The Burke–Schumann Diffusion Flame Revisited—With Fuel Spray Injection

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The classical Burke-Schumann diffusion flame model is reexamined assuming the fuel is introduced in the form of liquid droplets suspended homogeneously in an inert gas stream. A sectional approach and the concept of a quasi-monodisperse spray is employed to compactly describe downstream changes in droplet size due to vaporization. A Damkohler-type number for vaporization is identified as one of the dimensionless parameters of the model. Analytic solutions are presented for flat and cylindrical flames, under various assumptions concerning the motion of the droplets and their gaseous environment. A detailed parametric analysis for *n*-decane fuel reveals that under certain circumstances the use of the spray can quite markedly effect the existence, shape, and characteristics of the diffusion flame.

NOMENCLATURE

b	defined in Eq. 32
c	L/R
$\bar{C}$	sectional vaporization constant
D_g	diffusion coefficient for gases
D	diffusion coefficient for droplets
$\tilde{D}, D_d$	sectional diffusion coefficients
E_i	frequency of monomer vaporization from an *i*-mer droplet
f, g_n	defined in Eqs. 29 and 30
h	total enthalpy
K_S, K_L	number of monomers in smallest and largest droplets of the spray
L	radius of inner duct
m	defined in Eq. 8
m_i	mass fraction of species i
m_d	mass fraction of droplets
$m_{totfuel}$	total inital fuel mass fraction
n_i	concentration of discrete *i*-mer droplet per unit volume
p	positive roots of $J_1(p) = 0$
Pe	Peclet number (Eq. 15)
Q	integral property of the spray
q_n	root defined in Eq. 27
R	radius of outer duct
S_i	source term for species i

T	temperature
V	dimensionless initial mass fraction of oxygen
$\bar{V}$	volume of droplets
V_S, V_L	volume of smallest and largest droplets in the spray
v	velocity
x, y	coordinates

Greek Symbols

$\bar{\alpha}$	constant of proportionality
β	parameter associated with spray property
γ, γ_d	normalized mass fractions for gases and droplets
δ	initial fraction of liquid fuel
Δ	Damkohler number for vaporization
ξ, η	nondimensional coordinates
θ	defined in Eq. 16
ν	stoichiometric coefficient

INTRODUCTION

The classical Burke–Schumann diffusion flame model [1] has been adopted quite extensively as

the basis for a variety of diffusion flame studies. For example, Clarke [2,3] has relaxed the fundamental assumption of instantaneous reaction and has obtained a description of the finite flame region, using matched asymptotic expansions. On the other hand Chung and Law [4] have recently introduced the hitherto neglected streamwise diffusion into the original model, and, in addition, have investigated the effects of near unity Lewis numbers on the flame temperature and characteristics.

However, all the aforementioned works have considered only *gaseous* fuel and oxidant. In more realistic settings diffusion flames are formed when the fuel is inserted using jet atomizers, that is, fuel is introduced in *droplet form*. Due to the rapid development of more sophisticated measurement techniques experimental investigations of spray characteristics are increasing (see, e.g., Refs. 5–11). In parallel, theoretical modeling of spray vaporization and combustion has received inpetus using both "heavy" computational techniques and more elegant group or sectional approaches [12–14].

Returning specifically to diffusion flames, a confined laminar spray diffusion flame with iso-octane as the fuel was investigated experimentally and theoretically by Moore and Moore [15]. A fixed fraction, α, of the fuel vapour was assumed to diffuse, and liquid fuel droplets were supposed to move with the local gas velocity. The flame was taken to be located at the surface where the flow had a fixed equivalence ratio. The approach of Patankar and Spalding [16] was exploited to compute theoretical results and agreement with experimental data was generated by altering the parameter α.

Recently, an *analytical* study of a diffusion flame of a coflowing liquid jet with oxidizing gas has been presented by Kim and Sichel [17]. In their model the liquid surface maintains the same level even though evaporation occurs. In addition, the effect of spray droplet distribution is not accounted for. However, their results are in qualitative agreement with experimental evidence.

In the present work the Burke–Schumann type of flame is examined under the assumption that

fuel is introduced, at least partially, in droplet form. A "quasi-monodisperse" spray model is used in order to describe vaporization in a simple and compact form. Nondimensionalization of the governing two-phase equations leads to the introduction of a new Damkohler number for vaporization, which enables the effects of fuel droplets on flame characteristics to be isolated and hence analyzed in a precise fashion. In the final section the influence of various flow and droplet-related parameters on the nature of diffusion flames is investigated, based upon computed data.

