AI for Biotechnology
Exercise 1

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Exercise E1.1

Have a look at the following Python code:

```python
import numpy as np
x = np.array([1,2,3], [4,5,6])
```

Why does this code snippet not create a 2-dimensional array?

Exercise E1.2

Have a look at the following Python code:

```python
import numpy as np
x = np.array([1,2,3])
y = np.array([[1,2,3]])
```

What is the difference between the `x` and `y`?

Exercise E1.3

Execute the following code:

```python
import numpy as np
X = np.linspace(1,96,96).reshape(3,8,4)
```

Answer the following questions using Python:

a) How many dimensions has the array `X`, how many elements does the array have and what is the size of each dimension?

b) Use indexing or slicing to obtain the element 44.0

c) Use indexing or slicing to obtain `[13., 14., 15., 16.]`

d) Use indexing or slicing to obtain `[2., 6., 10., 14., 18., 22., 26., 30.]`

e) Use indexing or slicing to obtain the 3×3 array

```
[[74., 75., 76.],
 [78., 79., 80.],
 [82., 83., 84.]]
```

f) Use indexing or slicing to obtain the 3×4 array

```
[[ 9., 10., 11., 12.],
 [17., 18., 19., 20.],
 [25., 26., 27., 28.]]
```

g) Use indexing or slicing to obtain the 2×2 array

```
[[ 9., 10.],
 [41., 42.]]
```

Exercise E1.4

Given is the following Python code to draw 100 random samples from a Gaussian distribution with zero mean ($\mu = 0$) and a standard deviation of $\sigma = 10$:

```
np.random.seed(41)
x = np.random.normal(0,10,100)
```

Use boolean indexing to return a sub-array that only contains values that are less or equal than 1. What is the size of the new sub-array?

Exercise E1.5

The Gaussian function with mean μ and standard deviation σ is defined as follows:

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{1}$$

Write a Python function with the name `gaussian` to calculate the Gaussian function for a given mean, standard deviation and a list of x-values. Further, compute and plot the distributions of the Gaussians for $\mu = 0$ and four different standard deviations $\sigma \in \{1, 3, 5, 10\}$. Use a grid of 10'000 x-values in the interval of $-20 \le x \le 20$.

Exercise E1.6

In this exercise we will use the Python library `scipy`. This library contains additional functionality important for scientific computing, e.g. advanced solvers for optimisation problems. For this exercise we will just use the library to load a RGB image. Images are stored in a simple 3-dimensional array $m \times n \times 3$. Each of the three $m \times n$ arrays represents one of the three colour planes, as illustrated in Figure 1.

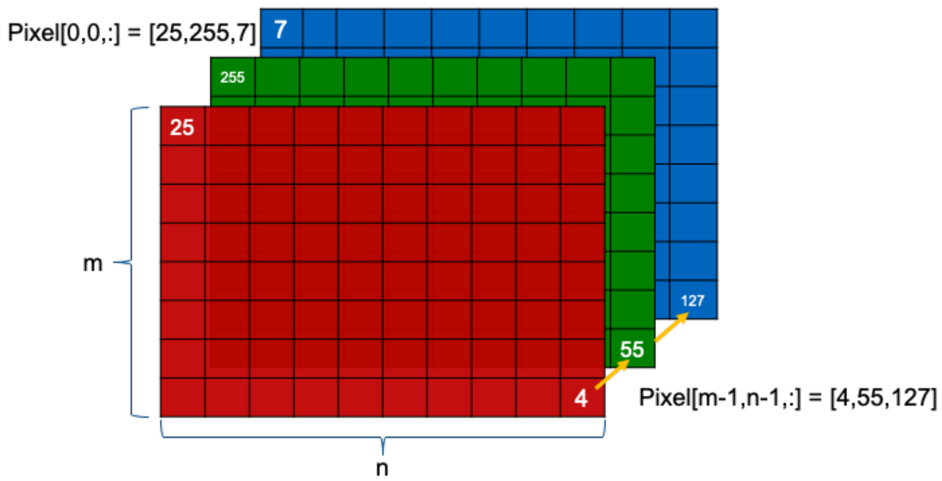

Figure 1: Representation of an RGB image as NumPy array

Values within the array range between 0 and 255, representing the color intensity of the pixel at position (i,j). The following code will load and plot the image, as illustrated in Figure 2.

```
import scipy.misc as misc
%matplotlib inline
import pylab as pl
im = misc.face()
pl.imshow(im)
```

