ENED 1091: Homework #4 Solutions

INCLUDE UNITS WHEREVER APPLICABLE IN YOUR ANSWERS!

Problem 1: The linear velocity of a piston connected to a crank of radius, r, with a rod of length, L, is given by:

$$v(t) = -r\omega\sin(\theta) - \frac{r^2\omega\sin(2\theta)}{2L}$$

ω is the angular velocity of the crank (radians/sec) and θ is the crank angle. Suppose r = 0.02 m, L = 0.4 m, and ω = 120 r.p.m.

(a) Convert the angular velocity, ω, to radians per second. **Show Calculations.**

$$\omega = 120\frac{\text{rev}}{\text{min}} * \frac{1\text{min}}{60\text{sec}} * \frac{2\pi\ \text{radians}}{\text{rev}} = 12.57\ \text{radians/sec}$$

(b) Plug the values for r, L, and ω into the linear velocity equation. Using MATLAB, take the first derivative of velocity with respect to the crank angle, θ. Then find all valid angle values (CriticalPoints) which make the first derivative zero.

MATLAB Commands:

Critical Points (Angles): **Choose real angles only (either radians or degrees is fine)**

$$\theta = \pm 1.521\ radians\ or\ \pm 87.15^o$$

(c) Apply the 2nd derivative test to prove which crank angle results in a maximum linear velocity and which crank angle results in a minimum linear velocity.

MATLAB Commands:

Angle at which Velocity is a Maximum: –1.521 radians or –87.15°

Angle at which Velocity is a Minimum: 1.521 radians or 87.15°

(d) Plug the angles derived in part (c) into the velocity equation to find the maximum and minimum linear velocity of the piston.

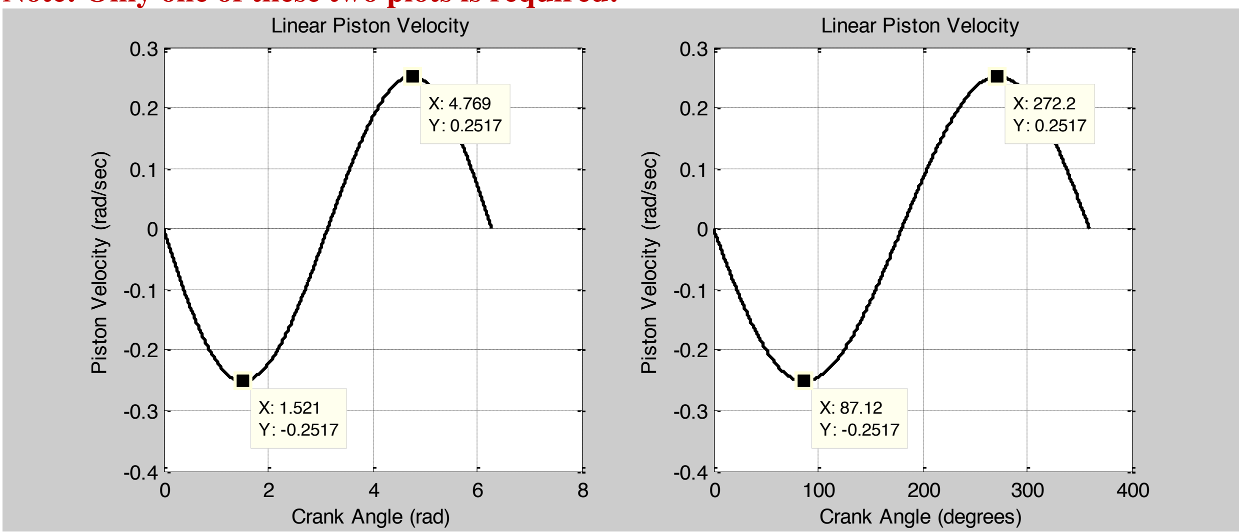
Maximum Velocity: 0.2517 radians/sec

Minimum Velocity: –0.2517 radians/sec

(e) Verify your results graphically by plotting linear velocity vs. crank angle. The crank angle, θ, should go on the x-axis and vary from 0 to 2π radians or 0 to 360° (your choice). Mark the maximum and minimum linear velocities using the data cursor tool. Make sure your plot is labeled and includes units.

