Final Report:

Mach-Zehnder Interferometer

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Introduction

The Mach-Zehnder Interferometer (MZI) is a well-known optical device used for measuring phase differences in light waves. It works by splitting light into two paths, allowing it to travel through different optical lengths, and then recombining it to create an interference pattern. MZI is commonly used in applications such as optical communication, sensing, and quantum optics.

In this project, we designed various MZI structures to obtain the group index of waveguides by measuring the Free Spectral Range (FSR).

Theory

MZI consists of two Y-branches and one waveguide for each arm of the interferometer.

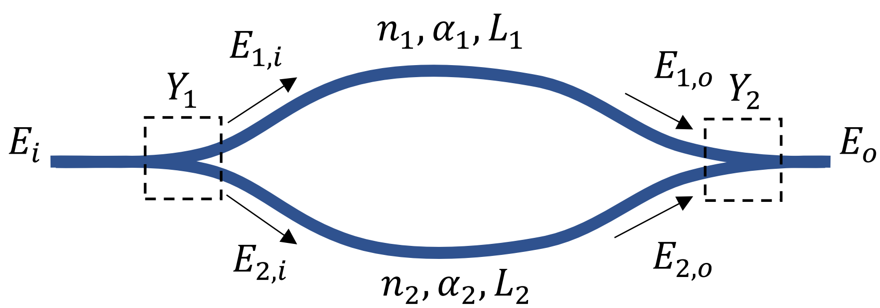


Fig1: Schematics of MZI. n: effective refractive index, α: attenuation constant, L: waveguide Length, $Y_{1,2}$: 50:50 Y-branches.

Derivation of interference pattern:

$$\xrightarrow{Y_1} E_{1,i} = E_{2,i} = \frac{E_i}{\sqrt{2}} \xrightarrow{Propagation} \begin{cases} E_{1,o} = E_{1,i} e^{j\frac{2\pi n_1}{\lambda}} e^{-\alpha_1 L_1} = \dfrac{E_i}{\sqrt{2}} e^{j\frac{2\pi n_1}{\lambda} L_1} e^{-\alpha_1 L_1} \\ E_{2,o} = E_{2,i} e^{j\frac{2\pi n_2}{\lambda}} e^{-\alpha_2 L_2} = \dfrac{E_i}{\sqrt{2}} e^{j\frac{2\pi n_2}{\lambda} L_2} e^{-\alpha_2 L_2} \end{cases}$$

By assuming lossless and similar waveguides ($\alpha_1 = \alpha_2 = 0, n_1 = n_2 = n$) we have:

$$E_{1,o} = \frac{E_i}{\sqrt{2}} e^{j\frac{2\pi n}{\lambda}L_1}, E_{2,o} = \frac{E_i}{\sqrt{2}} e^{j\frac{2\pi n}{\lambda}L_2}$$

$$\xrightarrow{Y_2} E_o = \frac{1}{\sqrt{2}} E_{1,o} + \frac{1}{\sqrt{2}} E_{2,o} = \frac{E_i}{2}\left(e^{j\frac{2\pi n}{\lambda}L_1} + e^{j\frac{2\pi n}{\lambda}L_2}\right)$$

$$\Rightarrow I_o = |E_o|^2 = \frac{|E_i|^2}{4}\left|e^{j\frac{2\pi n}{\lambda}L_1} + e^{j\frac{2\pi n}{\lambda}L_2}\right|^2 = \frac{I_i}{2}\left(1 + \cos\left[\frac{2\pi n}{\lambda}(L_2 - L_1)\right]\right)$$

$$\Rightarrow \boxed{\frac{I_o}{I_i} = \frac{1}{2}\left(1 + \cos\left(\frac{2\pi n_{(\lambda)}}{\lambda}\Delta L\right)\right)}$$