FORMULATION OF GOVERNING EQUATIONS

Assumptions of the Model

The basic diffusion flame under consideration here is formed as a result of diffusive mixing of two gaseous streams exiting from an outer and inner duct (Fig. 1). The outer duct contains air whereas the inner one exudes a mixture of an inert gas (nitrogen here), gaseous fuel, and fuel droplets.
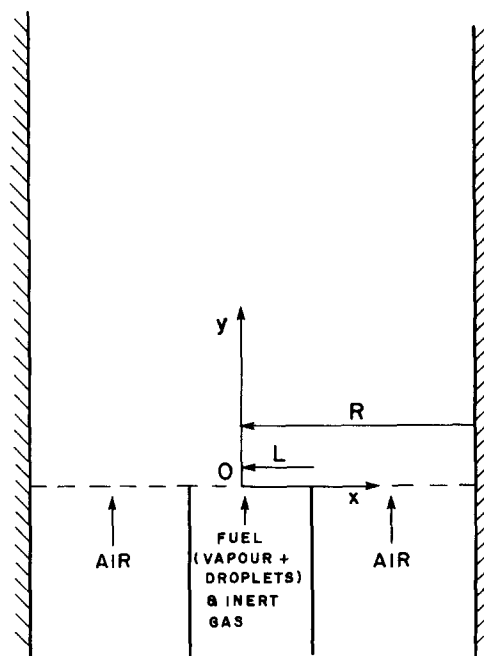

Fig. 1. Configuration for diffusion flame formation.

The assumption of Burke–Schumann that the velocity of the streams is equal and approximately constant is retained. The fuel droplets are presumed to be injected into the inner duct at a sufficiently large distance upstream so that on reaching its exit they have already undergone initial breakup and partial vaporization and have settled into motion at a velocity equal to that of the gas stream in which they are homogeneously suspended. This assumption is commonly adopted in analyzing spray characteristics and finds supportive evidence in experimental data [9] and theoretical and experimental comparisons [18]. The diffusion coefficients of the two gas streams are taken as equal but not necesarily equal to that of the droplets. The Lewis number is taken to be unity.

The most important assumption concerns the flame front that is defined as the locus of those points where the gaseous fuel and oxidant are in stoichiometric ratio and where instantaneous chemical reaction occurs.

Axial diffusion is permitted to play a role, although its realm of relevance will be examined later.

Spray Equations

In order to model the fuel droplets' behaviour the sectional approach of Tambour [18] is utilized. If $n_i(x, y)$ is the concentration of discrete droplets of various sizes per unit volume (each droplet being defined in terms of an integer multiple of monomers, where a monomer can be described as a single molecule of the species comprising the droplet), then the conservation equations for the spray in the current context are given by

$$v \frac{\partial n_i}{\partial y} = D \left[\frac{\partial^2 n_i}{\partial x^2} + \frac{\partial^2 n_i}{\partial y^2} \right]$$
$$- E_i n_i + E_{i+1} n_{i+1}, \quad i = 1, 2 \cdots, \quad (1)$$

where E_i is the frequency of molecule evaporation from an i-mer droplet. It is possible to obtain functional forms of E through expressions of vaporization rates of single droplets or correlations with experimental results [19].

Because the total number of discrete droplet sizes is prohibitively large a simplified model is extracted from Eq. 1 in the following fashion. Consider all droplets to be found in a *single section*, with K_S and K_L being the number of monomers in the smallest and largest droplets of the spray, respectively.

An integral sectional quantity of the spray is then defined:

$$Q(x, y) = \sum_{i=K_S}^{K_L} \bar{\alpha} i^\beta n_i(x, y), \quad (2)$$

and use of the fundamental approximation inherent in sectional representations leads to the following *single* equation that replaces Eq. 1:

$$v \frac{\partial Q}{\partial y} = \tilde{D} \left[\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2} \right] - \bar{C} Q, \quad (3)$$

where the sectional coefficients of diffusion and vaporization are described by

$$\bar{C} = \frac{E(V_S, T)}{V_L - V_S} + \frac{1}{V_L - V_S} \int_{V_S}^{V_L} \frac{1}{V^\beta} E(V, T) \, d\bar{V}^\beta \quad (4)$$

and

$$\tilde{D} = \left(\frac{1}{V_S} \right)^\beta \frac{1}{2} \frac{[D(V_S) + D(V_L)]}{V_L - V_S}. \quad (5)$$

For details of the derivation, see Ref. 18.

Within the framework of this simplified model attention is focused on initially monodispersed sprays only. Use of the aforedescribed monosectional approach implies that although redistribution of the monodisperse spray does occur downstream the integral quantity Q can nevertheless be used to describe *overall* spray properties and their effects on flame structure. In this sense the spray will be dubbed "quasi-monodisperse": see also Greenberg, et al. [20].

In the current work Eq. 3 is translated into the droplet mass fraction conservation equation, viz.

$$v \frac{\partial m_d}{\partial y} = D_d \left[\frac{\partial^2 m_d}{\partial x^2} + \frac{\partial^2 m_d}{\partial y^2} \right] - \bar{C} m_d, \quad (6)$$

where m_d is the mass fraction of droplets.