Plot of Velocity with Max and Min Points Marked:

Note: Only one of these two plots is required.



<u>Problem 2</u>: In Lab 1, you determined the angular paths for the two-link robot arm given starting coordinates of (−8.5, 0), ending coordinates of (6, 2), links lengths of 4 and 5, and a final time of 10 seconds.

$$Th1 = Th1_{start} + k_1t^5 + k_2t^4 + k_3t^3$$

$$Acc1 = 20 * k_1t^3 + 12 * k_2 * t^2 + 6 * k_3 * t$$

Th1_start = `158.4578`
k1 = `-0.00527`
k2 = `0.13171`
k3 = `-0.87807`

In lab, you were asked to estimate the times at which velocity and acceleration were maximized and minimized using your graphs of velocity and acceleration. In this problem, you will apply calculus to mathematically determine the times at which acceleration is maximized and minimized.

(a) Find all times, t, (CriticalPoints) at which acceleration is either maximized or minimized.

MATLAB Commands and/or

Times (CriticalPoints) at which Acceleration is Maximized or Minimized:
2.1135 seconds and 7.8835 seconds

(b) Apply the 2nd derivative test to determine the time at which maximum acceleration occurs and the time at which minimum acceleration occurs.

MATLAB Commands and/or Work:

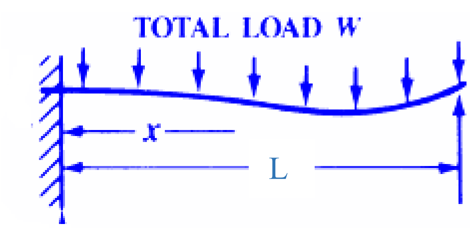
Time at which Acceleration is a Maximum: 7.8835 seconds
Time at which Acceleration is a Minimum: 2.1135 seconds

(c) Plug the times into the acceleration equation to determine maximum and minimum acceleration.

Maximum Acceleration = 5.0537 deg/s^2
Minimum Acceleration = –5.0699 deg/s^2

Note: You can check your results by looking at your acceleration graph for the base angle from Lab 1.

Problem 3: Consider the diagram of the beam below. The beam is fixed at one end, supported at the other end, and has a uniform load of W, Newtons.



Modified version of the diagram from this site:
http://www.engineersedge.com/beam_bending/beam_bending11.htm

The stress, S (N/m^2), at any point on the beam is given by:

$$S = \frac{W}{2ZL} * (L - x) * (0.25L - x)$$

W is the load, Z is the section modulus of the cross-section of the beam, L is the length of the beam, and x is the distance from the fixed end.

(a) Suppose W = 20,000 N and Z = 1 m^3. Find the distance, x, from the fixed end of the beam at which the stress will be minimized. Note: your answer will be a function of beam length, L.

MATLAB Commands and/or Work:

Distance x at which stress is minimized: 0.625*L m

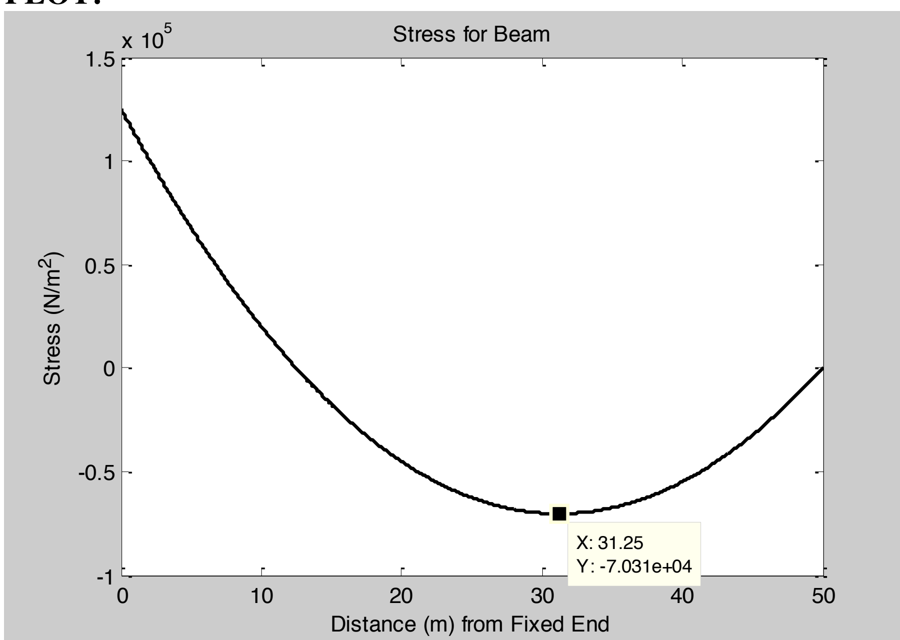
(b) Assume L = 50m. Find the distance x from the fixed end of the beam at which stress is minimized and calculate the minimum stress.