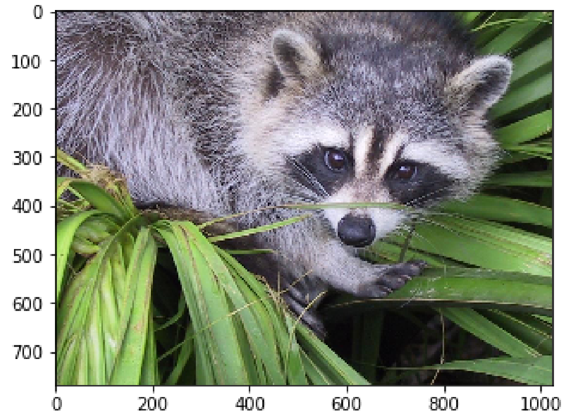

Figure 2: RGB image plotted using matplotlib

Answer the following questions and solve the tasks:

a) What is the shape of the image `im`?

b) How many pixels has the image?

c) Use indexing and slicing to crop the image, such that you get a closer look at the face of the animal. Use the plotting functionality to check your results!

d) Create a copy of your image using the command `new_im = im.copy()`. Set all intensity values in the copy to zero, except for the green plane and visualise your image.

e) Again, create a copy of your original image using the command `new_im = im.copy()`. Use boolean indexing to set all values for the red plane to 255 that are greater than 127.

Exercise E1.7

Given are the two matrices $\boldsymbol{A} \in \mathbb{R}^{3\times2}$ and $\boldsymbol{B} \in \mathbb{R}^{3\times2}$:

$$\boldsymbol{A} = \begin{pmatrix} -2 & 3 \\ 4 & 1 \\ -1 & 5 \end{pmatrix} \text{ und } \boldsymbol{B} = \begin{pmatrix} 1 & 4 \\ 0 & -2 \\ 3 & 5 \end{pmatrix}$$

and the two vectors $\boldsymbol{x} = (8, -5)^\top$ und $\boldsymbol{y} = (3, 2)^\top$. Compute the following expressions using the `NumPy` library:

a) $\boldsymbol{A} + \boldsymbol{B}$

b) $\boldsymbol{A}(\boldsymbol{x} + \boldsymbol{y})$

c) $(\boldsymbol{A} + \boldsymbol{B})\boldsymbol{x}$

d) $\boldsymbol{x}^\top \boldsymbol{y}$

e) $5\boldsymbol{A}\boldsymbol{B}^\top$

Exercise E1.8

Find out yourself how to compute the inverse of a matrix using the `NumPy` library and invert the following matrices:

a) $\boldsymbol{A} = \begin{pmatrix} 1 & 4 & -1 \\ -1 & -3 & 5 \\ 5 & 19 & -8 \end{pmatrix}$

b) $\boldsymbol{B} = \begin{pmatrix} 0 & 1 & -2 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}$

Exercise E1.9

Given are the two matrices $\boldsymbol{X} \in \mathbb{C}^{3\times3}$ and $\boldsymbol{Y} \in \mathbb{C}^{3\times3}$:

$$\boldsymbol{X} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{i}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \\ 1 & 0 & 0 \end{pmatrix} \text{ und } \boldsymbol{Y} = \begin{pmatrix} 2 & i & 0 \\ -i & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Compute the following expression using the `NumPy` library (Hint: complex numbers have the type `np.complex` and complex numbers are written as `1.j` in `NumPy`):

$$\boldsymbol{R} = \overline{\boldsymbol{X}}^\top \boldsymbol{Y} \boldsymbol{X},$$

where $\overline{\boldsymbol{X}}$ is the complex conjugate of $\boldsymbol{X}$. Print the matrix $\boldsymbol{R}$, the real values of $\boldsymbol{R}$ and the diagonal of the real values of $\boldsymbol{R}$.