

Ex 12 - Cop 3

Sejam U_1, U_2, \dots v.a. iid $U(0,1)$.

Defina,

$$N = \min \left\{ n : \sum_{i=1}^n U_i \geq 1 \right\}$$

Estime $E(N)$.

N_1, N_2, \dots, N_n iid

$$\frac{\sum_{i=1}^n N_i}{n} \rightarrow E(N)$$

Ex 13 - Cop 3

Sejam U_1, U_2, \dots v.a. iid $U(0,1)$

Defina

$$M = \max \left\{ n : \prod_{i=1}^n U_i \geq e^{-3} \right\}$$

Estime $E(M)$.

$$\begin{aligned} U_1 &\geq e^{-3} \\ U_1 U_2 &\geq e^{-3} \\ U_1 U_2 U_3 &\geq e^{-3} \\ U_1 U_2 U_3 U_4 &< e^{-3} \end{aligned}$$

N_1

N_2

N_3

1) Estimare π

Algoritmo:

1) $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)$

$X_i \sim U(0,1)$ indep.

$Y_i \sim U(0,1)$

2) $g(X_i, Y_i) = \begin{cases} 1 & \text{se } X_i^2 + Y_i^2 \leq 1 \\ 0 & \text{c.c.} \end{cases}, i=1, \dots, n$

3)

$$\sum_{i=1}^n \frac{g(X_i, Y_i)}{n} \xrightarrow{\text{p.e.}} \frac{\pi}{4}$$

$$\hat{\pi} = 4 \cdot \sum_{i=1}^n \frac{g(X_i, Y_i)}{n}$$

n grande!!