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1) $F(x,w) = \phi(x) \cdot w$ (column vector) ; $C = \frac{1}{2N} \sum_{n=1}^{N} \| F^n(x^n, w) - y^n \|^2$

$0 = \frac{\partial C}{\partial w_j} = \frac{\cancel{2}}{\cancel{2}N} \sum_{n=1}^{N} (F^n(x^n, w) - y^n) \frac{\partial}{\partial w_j} \sum_k (\phi_k^n(x) w_k)$

$\frac{\partial}{\partial w_j} \sum_k (\phi_k^n(x) w_k)$ is zero for $\underline{k \neq j}$ so

$= \phi_j^n$

$0 = \frac{1}{N} \sum_{n=1}^{N} (F^n(x^n, w) - y^n) \phi_j^n = \langle F^n - y^n \rangle \phi_j^n$

$\equiv \langle F^n \phi_j^n \rangle = \langle y^n \phi_j^n \rangle$

$\langle y^n, \phi_j^n \rangle = \frac{1}{N} \sum_{n=1}^{N} y^n \cdot \phi_j^n = \frac{1}{N}(y^1 \cdot \phi_j^1 + y^2 \cdot \phi_j^2 \dots)$

$= \frac{1}{N} \begin{pmatrix} \phi_0(x^1) & \phi_0(x^2) & \dots \\ \phi_1(x^1) & & \\ & \ddots & \\ & & \phi_{h-1}(x^n) \end{pmatrix} y = \frac{1}{N} \Phi^T y$

$\langle F^n \phi_j^n \rangle = \frac{1}{N} \sum_{n=1}^{N} F^n \cdot \phi_j^n = \frac{1}{N} (F^1 \cdot \phi_j^1 + F^2 \cdot \phi_j^2 \dots)$

$F^n = \phi(x^n) \cdot w^n$

So $\equiv \frac{1}{N} (\phi_j(x^1) \cdot w_j^1 \cdot \phi_j^1 + \phi(x^2) \cdot w^2 \cdot \phi_j^n \dots)$

$= \frac{1}{N} (\phi(x^1)\phi_j^1 + \phi(x^2)\phi_j^2 + \dots) w^*$

$= \frac{1}{N} \Phi^T \Phi w^*$

Thus $\cancel{\frac{1}{N}}\Phi^T\Phi w^* = \cancel{\frac{1}{N}}\Phi^T y$

$$\Rightarrow \boxed{w^* = (\Phi^T\Phi)^{-1}\Phi^T y}$$