Gas Phase Equations

The conservation equations for the mass fractions of the gaseous phase are standard:

$$v \frac{\partial m_i}{\partial y} = D_g \left[\frac{\partial^2 m_i}{\partial x^2} + \frac{\partial^2 m_i}{\partial y^2} \right] + S_{i,react} + S_{i,vapor},$$

$$(7)$$

where there is one equation of the above form for each chemical species present. The source terms stem from chemical reaction and vaporization: see Eq. 6. Following Burke–Schumann the following quantity is defined:

$$m = m_{g,fuel} - m_{02}/\nu, \qquad (8)$$

leading to the extended diffusion plus vaporization equation:

$$v \frac{\partial m}{\partial y} = D_g \left[\frac{\partial^2 m}{\partial x^2} + \frac{\partial^2 m}{\partial y^2} \right] + S_{fuel,vapor}. \qquad (9)$$

If the contribution of the latent heat of vaporization to the enthalpy is neglected the energy equation becomes

$$v \frac{\partial h}{\partial y} = k \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right]. \qquad (10)$$

Boundary Conditions

The boundary conditions can be written as follows:

$$y = 0: \quad m = \begin{cases} m_{g,fuel} - \dfrac{D_g}{\nu} \dfrac{\partial m_{g,fuel}}{\partial y} \\ \qquad\qquad 0 \leq x \leq L \\ -\dfrac{m_{02}}{\nu} + \dfrac{D_g}{\nu} \dfrac{\partial m_{02}}{\partial y} \\ \qquad\qquad L \leq x \leq R \end{cases}$$

$$h = \begin{cases} h_1(x, 0) & 0 \leq x \leq L \\ h_2(x, 0) & L \leq x \leq R \end{cases}$$

$$m_d = \begin{cases} m_d - \dfrac{D_d}{\nu} \dfrac{\partial m_d}{\partial y} & 0 \leq x \leq L \\ 0 & L \leq x \leq R \end{cases}$$

$$y \geq 0: \quad \frac{\partial m}{\partial x} = \frac{\partial h}{\partial x} = \frac{\partial m_d}{\partial x} = 0 \quad \text{for } x = 0, R.$$

$$y \to \infty: \quad \frac{\partial m}{\partial y} = \frac{\partial h}{\partial y} = \frac{\partial m_d}{\partial y} \to 0 \quad \text{for } 0 \leq x \leq R.$$

Nondimensionalization of Governing Equations

In order to clarify the ensuing analysis the equations will be nondimensionalized in the following fashion:

$$(\xi, \eta, c) = \left(\frac{x}{R}, \frac{yD_g}{\nu R^2}, \frac{L}{R} \right), \qquad (11)$$

$$(\gamma, \gamma_d, V) = (m, m_d, m_{02}(x, 0)/\nu)/m_{totfuel}, \qquad (12)$$

where $m_{totfuel}$ is the *total* mass fraction of fuel (i.e., droplets plus vapors) at the exit of the inner duct. If δ is taken to be the fraction of liquid fuel in this total then the governing two-phase equations (Eqs. 6 and 9) become:

$$\frac{\partial \gamma_d}{\partial \eta} = \theta \frac{\partial^2 \gamma_d}{\partial \xi^2} + \frac{\theta}{(Pe)^2} \frac{\partial^2 \gamma_d}{\partial \eta^2} - \Delta\gamma_d, \qquad (13)$$

$$\frac{\partial \gamma}{\partial \eta} = \frac{\partial^2 \gamma}{\partial \xi^2} + \frac{1}{(Pe)^2} \frac{\partial^2 \gamma}{\partial \eta^2} + \Delta\gamma_d, \qquad (14)$$

where the following nondimensional parameters have been introduced:

$$Pe = \nu R/D_g \quad \text{(Peclet Number)}, \qquad (15)$$

$$\theta = D_d/D_g \quad \text{(ratio of diffusion coefficients)}, \qquad (16)$$

$$\Delta = \bar{C}R^2/D_g. \qquad (17)$$

The latter quantity represents the ratio of a typical diffusion time to a characteristic vaporization time and as such will be referred to as the Damkohler number for vaporization.

In a similar manner the boundary conditions become

$$\eta = 0: \ \gamma - \frac{1}{(Pe)^2} \frac{\partial \gamma}{\partial \eta} = \begin{cases} (1 - \delta) & 0 \leq \xi \leq c \\ -V & c \leq \xi \geq 1 \end{cases},$$

(18)

$$\eta > 0, \ \xi = 0, \ 1: \frac{\partial \gamma}{\partial \xi} = 0,$$

(19)

$$\eta \to \infty, \ 0 \leq \xi \leq 1: \frac{\partial \gamma}{\partial \eta} \to 0,$$

(20)

$$\eta = 0: \ \gamma_d - \frac{\theta}{(Pe)^2} \frac{\partial \gamma_d}{\partial \eta} = \begin{cases} \delta & 0 \leq \xi \leq c \\ 0 & c \leq \xi \leq 1 \end{cases},$$

(21)

$$\eta > 0, \ \xi = 0, \ 1: \frac{\partial \gamma_d}{\partial \xi} = 0,$$

(22)

$$\eta \to \infty, \ 0 \leq \xi \leq 1: \frac{\partial \gamma_d}{\partial \eta} \to 0.$$

(23)

