Univariate Time Series Analysis

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SS 2014

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Introduction

Time series analysis:

- Focus: Univariate Time Series and Multivariate Time Series Analysis.
- A lot of theory and many empirical applications with real data
- Organization:
  - 08.04. - 27.05.: Univariate Time Series Analysis, seven lectures (Klaus Wohlrabe) (no lecture 22 April)
  - 03.06. - End of Semester: Multivariate Time Series Analysis (Stefan Mittnik)
  - 14.04. - 26.05. (mondays): Tutorial (Univariate) Malte Kurz 25.04.-30.05. (fridays): Tutorial (Univariate) Andreas Fuest
  - 30.05. - End of Semester: Tutorial (Multivariate): Fabian Spanhel
- $\Rightarrow$ **Lectures and Tutorials are complementary!**

Introduction

Tutorials:

- Next Week (16): Only Monday!
- In two weeks (17): Only Friday!
- in four Weeks (19): Only Monday!

Tutorials

- Login: tsa, Password: armagarch
- Mixture between theory and R - Examples

Literature

- **Shumway and Stoffer (2010): Time Series Analysis and Its Applications: With R Examples**
- Box, Jenkins, Reinsel (2008): Time Series Analysis: Forecasting and Control
- Lütkepohl (2005): Applied Time Series Econometrics.
- Hamilton (1994): Time Series Analysis.
- Lütkepohl (2006): New Introduction to Multiple Time Series Analysis
- Chatham (2003): The Analysis of Time Series: An Introduction
- Neusser (2010): Zeitreihenanalyse in den Wirtschaftswissenschaften

Exam

- Evidence of academic achievements: Two hour written exam both for the univariate and multivariate part
- Schedule for the Univariate Exam: tba (maybe last week of the lecture)

Prerequisites

- Basic Knowledge (ideas) of OLS, maximum likelihood estimation, heteroscedasticity, autocorrelation.
- Some algebra

Software

With costs:

- STATA
- Eviews
- Matlab

Free software:

- R (www.r-project.org)
- Jmulti (www.jmulti) (Based on the book by Lütkepohl (2005))

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Outline

Goals

After the lecture you should be able to ...

- ... identify time series characteristics and dynamics
- ... build a time series model
- ... estimate a model
- ... forecast
- ... check a model
- ... understand financial time series

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Goals and methods of time series analysis

The following section partly draws upon Levine, Stephan, Krehbiel, and Berenson (2002), *Statistics for Managers*.

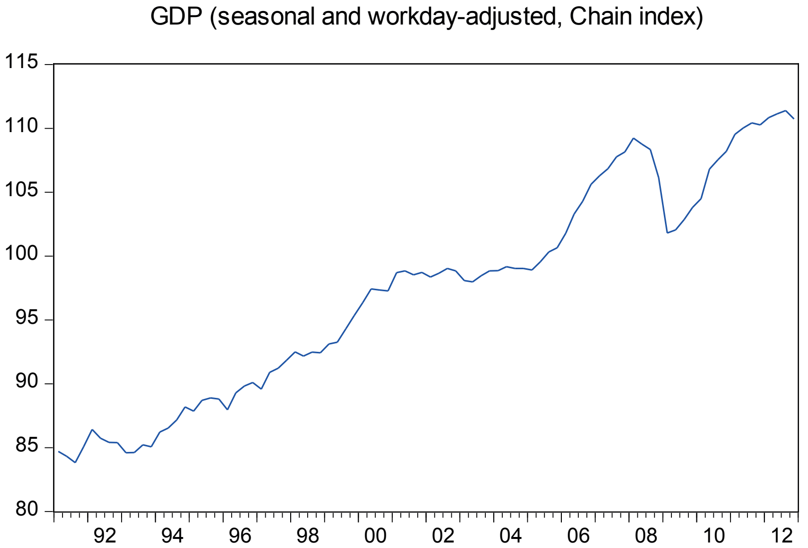
Goals and methods of time series analysis

- understanding time series characteristics and dynamics
- necessity of (economic) forecasts (for policy)
- time series decomposition (trends vs. cycle)
- smoothing of time series (filtering out noise)
  - moving averages
  - exponential smoothing

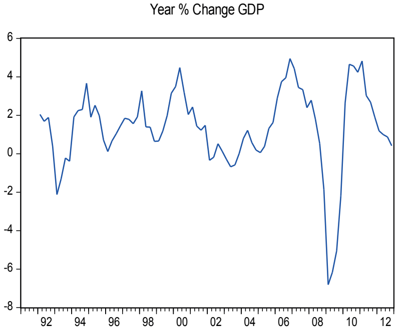
Time Series

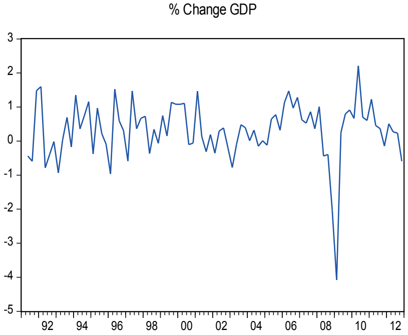
- A time series is timely ordered sequence of observations.
- We denote y_t as an observation of a specific variable at date t.
- A time series is list of observations denoted as $\{y_1, y_2, \dots, y_T\}$ or in short $\{y_t\}_{t=1}^{T}$.
- **What are typical characteristics of times series?**

