Math 3398 Homework – Recurrence relation (Part 2)

1. Solve each of the following recurrence relation with the given initial conditions.

   (1) $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ for $n \geq 3$,
   $a_0 = 3, \quad a_1 = 6 \quad a_2 = 0$

   (2) $a_n = 7a_{n-2} + 6a_{n-3}$ for $n \geq 2$,
   $a_0 = 9, \quad a_1 = 10, \quad a_2 = 32$

   (3) $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ for $n \geq 3$,
   $a_0 = 7, \quad a_1 = -4 \quad a_2 = 8$

   (4) $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$ for $n \geq 3$,
   $a_0 = -4, \quad a_1 = 4 \quad a_2 = 8$

   (5) $a_n = -3a_{n-1} + a_{n-2} + 3a_{n-3}$ for $n \geq 3$,
   $a_0 = 5, \quad a_1 = -9 \quad a_2 = 15$

   (6) $a_n = -3a_{n-1} + 4a_{n-3}$ for $n \geq 3$,
   $a_0 = 1, \quad a_1 = 2 \quad a_2 = -3$

   (7) $a_n = 9a_{n-1} - 27a_{n-2} + 27a_{n-3}$ for $n \geq 3$,
   $a_0 = 2, \quad a_1 = 6 \quad a_2 = -18$

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2. What is the general form of the particular solution to the linear nonhomogeneous recurrence relation

$$a_n = -7a_{n-1} - 12a_{n-2} + F(n)$$

   if

   (1) $F(n) = 2n + 4$

   (2) $F(n) = n^2 - 1$

   (3) $F(n) = 2^n$

   (4) $F(n) = (-5)^n$

   (5) $F(n) = (-3)^n$

   (6) $F(n) = (2n + 4)7^n$

3. Find the particular solution to the linear nonhomogeneous recurrence relation

$$a_n = a_{n-1} + 2a_{n-2} + F(n)$$

   if

   (1) $F(n) = 7$

   (2) $F(n) = 2n + 4$

   (3) $F(n) = n^2 - 1$

   (4) $F(n) = 3^n$

   (5) $F(n) = 2^n$

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4. Find the particular solution to the linear nonhomogeneous recurrence relation

$$a_n = 2a_{n-1} + 8a_{n-2} + F(n)$$

   if

   (1) $F(n) = 18$

   (2) $F(n) = 3n - 2$

   (3) $F(n) = n^2$

   (4) $F(n) = 7^n$

   (5) $F(n) = 4^n$

5. Solve each of the following recurrence relation with the given initial condition(s).

(1) $a_n = 2a_{n-1} + 3^n$ for $n \geq 2$,
$a_1 = 6$

(2) $a_n = 2a_{n-1} + 2n^2$ for $n \geq 2$,
$a_1 = 4$

(3) $a_n = a_{n-1} + 20a_{n-2} + 16$ for $n \geq 2$,
$a_0 = \frac{1}{5}, \quad a_1 = \frac{6}{5}$

(4) $a_n = a_{n-1} + 6a_{n-2} - 2n$ for $n \geq 2$,
$a_0 = 3, \quad a_1 = 6$

(5) $a_n = a_{n-1} + 2a_{n-2} + 3^n$ for $n \geq 2$,
$a_0 = \frac{1}{4}, \quad a_1 = \frac{7}{4}$

(6) $a_n = a_{n-1} + 6a_{n-2} + 5^n$ for $n \geq 2$,
$a_0 = 3, \quad a_1 = 6$

(7) $a_n = 6a_{n-1} - 9a_{n-2} + 5^n$ for $n \geq 2$,
$a_0 = 0, \quad a_1 = 2$

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6. Word problems:

<u>6th edition:</u>

(1) p. 457 #11 - 21 odd, 29

(2) p. 471 #5 - 9, 47

<u>7th edition:</u>

(1) p. 511 #3 - 13 odd, 19

(2) p. 524 #5 - 9, 47

KEY

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1 Solve each of the following recurrence relation with the given initial conditions.

(1) $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ for $n \geq 3$,
$a_0 = 3, \quad a_1 = 6 \quad a_2 = 0$

Answer: $a_n = 6 - 2(-1)^n - 2^n, \quad n \geq 0$

(2) $a_n = 7a_{n-2} + 6a_{n-3}$ for $n \geq 2$,
$a_0 = 9, \quad a_1 = 10, \quad a_2 = 32$

Answer: $a_n = 8(-1)^n - 3(-2)^n + 4 \cdot 3^n, \quad n \geq 0$

(3) $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ for $n \geq 3$,
$a_0 = 7, \quad a_1 = -4 \quad a_2 = 8$

Answer: $a_n = 5 + 3(-2)^n - 3^n, \quad n \geq 0$

(4) $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$ for $n \geq 3$,
$a_0 = -4, \quad a_1 = 4 \quad a_2 = 8$

Answer: $a_n = (-3n^2 + 9n - 4)2^n, \quad n \geq 0$

(5) $a_n = -3a_{n-1} + a_{n-2} + 3a_{n-3}$ for $n \geq 3$,
$a_0 = 5, \quad a_1 = -9 \quad a_2 = 15$

Answer: $a_n = -\frac{3}{4} + \frac{9}{2}(-1)^n + \frac{5}{4}(-3)^n, \quad n \geq 0$

(6) $a_n = -3a_{n-1} + 4a_{n-3}$ for $n \geq 3$,
$a_0 = 1, \quad a_1 = 2 \quad a_2 = -3$

