# NCERT Solutions for Class 10

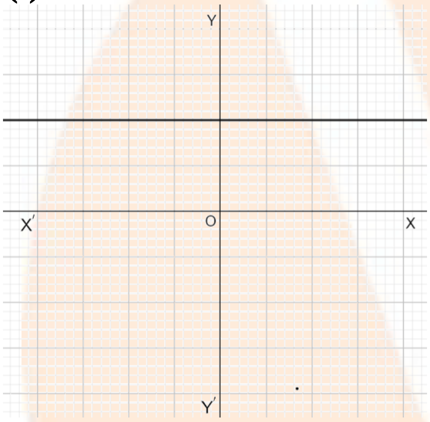
# Mathematics

# Chapter 2 – Polynomials

## Exercise 2.1

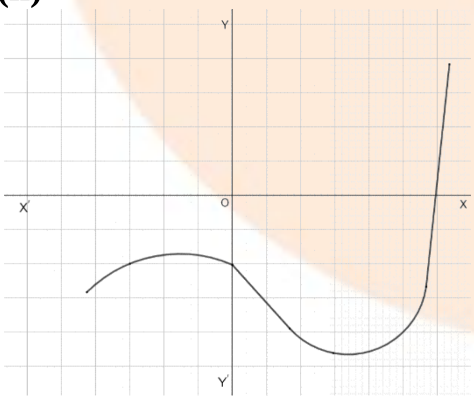
**1. The graphs of $y=p(x)$ are given in following figure, for some Polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.**