Economic Time Series: GDP I

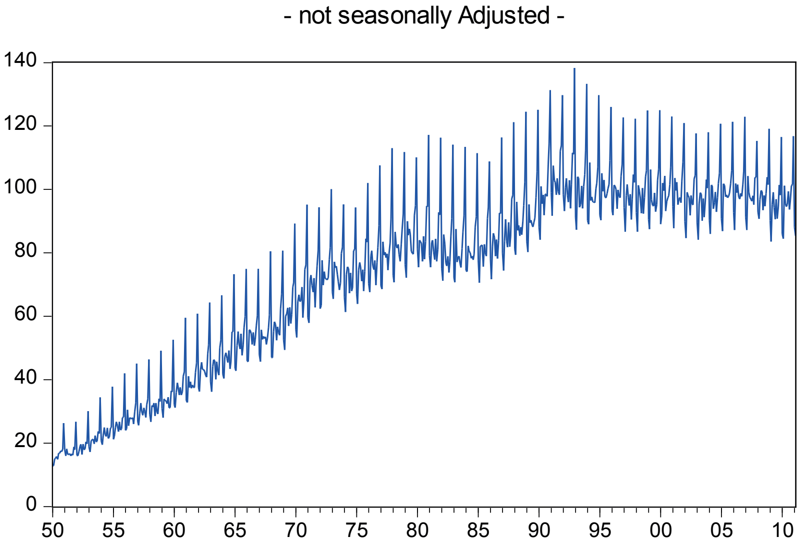


Economic Time Series: GDP II



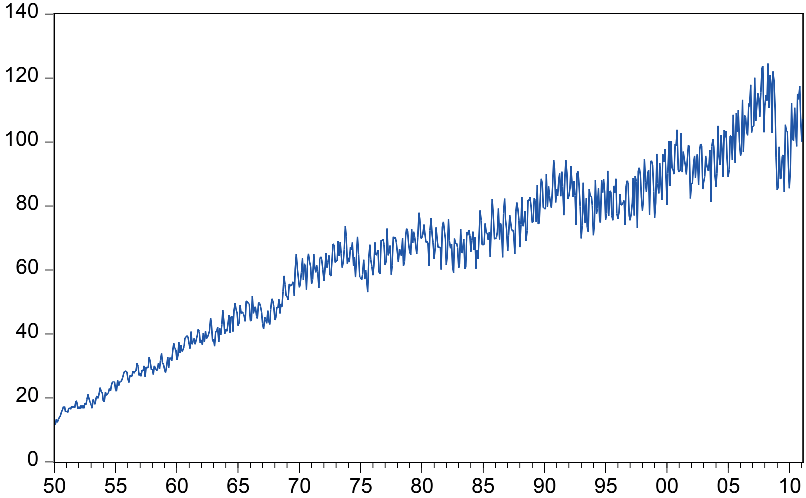


Economic Time Series: Retail Sales

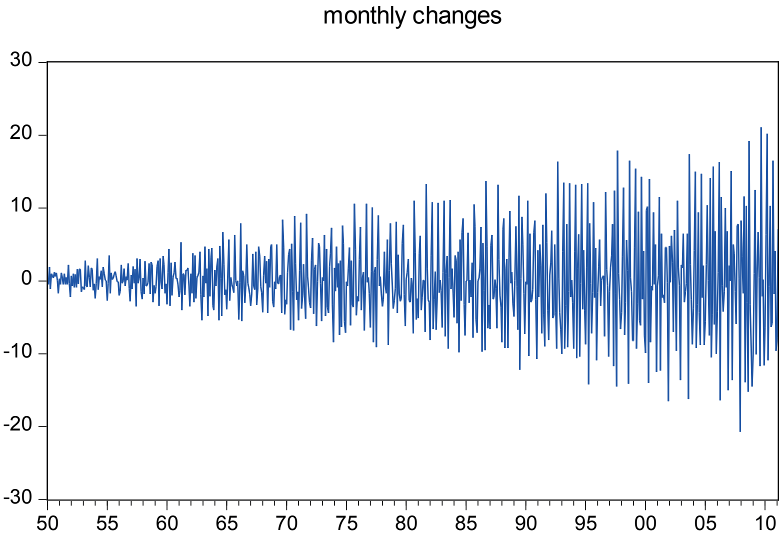


Economic Time Series: Industrial Production

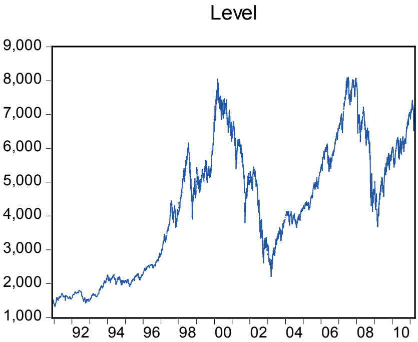
- not seasonally adjusted -

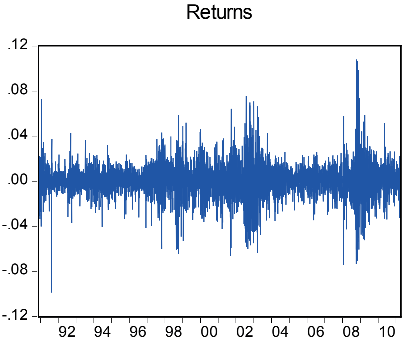


Economic Time Series: Industrial Production

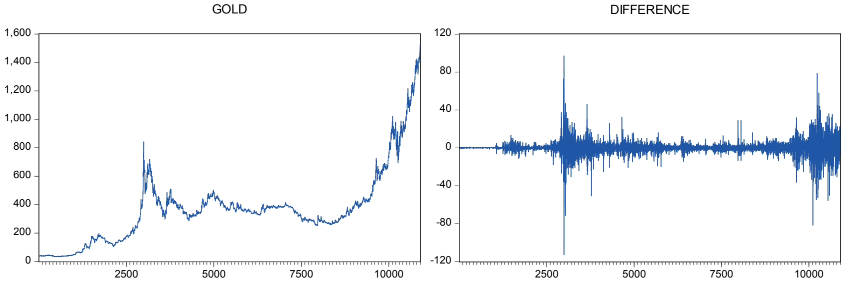


Economic Time Series: The German DAX

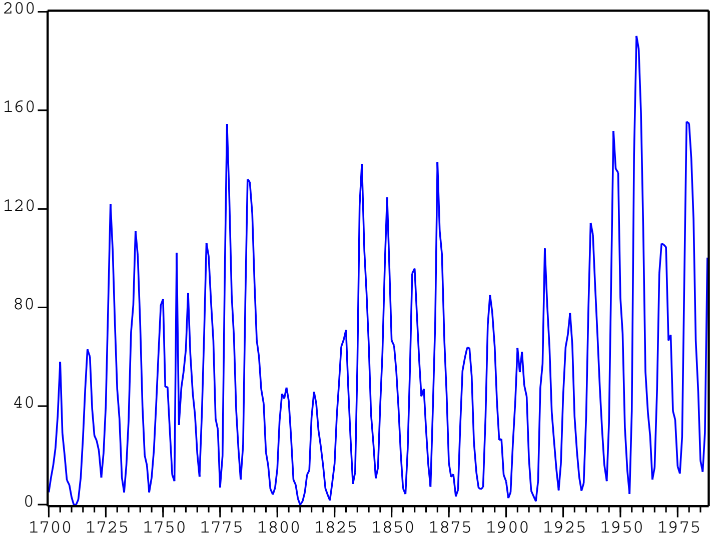




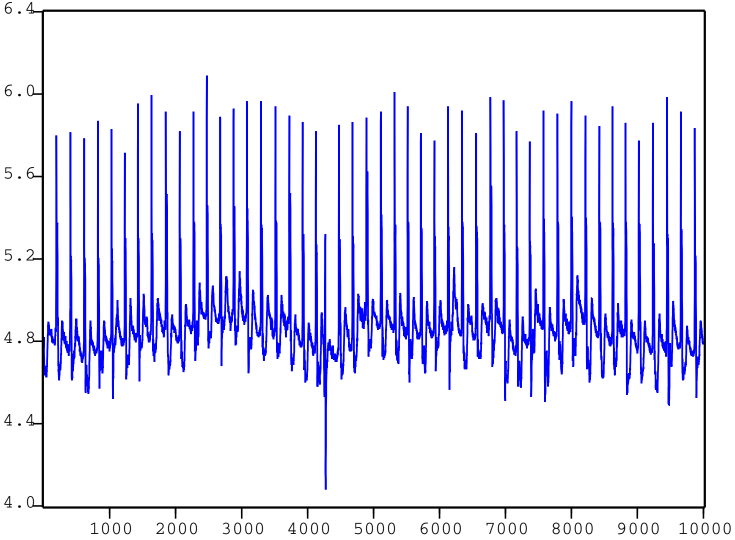
Economic Time Series: Gold Price



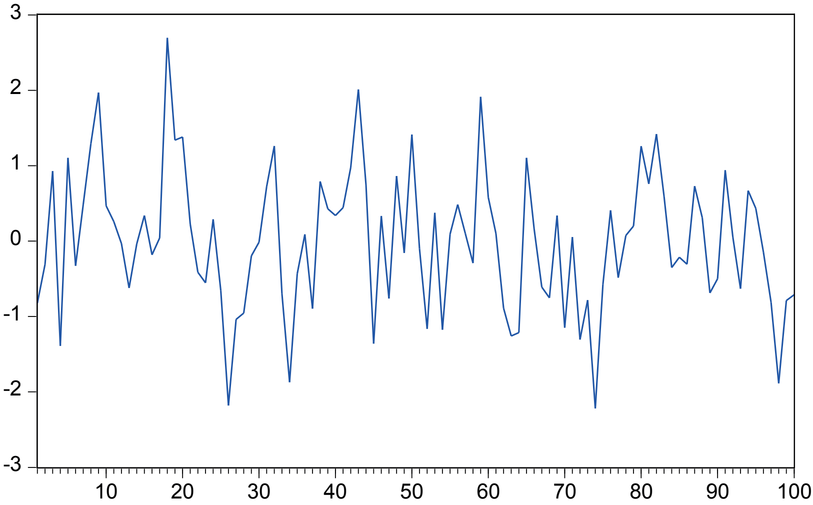
Further Time Series: Sunspots



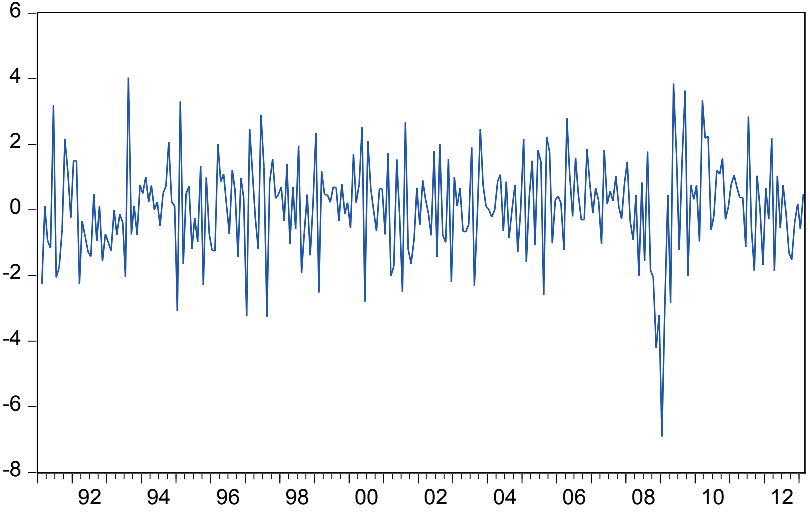
Further Time Series: ECG



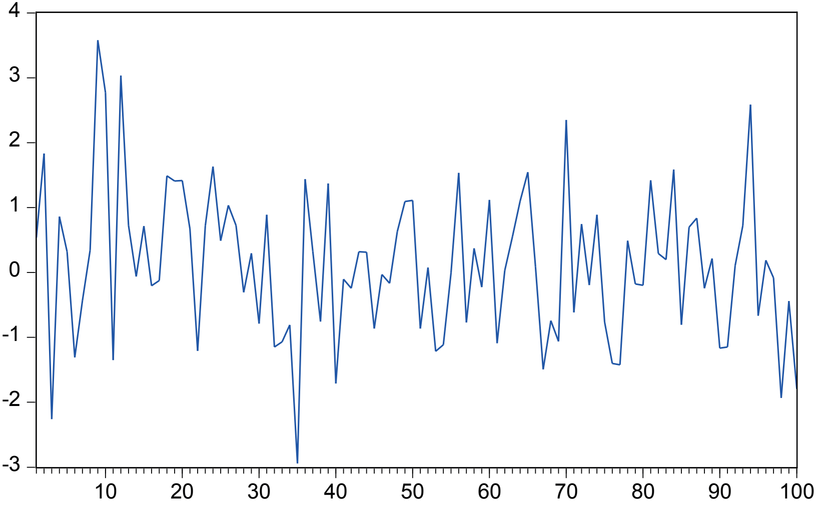
Further Time Series: Simulated Series: AR(1)



Further Time Series: Chaos or a real time series?



Further Time Series: Chaos?



Characteristics of Time series

- Trends
- Periodicity (cyclicly)
- Seasonality
- Volatility Clustering
- Nonlinearities
- Chaos

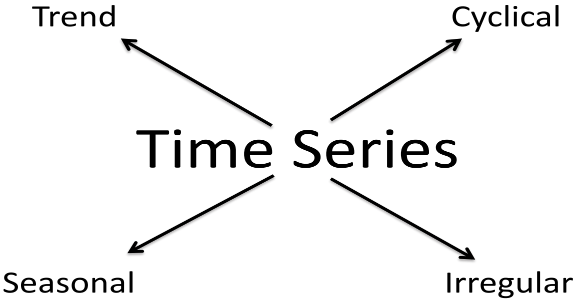
Necessity of (economic) Forecasts

- For political actions and budget control governments need forecasts for macroeconomic variables
  GDP, interest rates, unemployment rate, tax revenues etc.
- marketing need forecasts for sales related variables
  - future sales
  - product demand (price dependent)
  - changes in preferences of consumers

Necessity of (economic) Forecasts

- retail sales company need forecasts to optimize warehousing and employment of staff
- firms need to forecasts cash-flows in order to account of illiquidity phases or insolvency
- university administrations needs forecasts of the number of students for calculation of student fees, staff planning, space organization

Time series decomposition



Time series decomposition

Classical additive decomposition:

$$y_t = d_t + c_t + s_t + \epsilon_t \tag{1}$$

- d_t trend component (deterministic, almost constant over time)
