Dynamic Slipping

Oct 2020

Purpose of this ppt

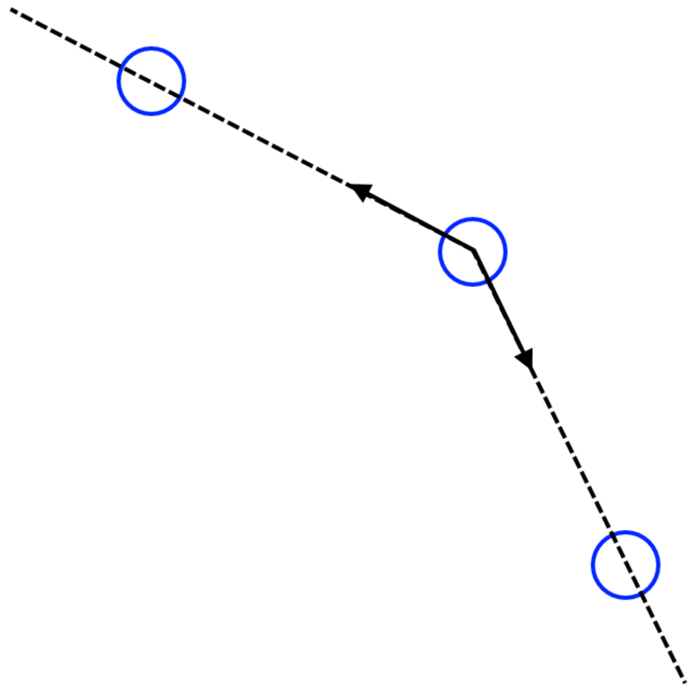
- This is an outline for the dynamic slipping part of the report.
- Michael: if you have time, you may start writing based on this ppt.

Terminology clarification

- Wall: The solid surface, usually no-flux.
- Interface: the liquid-gas interface.

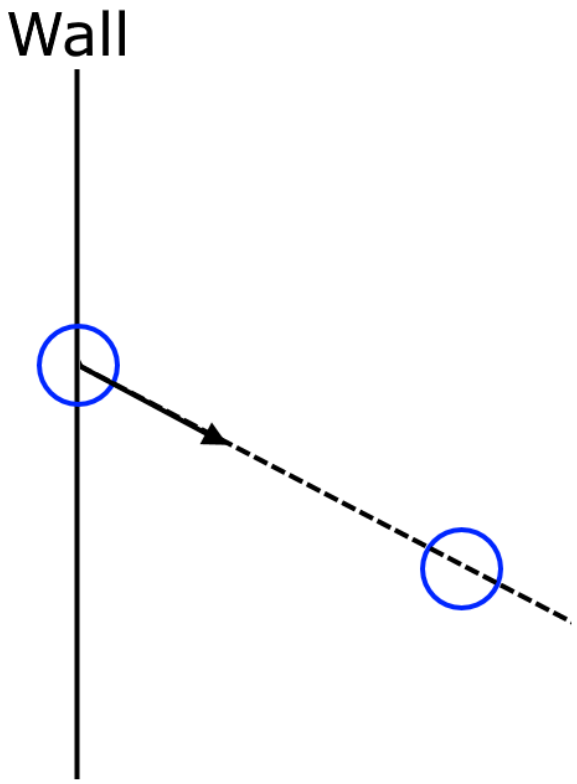
Our numerical method for tension force

- Each interface marker is pulled by its two neighbors at constant magnitude.
- Force is the gradient of energy. The 2D tension energy is proportional to the interface length. View that as the sum of all line segments. An attractive force with constant magnitude is precisely the gradient of the length of one line segment.
- Notice that the total force on a marker is always normal to the interface.



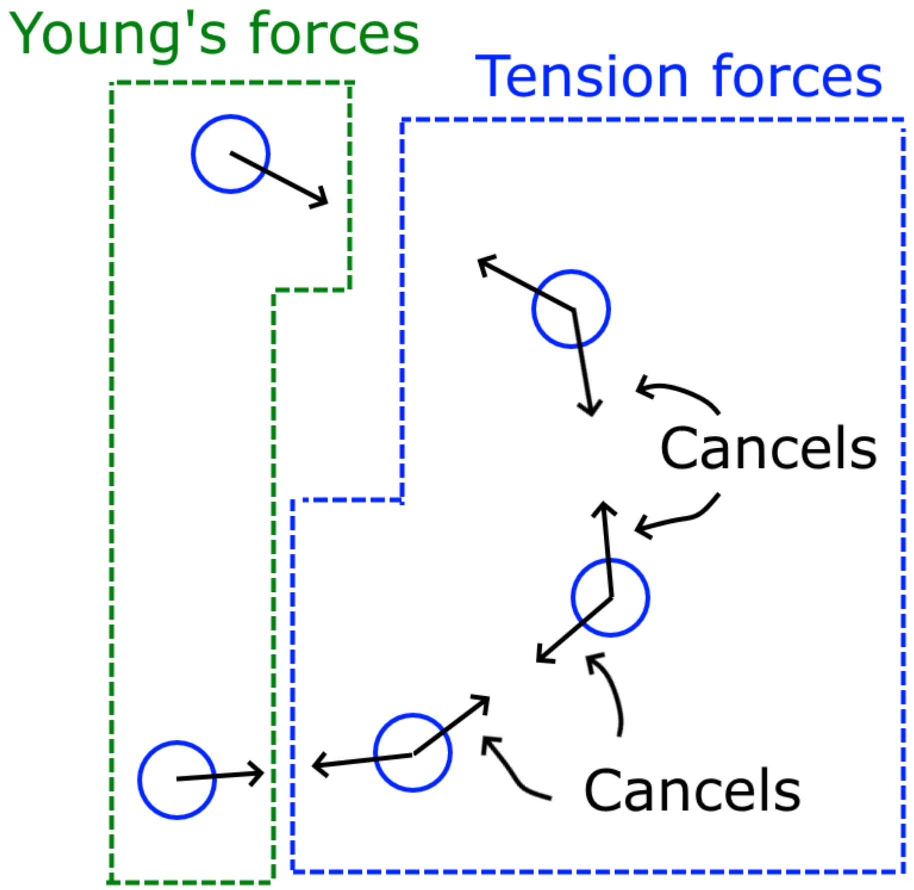
Natural extension: Young's force

- The same logic also simulates the unbalanced Young's force at a contact point.
- At the wall, there is one marker that has only one neighbor. The total force on this marker becomes tangent to the interface.
- However, its horizontal component is balanced by the no-flux force provided by the wall.
- Its vertical component turns out to equal the Young's force.



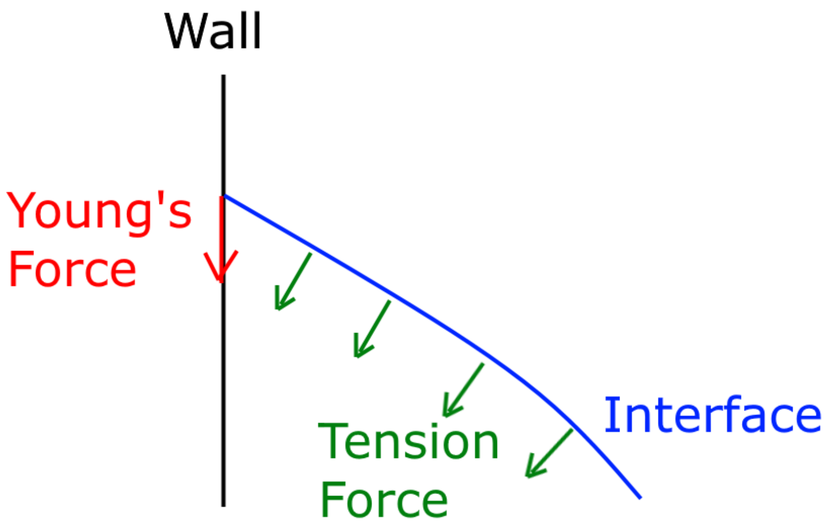
Beautiful fact

- Notice that the sum of the tension force on all markers is exactly the sum of Young's force on the two markers, just inverted in direction.
- We can view tension force and Young's force as a pair of action-reaction forces this way.



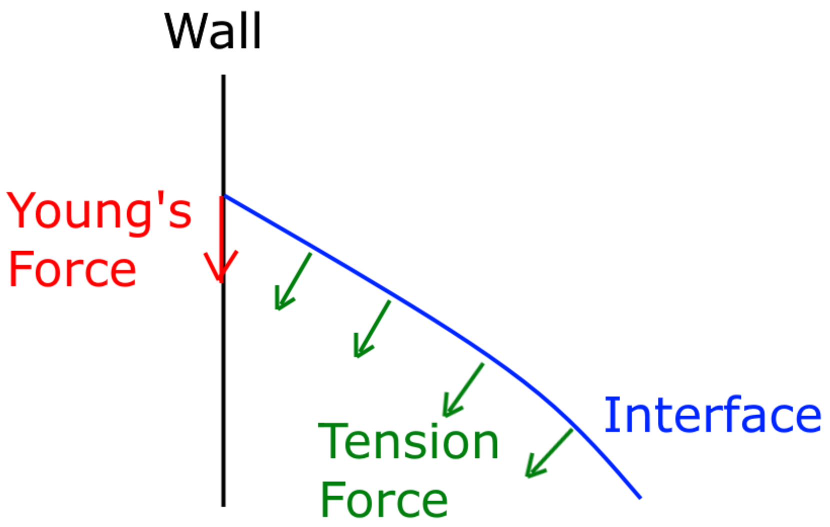
The two forces at the contact point

- Young’s force is tangent to the wall.
- The tension force is normal to the interface.



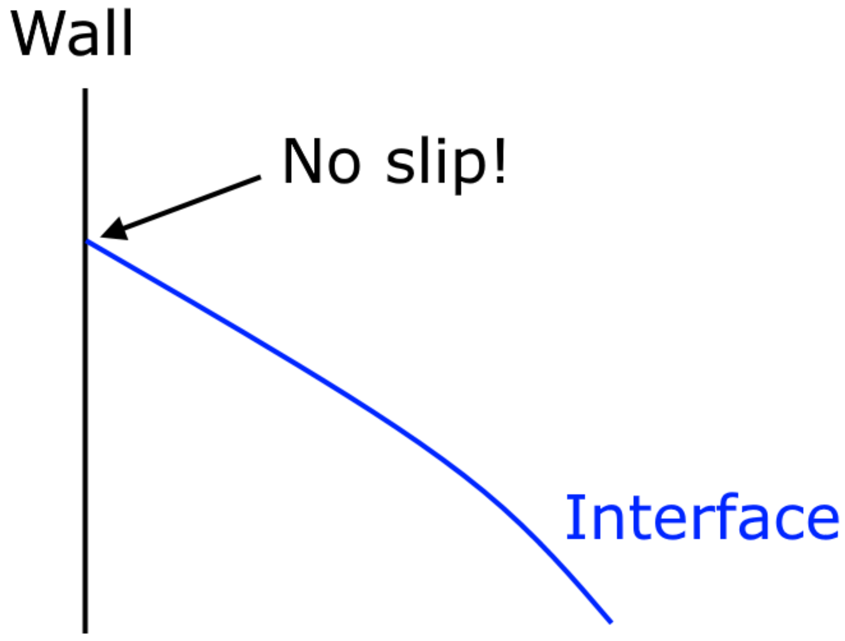
The two forces at the contact point

- Young's force has unit N, and is focused on one point.
- Tension force has unit N/cm, and is spread across the interface.
- Therefore, at the exact contact point, tension force is negligible compared to Young's force.



No-slip friction balances Young's force

- Now, view the no-slip boundary condition as a friction force, provided by the wall, exerted onto the fluid.
- No-slip simply means that this friction force is a *reactive force*, always balancing the sum of *active forces* at this location in the vertical direction.
- The only active force at the wall is Young's force. Therefore, the wall friction, if no-slip, equals the Young's force inverted in direction.



Clarifications about the last page

- Even not in hydrostatic equilibrium, the active forces are readily computed in a numerical simulation, so the friction is easy to solve.
- Most other active forces are also in unit N/cm or even N/cm^2, leaving Young's force to be the only relevant active force *at the wall*.
- "Friction" here is modelled similarly to the *viscous force*. Consider the viscous force to be the *friction* between neighboring "layers" of the fluid. Wall friction is simply the friction between the wall and the first layer of the fluid. Observe the units:

3D		**2D**	
Viscosity	$N/m^2 \cdot s$	Viscosity	$N/m^1 \cdot s$
Wall friction coefficient	$N/m^3 \cdot s$	Wall friction coefficient	$N/m^2 \cdot s$

Deep dive into how the hell a droplet can stay on a wall

<table>
<tr><th>Force</th><th>Exerted on</th><th>Supplied by</th><th>Direction</th><th>Pair of action-reaction forces?</th><th>Pair of balanced forces?</th></tr>
<tr><td>Gravity</td><td>Earth</td><td>Fluid</td><td>Up</td><td rowspan="2">Yes</td><td></td></tr>
<tr><td>Gravity</td><td>Fluid</td><td>Earth</td><td>Down</td><td rowspan="2">Yes</td></tr>
<tr><td>Total Tension</td><td>Fluid</td><td>Interface</td><td>Up</td><td rowspan="2">Yes</td></tr>
<tr><td>Young’s</td><td>Interface</td><td>Fluid</td><td>Down</td><td rowspan="2">Yes</td></tr>
<tr><td>Friction</td><td>Interface</td><td>Wall</td><td>Up</td><td rowspan="2">Yes</td></tr>
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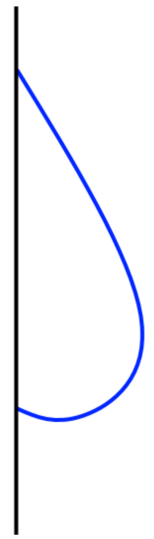

Dynamic slipping: we are changing these

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<tr><th>Force</th><th>Exerted on</th><th>Supplied by</th><th>Direction</th><th>Pair of action-reaction forces?</th><th>Pair of balanced forces?</th></tr>
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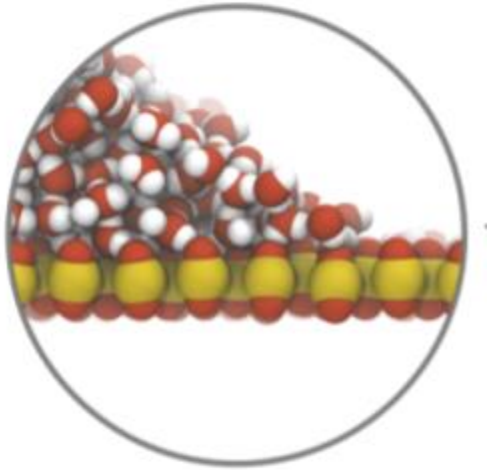
Dynamic Slipping

We have thoroughly looked at no-slip. Now we are ready to look at the dynamic slipping scheme.

Motivations

- In real life, we observe that small droplets stay on the wall as if no-slip, while bigger droplets roll down the wall.

- A 2018 molecular simulation study has partially revealed the microscopic model for water-hydrophilic surface interaction.



The macroscopic model

- The dynamic slipping scheme basically explains how we simulate a slip-with-friction wall.
- From here on we talk only about 2D.

$$f = min(|F_Y|, f_{limit}) \cdot \left(-\frac{F_Y}{|F_Y|}\right)$$

$$f_{limit} = f_{min} + \mu(\theta) \cdot |\vec{u} \cdot \hat{j}|$$

- f is the resulting wall friction. F_Y is the local Young’s force. f_{min} is a minimum friction force, an exogenous parameter. θ is the contact angle. μ is a function, giving a coefficient without unit. $\vec{\mu}$ is the local fluid velocity. $\hat{\jmath}$ is the vertical unit vector.

$\mu(\theta)$

- This function computes the friction coefficient given the contact angle.
- It is based on a regression of results yielded in the 2018 molecular simulations.

t (ns)	r (nm)	v (m/s)	θ	μ_f (μ)	f_{MKT}
2.5	50	14	$95°$	4.4 ± 0.5	0.19 ± 0.01
8.0	80	3.0	$64°$	9 ± 1	0.49 ± 0.01
12.5	90	1.7	$55°$	12 ± 2	0.49 ± 0.01

```
if angle > 1.117
  if angle > 2
    mu = 1.54;
  else
    mu = - 8.48 * angle + 18.5;
  end
else
  mu = - 19.1 * angle + 30.31;
end
f = - mu * v * FRICTION_ADJUST;
```

Macroscopic behaviors

$$f = min(|F_Y|, f_{limit}) \cdot \left(-\frac{F_Y}{|F_Y|}\right)$$

$$f_{limit} = f_{min} + \mu(\theta) \cdot |\vec{u} \cdot \hat{j}|$$

- When the contact angle is large enough so that $|F_Y| < f_{limit}$, the contact point does not slip, and we have $f = -F_Y$ which is identical to the no-slip case.
- Otherwise, the contact point moves. There is work done. Kinetic energy dissipates into heat during this process.

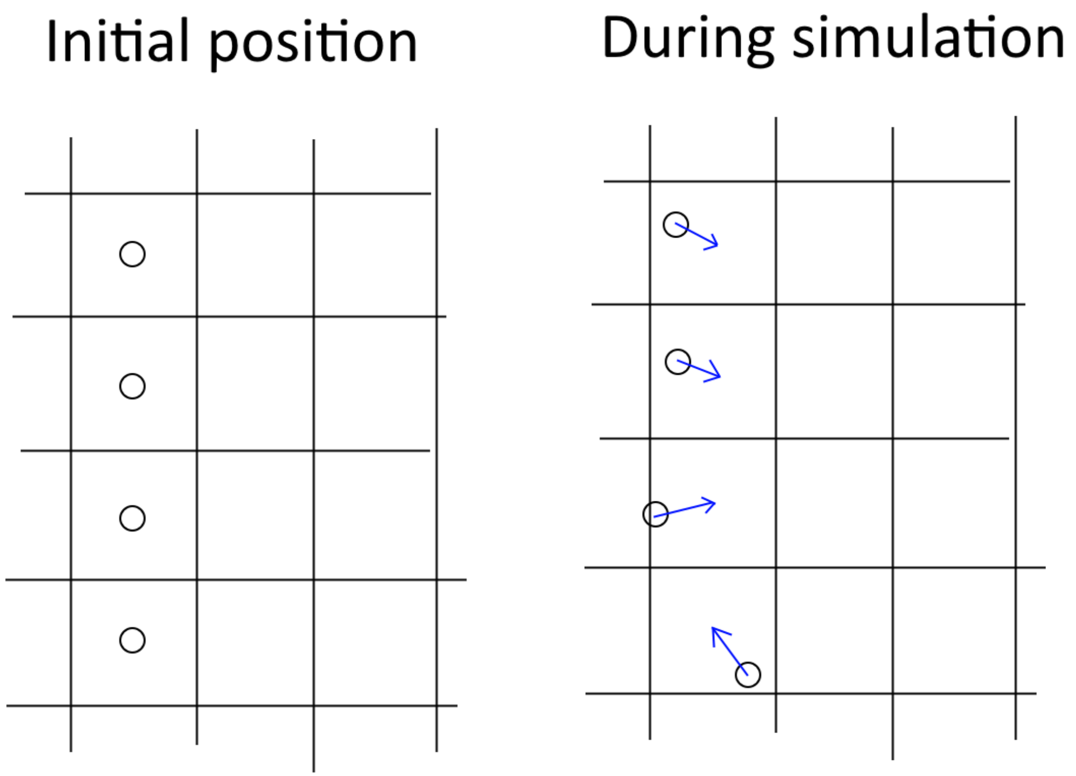
The corresponding numerical method

- It is well known that a no-slip wall can be simulated as an array of markers with the *penalty method*.
- To simulate a dynamic slipping wall, we use what we call the *incur-redeem-dismiss* penalty method. In this view, the baseline penalty method would be an *incur-redeem* penalty method.

	Used for	Incur?	Redeem?	Dismiss?
Baseline penalty method	No-flux no-slip wall	Yes	Yes	No
Ours	No-flux slip-with-friction wall	Yes	Yes	Yes

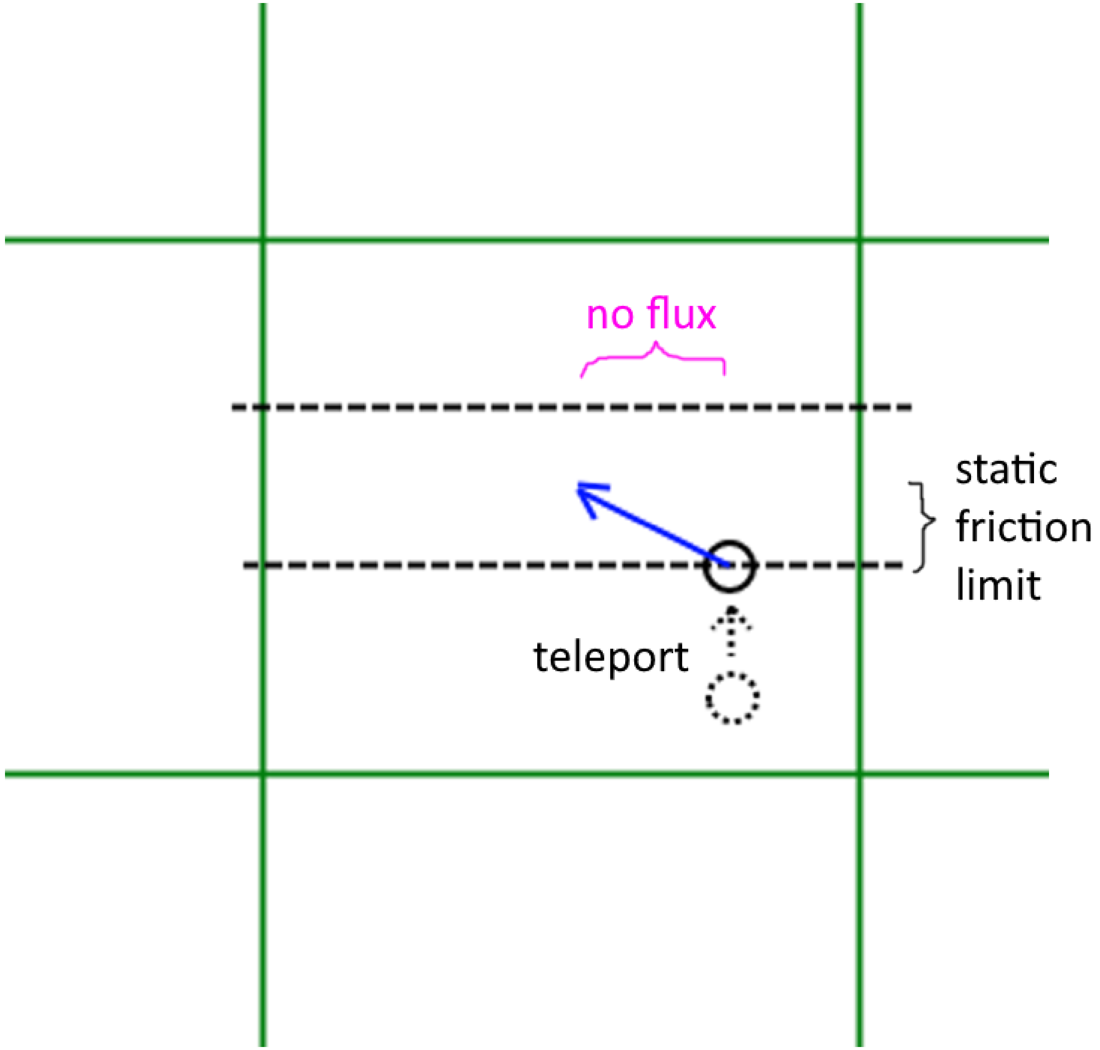
Baseline penalty method for no-slip wall

- Represent the wall with an array of markers. They follow the fluid flow.
- As they move away from their initial location, penalty is *incurred*.
- A penalty force is applied.
- The penalty will never be fully *redeemed* until a marker return to its correct location.
- This guarantees no-flux and no-slip.



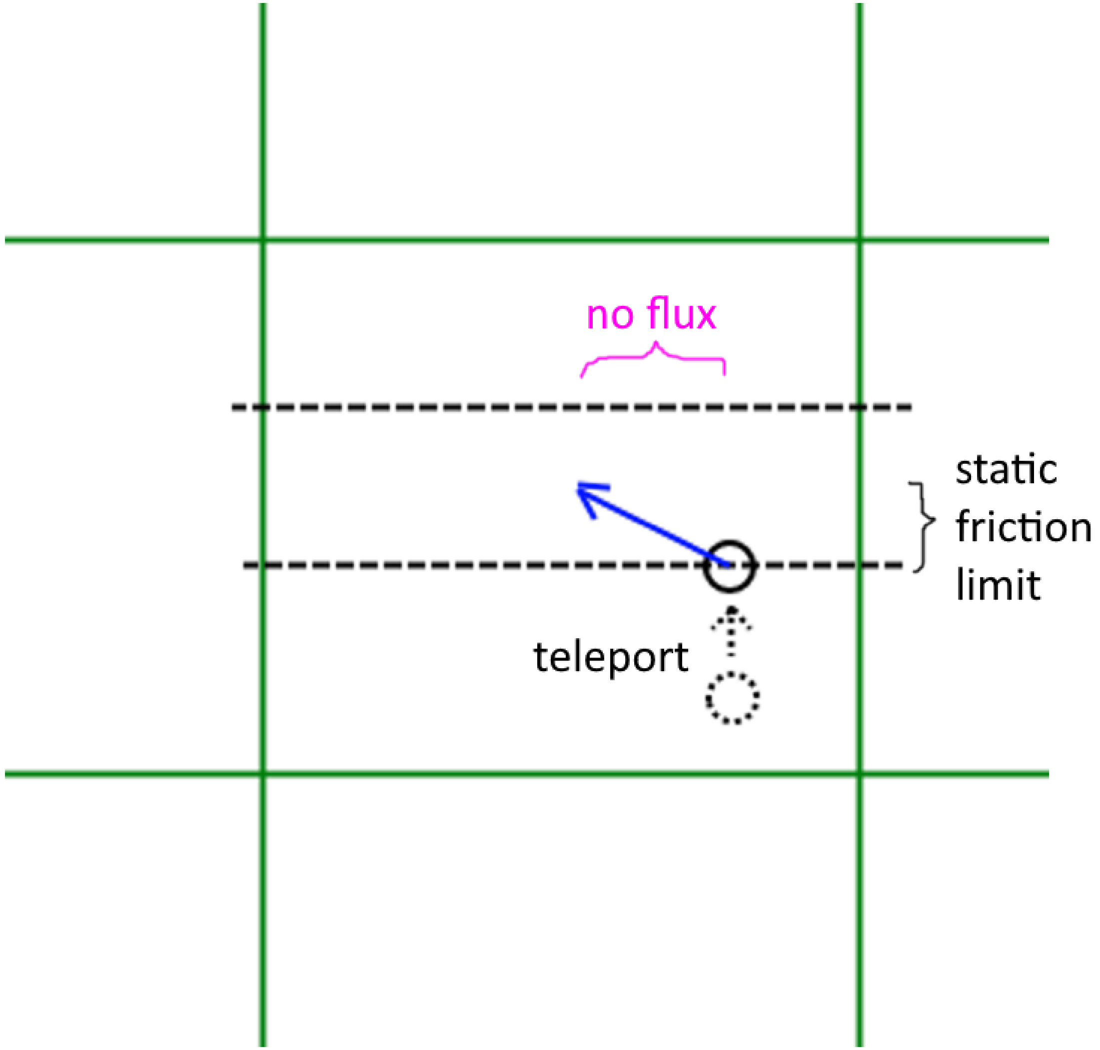
Incur-redeem-dismiss

- Previously:
  Fluid flow incurs penalty.
  Fluid flow redeems penalty.
- Now we introduce another way penalty can decrease: dismissal. No fluid flow is involved. Penalty is instantly decreased within a timestep by “teleporting” the marker.
- The amount of penalty dismissed, multiplied by friction force, gives the work done, i.e. the amount of heat dissipation.



Incur-redeem-dismiss

- Whenever the vertical penalty exceeds $\frac{f_{limit}}{K_{wall}}$, dismiss an amount of penalty to bring it to $\frac{f_{limit}}{K_{wall}}$.
- K_{wall} is the stiffness of the wall markers. $\frac{f_{limit}}{K_{wall}}$ is then the maximum penalty that static friction can hold.
- The horizontal penalty is never dismissed, so no-flux is preserved.



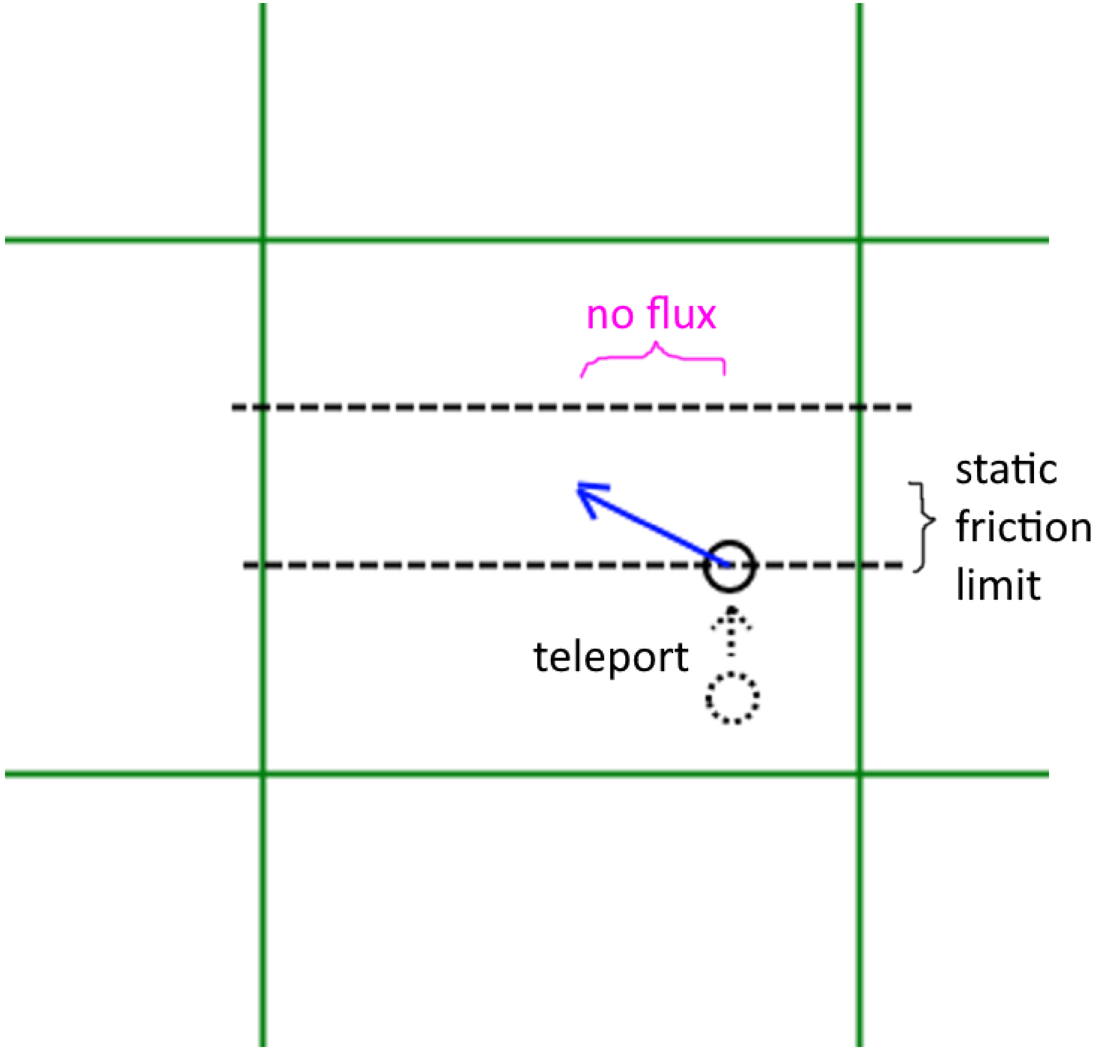
- [put pseudo code into paper here?]

Results

- Show some plots.
- Wait for Daniel to make those.

Problems

- Volume conservation



References for this ppt

- Molecular Origin of Contact Line Friction in Dynamic Wetting. Johansson and Hess. 2018.