \frac{dy}{dt} \\ \therefore \quad \frac{dy}{dt} &= \frac{dy}{dx} \cdot \frac{dx}{dt} \\ &= 10 \cos x \\ &= 10 \cos(10t) \end{aligned}$$