- c_t cyclical component (deterministic, periodic, medium term horizons)
- s_t seasonal component (deterministic, periodic; more than one possible)
- ϵ_t irregular component (stochastic, stationary)

Time series decomposition

Goal:

- Extraction of components d_t, c_t and s_t
- The irregular component

$$\epsilon_t = y_t - d_t - c_t - s_t$$

  should be stationary and ideally white noise.
- Main task is then to model the components appropriately.
- Data transformation maybe necessary to account for heteroscedasticity (e.g. log-transformation to stabilize seasonal fluctuations)

Time series decomposition

The multiplicative model:

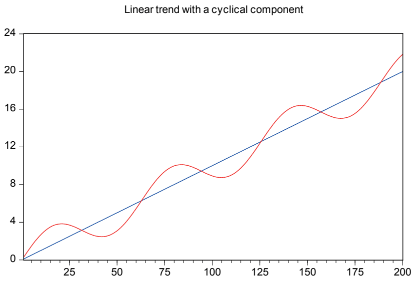
$$y_t = d_t \cdot c_t \cdot s_t \cdot \epsilon_t \tag{2}$$

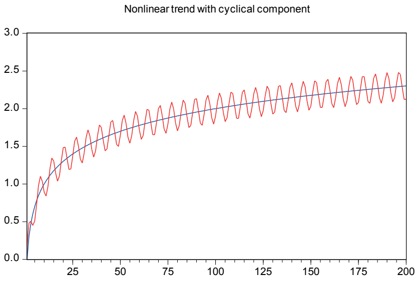
will be treated in the tutorial.

Trend Component

- positive or negative trend
- observed over a longer time horizon
- linear vs. non–linear trend
- smooth vs. non–smooth trends
- $\Rightarrow$ trend is 'unobserved' in reality

Trend Component: Example





Why is trend extraction so important?

The case of detrending GDP

- trend GDP is denoted as **potential output**
- The difference between trend and actual GDP is called the **output gap**
- Is an economy below or above the current trend? (Or is the output gap positive or negative?)
  $\Rightarrow$ consequences for economic policy (wages, prices etc.)
- Trend extraction can be highly controversial!

Linear Trend Model

Year	Time (x_t)	Turnover (y_t)
05	1	2
06	2	5
07	3	2
08	4	2
09	5	7
10	6	6

$$y_t = \alpha + \beta x_t$$

Linear Trend Model

Estimation with OLS

$$\hat{y}_t = \hat{\alpha} + \hat{\beta} x_t = 1.4 + 0.743 x_t$$

Forecast for 2011:

$$\hat{y}_{2011} = 1.4 + 0.743 \cdot 7 = 6.6$$

Quadratic Trend Model

Year	Time (x_t)	Time^2 (x_t^2)	Turnover (y_t)
05	1	1	2
06	2	4	5
07	3	9	2
08	4	16	2
09	5	25	7
10	6	36	6

$$y_t = \alpha + \beta_1 x_t + \beta_2 x_t^2$$

Quadratic Trend Model

$$\hat{y}_t = \hat{\alpha} + \hat{\beta} x_t = 3.4 - 0.757143 x_t + 0.214286 y_t^2$$