**(i)**



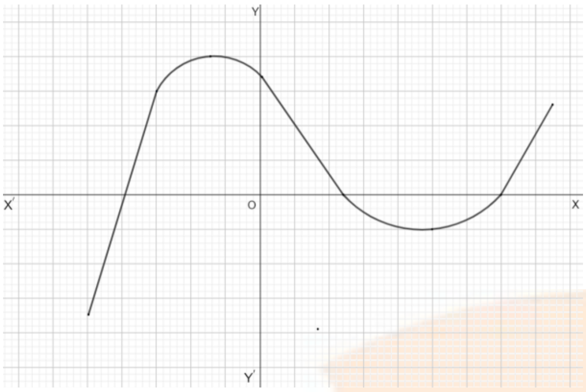

**Ans:** The graph does not intersect the $x$-axis at any point. Therefore, it does not have any zeroes.

**(ii)**



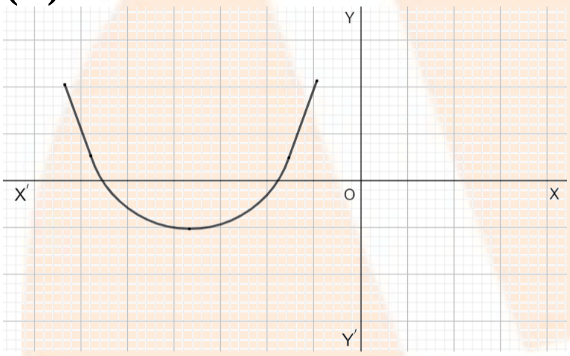

**Ans:** The graph intersects at the $x$-axis at only $1$point. Therefore, the number of zeroes is $1$.

**(iii)**



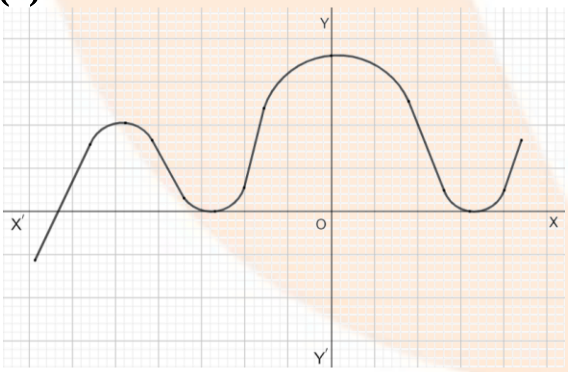

**Ans:** The graph intersects at the x-axis at 3 points. Therefore, the number of zeroes is 3.

**(iv)**



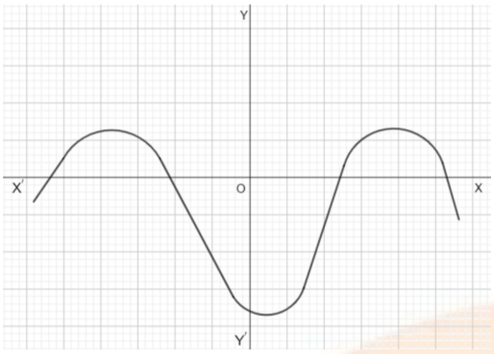

**Ans:** The graph intersects at the x-axis at 2 points. Therefore, the number of zeroes is 2.

**(v)**




**Ans:** The graph intersects at the x-axis at 4 points. Therefore, the number of zeroes is 4.

**(vi)**




**Ans:** The graph intersects at the $x$-axis at $3$ points. Therefore, the number of zeroes is $3$.

**Exercise 2.2**

**1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients. $x^2-2x-8$**

Given: $x^2-2x-8$.

Now factorize the given polynomial to get the roots.

$\Rightarrow (x-4)(x+2)$

**Ans:** The value of $x^2-2x-8$ is zero.

when $x-4=0$ or $x+2=0$. i.e., $x=4$ or $x=-2$

Therefore, the zeroes of $x^2-2x-8$ are $4$ and $-2$.

Now, Sum of zeroes $=4-2=2=-\frac{2}{1}=-\frac{\text{Coefficient of x}}{\text{Coefficient of } x^2}$

$\therefore$ Sum of zeroes $=-\frac{\text{Coefficient of x}}{\text{Coefficient of } x^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\therefore$ Product of zeroes $=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$.

**(i) $4s^2-4s+1$**

**Ans:** Given: $4s^2-4s+1$

Now factorize the given polynomial to get the roots.

$\Rightarrow (2s-1)^2$

The value of $4s^2-4s+1$ is zero.

when $2s-1=0$, $2s-1=0$. i.e., $s=\frac{1}{2}$ and $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2-4s+1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Now, Sum of zeroes $=\frac{1}{2}+\frac{1}{2}=1=\frac{(-4)}{4}=\frac{-(\text{Coefficient of s})}{\text{Coefficient of s}^2}$

$\therefore$ Sum of zeroes $=\frac{-(\text{Coefficient of s})}{\text{Coefficient of s}^2}$

Product of zeroes $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of s}^2}$

$\therefore$ Product of zeroes $=\frac{\text{Constant term}}{\text{Coefficient of s}^2}$.

**(ii) $6x^2-3-7x$**

**Ans:**Given: $6x^2-3-7x$

$\Rightarrow 6x^2-7x-3$

Now factorize the given polynomial to get the roots.

$\Rightarrow (3x+1)(2x-3)$

The value of $6x^2-3-7x$ is zero.

when $3x+1=0$ or $2x-3=0$. i.e., $x=\frac{-1}{3}$ or $x=\frac{3}{2}$.

Therefore, the zeroes of $6x^2-3-7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Now, Sum of zeroes $=\frac{-1}{3}+\frac{3}{2}=\frac{7}{6}=\frac{-(-7)}{6}=\frac{-(\text{Coefficient of x})}{\text{Coefficient of x}^2}$

$\therefore$ Sum of zeroes$=\frac{-(\text{Coefficient of x})}{\text{Coefficient of x}^2}$

Product of zeroes $=\frac{-1}{3}\times\frac{3}{2}=\frac{-3}{6}=\frac{\text{Constant term}}{\text{Coefficient of x}^2}$

$\therefore$ Product of zeroes$=\frac{\text{Constant term}}{\text{Coefficient of x}^2}$

**(iii) $4u^2+8u$**

**Ans:**Given: $4u^2+8u$

$\Rightarrow 4u^2+8u+0$

$\Rightarrow 4u(u+2)$

The value of $4u^2+8u$ is zero.

when $4u=0$ or $u+2=0$. i.e., $u=0$ or $u=-2$

Therefore, the zeroes of $4u^2+8u$ are $0$ and $-2$.

Now, Sum of zeroes $=0+(-2)=-2=\frac{-8}{4}=\frac{-(\text{Coefficient of u})}{\text{Coefficient of u}^2}$

$\therefore$ Sum of zeroes $=\frac{-(\text{Coefficient of u})}{\text{Coefficient of u}^2}$

Product of zeroes $=0\times(-2)=0=\frac{0}{4}=\frac{\text{Constant term}}{\text{Coefficient of u}^2}$

$\therefore$ Product of zeroes $=\frac{\text{Constant term}}{\text{Coefficient of u}^2}$

**(iv) $t^2-15$**

**Ans:** Given: $t^2-15$

$\Rightarrow t^2-0t-15$

Now factorize the given polynomial to get the roots.

