Math 3398 Homework – Generating Functions (Part 1)

1. Find the power series for each of the following functions.

(1) $f(x) = (1+2x)^7$

(2) $f(x) = \dfrac{1}{1-3x}$

(3) $f(x) = \dfrac{2}{1+4x}$

(4) $f(x) = \dfrac{x}{1+5x}$

(5) $f(x) = \dfrac{x^4}{1-2x}$

(6) $f(x) = \dfrac{2}{4-3x}$

(7) $f(x) = \dfrac{x}{2x-1}$

(8) $f(x) = \dfrac{1}{1-x^3}$

(9) $f(x) = \dfrac{1}{x-3x^2}$

(10) $f(x) = \dfrac{1}{(1-3x)^2}$

(11) $f(x) = \dfrac{x}{(1+2x)^2}$

(12) $f(x) = \dfrac{1}{(1-5x)^3}$

(13) $f(x) = \dfrac{1}{(2-x)^2}$

(14) $f(x) = \dfrac{1}{(x+2)(x-3)}$

(15) $f(x) = \dfrac{x}{12-4x-x^2}$

(16) $f(x) = e^{2x}$

(17) $f(x) = e^{-3x}$

(18) $f(x) = e^{2x^2}$

(19) $f(x) = \ln(1+2x)$

(20) $f(x) = \ln(1-4x)^2$

(21) $f(x) = \ln[(1+2x)(1+3x)]$

(22) $f(x) = \ln \dfrac{1+2x}{1+3x}$

Answers:

(1) $\displaystyle\sum_{n=0}^{7} C(7,n)2^n x^n$

(2) $\displaystyle\sum_{n=0}^{\infty} 3^n x^n$

(3) $\displaystyle\sum_{n=0}^{\infty} 2(-4)^n x^n$

(4) $\displaystyle\sum_{n=1}^{\infty} (-5)^{n-1} x^n$

(5) $\displaystyle\sum_{n=4}^{\infty} 2^{n-4} x^n$

(6) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{3}{4}\right)^n x^n$

(7) $\displaystyle -\sum_{n=1}^{\infty} 2^{n-1} x^n$

(8) $\displaystyle\sum_{n=0}^{\infty} x^{3n}$

(9) $\displaystyle\sum_{n=0}^{\infty} 3^n x^{n-1}$

(10) $\displaystyle\sum_{n=0}^{\infty} (n+1)3^n x^n$

(11) $\displaystyle\sum_{n=0}^{\infty} (n+1)(-2)^n x^{n+1}$

(12) $\displaystyle\sum_{n=0}^{\infty} \frac{(n+2)(n+1)}{2}\, 5^n x^n$

(13) $\displaystyle\sum_{n=0}^{\infty} (n+1)\left(\frac{1}{2}\right)^{n-2} x^n$

(14) $\displaystyle\sum_{n=0}^{\infty} -\frac{1}{5}\left(\left(-\frac{1}{2}\right)^{n+1}+\left(\frac{1}{3}\right)^{n+1}\right)x^n$

(15) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{8}\left(\left(\frac{1}{2}\right)^{n+1}-\left(-\frac{1}{6}\right)^{n+1}\right)x^n$

(16) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}\ 2^n x^n$

(17) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}\ (-3)^n x^n$

(18) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}\ 2^n x^{2n}$

(19) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\ 2^n x^n$

(20) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\ 2(-4)^n x^n$

(21) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\ (2^n+3^n)x^n$

(22) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\ (2^n-3^n)x^n$