Forecast for 2011:

$$\hat{y}_{2011} = 3.4 - 0.757143 \cdot 7 + 0.214286 \cdot 7^2 = 8.6$$

Exponential Trend Model

Year	Time (x_t)	Turnover (y_t)
05	1	2
06	2	5
07	3	2
08	4	2
09	5	7
10	6	6

$$y_t = \alpha\beta_1^{x_t}$$

$\Rightarrow$ Non-linear Least Squares (NLS) or
Linearize the model and use OLS:

$$\log y_t = \log\alpha + \log(\beta_1)x_t$$

$\Rightarrow$ 'relog' the model

Exponential Trend Model

Estimation via **NLS**:

$$\hat{y}_t = \hat{\alpha} + \hat{\beta}_1^{\,x_t} = 0.08 \cdot 1.93^{x_t}$$

Forecast for 2011:

$$\hat{y}_{2011} = 0.08 \cdot 1.93^7 = 15.4$$

Logarithmic Trend Model

Year	Time (x_t)	log(*Time*)	Turnover (y_t)
05	1	log(1)	2
06	2	log(2)	5
07	3	log(3)	2
08	4	log(4)	2
09	5	log(5)	7
10	6	log(6)	6

Logarithmic Trend:

$$y_t = \alpha + \beta_1 \log x_t$$

Logarithmic Trend Model

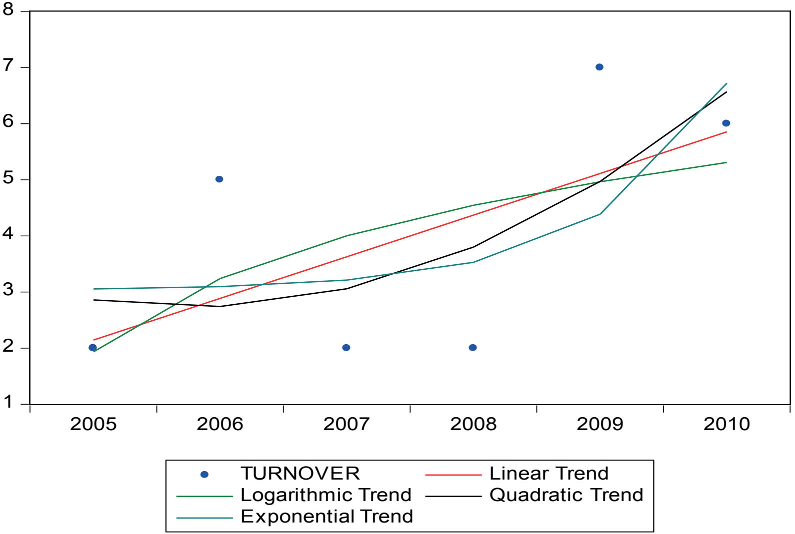
Estimation via **OLS**:

$$\hat{y}_t = \hat{\alpha} + \hat{\beta}_1 \log x_t = 1.934675 + 1.883489 \cdot \log y_t$$

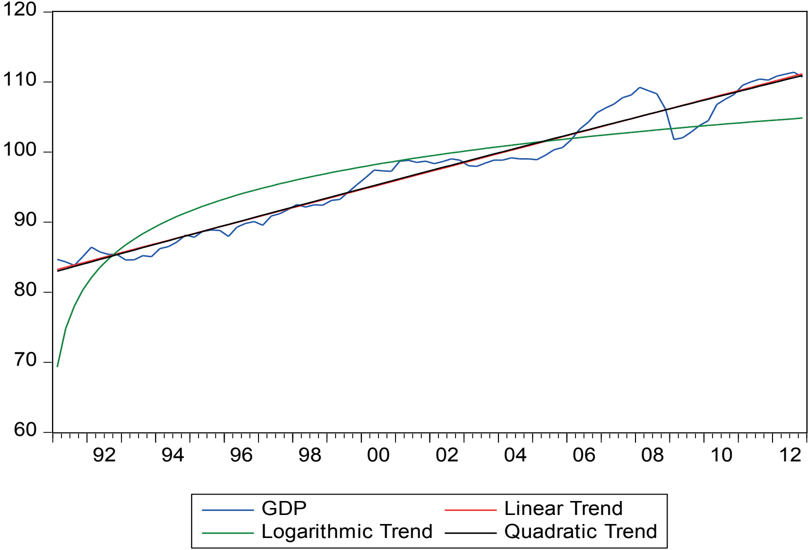
Forecast for 2011:

$$\hat{Y}_{2011} = 1.934675 + 1.883489 \cdot \log(7) = 5.6$$

Comparison of different trend models



Detrending GDP



Which trend model to choose?

- Linear Trend model, if the first differences

$$y_t - y_{t-1}$$

  are stationary

- Quadratic trend model, if the second differences

$$(y_t - y_{t-1}) - (y_{t-1} - y_{t-2})$$

  are stationary

- Logarithmic trend model, if the relative differences

$$\frac{y_t - y_{t-1}}{y_t}$$

  are stationary

The Hodrick-Prescott-Filter (HP)

The HP extracts a flexible trend. The filter is given by

$$\min_{\mu_t} \sum_{t=1}^{T} [(y_t - \mu_t)^2 + \lambda \sum_{t=2}^{T-1} \{(\mu_{t+1} - \mu_t) - (\mu_t - \mu_{t-1})\}^2] \quad (3)$$

where μ_t is the flexible trend and λ a smoothness parameter chosen by the researcher.

- As λ approaches infinity we obtain a linear trend.
- Currently the most popular filter in economics.

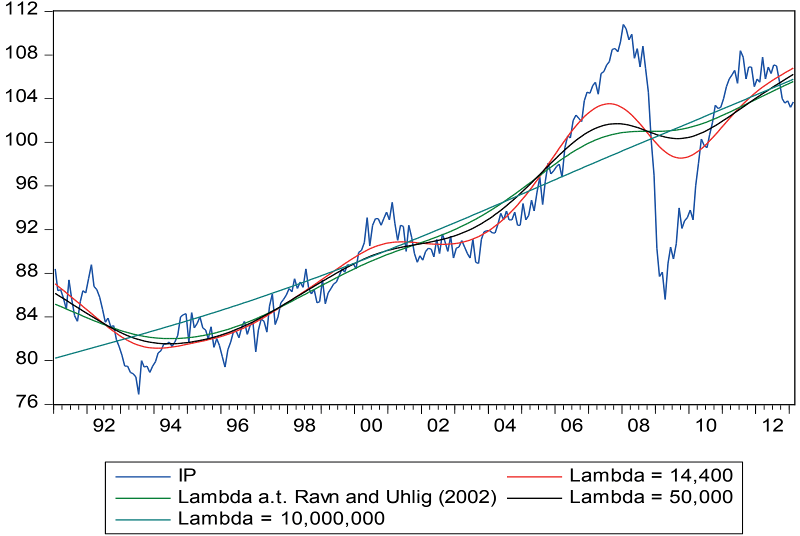
The Hodrick-Prescott-Filter (HP)

How to choose λ?
Hodrick-Prescot (1997) recommend:

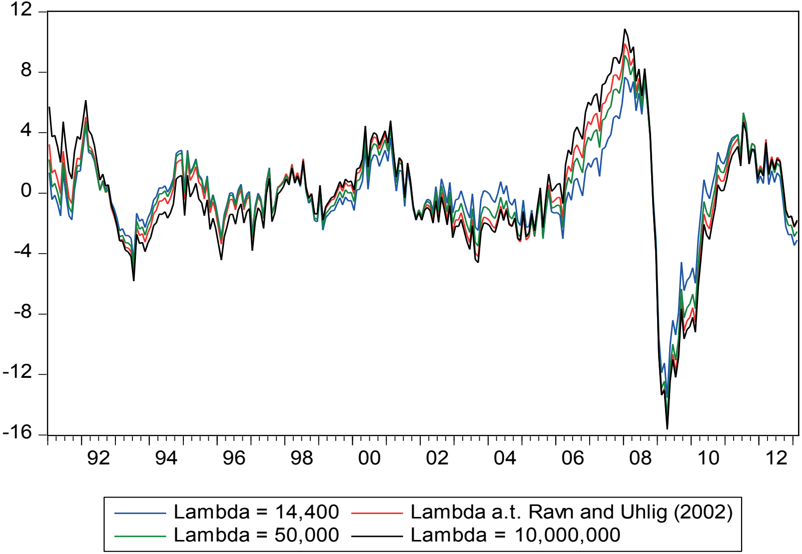
$$\lambda = \begin{cases} 100 \text{ for annual data} \\ 1600 \text{ for quarterly data} \\ 14400 \text{ for monthly data} \end{cases} \tag{4}$$

