Grandfather Paradox Solved with Mixed Strategy Equilibrium

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1. Motivation

Many time travel paradoxes can be interpreted as *instability problems*.[1]

In Game Theory, games like rock-paper-scissors and heads-or-tails does not have a Nash equilibrium.[2] However, those games do have mixed strategy equilibria. By introducing probability, we can achieve equilibria in otherwise unstable games.

Can we apply the same logic to solve a time travel paradox?

2. Grandfather Paradox, Baseline

Let x denote whether Jane would kill her grandpa. $x = 1$ if she would.

Let y denote whether her grandpa was alive. $y = 1$ if he was.

Therefore, evaluating the timeline once updates y to y':

$$y' = f(y) = 1 - xy$$

The paradox emerges when Jane decides to kill her grandpa: $x = 1$.

$$\therefore y' = f(y) = 1 - y$$

Evaluation of the timeline would toggle y between 0 and 1. It is analogous to an unstable game in Game Theory.

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[1] For example, the grandfather paradox, vacuum fluctuation feedback amplification, etc.

[2] If you play rock, I will play paper. You would then want to play scissor, etc. The game does not converge to an equilibrium.

3. Grandfather Paradox, Solution

Let x denote the *probability* that Jane would kill her grandpa, given the chance.

Let y denote the *probability* that her grandpa was alive.

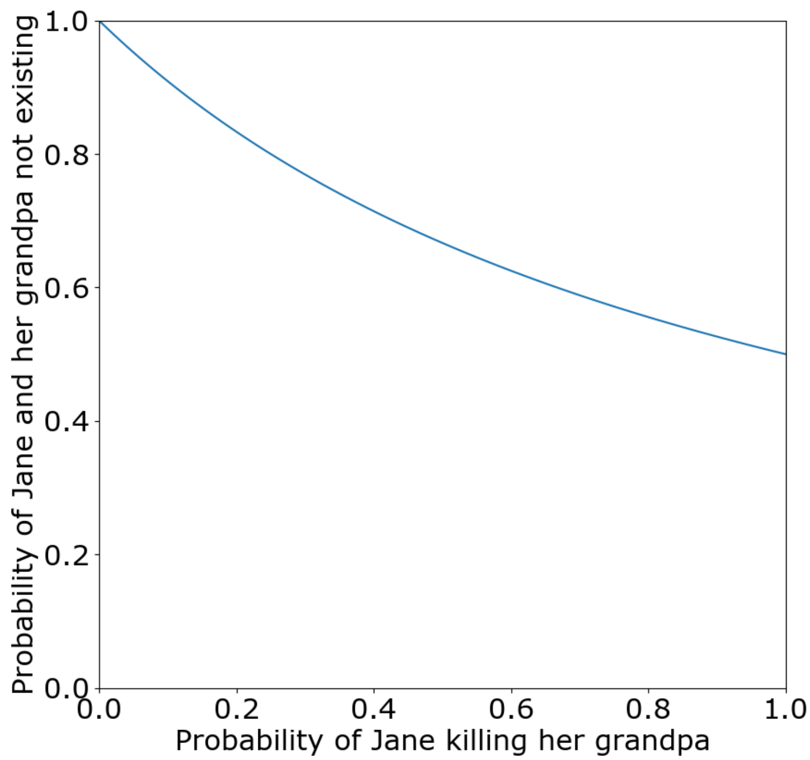
Therefore, evaluating the timeline once updates y to y':

$$y' = f(y) = 1 - xy$$

We can solve for $f(y) = y$:

$$y = 1 - xy$$

$$\therefore y = \frac{1}{x+1}$$



For stability,

$$y + dy' = 1 - x(y + dy)$$

Convergence requires $|dy'| < |dy|$.

$$\because y + dy' = 1 - x(y + dy)$$

$$\therefore y + dy' = 1 - xy - xdy$$

$$\therefore (1 + x)y + dy' = 1 - xdy$$

$$\because (1 + x)y = 1$$

$$\therefore dy' = -xdy$$

$$\because |dy'| < |dy|, \qquad 0 \leq x \leq 1$$

$$\therefore 0 \leq x < 1$$

It turns out that $x = 1$ is the only setup that leads to instability.

Conclusion: if Jane wants to maximize the probability of herself not existing, she should kill her grandpa with a probability slightly lower than 1.

4. Future Work

Can we solve the problem of vacuum fluctuation feedback amplification in a similar way?