Deriving FSR:

$$if: \frac{2\pi n_{(\lambda_{m+1})}}{\lambda_{m+1}}\Delta L - \frac{2\pi n_{(\lambda_m)}}{\lambda_m}\Delta L = 2\pi, \qquad then: \lambda_m - \lambda_{m+1} = \Delta\lambda = FSR \ll \lambda_m$$

$$\Rightarrow \frac{n_{(\lambda_{m+1})}}{\lambda_{m+1}} - \frac{n_{(\lambda_m)}}{\lambda_m} = \frac{1}{\Delta L} \xRightarrow{n_{(\lambda_{m+1})} \approx n_{(\lambda_m)} - \frac{dn}{d\lambda}\Delta\lambda} \frac{n_{(\lambda_m)} - \frac{dn}{d\lambda}\Delta\lambda}{\lambda_m - \Delta\lambda} - \frac{n_{(\lambda_m)}}{\lambda_m} = \frac{1}{\Delta L}$$

$$\xRightarrow{Taylor\ expansion} \frac{1}{\Delta L} \approx \frac{n_{(\lambda)}}{\lambda}\left(1 - \frac{1}{n_{(\lambda)}}\frac{dn}{d\lambda}\Delta\lambda\right)\left(1 + \frac{\Delta\lambda}{\lambda}\right) - \frac{n_{(\lambda)}}{\lambda} \approx -\frac{\Delta\lambda}{\lambda}\frac{dn}{d\lambda} + \frac{n_{(\lambda)}}{\lambda^2}\Delta\lambda = \frac{\Delta\lambda}{\lambda^2}\left(n - \lambda\frac{dn}{d\lambda}\right)$$

$$\xRightarrow{n_g \triangleq n - \lambda\frac{dn}{d\lambda}} \frac{1}{\Delta L} = \frac{\Delta\lambda}{\lambda^2} n_g \Rightarrow \boxed{FSR = \Delta\lambda = \frac{\lambda^2}{n_g \Delta L}}$$

Modeling and Simulation

Waveguide:

In this design, we use strip waveguide with 500nm width and 220nm height (Fig2).

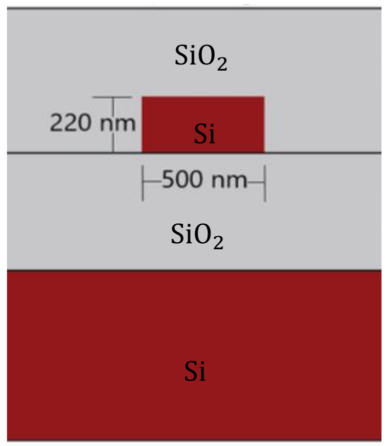


Fig2: Cross-section of Strip waveguide

I have utilized Lumerical Mode to simulate our waveguide. The simulated TE mode profile, Group, and effective index are as follows (Fig 3 and 4):

