

# NUMBER OF ARRANGEMENTS: ONE POOL

WITH REPETITION

LIMITED REPETITION

WITHOUT REPETITION

ORDER-CONSCIOUS

WORDS

$$S(N,k) = N^k$$

N=5

n=∞

n <sub>1</sub> =∞	A	A	A	A	...
n <sub>2</sub> =∞	B	B	B	B	...
n <sub>3</sub> =∞	C	C	C	C	...
n <sub>4</sub> =∞	D	D	D	D	...
n <sub>5</sub> =∞	E	E	E	E	...

$$A(n_1, \dots, n_N, k) = \sum_{\substack{\sum k_i = k \\ k_i \leq n_i}} \frac{k!}{\prod k_i!}$$

N=5

n=15

n <sub>1</sub> =5	A	A	A	A	A
n <sub>2</sub> =3	B	B	B		
n <sub>3</sub> =4	C	C	C	C	
n <sub>4</sub> =1	D				
n <sub>5</sub> =2	E	E			

PARTIAL PERMUTATIONS

$$P(N,k) = \frac{N!}{(N-k)!}$$

N=5

n=5

n <sub>1</sub> =1	A
n <sub>2</sub> =1	B
n <sub>3</sub> =1	C
n <sub>4</sub> =1	D
n <sub>5</sub> =1	E

PARTIALLY O.C.  
(indisting. items not O.C.)

N=3

n=∞

n <sub>1</sub> =∞	A	A	A	A	...
	A	A	A	A	...
n <sub>2</sub> =∞	B	B	B	B	...
	B	B	B	B	...
n <sub>3</sub> =∞	C	C	C	C	...

N=3

n=15

n <sub>1</sub> =8	A	A	A	A	A
	A	A	A		
n <sub>2</sub> =5	B	B	B	B	
	B				
n <sub>3</sub> =2	C	C			

N=3

n=5

n <sub>1</sub> =2	A
	A
n <sub>2</sub> =2	B
	B
n <sub>3</sub> =1	C

MULTICOMBINATIONS

$$M(N,k) = \left( \binom{N}{k} \right)$$

$$= \frac{(N+k-1)!}{k! (N-1)!}$$

N=5

n=∞

n <sub>1</sub> =∞	A	A	A	A	...
n <sub>2</sub> =∞	B	B	B	B	...
n <sub>3</sub> =∞	C	C	C	C	...
n <sub>4</sub> =∞	D	D	D	D	...
n <sub>5</sub> =∞	E	E	E	E	...

$$\text{coef}(n_1, \dots, n_N, k)$$

N=5

n=15

n <sub>1</sub> =5	A	A	A	A	A
n <sub>2</sub> =3	B	B	B		
n <sub>3</sub> =4	C	C	C	C	
n <sub>4</sub> =1	D				
n <sub>5</sub> =2	E	E			

COMBINATIONS

$$C(N,k) = \binom{N}{k}$$

$$= \frac{N!}{k! (N-k)!}$$

N=5

n=5

n <sub>1</sub> =1	A
n <sub>2</sub> =1	B
n <sub>3</sub> =1	C
n <sub>4</sub> =1	D
n <sub>5</sub> =1	E

NOT ORDER-CONSCIOUS

N=3

n=∞

n <sub>1</sub> =∞	A	A	A	A	...
	A	A	A	A	...
n <sub>2</sub> =∞	B	B	B	B	...
	B	B	B	B	...
n <sub>3</sub> =∞	C	C	C	C	...

N=3

n=15

n <sub>1</sub> =8	A	A	A	A	A
	A	A	A		
n <sub>2</sub> =5	B	B	B	B	
	B				
n <sub>3</sub> =2	C	C			

N=3

n=5

n <sub>1</sub> =2	A
	A
n <sub>2</sub> =2	B
	B
n <sub>3</sub> =1	C