Now, the coefficient of vaporization $\bar{C}$ is a complicated function of droplet diameter, the temperature differential between the droplets and the surrounding gas, and the diffusivity and other properties of the fuel and its surroundings, and consideration of all these parameters renders the *analytic* solution of the problem at handle totally unfeasible. Thus, in order to simplify the analysis, while bearing in mind the restrictive nature of such a step, the Damkohler number will be assumed to be constant. Justification for this assumption is anchored in the use of the d^2-law for a description of the vaporization coefficient. Reasonably accurate estimates of droplet size and vaporization time do provide some evidence of the validity of this law even under transient temperature conditions [21–23]. In addition, Labowsky [23] has shown that the d^2-law predicts the actual vaporization history of an *interacting* droplet, especially in the initial period of combustion. Adopting this law in the quasi-monodisperse sectional model of the spray then leads to a constant *averaged* value of the vaporization coefficient. Under these circumstances, and with the specific purpose of attempting to obtain some *qualitative* indication of the effects of the spray on combustion within the

framework of a simplified model, use of the constant Damkohler number assumption seems to be vindicated. (Clearly, the assumption would be totally unnecessary if a *numerical* solution of the governing equations were under consideration. Then, any valid model for the vaporization coefficient could be readily accommodated.)

It should be noted that the energy equation will not be solved here, as the solution for the temperature can be extracted from an equation analogous to Eq. 14 (see Williams [24]).

SOLUTIONS OF BURKE-SCHUMANN FLAME WITH FUEL SPRAY DROPLETS

Several different solutions will be presented for the problem at hand, all of which were obtained using a finite Fourier cosine transform method.

Case I

Fuel droplet diffusion will be assumed negligible (i.e., $\theta = 0$), as is axial diffusion of gases (i.e., $Pe \to \infty$). Under these circumstances the solution of Eq. 13 is simply

$$\gamma_d(\xi, \eta) = \delta \exp(-\Delta \eta).$$

(24)

Note that in this case the droplets remain confined to the region bounded by $0 \leq \xi \leq c$. Substitution from Eq. 24 into Eq. 14 leads to the following solution for the quantity γ:

$$\gamma(\xi, \eta) = c[(1 - \delta) + V] - V + c\delta\{1 - e^{-\Delta \eta}\}$$

$$+ \frac{2}{\pi} [(1 - \delta) + V] \sum_{n=1}^{\infty} \frac{1}{n} \sin(n\pi c)$$

$$\times \cos(n\pi \xi) \exp[-(n\pi)^2 \eta]$$

$$+ \frac{2\delta}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \frac{\Delta}{[(n\pi)^2 - \Delta]}$$

$$\times \sin(n\pi c) \cos(n\pi \xi) \{\exp(-\Delta \eta)$$

$$- \exp[-(n\pi)^2 \eta]\}.$$

(25)

It is not difficult to show that if *only* gaseous fuel is supplied through the central orifice (i.e., $\delta = 0$)

or if $\Delta \to \infty$ then this solution reduced identically to that of Burke–Schumann [1].

Case II

Once again fuel droplet diffusion is negligible. However, axial diffusion of the gaseous components is permitted. The solution for $\gamma(\xi, \eta)$ is then

$$\gamma(\xi, \eta) = (1 + V)c - V - \delta c e^{-\Delta\eta} \left[1 + \frac{\Delta}{(Pe)^2}\right]^{-1}$$

$$+ \frac{2}{\pi}[1 - \delta + V] \sum_{n=1}^{\infty} \frac{1}{n} \left[1 - \frac{q_n^2}{(Pe)^2}\right]^{-1}$$

$$\times \sin(n\pi c) \cos(n\pi\xi) e^{q_n\eta}$$

$$+ \frac{2\delta}{\pi} \left\{ \sum_{n=1}^{\infty} \frac{1}{n} \left[\left[1 - \frac{q_n^2}{(Pe)^2}\right]^{-1} \right. \right.$$

$$\cdot \frac{\sin(n\pi c) \cos(n\pi\xi)}{\left[1 - \frac{(n\pi)^2}{\Delta(1 + \Delta/(Pe)^2)}\right]} \cdot e^{q_n\eta}$$

$$- \sin(n\pi c) \cos(n\pi\xi)$$

$$\left. \times \left[1 + \frac{\Delta}{(Pe)^2} - \frac{(n\pi)^2}{\Delta}\right]^{-1} e^{-\Delta\eta} \right] \right\},$$

$$(26)$$

where q_n is defined by

$$q_n = \{1 - \sqrt{1 + 4(n\pi)^2/(Pe)^2}\} \ (Pe)^2/2. \quad (27)$$

This solution can be shown (after some algebraic manipulation) to collapse to that of Chung and Law [4] if $\delta = 0$ or if $\Delta \to \infty$. Also, if $Pe \to \infty$ the solution of Burke–Schumann is reclaimed. Once again it should be noted that in this case the droplets cannot stray beyond the limits $(0, c)$ in the radial direction because only their (streamwise) convection is permitted.