$\Rightarrow (t-\sqrt{15})(t+\sqrt{15})$

The value of $t^2-15$ is zero.

when $t-\sqrt{15}=0$ or $t+\sqrt{15}=0$, i.e., $t=\sqrt{15}$ or $t=-\sqrt{15}$

Therefore, the zeroes of $t^2-15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Now, Sum of zeroes $=\sqrt{15}+(-\sqrt{15})=0=\frac{-0}{1}=\frac{-(\text{Coefficient of t})}{\text{Coefficient of t}^2}$

$\therefore$ Sum of zeroes $=\frac{-(\text{Coefficient of t})}{\text{Coefficient of t}^2}$

Product of zeroes $=\left(\sqrt{15}\right)\times\left(-\sqrt{15}\right)=-15=\frac{-15}{1}=\frac{\text{Constant term}}{\text{Coefficient of t}^2}$

$\therefore$ Product of zeroes $=\frac{\text{Constant term}}{\text{Coefficient of t}^2}$.

**(v) $3x^2-x-4$**

**Ans:** Given: $3x^2-x-4$

Now factorize the given polynomial to get the roots.

$\Rightarrow (3x-4)(x+1)$

The value of $3x^2-x-4$ is zero.

when $3x-4=0$ or $x+1=0$, i.e., $x=\frac{4}{3}$ or $x=-1$

Therefore, the zeroes of $3x^2-x-4$ are $\frac{4}{3}$ and -1.

Now, Sum of zeroes $=\frac{4}{3}+(-1)=\frac{1}{3}=\frac{-(-1)}{3}=\frac{-(\text{Coefficient of x})}{\text{Coefficient of } x^2}$

$\therefore$ Sum of zeroes$=\frac{-(\text{Coefficient of x})}{\text{Coefficient of } x^2}$

Product of zeroes $=\frac{4}{3}\times(-1)=\frac{-4}{3}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\therefore$ Product of zeroes $=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$.

**2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.**

**(i)** $\mathbf{\frac{1}{4},-1}$

**Ans**:Given: $\frac{1}{4}$,-1

Let the zeroes of polynomial be $\alpha$ and $\beta$.

Then,

$\alpha+\beta=\frac{1}{4}$

$\alpha\beta=-1$

Hence, the required polynomial is $x^2-(\alpha+\beta)x+\alpha\beta$.

$\Rightarrow x^2-\frac{1}{4}x-1$

$\Rightarrow 4x^2-x-4$

Therefore, the quadratic polynomial is $4x^2-x-4$.

**(ii)** $\mathbf{\sqrt{2},\frac{1}{3}}$

**Ans:** Given: $\sqrt{2},\frac{1}{3}$

Let the zeroes of polynomial be $\alpha$ and $\beta$.

Then,

$\alpha+\beta=\sqrt{2}$

$\alpha\beta=\frac{1}{3}$

Hence, the required polynomial is $x^2-(\alpha+\beta)x+\alpha\beta$.

$\Rightarrow x^2-\sqrt{2}x+\frac{1}{3}$

$\Rightarrow 3x^2-3\sqrt{2}x+1$

Therefore, the quadratic polynomial is $3x^2-3\sqrt{2}x+1$.

**(iii) $0, \sqrt{5}$**

**[here, root is missing]**

**Ans:** Given: $0, \sqrt{5}$

Let the zeroes of polynomial be $\alpha$ and $\beta$.

Then,

$\alpha+\beta = 0$

$\alpha\beta=\sqrt{5}$

Hence, the required polynomial is $x^2-(\alpha+\beta)x+\alpha\beta$.

$\Rightarrow x^2-0x+\sqrt{5}$

$\Rightarrow x^2+\sqrt{5}$

Therefore, the quadratic polynomial is $x^2+\sqrt{5}$.

**(iv) 1,1**

**Ans:** Given: 1,1

Let the zeroes of polynomial be $\alpha$ and $\beta$.