Alternative: Ravn and Uhlig (2002)

The Hodrick-Prescott-Filter (HP)



The Hodrick-Prescott-Filter (HP)



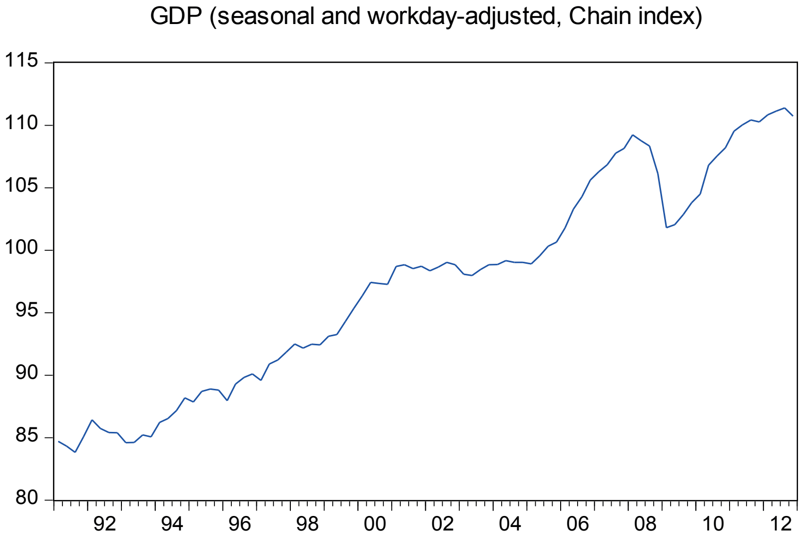
Problems with the HP-Filter

- λ is a 'tuning' parameter
- end of sample instability
  $\Rightarrow$ AR-forecasts

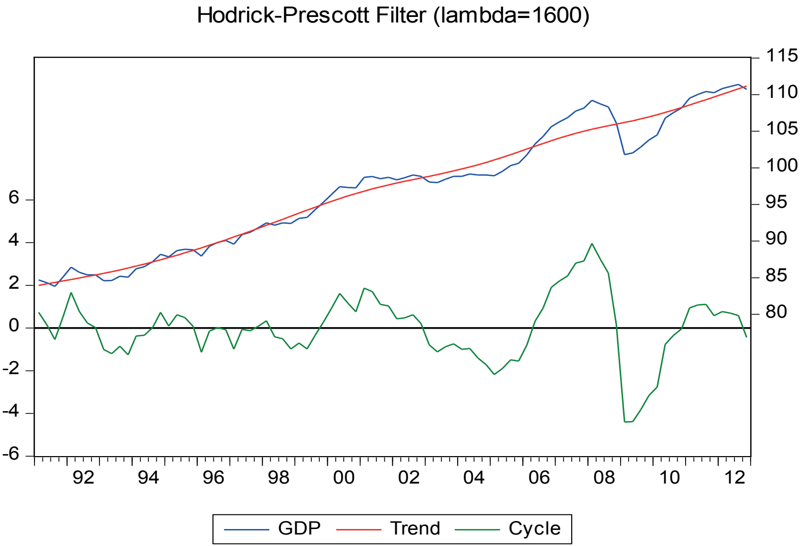
Alternatives

- Frequency filters
  Roughly speaking, the band-pass filter is a linear filter that takes a two-sided weighted moving average of the data where cycles in a “band”, given by a specified lower and upper bound, are “passed” through, or extracted, and the remaining cycles are “filtered” out.
- Baxter-King
- Christiano-Fitzgerald

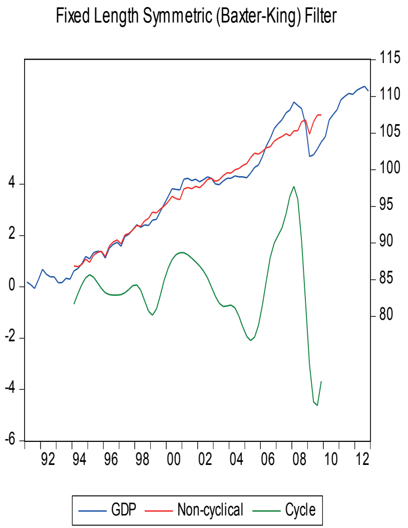
Case study for German GDP: Where are we now?

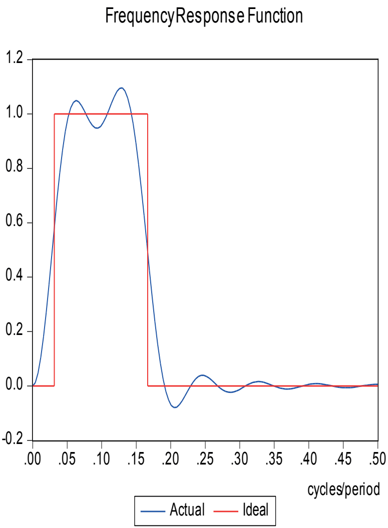


HP-Filter

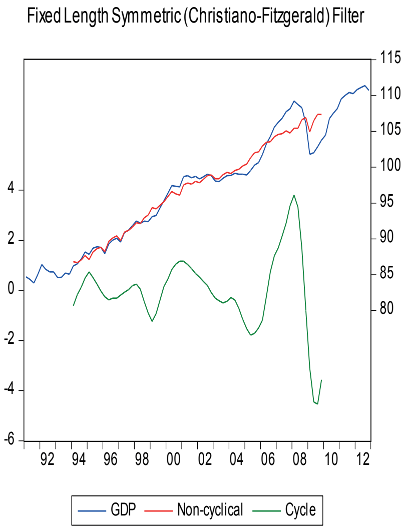

Hodrick-Prescott Filter (lambda=1600)
115
110
105
100
95
90
85
80
6
4
2
0
-2
-4
-6
92
94
96
98
00
02
04
06
08
10
12
GDP
Trend
Cycle

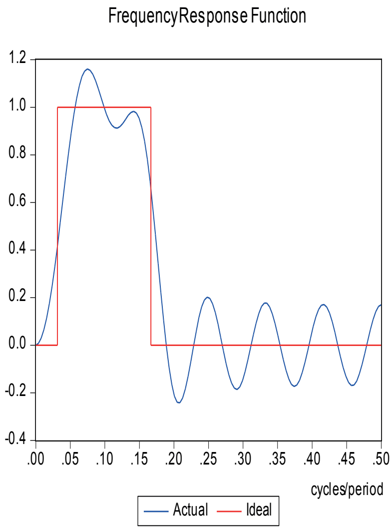
Baxter-King-Filter



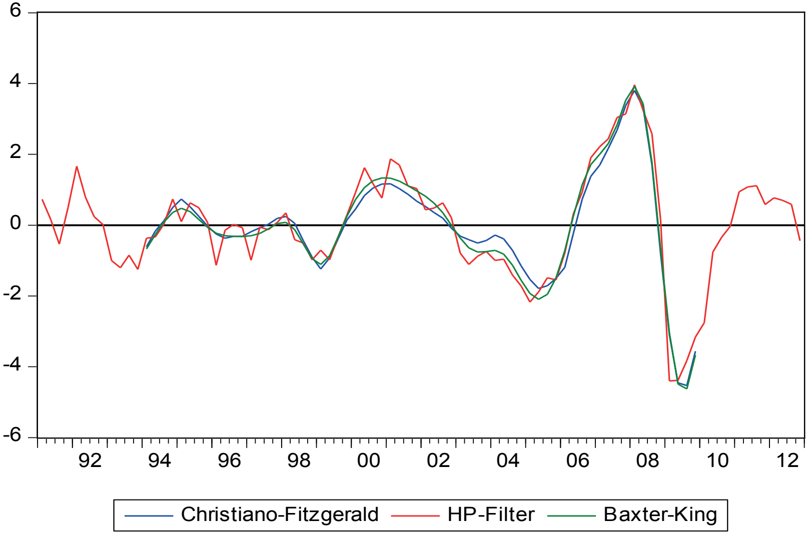


Christiano-Fitzgerald-Filter





Comparison of Cycles



Can we test for a trend?