Case III

No assumptions are adopted in this case. Both the gaseous and liquid phases can diffuse axially and radially. The solution for the droplets' mass

fraction is found to be

$$\gamma_d(\xi, \eta) = \left[1 - \frac{\theta f}{(Pe)^2}\right]^{-1} \delta c e^{f\eta}$$

$$+ \frac{2\delta}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left[1 - \frac{\theta g_n}{(Pe)^2}\right]^{-1}$$

$$\times \sin(n\pi c) \cos(n\pi\xi) e^{g_n\eta}, \quad (28)$$

where the functions g_n and f are given by

$$g_n = [1 - \sqrt{1 + 4[(n\pi)^2\theta + \Delta]\theta/(Pe)^2}](Pe)^2/2\theta,$$

$$(29)$$

$$f = g_0. \quad (30)$$

By substituting from Eq. 28 into Eq. 14 the solution for γ is

$$\gamma(\xi, \eta) = [(1 - \delta + V)c - V - b/f]$$

$$- b[f^2/(Pe)^2 - f]^{-1} e^{f\eta}$$

$$+ \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(n\pi c) \cos(n\pi\xi)$$

$$\times \left[\frac{1}{1 - q_n/(Pe)^2} \left\{ (1 - \delta + V) \right. \right.$$

$$\left. - \frac{\delta\Delta[1 - \theta g_n/(Pe)^2]^{-1}}{g_n \left[1 - \frac{(n\pi)^2}{g_n^2/(Pe)^2 - g_n}\right]} \right\} e^{q_n\eta}$$

$$\left. - \frac{\delta\Delta[1 - \theta g_n/(Pe)^2]^{-1}}{g_n^2/(Pe)^2 - g_n - (n\pi)^2} e^{g_n\eta} \right], \quad (31)$$

where

$$b = \delta c \Delta[1 - \theta f/(Pe)^2]^{-1}. \quad (32)$$

In contrast to cases I and II, the source term relating to the production of vapors is defined here for the range of $(0, 1)$ in the radial direction, due to the radial diffusion of droplets. For all three cases the flame location will be found by determining those coordinates for which γ disappears.

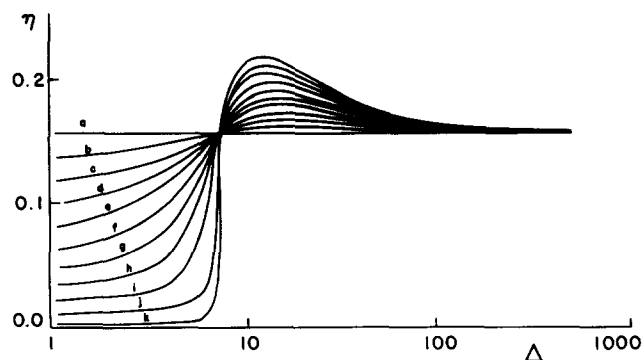In their original work Burke and Schumann [1]

Fig. 2. Case I: Effect of Damkohler number on flame height for various initial percentages of liquid fuel; $c = 0.166$, $V = 0.306$. Key to values of δ: a to k corresponds to $\delta = 0$ to $\delta = 1$ in increments of 0.1.

also presented results for the case of the diffusion flame formed by intermixing two streams of fuel and oxidant issuing from two concentric cylindrical tubes. The previously described results can be similarly developed from the governing equations and boundary conditions written in cylindrical coordinates, and the use of a Hankel transform. The solution for case I in this (more realistic) setup turns out to be

$$\gamma(\xi, \eta) = c^2(1 - \delta + V) - V + c^2\delta[1 - e^{-\Delta\eta}]$$

$$+ 2c(1 - \delta + V) \sum_{p>0} \frac{J_1(pc)J_0(p\xi)e^{-p^2\eta}}{pJ_0^2(p)}$$

$$+ 2c\delta \sum_{p>0} \frac{J_1(pc)J_0(p\xi)}{p\left(\dfrac{p^2}{\Delta} - 1\right)J_0^2(p)}$$

$$\times (e^{-\Delta\eta} - e^{-p^2\eta}), \tag{33}$$

where p assumes the values of all the positive roots of the equation $J_1(p) = 0$, and J_0 and J_1 are Bessel functions of the first kind. (The parallel solutions for cases II and III will not be brought here.)

RESULTS AND DISCUSSION

The chemical kinetic scheme employed concerns the burning of n-decane. The global chemistry is described as

$$C_{10}H_{22} + \frac{31}{2} O_2 \rightarrow 10\, CO_2 + 11\, H_2O. \tag{34}$$
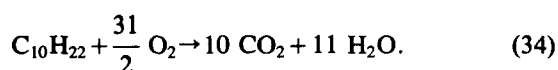
For most of the results to be described the value of V (see Eq. 12) was taken to be 0.306. The total amount of fuel entering via the inner duct was taken to be 20%, with the fraction of droplets, δ, running between 0 and 1. The ratio of the radii of the two ducts, c, was given the value 1/6. For case III the parameter θ was arbitrarily set equal to unity. In all cases the purpose of the investigation was to examine the effects of the nondimensional parameters Δ, Pe, and δ upon the flame height and shape.