Then,

$\alpha+\beta = 1$

$\alpha\beta = 1$

Hence, the required polynomial is $x^2-(\alpha+\beta)x+\alpha\beta$.

$\Rightarrow x^2-1x+1$

Therefore, the quadratic polynomial is $x^2-x+1$.

**(v) $-\frac{1}{4}, \frac{1}{4}$**

**Ans:** Given: $-\frac{1}{4}, \frac{1}{4}$

Let the zeroes of polynomial be $\alpha$ and $\beta$.

Then,

$\alpha+\beta=-\frac{1}{4}$

$\alpha\beta = \frac{1}{4}$

Hence, the required polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$.

$\Rightarrow x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4}$

$\Rightarrow 4x^2 + x + 1$

Therefore, the quadratic polynomial is $4x^2 + x + 1$.

**(vi) 4,1**

**Ans:** Given: 4,1

Let the zeroes of polynomial be $\alpha$ and $\beta$.

Then,

$\alpha + \beta = 4$

$\alpha\beta = 1$

Hence, the required polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$.

$\Rightarrow x^2 - 4x + 1$

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.

## Exercise 2.3

**1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:**

**(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$**

**Ans:** Given: $p(x) = x^3 - 3x^2 + 5x - 3$ and $g(x) = x^2 - 2$

Then, divide the polynomial $p(x)$ by $g(x)$.

$$
\begin{array}{r|l}
 & \quad\quad\quad\quad x - 3 \\
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & x^3 \quad\quad\; - 2x \\
 & - \quad\quad\quad + \\
\hline
 & \quad - 3x^2 + 7x - 3 \\
 & \quad - 3x^2 \quad\quad + 6 \\
 & \quad + \quad\quad\quad - \\
\hline
 & \quad\quad\quad 7x - 9
\end{array}
$$

Therefore, Quotient $= x - 3$ and Remainder $= 7x - 9$.

**(ii) $p(x) = x^4 - 3x^2 + 4x + 5, g(x) = x^2 + 1 - x$**

**Ans:** Given: $p(x) = x^4 - 3x^2 + 4x + 5 \Rightarrow x^4 + 0x^3 - 3x^2 + 4x + 5$

$g(x) = x^2 + 1 - x \Rightarrow x^2 - x + 1$

Then, divide the polynomial $p(x)$ by $g(x)$.

$$
\begin{array}{rl}
 & \qquad\qquad\qquad x^2+x-3 \\
x^2-x+1 & \overline{)\,x^4+0x^3-3x^2+4x+5} \\
 & x^4-x^3+x^2 \\
 & -\quad+\quad- \\
\hline
 & \qquad x^3-4x^2+4x+5 \\
 & \qquad x^3-x^2+x \\
 & \qquad -\quad+\quad- \\
\hline
 & \qquad\quad -3x^2+3x+5 \\
 & \qquad\quad -3x^2+3x-3 \\
 & \qquad\quad +\quad-\quad+ \\
\hline
 & \qquad\qquad\qquad 8
\end{array}
$$

Therefore, Quotient $= x^2 + x - 3$ and Remainder $= 8$.

**(iii) $p(x) = x^4 - 5x + 6, g(x) = 2 - x^2$**

**Ans:** Given: $p(x) = x^4 - 5x + 6 \Rightarrow x^4 + 0x^3 + 0x^2 - 5x + 6$

$g(x) = 2 - x^2 \Rightarrow -x^2 + 2$

Then, divide the polynomial $p(x)$ by $g(x)$.

$$
\begin{array}{rl}
 & \qquad\qquad\qquad -x^2-2 \\
-x^2+2 & \overline{)\,x^4+0x^3+0x^2-5x+6} \\
 & x^4\qquad -2x^2 \\
 & -\qquad + \\
\hline
 & \qquad\quad 2x^2-5x+6 \\
 & \qquad\quad 2x^2\qquad -4 \\
 & \qquad\quad -\qquad\quad + \\
\hline
 & \qquad\qquad -5x+10
\end{array}
$$

Therefore, Quotient $= -x^2 - 2$ and Remainder $= -5x + 10$

**2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:**

**(i)** $\mathbf{t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12}$

**Ans:**Given: $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Let us take first polynomial is $t^2 - 3 \Rightarrow t^2 + 0t - 3$.