- Yes and no
- If the trend component significant?
- several trends can be significant
- Trend might be spurious
- Is it plausible that there is a trend?
- A priori information by the researcher
- unit roots

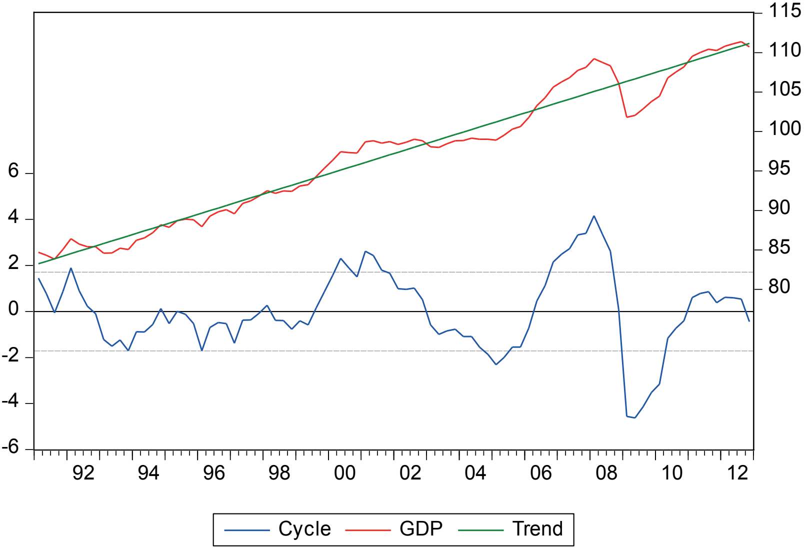
Cyclical Component

- is not always present in time series
- Is the difference between the observed time series and the estimated trend

In economics

- characterizes the Business cycle
- different length of cycles (3-5 or 10-15 years)

Cyclical Component: Example

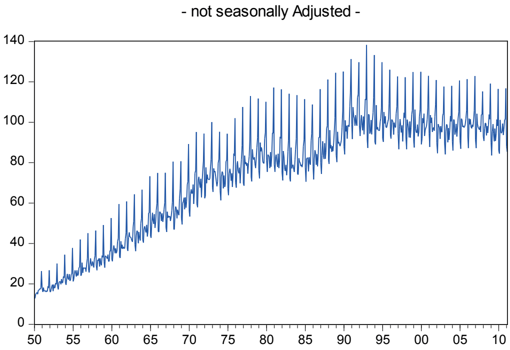


Can we test for a cyclical component?

- Yes and no
- see the trend section
- Is a cycle possible?

Seasonal Component

- similar upswings and downswings in a fixed time interval
- regular pattern, i.e. over a year



Types of Seasonality

- A: $y_t = m_t + S_t + \epsilon_t$
- B: $y_t = m_t S_t + \epsilon_t$
- C: $y_t = m_t S_t \epsilon_t$

Model A is additive seasonal, Models B and C contains multiplicative seasonal variation

Types of Seasonality

- if the seasonal effect is constant over the seasonal periods
  $\Rightarrow$ additive seasonality (Model A)
- if the seasonal effect is proportional to the mean
  $\Rightarrow$ multiplicative seasonality (Model A and B)
- in case of multiplicative seasonal models use the logarithmic transformation to make the effect additive

Seasonal Adjustment

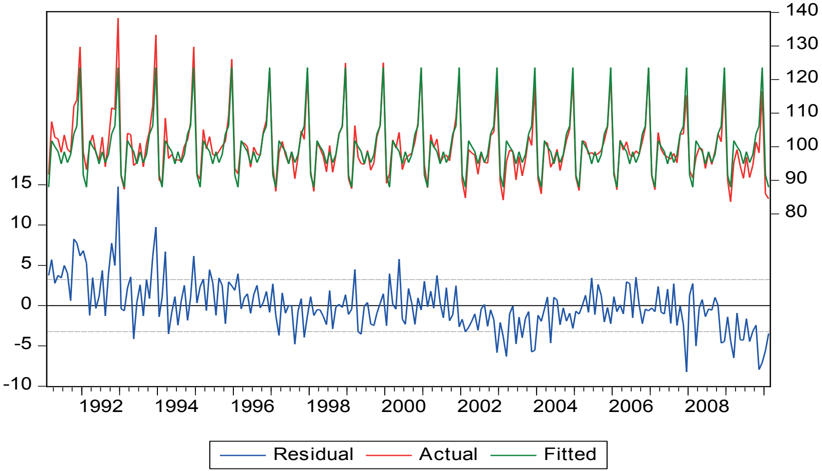
Simplest Approach to seasonal adjustment:

- Run the time series on a set of dummies without a constant (Assumes that the seasonal pattern is constant over time)
- the residuals of this regression are seasonal adjusted
- Example: Monthly data

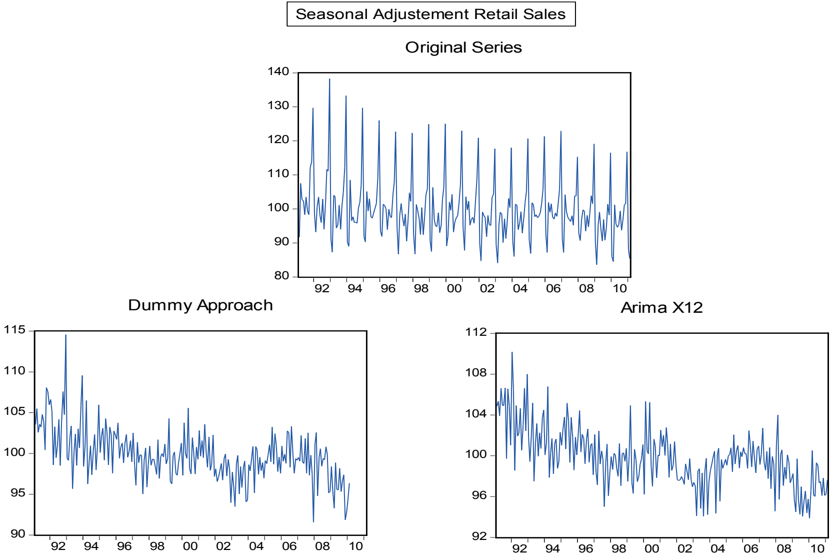
$$\begin{aligned} y_t &= \sum_{i=1}^{12} \beta_i D_i + \epsilon_t \\ \epsilon_t &= y_t - \sum_{i=1}^{12} \hat{\beta} D_i \\ y_{t,sa} &= \epsilon_t + mean(y_t) \end{aligned}$$