In Fig. 2 the effect of the Damkohler number on flame height is illustrated for various initial droplet fractions. For values of Δ less than about 8 the flame height appears to be determined mainly by the initial amount of *gaseous* fuel, for as the fraction of droplets increases the flame height decreases. For example, if initially 40% of the fuel is liquid and $\Delta = 1$, the flame height is half of that obtained when all the fuel is vapor (corresponding to $\delta = 0$). Evidently in this range of values of Δ the vaporization does not occur rapidly enough to significantly contribute to the flame properties. A similar effect was found in Ref. 20. On the other hand, for larger Damkohler numbers ($\Delta > 8$) the vaporization of the droplets *does* seem to play a role in determining the flame height. For the limiting case of *only* liquid fuel initially emitting from the inner pipe ($\delta = 1$) the flame height is found to maximally be some 27% greater than the height of the corresponding Burke–Schumann flame. In general, the greater the fraction of droplets initially present the greater the flame
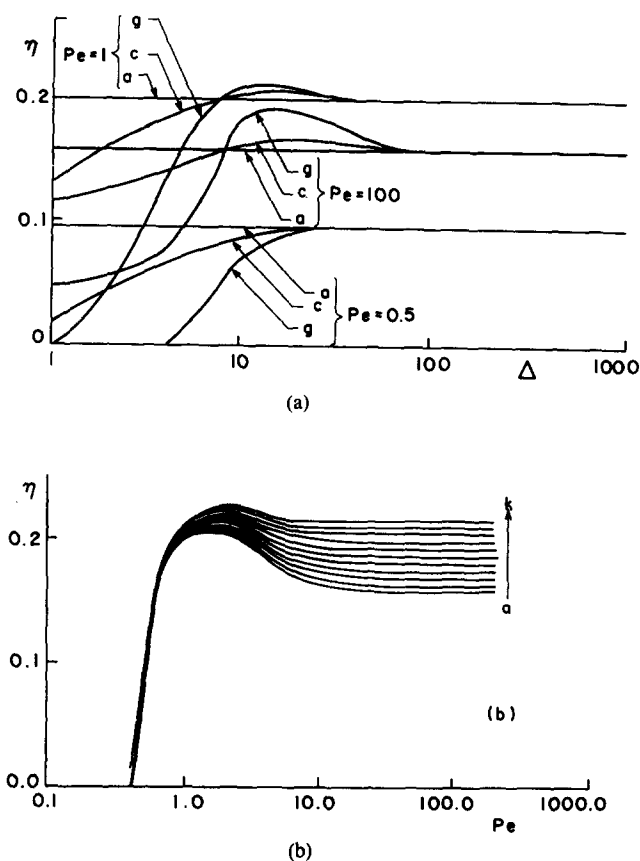
Fig. 3. Case II: (a) Effect of Damkohler number on flame height for various initial percentages of liquid fuel and Peclet numbers. (b) Effect of Peclet number on flame height for various initial percentages of liquid fuel; $\Delta = 15$. Data: $c = 0.166$, $V = 0.306$. Key to values of δ: a to k corresponds to $\delta = 0$ to $\delta = 1$ in increments of 0.1.

height. Note that for very large Damkohler numbers the vaporization occurs so rapidly that the Burke–Schumann flame is essentially reproduced. A point of interest is that corresponding to $\Delta \sim 8$. Here the Burke–Schumann flame height is reattained apparently irrespective of the initial precentage of liquid fuel present. It is surmised that this phenomenon may be a function of the flame sheet model employed here (and, hence, the form of the solution). To investigate this hypothesis finite rate chemistry should be included (this will be reported in a forthcoming article).

Selected calculated results for case II are illustrated in Fig 3a, where the effect of Damkohler number on flame height is shown for various values of δ and for a variety of Peclet numbers.

For low Peclet number it is observed that the flame height never exceeds that of the Burke–Schumann flame. In all instances the introduction of fuel in droplet form reduces the flame height, except, of course, for large Damkohler numbers. For low values of Δ the dominant mechanism in determining the flame characteristics is axial diffusion of the initial fuel vapor. For a Peclet number of unity the Burke–Schumann flame is twice the height of that of Pe $= 0.5$. Once again the possibility of obtaining taller flames occurs, although the increase is of the order of 1%–5% for $10 \leq \Delta \leq 50$. As the Peclet number increases to 100 the picture tends to that obtained in case I. The *ratio* of the maximum flame height to that of the Burke–Schumann flame *increases* with increasing Peclet
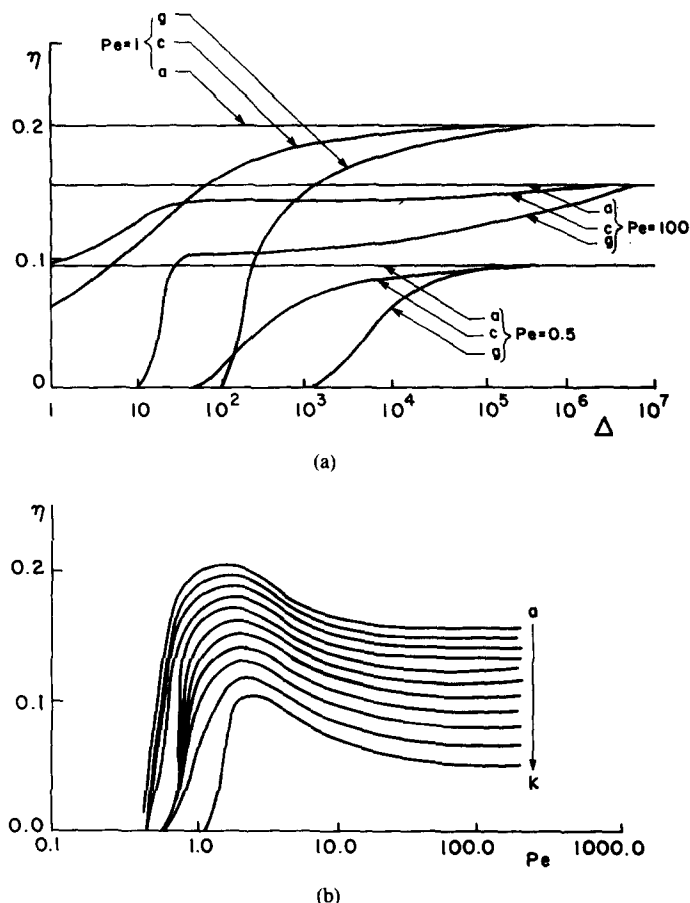
(a)