And second polynomial is $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

Now, divide the second polynomial by first polynomial.

$$
\begin{array}{r|l}
 & \quad\quad\quad\quad 2t^2 + 3t + 4 \\
\hline
t^2 + 0t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 + 0t^3 - 6t^2 \\
 & - \quad - \quad + \\
\hline
 & \quad 2t^3 + 4t^2 - 9t - 12 \\
 & \quad 3t^3 + 0t^2 - 9t \\
 & \quad - \quad - \quad + \\
\hline
 & \quad\quad 4t^2 + 0t - 12 \\
 & \quad\quad 4t^2 + 0t - 12 \\
\hline
 & \quad\quad\quad 0 \\
\hline
\end{array}
$$

Since the remainder is $0$

Therefore, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

**(ii)** $\mathbf{x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2}$

**Ans:**Given: $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

Let us take first polynomial is $x^2 + 3x + 1$.

And second polynomial is $3x^4 + 5x^3 - 7x^2 + 2x + 2$

Now, divide the second polynomial by first polynomial.

$$
\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\
\cline{2-2}
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & - \quad - \quad - \\
\cline{2-2}
 & \quad -4x^3 - 10x^2 + 2x + 2 \\
 & \quad -4x^3 - 12x^2 - 4x \\
 & \quad + \quad + \quad + \\
\cline{2-2}
 & \qquad\qquad 2x^2 + 6x + 2 \\
 & \qquad\qquad 2x^2 + 6x + 2 \\
 & \qquad\qquad - \quad - \quad - \\
\cline{2-2}
 & \qquad\qquad 0 \\
\cline{2-2}
\end{array}
$$

Since the remainder is $0$

Therefore, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

**(iii) $x^2 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$**

**Ans:** Given: $x^2 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$.

Let us take first polynomial is $x^2 - 3x + 1$.

And second polynomial is $x^5 - 4x^3 + x^2 + 3x + 1$.

Now, divide the second polynomial by first polynomial.

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\cline{2-2}
x^2 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 - 3x^3 + x^2 \\
 & - \quad + \quad - \\
\cline{2-2}
 & \quad -x^3 \qquad + 3x + 1 \\
 & \quad -x^3 \qquad + 3x - 1 \\
 & \quad + \qquad - \quad + \\
\cline{2-2}
 & \qquad\qquad 2 \\
\cline{2-2}
\end{array}
$$

Since the remainder $\neq 0$.

Therefore, $x^2 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

**3. Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$**

**Ans:** Let us assume, $p(x)=3x^4+6x^3-2x^2-10x-5$

Then, given two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$\therefore \left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$

Now, divide the given polynomial by $x^2-\frac{5}{3}$

$$
\begin{array}{r|l}
 & 3x^2+6x+3 \\
\hline
x^2+0x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4+0x^3-5x^2 \\
 & -\quad -\quad + \\
\hline
 & 6x^3+3x^2-10x-5 \\
 & 6x^3+0x^2-10x \\
 & -\quad -\quad + \\
\hline
 & 3x^2+0x-5 \\
 & 3x^2+0x-5 \\
 & -\quad -\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

$\therefore 3x^4+6x^3-2x^2-10x-5=\left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right)$

$\Rightarrow 3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)$

$\Rightarrow \left(3x^2-5\right)\left(x^2+2x+1\right)$

Now, factorize the polynomial $x^2+2x+1$

$\Rightarrow (x+1)^2$

Hence, its zero is given by $x+1=0$

$\Rightarrow x=-1$

As it has the term $(x+1)^2$, then, there will be $2$ zeroes at $x=-1$.

Therefore, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and $-1$.

**4. On dividing $x^3-3x^2+x+2$ by a polynomial $g(x)$, the quotient and remainder were $x-2$ and $-2x+4$, respectively. Find $g(x)$.**

**Ans:** Let us take the dividend as $p(x)$. Then, $p(x)=x^3-3x^2+x+2$

And the divisor is $g(x)$. Then, find the value of $g(x)$.