Answer: $a_n = n(-2)^{n-1} + 1, \quad n \geq 0$

(7) $a_n = 9a_{n-1} - 27a_{n-2} + 27a_{n-3}$ for $n \geq 3$,
$a_0 = 2, \quad a_1 = 6 \quad a_2 = -18$

Answer: $a_n = (2 + 4n - 3n^2) \cdot 3^n, \quad n \geq 0$

2 What is the general form of the particular solution to the linear nonhomogeneous recurrence relation

$$a_n = -7a_{n-1} - 12a_{n-2} + F(n)$$

if

(1) $F(n) = 2n + 4$

Answer: $a_n^{(p)} = an + b$

(2) $F(n) = n^2 - 1$

Answer: $a_n^{(p)} = an^2 + bn + c$

(3) $F(n) = 2^n$

Answer: $a_n^{(p)} = a \cdot 2^n$

(4) $F(n) = (-5)^n$

Answer: $a_n^{(p)} = a \cdot (-5)^n$

(5) $F(n) = (-3)^n$

Answer: $a_n^{(p)} = an \cdot (-3)^n$

(6) $F(n) = (2n + 4)7^n$

Answer: $a_n^{(p)} = (an + b) \cdot 7^n$

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3 Find the particular solution to the linear nonhomogeneous recurrence relation

$$a_n = a_{n-1} + 2a_{n-2} + F(n)$$

if

(1) $F(n) = 7$

Answer: $a_n^{(p)} = -\frac{7}{2}$

(2) $F(n) = 2n + 4$

Answer: $a_n^{(p)} = -n - \frac{9}{2}$

(3) $F(n) = n^2 - 1$

Answer: $a_n^{(p)} = -\frac{1}{2}n^2 - \frac{5}{2}n - \frac{7}{2}$

(4) $F(n) = 3^n$

Answer: $a_n^{(p)} = \frac{1}{4} \cdot 3^{n+2}$

(5) $F(n) = 2^n$

Answer: $a_n^{(p)} = \frac{1}{3}\, n \cdot 2^{n+1}$

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4 Find the particular solution to the linear nonhomogeneous recurrence relation

$$a_n = 2a_{n-1} + 8a_{n-2} + F(n)$$

if

(1) $F(n) = 18$

Answer: $a_n^{(p)} = -2$

(2) $F(n) = 3n - 2$

Answer: $a_n^{(p)} = -\frac{1}{3}n - \frac{4}{9}$

(3) $F(n) = n^2$

Answer: $a_n^{(p)} = \frac{13}{45}n^2 - \frac{4}{5}n - \frac{206}{405}$

(4) $F(n) = 7^n$

Answer: $a_n^{(p)} = \frac{1}{27} \cdot 7^{n+2}$

(5) $F(n) = 4^n$

Answer: $a_n^{(p)} = \frac{2}{3}\, n \cdot 4^n$

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5 Solve each of the following recurrence relation with the given initial condition(s).

(1) $a_n = 2a_{n-1} + 3^n$ for $n \geq 2$,
$a_1 = 6$

Answer:

$$a_n = -\frac{3}{2} \cdot 2^n + 3^{n+1}, \quad n \geq 1$$

Or

$$a_n = -3 \cdot 2^{n-1} + 3^{n+1}, \quad n \geq 1$$

(2) $a_n = 2a_{n-1} + 2n^2$ for $n \geq 2$,
$a_1 = 4$

Answer:

$$a_n = 13 \cdot 2^n - 2n^2 - 8n - 12 \quad n \geq 1$$

(3) $a_n = a_{n-1} + 20a_{n-2} + 16$ for $n \geq 2$,
$a_0 = \frac{1}{5}, \quad a_1 = \frac{6}{5}$

Answer:

$$a_n = \frac{2}{3} \cdot 5^n + \frac{1}{3}(-4)^n - \frac{4}{5}, \quad n \geq 0$$

(4) $a_n = a_{n-1} + 6a_{n-2} - 2n$ for $n \geq 2$,
$a_0 = 3, \quad a_1 = 6$

Answer:

$$a_n = \frac{19}{10} \cdot 3^n + \frac{17}{45}(-2)^n + \frac{13}{n} + \frac{13}{18}, \quad n \geq 0$$

(5) $a_n = a_{n-1} + 2a_{n-2} + 3^n$ for $n \geq 2$,
$a_0 = \frac{1}{4}, \quad a_1 = \frac{7}{4}$

Answer:

$$a_n = -\frac{7}{3} \cdot 2^n + \frac{1}{3}(-1)^n + \frac{9}{4} \cdot 3^n, \quad n \geq 0$$

(6) $a_n = a_{n-1} + 6a_{n-2} + 5^n$ for $n \geq 2$,
$a_0 = 3, \quad a_1 = 6$

Answer:

$$a_n = -\frac{1}{10} \cdot 3^n + \frac{46}{35}(-2)^n + \frac{25}{14} \cdot 5^n, \quad n \geq 0$$

(7) $a_n = 6a_{n-1} - 9a_{n-2} + 5^n$ for $n \geq 2$,
$a_0 = 0, \quad a_1 = 2$

Answer:

$$a_n = \left(-\frac{25}{4} - \frac{7}{2}n\right) \cdot 3^n + \frac{1}{4} \cdot 5^{n+2}, \quad n \geq 0$$