x = 0.625*50 = 31.25 m

Minimum S = –70313 N/m²

(c) Verify your results graphically by plotting stress, S, on the y-axis versus distance, x. Use the data cursor to mark the point of minimum stress.

MATLAB PLOT:



MATLAB Commands for Plot:

<u>Problem 4</u>: Consider the same beam from Problem 3. The deflection, y (m), at any point on the beam is given by:

$$y = \frac{W}{48E \cdot I \cdot L} * x^2(L - x)(3L - 2x)$$

W is the load, E is the modulus of elasticity, I is the moment of inertia, L is the length of the beam, and x is the distance from the fixed end of the beam.

(a) Assume W = 20,000 N, $E = 5*10^{10}$ N/m², and I = 0.5m⁴. Find all points, x, at which the deflection of the beam is either minimized or maximized. Note: x may again be a function of beam length, L.

MATLAB commands and/or Work:

Values of x for minimum or maximum deflection: 0 and 0.5785*L meters

(b) Use the 2nd derivative test to determine where minimum and maximum deflection occur.

Value of x for minimum deflection = 0 m
Value of x for maximum deflection = 0.5785*L m

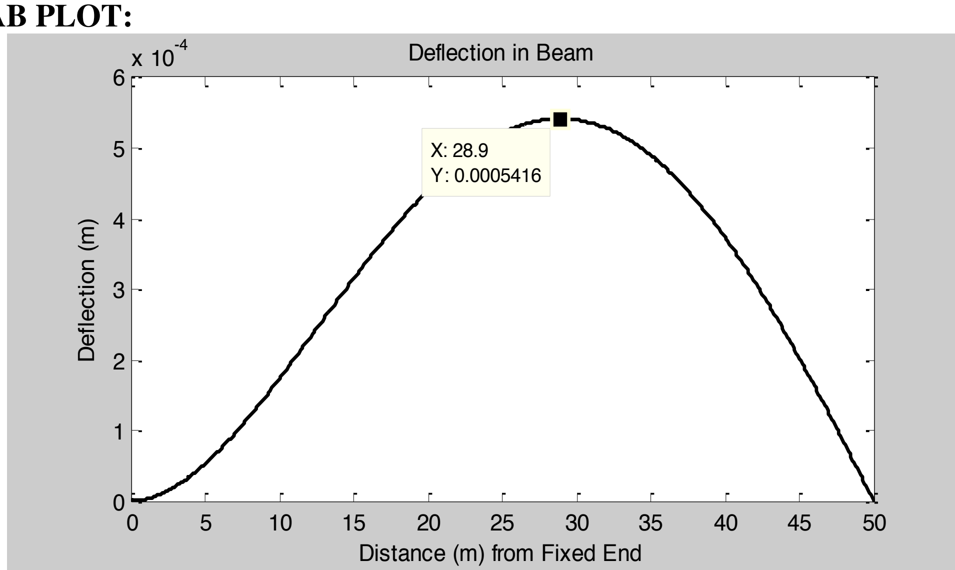
MATLAB commands and/or Work:

(c) Assume L = 50m. Find the point on the beam where maximum deflection occurs and calculate the maximum deflection.

Distance x at which deflection is a maximum = 0.5785*50 = 28.925 m
Maximum Deflection = 5.4161e-04 m = 0.54161 mm

(d) Verify your results graphically by plotting deflection on the y-axis versus distance x. Use the data cursor tool to mark the point of maximum deflection.

MATLAB PLOT:



MATLAB Commands for Plot: