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Game Theory
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OPEC Auction

Goal: Calculate the possible profits for each country under different potential games made in the next eleven/thirteen rounds of games.

Potential Games: Cournot Competition, Cooperative production reduction with no cheating, and Cooperative production with cheating.

Cournot Competition

The equilibrium production for Saudi Arabia, Iran, Venezuela, and Iraq is 9534.41, 8983.75, 8433.09, and 8433.09 respectively, when the production limit for each country is not considered. The step-by-step calculation can be found in the appendix.

From this, we can see that other than Saudi Arabia, the other countries' equilibrium production exceeds their full capacity, so the Nash equilibrium output of the Cournot game for these three countries is to produce at full capacity. When the other three countries produce at their capacity, Saudi Arabia's optimal output becomes 22658.89, which also exceeds its full capacity. Therefore, when the Cournot Competition reaches equilibrium, all four countries produce at their full capacity.

Calculating the profits for each country when operating at full capacity in Cournot Competition: The demand curve is given to be

$$P = 87.57248 - 0.0018161 \times q$$

When all countries are producing at maximum capacity, the total quantity produced by the OPEC countries is 24700, which yields a price of 42.7148 when plugged into the demand function above. From this the total profits for the countries can be calculated (Period 1-11 + 0.5 * Period 12 + 0.25 * Period 13).

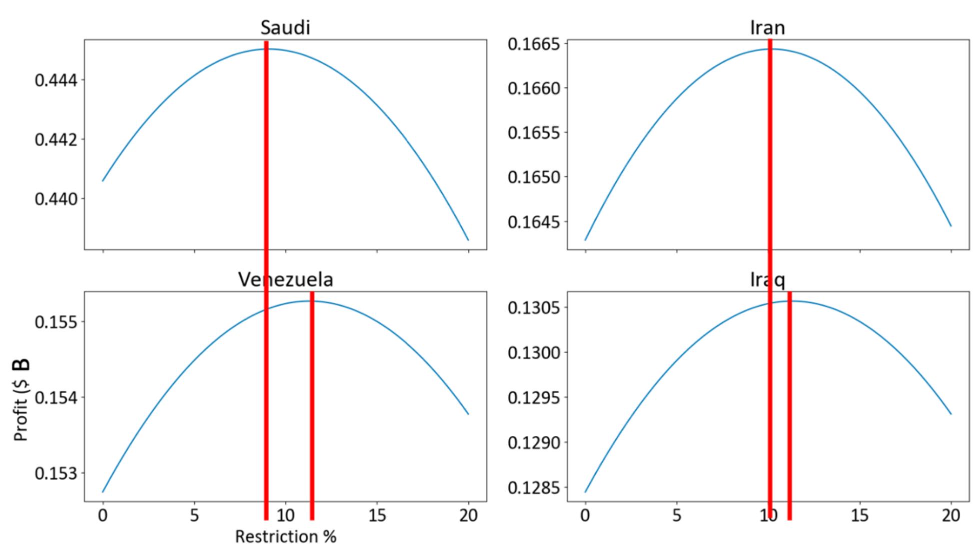
	Saudi Arabia	Iran	Venezuela	Iraq	OPEC Total
Production Capacity (Thousands)	12000	4600	4400	3700	**24700**
Marginal Cost	6	7	8	8	
Profit per period (Thousands)	440577.72	164288.126	152745.164	128444.797	
Total Profit (Thousands)	5176788.21	1930385.48	1794755.68	1509226.36	

Cooperative Production Reduction with No Cheating

Suppose that the four OPEC countries reduce their production by equal proportion. The following table shows the profits of each period for the countries.

Percentage Reduction (%)	Price	Saudi	Iran	Venezuela	Iraq
1	43.1658	0.4415	0.1647	0.1532	0.1288
2	43.6144	0.4423	0.1651	0.1536	0.1291
3	44.0629	0.4431	0.1654	0.1539	0.1294
4	44.5115	0.4437	0.1657	0.1542	0.1297
5	44.9600	0.4441	0.1659	0.1545	0.1299
6	45.4086	0.4445	0.1661	0.1547	0.1301
7	45.8571	0.4448	0.1662	0.1549	0.1303
8	46.3057	0.4450	0.1663	0.1551	0.1304
9	46.7542	0.4450	0.1664	0.1552	0.1305
10	47.2028	0.4450	0.1664	0.1552	0.1305
11	47.6514	0.4448	0.1664	0.1553	0.1306
12	48.0999	0.4446	0.1664	0.1553	0.1306
13	48.5485	0.4442	0.1663	0.1552	0.1305
14	48.9970	0.4437	0.1661	0.1551	0.1305
15	49.4456	0.4431	0.1660	0.1550	0.1303
16	49.8941	0.4425	0.1657	0.1548	0.1302
17	50.3427	0.4417	0.1655	0.1546	0.1300
18	50.7912	0.4407	0.1652	0.1544	0.1298
19	51.2398	0.4397	0.1648	0.1541	0.1296
20	51.6883	0.4386	0.1645	0.1538	0.1293

From this, we can find the profit-maximizing production reduction point for each country, and the following image shows the optimal point for each country.



Cooperative Production Reduction with Cheating

When cheating is involved, it is necessary to make assumptions regarding the appropriate punishment to take place. In our case, we assume that if one country deviates from the agreed production restrictions, then Cournot Competition will resume in all subsequent periods. Evidently, in the period where one country cheats, the cheating country will gain while the other countries will be worse off by comparison. For example, when the agreed reduction rate is 9%, but Saudi Arabia chooses to deviate, its profit for the one period is shown in the "ΔProfit Saudi Arabia" column. Change in profits for the other three countries is shown in the last three columns.

Saudi Arabia Actual Reduction (%)	Saudi Arabia Total Production	OPEC Country Production	Price	Saudi Arabia	Iran	Venezuela	Iraq	ΔRevenue Saudi Arabia	ΔRevenue Iran	ΔRevenue Venezuela	ΔRevenue Iraq
1	11880	11557	45.0109	0.4634	0.1591	0.1482	0.1246	0.0184	-0.0073	-0.0070	-0.0059
2	11760	11557	45.2288	0.4613	0.1600	0.1491	0.1253	0.0163	-0.0064	-0.0061	-0.0051
3	11640	11557	45.4467	0.4592	0.1609	0.1499	0.1261	0.0141	-0.0055	-0.0052	-0.0044
4	11520	11557	45.6646	0.4569	0.1619	0.1508	0.1268	0.0119	-0.0046	-0.0044	-0.0037
5	11400	11557	45.8826	0.4547	0.1628	0.1517	0.1276	0.0096	-0.0036	-0.0035	-0.0029
6	11280	11557	46.1005	0.4523	0.1637	0.1526	0.1283	0.0073	-0.0027	-0.0026	-0.0022
7	11160	11557	46.3184	0.4500	0.1646	0.1534	0.1290	0.0049	-0.0018	-0.0017	-0.0015
8	11040	11557	46.5363	0.4475	0.1655	0.1543	0.1298	0.0025	-0.0009	-0.0009	-0.0007
9	10920	11557	46.7542	0.4450	0.1664	0.1552	0.1305	0.0000	0.0000	0.0000	0.0000

Bidding Strategy

The last bidder has no one to compete with, so it must bid 100 million to acquire the last country, which is Iraq. With this information, this player's minimum payoff under Cournot Competition can be calculated as

$$(42.71481 - 8) \times 3{,}700{,}000 \times 11.75 - 100{,}000{,}000 = 1409226364.75$$

Therefore, when bidding for the previous countries, the maximum bid can be determined by subtracting each country's total profits under Cournot Competition by 1409226364.75, as shown in the table below.

	Saudi Arabia	Iran	Venezuela	Iraq
Production Capacity	12000	4600	4400	3700
Marginal Cost	6	7	8	8
Total Profit	5176788210	1930385480.50	1794755677.00	1509226364.75
Maximum Bid	3767561845	521159115.8	385529312.3	100000000

Appendix

Cournot Competition with no production restriction

Cournot competition

Let q_1^*, q_2^*, q_3^*, q_4^* be the equilibrium quantity of Saudi Arabia, Iran, Venezuela and Iraq. Assume the inverse demand function is $P = a - bQ$. Saudi Arabia's profit maximization equation is as follows:

$$\max \ \pi_S = q_1 (a - b \cdot (q_1 + q_2^* + q_3^* + q_4^*) - c_1)$$

where c_1 is the marginal cost of Saudi Arabia.

The First Order Condition is $a - c_1 - bq_1 = b(q_1 + q_2^* + q_3^* + q_4^*)$ where the solution to q_1 is q_1^*.

$$a - c_1 - bq_1^* = b(q_1^* + q_2^* + q_3^* + q_4^*) \qquad (1)$$

Similarly, other countries profit maximization equations lead to the following equations:

$$a - c_2 - bq_2^* = b(q_1^* + q_2^* + q_3^* + q_4^*) \qquad (2)$$

$$a - c_1 - bq_3^* = b(q_1^* + q_2^* + q_3^* + q_4^*) \qquad (3)$$

$$a - c_1 - bq_4^* = b(q_1^* + q_2^* + q_3^* + q_4^*) \qquad (4)$$

where c_2, c_3, c_4 are marginal costs of Iran, Venezuela, and Iraq.

Solving (1) ~ (4), we get $q_i^* = \frac{a - c_i}{5b} + \frac{\sum_{j=1}^{4} c_j - 4c_i}{5b}$ $(i = 1, 2, 3, 4)$

The value of a and b is the given demand function which is $a = 87.57248$, $b = 0.001816$.

$c_1 = 6$, $c_2 = 7$, $c_3 = c_4 = 8$

Thus, $q_1^* = 9534.41$

$q_2^* = 8983.75$

$q_3^* = q_4^* = 8433.09$