Quotient $=(x-2)$

Remainder $=(-2x+4)$

Dividend = Divisor × Quotient + Remainder

$x^3-3x^2+x+2=g(x)\times(x-2)+(-2x+4)$

$x^3-3x^2+x+2+2x-4=g(x)\times(x-2)$

$x^3-3x^2+3x-2=g(x)\times(x-2)$

Hence, $g(x)$ is the quotient when we divide $(x^3-3x^2+3x-2)$ by $(x-2)$.

$$
\begin{array}{r|l}
 & x^2-x+1 \\
\hline
x-2 & x^3-3x^2+3x-2 \\
 & x^3-2x^2 \\
 & -\quad + \\
\hline
 & -x^2+3x-2 \\
 & -x^2+2x \\
 & +\quad - \\
\hline
 & x-2 \\
 & x-2 \\
 & -\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

$\therefore g(x)=(x^2-x+1)$

**5. Give examples of polynomial $p(x), g(x), q(x)$ and $r(x)$, which satisfy the division algorithm and**

**(i) deg p(x) = deg q(x)**

**Ans:** According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$,

Where $r(x)=0$ or degree of $r(x)<$ degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

Given: $\deg p(x) = \deg q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by $2$.

Here, $p(x) = 6x^2 + 2x + 2$

$g(x) = 2$

$q(x) = 3x^2 + x + 1$ and $r(x) = 0$

Degree of $p(x)$ and $q(x)$ is the same i.e., $2$.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$6x^2 + 2x + 2 = 2\left(3x^2 + x + 1\right)$

$6x^2 + 2x + 2 = 6x^2 + 2x + 2$

Therefore, the division algorithm is satisfied.

**(ii) deg q(x) = deg r(x)**

**Ans:** Given: $\deg q(x) = \deg r(x)$

Let us assume the division of $x^3 + x$ by $x^2$

Here, $p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same, i.e.,

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = \left(x^2\right) \times x + x$

$x^3 + x = x^3 + x$

Therefore, the division algorithm is satisfied.

**(iii) $\deg r\left(x\right) = 0$**

**Ans:** Given: $\deg r(x) = 0$

Degree of remainder will be $0$ when remainder comes to a constant.

Let us assume the division of $x^3+1$ by $x^2$.

Here, $p(x) = x^3+1$

$g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is $0$

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3+1 = \left(x^2\right) \times x + 1$

$x^3+1 = x^3+1$

Therefore, the division algorithm is satisfied.

## Exercise 2.4

**1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:**

**(i) $2x^3 + x^2 - 5x + 2;\ \frac{1}{2}, 1, -2$**

**Ans:** Let us assume $p(x) = 2x^3+x^2-5x+2$

And zeroes for this polynomial are $\frac{1}{2}, 1, -2$. Then,

$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$

$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$

$= 0$

$p(1) = 2\times 1^3 + 1^2 - 5\times 1 + 2$

$= 0$

$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$

$= -16 + 4 + 10 + 2$

$= 0$

Therefore, $\frac{1}{2}, 1$ and $-2$ are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3+bx^2+cx+d$ to get, $a = 2, b = 1, c = -5$, and $d = 2$.

Let us take $\alpha = \frac{1}{2}, \beta = 1,$ and $\gamma = -2$.

Then,

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\therefore \alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\therefore \alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$$

$$\therefore \alpha\beta\gamma = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

**(ii)** $\mathbf{x^3 - 4x^2 + 5x - 2; 2,1,1}$

**Ans:** Let us assume $p(x) = x^3 - 4x^2 + 5x - 2$

And zeroes for this polynomial are $2,1,1$. Then,

$$p(2) = 2^3 - 4(2^2) + 5(2) - 2$$
$$= 8 - 16 + 10 - 2$$
$$= 0$$
$$p(1) = 1^3 - 4(1^2) + 5(1) - 2$$
$$= 1 - 4 + 5 - 2$$
$$= 0$$

Therefore, $2, 1,$ and $1$ are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$ to get, $a = 1, b = -4, c = 5,$ and $d = -2$.

Let us take $\alpha = 2, \beta = 1,$ and $\gamma = 1$.

Then,

$$\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

$$\therefore \alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = (2)(1) + (1)(1) + (2)(1)$$

$= 2+1+2 = 5 = \frac{(5)}{1} = \frac{c}{a}$

$\therefore \alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$

$\alpha\beta\gamma = 2\times1\times1 = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$

$\therefore \alpha\beta\gamma = \frac{-d}{a}$

Therefore, the relationship between the zeroes and the coefficients is verified.