(b)

Fig. 4. Case II: (a) Effect of Damkohler number on flame height for various initial percentages of liquid fuel and Peclet numbers. (b) Effect of Peclet number on flame height for various initial percentages of liquid fuel; $\Delta = 200$. Data: $c = 0.166$, $V = 0.306$. Key to values of δ: a to k corresponds to $\delta = 0$ to $\delta = 1$ in increments of 0.1.

number, for a Damkohler number of, say, 15. However, the *actual* height of all flames *increases* for $0 \leq Pe \leq 2$ with a subsequent *decrease* to the limiting value as $Pe \rightarrow \infty$. This fact is illustrated in Fig. 3b.

Figure 4a is parallel to Fig. 3a, but for case III. The picture that emerges now is quite different from those observed previously. For all values of Pe the presence of droplets does not result in a flame greater in height than that of Burke–Schumann. As droplet diffusion is permitted in this case the liquid fuel is transported beyond the radial confines of the inner orifice. The vapor-producing source terms can be active for all values of ξ, η, as long as the droplets persist. This total

intermixing of both the liquid and vapor fuel qualitatively explains the differences between the results of cases I and II and case III. In case III the actual existence of *spray* diffusion flames for Damkohler numbers of up to 100 involves a complex interaction between the flow rate (expressed through Pe) and the initial droplet fraction. For the aforementioned range of values of Δ the greatest initial fraction of droplets that helps sustain a flame was found to increase from 0.2 for $Pe = 0.5$ to 1.0 for $Pe = 100$. In Fig. 4b the effect of Peclet number on flame height is shown for $\Delta = 200$. Qualitatively the behavior of the curves is similar to that seen for case II. However, here for Peclet numbers above about 3 the ratio of
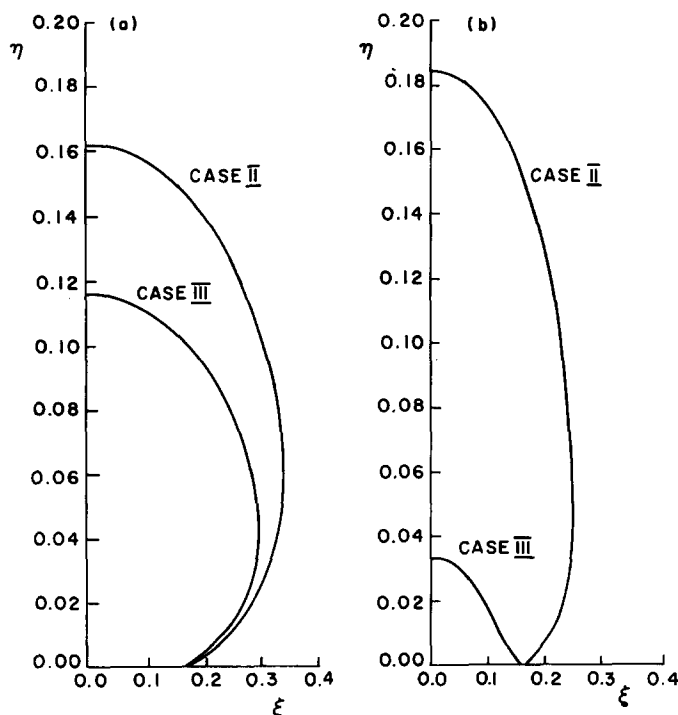
Fig. 5. Shape of spray diffusion flames. (a) $c = 0.166$, $V = 0.306$, $\Delta = 100$, Pe $= 100$, $\delta = 0.5$ (b) $c = 0.166$, $V = 0.306$, $\Delta = 10$, Pe $= 10$, $\delta = 0.5$.

the *spray* diffusion flame height to that of Burke–Schumann is roughly proportional to $(1 - \delta/2)$.