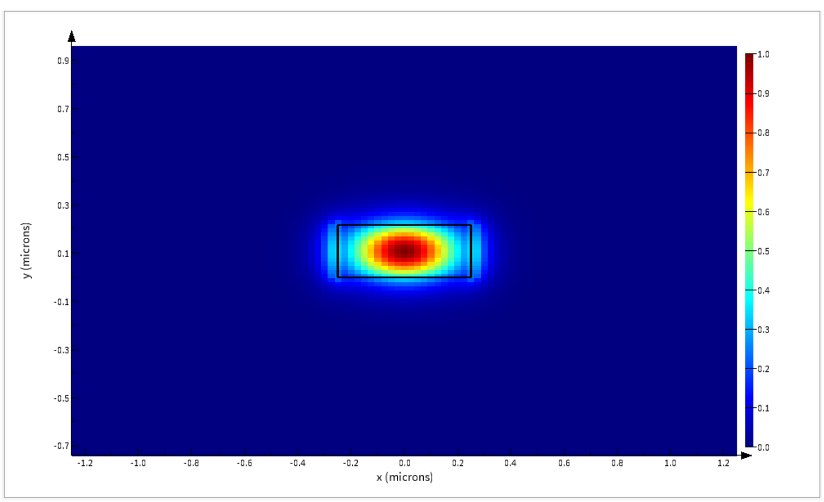


Fig 3: TE mode profile

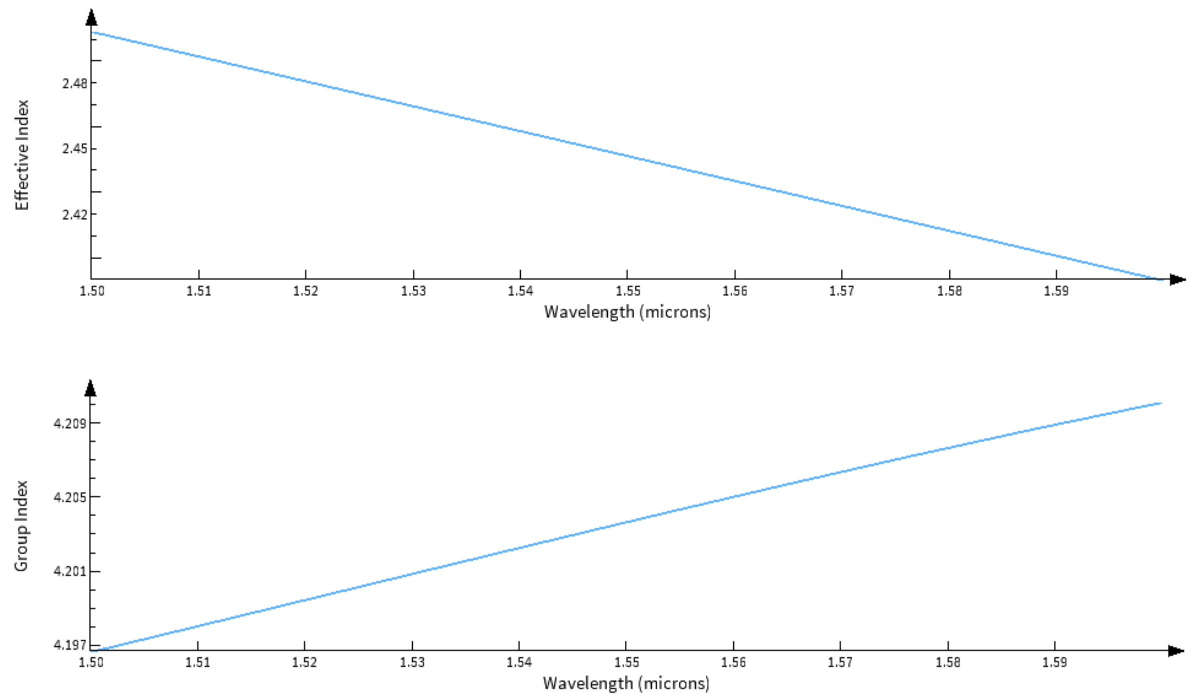


Fig 4: Group and effective index

Using Matlab, the compact model of the waveguide is:

$$n(\lambda) = 2.447 - 1.133(\lambda - \lambda_0) - 0.044(\lambda - \lambda_0)^2$$

Mach-Zehnder Interferometer

We have considered two different path length differences (50 μm ,100 μm) each with and without a bend. Here are the layouts:

