

NUMBER OF ARRANGEMENTS: ONE POOL

ORDER-CONSCIOUS

PARTIALLY O.C.
(indistinguishable items)

NOT ORDER-CONSCIOUS

REPETITION

LIMITED REPETITION

NO REPETITION

PERMUTATIONS
WITH REPETITION

$$S(N,k) = N^k$$

N=5 n=∞

n ₁ =∞	A	A	A	A	...
n ₂ =∞	B	B	B	B	...
n ₃ =∞	C	C	C	C	...
n ₄ =∞	D	D	D	D	...
n ₅ =∞	E	E	E	E	...

$$A(n_1, \dots, n_N, k)$$

$$= \sum_{\substack{\sum k_i = k \\ k_i \leq n_i}} k! / \prod k_i!$$

N=5 n=15

n ₁ =5	A	A	A	A	A
n ₂ =3	B	B	B		
n ₃ =4	C	C	C	C	
n ₄ =1	D				
n ₅ =2	E	E			

PARTIAL PERMUTATIONS

$$P(N,k) = \frac{N!}{(N-k)!}$$

N=5 n=5

n ₁ =1	A
n ₂ =1	B
n ₃ =1	C
n ₄ =1	D
n ₅ =1	E

N=3 n=∞

n ₁ =∞	A	A	A	A	...
	A	A	A	A	...
n ₂ =∞	B	B	B	B	...
	B	B	B	B	...
n ₃ =∞	C	C	C	C	...

N=3 n=15

n ₁ =8	A	A	A	A	A
	A	A	A		
n ₂ =5	B	B	B	B	
	B				
n ₃ =2	C	C			

N=3 n=5

n ₁ =2	A
	A
n ₂ =2	B
	B
n ₃ =1	C

COMBINATIONS
WITH REPETITION

$$M(N,k) = \binom{N+k-1}{k}$$

$$= \frac{(N+k-1)!}{k! (N-1)!}$$

N=5 n=∞

n ₁ =∞	A	A	A	A	...
n ₂ =∞	B	B	B	B	...
n ₃ =∞	C	C	C	C	...
n ₄ =∞	D	D	D	D	...
n ₅ =∞	E	E	E	E	...

$$\text{coef}(n_1, \dots, n_N, k)$$

N=5 n=15

n ₁ =5	A	A	A	A	A
n ₂ =3	B	B	B		
n ₃ =4	C	C	C	C	
n ₄ =1	D				
n ₅ =2	E	E			

COMBINATIONS

$$C(N,k) = \binom{N}{k}$$

$$= \frac{N!}{k! (N-k)!}$$

N=5 n=5

n ₁ =1	A
n ₂ =1	B
n ₃ =1	C
n ₄ =1	D
n ₅ =1	E

N=3 n=∞

n ₁ =∞	A	A	A	A	...
	A	A	A	A	...
n ₂ =∞	B	B	B	B	...
	B	B	B	B	...
n ₃ =∞	C	C	C	C	...

N=3 n=15

n ₁ =8	A	A	A	A	A
	A	A	A		
n ₂ =5	B	B	B	B	
	B				
n ₃ =2	C	C			

N=3 n=5

n ₁ =2	A
	A
n ₂ =2	B
	B
n ₃ =1	C