As far as flame *shape* is concerned typical results are illustrated in Fig. 5a for 50% initial droplet concentration, $\Delta = 100$ and Pe $= 100$. The difference in flame shape between cases II and III is quite clear. The latter flame is shorter and thinner, presumably due to the enhanced dispersion of the droplets as a result of diffusion. The case III flame is some 30% shorter than its counterpart and some 15% thinner. In Fig. 5b a similar comparison for lower values of Δ and Pe reveals an even more striking difference. The ratio of flame heights is now 0.18, with the case II flame being fatter than its counterpart by some 33%.

A different sort of flame can be obtained by enabling less oxygen to flow in the outer duct. A typical underventilated flame is shown in Fig. 6, for three initial droplet fractions (case III). For this problem the Burke–Schumann flame front stands closest to the wall of the outer duct, and is highest.

With 50% liquid fuel the flame front moves inwards and is lower than that of the Burke–Schumann front. If all the fuel is supplied as liquid a flame cannot be supported on the burner.

Finally, the effect of Damkohler number on flame height for the cylindrical case is shown in Fig. 7 (case I). The results are qualitatively similar to those of the flat flames, although for the data used the flames are lower. Furthermore a greater vaporization rate is required in order that the spray flame be taller than its Burke–Schumann counterpart. Whereas for the flat flames the "overshoot" occurred for $\Delta > 8$ the current setup exhibits this phenomenon for $\Delta > 30$.

CONCLUSIONS

The classical Burke–Schumann diffusion flame model has been extended to permit injection of fuel in droplet form. A parametric study reveals the role of a Damkohler number for vaporization in determining flame characteristics. Results point
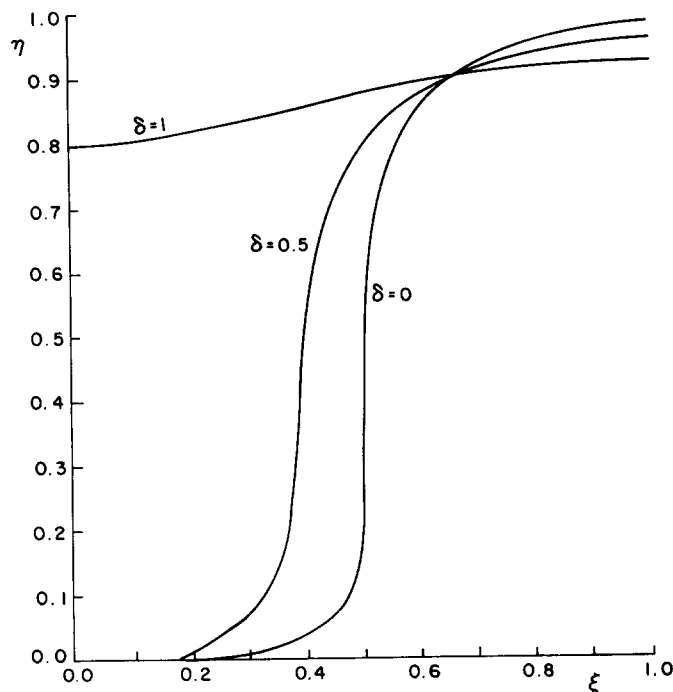
Fig. 6. Shape of spray diffusion flames; case III, $c = 0.166$, $V = 0.2$, $\Delta = 10$, $Pe = 10$.

to the possibility of the influence of fuel evaporation in the flame being at least as large as the role of diffusion, under certain circumstances. Further effects due to flow rates and axial diffusion (via Peclet number) and initial droplet concentration, and the interaction between these parameters, have also been studied. No exhaustive experimental results have been found in the literature for comparison. Thus, the need for experimental data to complement the present theoretical results is of paramount importance.
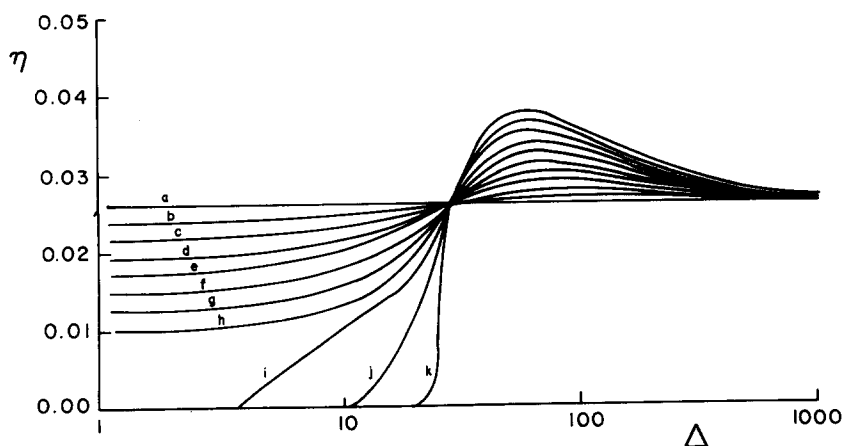
Fig. 7. Effect of Damkohler number on cylindrical flame height for various initial percentages of liquid fuel; $c = 0.166$, $V = 0.306$. Key to values of δ: a to k corresponds to $\delta = 0$ to $\delta = 1$ in increments of 0.1.

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