- The most well known seasonal adjustment procedure: CENSUS X12 ARIMA

Seasonal Adjustment: Dummy Regression Example



Seasonal Adjustment: Example



Seasonal Moving Averages

For monthly data one can employ the filter

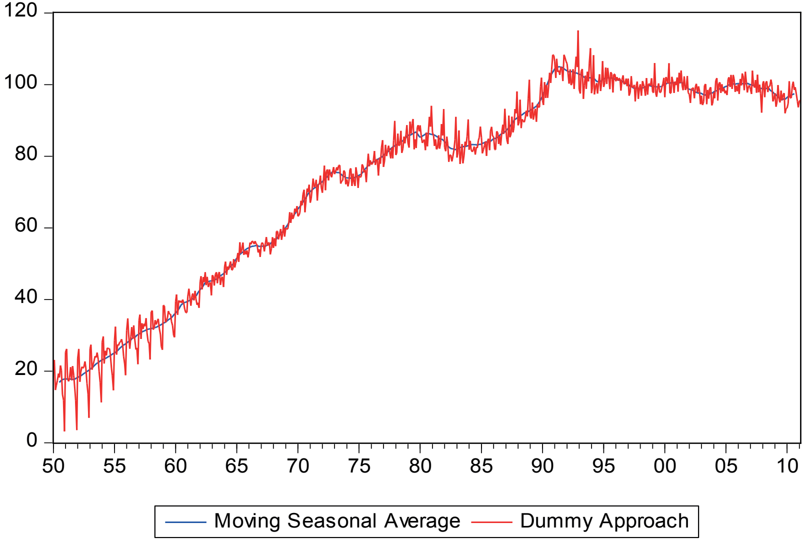
$$SMA(y_t) = \frac{\frac{1}{2}y_{t-6} + y_{t-5} + y_{t-4} + \ldots + y_{t+6} + \frac{1}{2}y_{t+6}}{12}$$

or for quarterly data

$$SMA(y_t) = \frac{\frac{1}{2}y_{t-2} + y_{t-1} + y_t + y_{t+1} + \frac{1}{2}y_{t+2}}{4}$$

- Note: The weights add up to one!
- Standard moving average not applicable

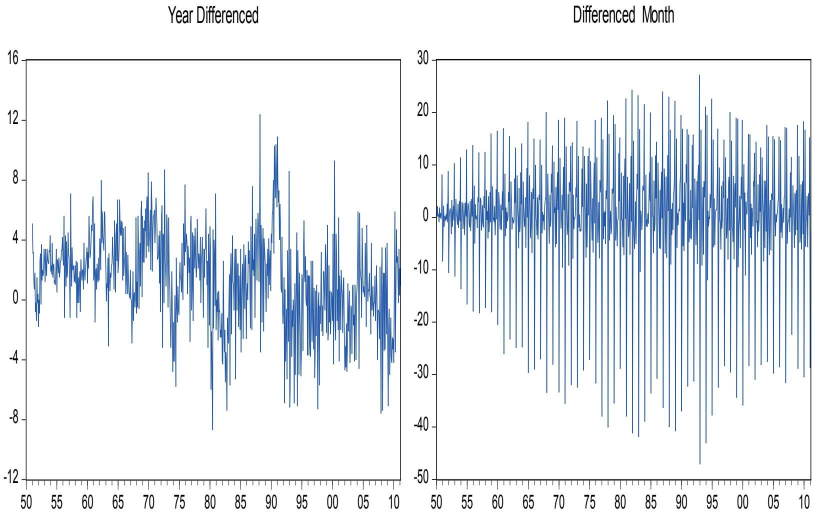
Seasonal Moving Averages: Retail Sales Example



Seasonal Differencing

- seasonal effect can be eliminated using the a simple linear filter
- in case of a monthly time series: $\Delta_{12} y_t = y_t - y_{t-12}$
- in case of a quarterly time series: $\Delta_4 y_t = y_t - y_{t-4}$

Seasonal Differencing: Retail Sales Example



Can we test for seasonality?

- Yes and no
- Does seasonality makes sense?
- Compare the seasonal adjusted and unadjusted series
- look into the ARIMA X12 output

Irregular Component

- erratic, non-systematic, random "residual" fluctuations due to random shocks
  - in nature
  - due to human behavior
- no observable iterations

Can we test for an irregular component?

- YES
- several tests available whether the irregular component is a white noise or not

Linear Filters

A linear filter converts one times series (x_T) into another (y_t) by the linear operation

$$y_t = \sum_{r=-q}^{+s} a_r x_{t+r}$$

where a_r is a set of weights. In order to smooth local fluctuation one should chose the weight such that

$$\sum a_r = 1$$

Moving Average

- Used for time series smoothing.
- Consists of a series of arithmetic means.
- Result depends on the window size L (number of included periods to calculate the mean).
- In order to smooth the cyclical component, L should exceed the cycle length
- L should be uneven (avoids another cyclical component)

Moving Average

$$
\begin{aligned}
MA(y_t) &= \frac{1}{2q+1} \sum_{r=-q}^{+q} y_{t+r} \\
L &= 2q+1
\end{aligned}
$$

where the weights are given by

$$a_r = \frac{1}{2q+1}$$

Moving Average

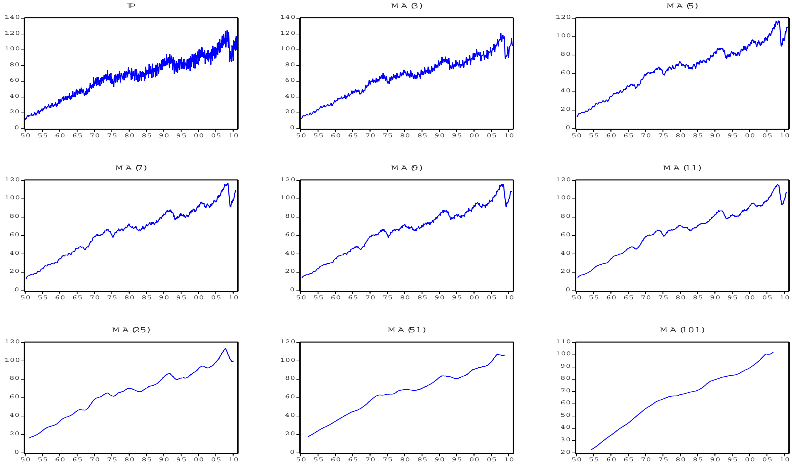
Example: Moving Average (MA) over 3 Periods

- First MA term: $MA_1(3) = \frac{y_1+y_2+y_3}{3}$
- Second MA term: $MA_2(3) = \frac{y_2+y_3+y_4}{3}$

Moving Average

Year	Projects	MA(3) L=3
2005	2	
2006	5	3
2007	2	3
2008	2	3.67
2009	7	5
2010	6	

Moving Average



⇒ the larger L the smoother and shorter the filtered series

Exponential Smoothing

- weighted moving averages
- latest observation has the highest weight compared to the previous periods

$$\hat{y}_t = wy_t + (1 - w)\hat{y}_{t-1}$$

Repeated substitution gives:

$$\hat{y}_t = w\sum_{s=0}^{t-1}(1 - w)^s y_{t-s}$$

$\Rightarrow$ that´s way it is called exponential smoothing, forecasts are the weighted average of past observations where the weights decline exponentially with time.