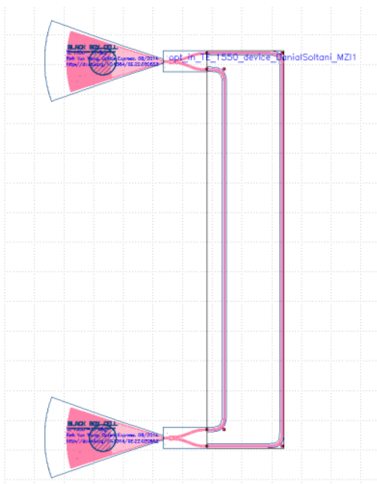


Fig 5: $\Delta L = 50\mu m$, without bend

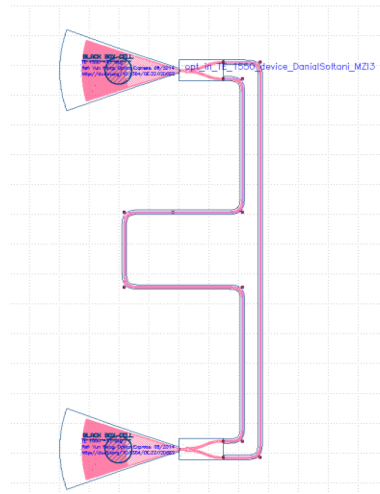


Fig 6: $\Delta L = 50\mu m$, with bend

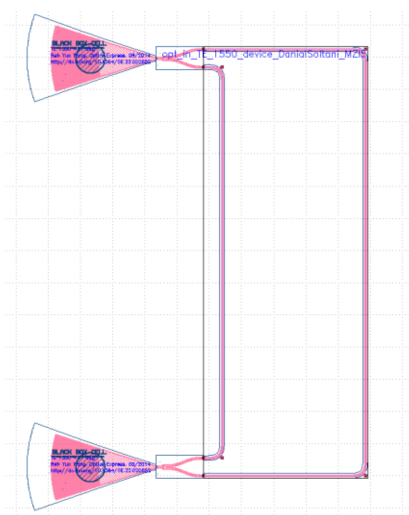


Fig 7: $\Delta L = 100\mu m$, without bend

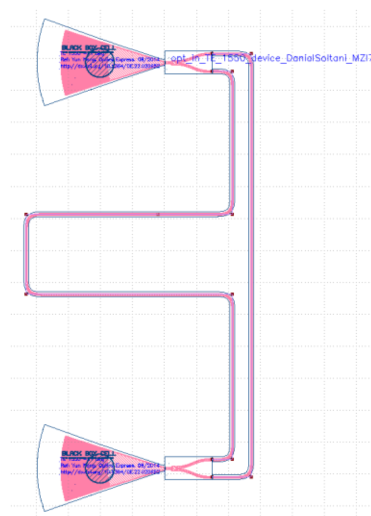


Fig 8: $\Delta L = 100\mu m$, with bend

Simulated gain results for these layouts:

