1. Find each of the following limits. Don't use the rules (comparisons of degrees of the numerator and denominator). Show your work in detail.

(1) $\lim_{x\to\infty} \frac{x+2}{x+3}$

(2) $\lim_{x\to\infty} \frac{2-x}{(x-1)^2}$

(3) $\lim_{x\to\infty} \sec x$

(4) $\lim_{x\to\infty} \frac{x^3+5x}{2x^3-x^2+4}$

(5) $\lim_{x\to\infty} \frac{2x^4+5x^2-3}{2x^6-3x^2+4x-10}$

(6) $\lim_{x\to-\infty} \frac{x^5-7x^3-9x^2+1}{2x^4-x^2+4}$

(7) $\lim_{x\to\infty} \frac{x+3}{\sqrt{x^2-9}}$

(8) $\lim\limits_{x\to\infty} \dfrac{(x^2+4)(x-2)}{(3x^4-9)(x+2)}$

(9) $\lim\limits_{x\to\infty} (x^3+5x^2-10x)$

(10) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^4+5x}}{2x^2-x+4}$

(11) $\lim\limits_{x\to\infty} (\sqrt{16x^2+3}-4x)$
(Hint: Rationalize the numerator.)

(12) $\lim\limits_{x\to\infty} (\sqrt{9x^2+x}-3x)$

(13) $\lim\limits_{x\to\infty} (x^2-\sqrt{x^4-x+5})$

(14) $\lim_{x\to\infty} \frac{1}{x-\sqrt{2x+1}}$

(15) $\lim_{x\to\infty} \frac{\sqrt{4x^3+7x^2-1}-x}{\sqrt{x^4-7x^3+25}-\sqrt{x}}$

(16) $\lim_{x\to\infty} \frac{\sqrt[3]{8x^3-4x}-9x}{5x-6}$

2. Use the definition to show each of the following:

(1) $3x^4-5x^3+7x^2-8=O(x^4)$

(2) $4x^3+9x^2+17x+25=O(x^4)$

(3) $\sqrt{3x^2-6x+1}+7x-1=O(x)$

(4) $\frac{3}{4x^2+8x-1}=O(1)$

3. Determine whether the following is true. If it is true, prove it. If it is false, explain why.

$$3x^4-5x^3+7x^2-8=O(x^5)$$