Exponential Smoothing

- Is used for smoothing and short–term forecasting
- Choice of w:
  - subjective or through calibration
  - numbers between 0 and 1
    Close to 0 for smoothing out unpleasant cyclical or irregular components
    Close to 1 for forecasting

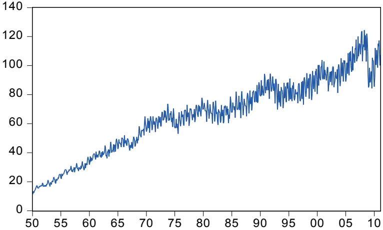
Exponential Smoothing

$$\hat{y}_t = wy_t + (1-w)\hat{y}_{t-1} \quad w = 0.2$$

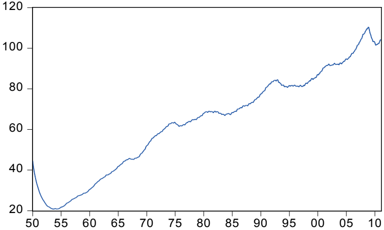
Year	Projects	Smothed Value	Forecast
2005	2	2	-
2006	5	0.2*5+0.8*2 =2.6	2.000
2007	2	0.2*2+0.8*2.6 =2.48	2.600
2008	2	0.2*2+0.8*2.48 =2.3684	2.480
2009	7	0.2*7+0.8*2.384=3.307	2.384
2010	6	0.2*6+0.8*3.307 =3.846	3.307

Exponential Smoothing

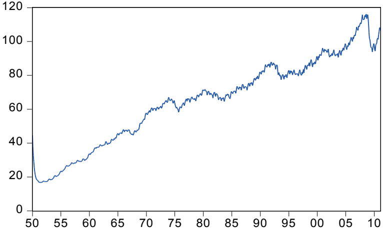
IP



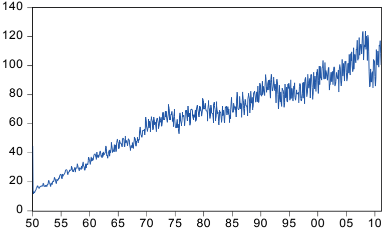
w=0.05



w=0.2



w=0.95



White Noise

A process $\{y_t\}$ is called **white noise** if

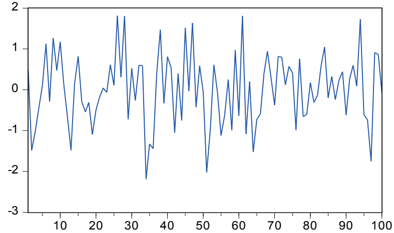
$$\begin{aligned} \mathrm{E}(y_t) &= 0 \\ \gamma(0) &= \sigma^2 \\ \gamma(h) &= 0 \text{ for } |h| > 0 \end{aligned}$$

$\Rightarrow$ all y_t's are uncorrelated. We write: $\{y_t\} \sim WN(0, \sigma^2)$

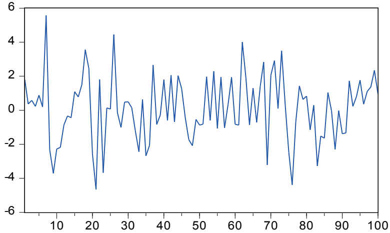
White Noise

White Noise

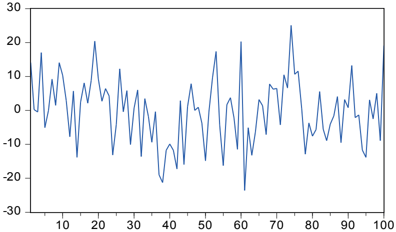
WN with Variance = 1



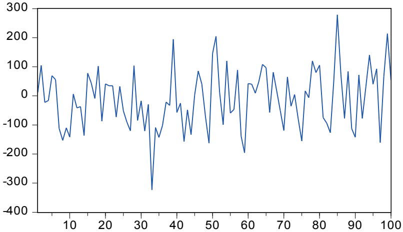
White Noise with Variance = 2



White Noise with Variance = 10



White Noise with Variance = 100



Random Walk (with drift)

A simple *random walk* is given by

$$y_t = y_{t-1} + \epsilon_t$$

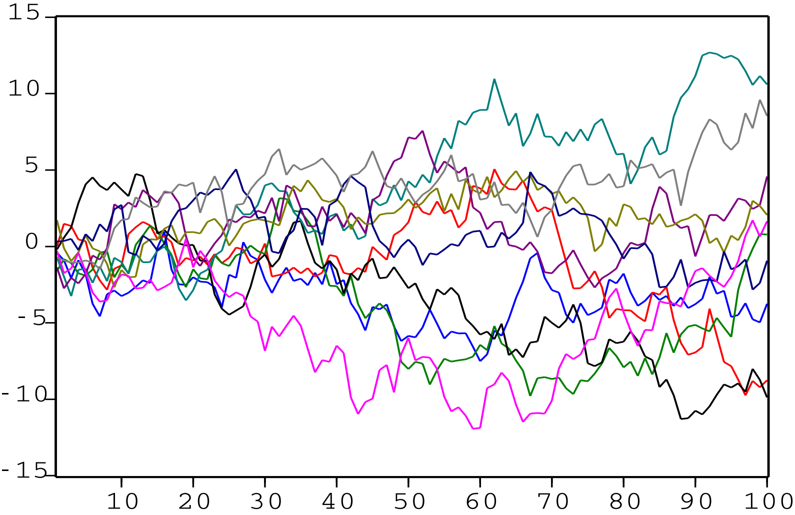
By adding a constant term

$$y_t = c + y_{t-1} + \epsilon_t$$

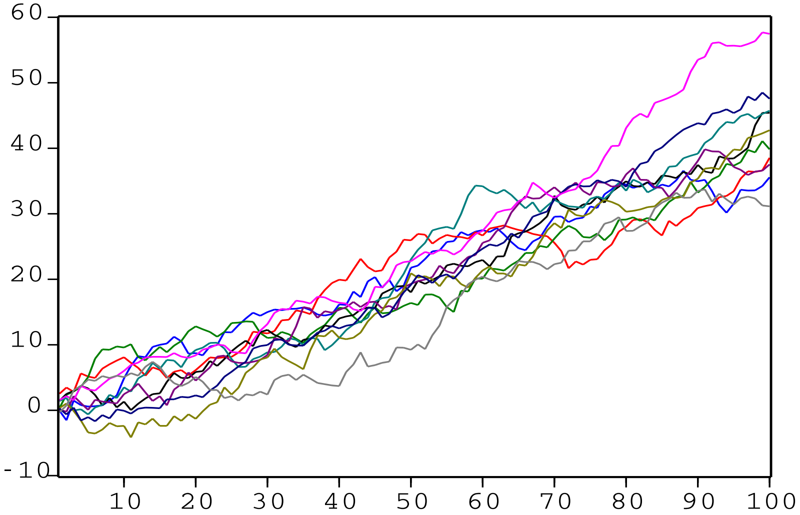
we get a *random walk with drift*. It follows that

$$y_t = ct + \sum_{j=1}^{t} \epsilon_j$$

Random Walk: Examples



Random Walk with Drift: Examples



Autoregressive processes

- especially suitable for (short-run) forecasts
- utilizes autocorrelations of lower order
  - 1st order: correlations of successive observations
  - 2nd order: correlations of observations with two periods in between
- Autoregressive model of order p

$$y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \ldots + \beta_p y_{t-p} + \epsilon_t$$

Autoregressive processes

Number of machines produced by a firm

Year	Units
2003	4
2004	3
2005	2
2006	3
2007	2
2008	2
2009	4
2010	6

$\Rightarrow$ Estimation of an AR model of order 2

$$y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \epsilon_t$$

Autoregressive processes

Estimation Table:

Year	Constant	y_t	y_{t-1}	y_{t-2}
2003	1	4		
2004	1	3	4	
2005	1	2	3	4
2006	1	3	2	3
2007	1	2	3	2
2008	1	2	2	3
2009	1	4	2	2
2010	1	6	4	2

$\Rightarrow$ OLS

$$\hat{y}_t = 3.5 + 0.8125 y_{t-1} - 0.9375 y_{t-2}$$

Autoregressive processes

Forecasting with an AR(2) model:

$$\begin{aligned}\hat{y}_t &= 3.5 + 0.8125 y_{t-1} - 0.9375 y_{t-2} \\ y_{2011} &= 3.5 + 0.8125 y_{2010} - 0.9375 y_{2009} \\ &= 3.5 + 0.8125 \cdot 6 - 0.9375 \cdot 4 \\ &= 4.625\end{aligned}$$