**2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as $2, -7, -14$ respectively.**

**Ans:** Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be $\alpha, \beta,$ and $\gamma$.

Then given that,

$\alpha+\beta+\gamma = \frac{2}{1} = \frac{-b}{a}$

$\alpha\beta+\beta\gamma+\alpha\gamma = \frac{-7}{1} = \frac{c}{a}$

$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$

If $a=1$, then $b = -2, c = -7, d = 14$

Therefore, the polynomial is $x^3 - 2x^2 - 7x + 14$.

**3. If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a-b, a, a+b$, find $a$ and $b$.**

**Ans:** Let us assume $p(x) = x^3 - 3x^2 + x + 1$.

And the zeroes are $a-b, a, a+b$.

Let us assume $\alpha = a-b, \beta = a$ and $\gamma = a+b$.

Comparing the given polynomial with $px^3 + qx^2 + rx + t$ to get,

$p = 1, q = -3, r = 1,$ and $t = 1$.

Then,

$\alpha+\beta+\gamma = a - b + a + a + b$

$\Rightarrow \frac{-q}{p} = 3a$

$\Rightarrow \frac{-(-3)}{1} = 3a$

$\Rightarrow 3 = 3a$

$\therefore a = 1$

Then, the zeroes are $1\text{-}b, 1+b$.

Now,

$\alpha\beta\gamma = 1(1\text{-}b)(1+b)$

$\Rightarrow \dfrac{-t}{p} = 1 - b^2$

$\Rightarrow \dfrac{-1}{1} = 1 - b^2$

$\Rightarrow 1 - b^2 = -1$

$\Rightarrow b^2 = 2$

$\therefore b = \pm\sqrt{2}$

Therefore, $a = 1$ and $b = \pm\sqrt{2}$.

**4. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.**

**Ans:** Given that $2+\sqrt{3}$ and $2-\sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right) = x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$

Hence, $x^2 - 4x + 1$ is a factor of the given polynomial.

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\ \hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\ \hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\ \hline
 & 0 \\ \hline
\end{array}
$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138x - 35 = \left(x^2 - 4x + 1\right)\left(x^2 - 2x - 35\right)$

Then, $x^2-2x-35$ is also a factor of the given polynomial.

And, $x^2-2x-35=(x-7)(x+5)$

Therefore, the value of the polynomial is also zero when $x-7=0$ Or $x+5=0$

Hence, $x=7$ or $-5$

Therefore, $7$ and $-5$ are also zeroes of this polynomial.

**5. If the polynomial $x^4-6x^3+16x^2-25x-10$ is divided by another polynomial $x^2-2x+k$, the remainder comes out to be $x+a$, find $k$ and $a$.**

**Ans:** Given: $x^4-6x^3+16x^2-25x-10$ and $x^2-2x+k$.

Then, the remainder is $x+a$

By division algorithm,

Dividend = Divisor×Quotient+Remainder

Dividend-Remainder= Divisor×Quotient

$x^4-6x^3+16x^2-25x-10-x-a \Rightarrow x^4-6x^3+16x^2-26x+10-a$ will be perfectly divisible by $x^2-2x+k$.

Let us divide $x^4-6x^3+16x^2-26x+10-a$ by $x^2-2x+k$

$$
\begin{array}{r|l}
 & x^2-4x+(8-k) \\
\hline
x^2-2x+k & x^4-6x^3+16x^2-26x+10-a \\
 & x^4-2x^3+kx^2 \\
 & -\ \ +\ \ - \\
\hline
 & -4x^3+(16-k)x^2-26x \\
 & -4x^3+\ \ 8x^2-4kx \\
 & +\ \ -\ \ + \\
\hline
 & (8-k)x^2-(26-4k)x+10-a \\
 & (8-k)x^2-(16-2k)x+(8k-k^2) \\
 & -\ \ +\ \ - \\
\hline
 & (-10+2k)x+(10-a-8k+k^2)
\end{array}
$$

Hence, the reminder $(-10+2k)x+(10-a-8k+k^2)$ will be $0$.

Then, $(-10+2k)=0$ and $(10-a-8k+k^2)=0$

For $(-10+2k)=0$

$2k=10$

$\therefore k=5$

For $\left(10-a-8k+k^2\right)=0$

$10-a-8\times5+25=0$
$10-a-40+25=0$
$-5-a=0$
$\therefore a=-5$

Hence, $k=5$ and $a=-5$.