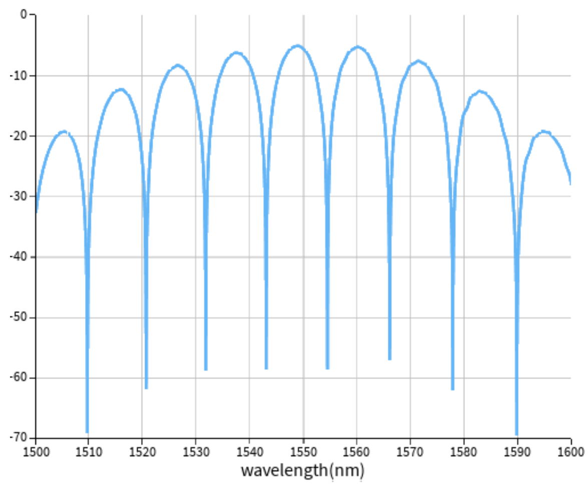


Fig 9: $\Delta L = 50\mu m$, without bend

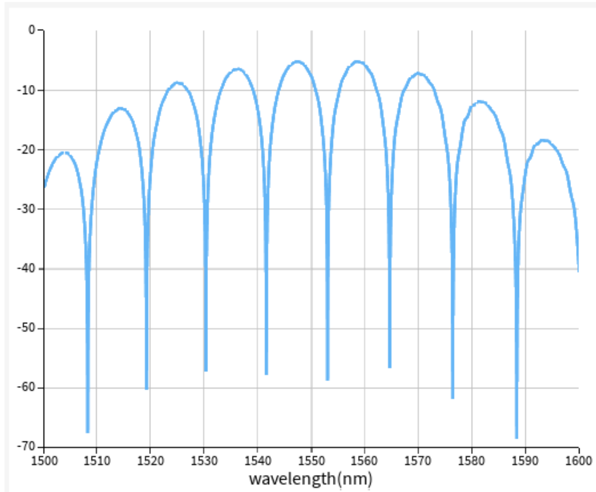


Fig 10: $\Delta L = 50\mu m$, with bend

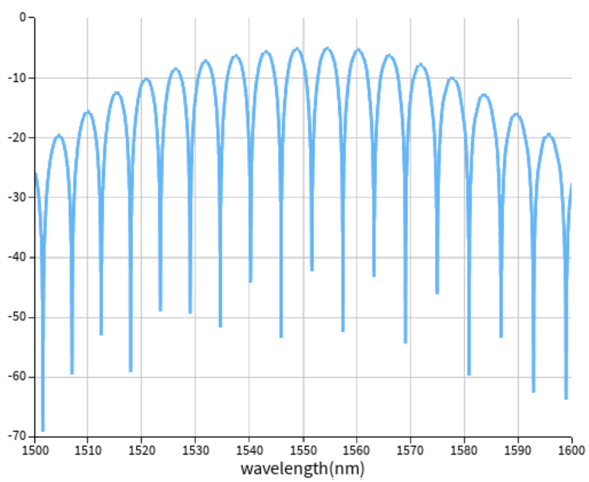


Fig 11: $\Delta L = 100\mu m$, without bend

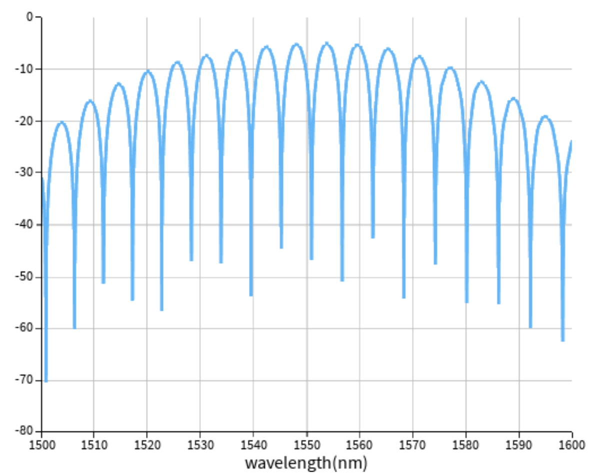


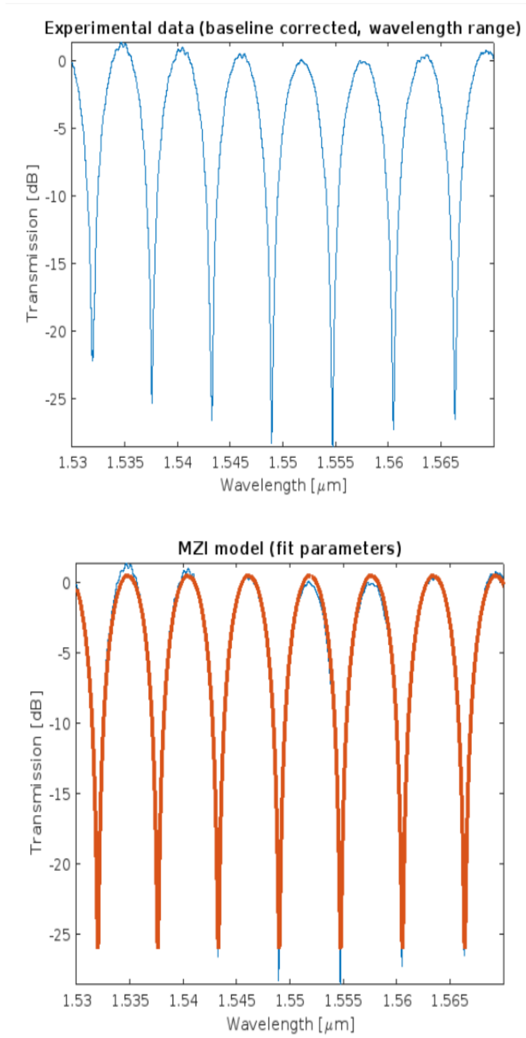
Fig 12: $\Delta L = 100\mu m$, with bend

FSR table:

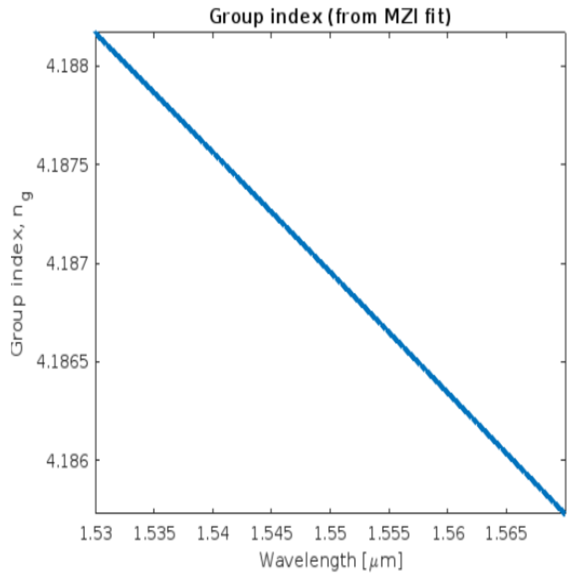
$\Delta L(\mu m)$	$FSR(nm)$
50	11.5
100	5.8

Data Analysis

Using baseline correction and the curve fitting method, I have analyzed experimental data for the 100 µm length difference (Fig. 7), and the results are as this:



And our measured group index for different wavelengths is as follows:



So, our measured average group index is 4.187, which is in the interval of our corner analysis (4.16 < group index < 4.21). Also, using the autocorrelation method, our measured FSR is 5.71 nm (100 µm length difference) for TE polarization at 1550 nm.