General identities researched and discovered when I was young. Probably all of them are known, that was the usual trend I found, and I doubt I'll have any use for them these days, but these are the forms I cut my *brute-force-formula-manipulating teeth* on, so to speak. Fun. No proofs are given. If nothing else they are formal identities.

1. $$(fg)^{(n)} = \sum_{0\le k\le n} \binom{n}{k} f^{(n-k)} g^k$$

2. $$(f\circ g)^{(n)} \text{ - index algorithm } (2,1,1)$$

3. $$(fg\ldots yz)' = f'g\ldots yz + fg'\ldots yz + \ldots + fg\ldots y'z + fg\ldots yz'$$

4. $$mn = \sum_{k=1}^{m+n} k - \left(\sum_{k=1}^{m} k + \sum_{k=1}^{n} k\right)$$

5. $$\int_{0^+}^{1} \frac{t^n}{(t^t)^w} dt = \sum_{k\ge 0} \frac{w^k}{(n+k+1)^{k+1}} = \frac{1}{n+1} + \frac{w}{(n+2)^2} + \frac{w^2}{(n+3)^3} + \ldots$$

6. $$F'(x) = f(x) \quad \Longrightarrow \quad \int f^{-1}(x)dx = xf^{-1}(x) - F(f^{-1}(x))$$

7. $$(n+1)! = \sum_{k=1}^{n} k!k$$

8. $$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$$

9. $$\frac{1+\sqrt{4x+1}}{2} = \sqrt{x+\sqrt{x+\sqrt{x+\sqrt{\ldots}}}}$$

10. $$\frac{1+\sqrt{4x-3}}{2} = \sqrt{x+\sqrt{x-\sqrt{x+\sqrt{\ldots}}}}$$

11. $$\ln x = \lim_{n\to 0} \frac{1}{n}\left[1-\left(\frac{1}{x}\right)^n\right]$$

12. $$1 = \sum_{k=0}^{n} (-1)^k \binom{n}{k}\binom{2n-k}{n}$$

13. $$S = 1 + 2(1) + 3(1+2(1)) + 4(1+2(1)+3(1+2(1))) + \ldots \quad \Longrightarrow \quad 2S_n = n!$$

14.

$$(x+1)\left(x-\frac{1}{3}\right)\left(x+\frac{1}{2}\right)(x+3) = x^4 + \frac{25}{6}x^3 + \frac{7}{2}x^2 - \frac{1}{6}x - \frac{1}{2} \quad \sim \quad \begin{array}{r|lllll} 1 & 1 & & & & \\ -\frac{1}{3} & 1 & 1 & & & \\ \frac{1}{2} & 1 & \frac{2}{3} & -\frac{1}{3} & & \\ 3 & 1 & \frac{7}{6} & 0 & -\frac{1}{6} & \\ & 1 & \frac{25}{6} & \frac{7}{2} & -\frac{1}{6} & -\frac{1}{2} \end{array}$$

15.

$$\int_a^b f(x)dx = f(c)(b-a) \quad \Longrightarrow \quad \int_{0^+}^1 x^x dx = c^c = 1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \dots$$

16.

$$\sqrt{a^2+4b} = a + \frac{2b}{a + \frac{b}{a+\frac{b}{a+\dots}}}$$

17.

$$\int_a^\infty \frac{dt}{(t^t)^w} \quad \leq \quad \frac{1}{(a^a)^w} \cdot \frac{1}{w \ln a}, \quad a > 1, \quad (\text{converges } a \to \infty)$$

18.

$$f = e^{ax},\ g = e^{bx} \quad \Longrightarrow \quad (a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

19.

$$\frac{\partial h}{\partial x} h(x,w) = e^{wp(x)} \quad \Longleftrightarrow \quad \int [p(x)]^n dx = \left(\frac{d}{dw}\right)^{(n)} h(x, w \to 0)$$

20.

$$a_n = b_n + c_n a_{n-d} \quad \Longrightarrow \quad a_n = b_n + \sum_{\ell=1}^{m} \left(\prod_{p=0}^{\ell-1} c_{n-dp}\right) b_{n-d\ell} + \left(\prod_{p=0}^{m} c_{n-dp}\right) a_{n-d(m+1)}$$

21.

$$f^{(n)}(x) \quad \exists \Longrightarrow \quad \left(\frac{d}{dx}\right)^{(n)} f(e^x) = \sum_{k=1}^{n} \left\{ {n \atop k} \right\} f^{(k-1)}(e^x) e^{kx}$$

22.

$$R(n, n-m) = \sum_{k=m}^{n} a_k R(k-1, k-m), \quad R(n,n) = 1$$

$$\Longrightarrow \quad R(n, n-m) = R(n-1, n-m-1) + a_n R(n-1, n-m)$$

$$\Longrightarrow \quad (x+a_1)(x+a_2)(\dots)(x+a_n) = R(n,n)x^n + R(n,n-1)x^{n-1} + \dots + R(n,1)x + R(n,0)$$

23.

$$F'(x) = f(x) \wedge f^{-1}(a) = b \quad \Longrightarrow \quad F(x) = xf(x) - ab - F(b) - \int_a^{f(x)} f^{-1}(t)dt$$

24.

$$a = 1 + xa \quad \Longrightarrow \quad \frac{x^{n+1}-1}{x-1} = 1 + x + x^2 + \dots + x^{n-1} + x^n$$

25.

$$1 < a \quad \Longrightarrow \quad \int_1^\infty \frac{dt}{t^t} \quad \leq \quad \int_1^a \frac{dt}{t^t} + \frac{1}{a^a \ln a}$$

26.

$$1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \dots \sim 0.783430510709$$

27.

$$1+\frac{1}{2^2}+\frac{1}{3^3}+\frac{1}{4^4}+\ldots\sim 1.29128599704$$

28.

$$a_n := \int x^n e^{x^2} dx \quad \Longrightarrow \quad a_n = \frac{1}{2}x^{n-1}e^{x^2} - \frac{n-1}{2}a_{n-2}$$

29.

$$\left(\frac{d}{dx}\right)^{(n)} \sin x = \sin(x+\pi n)$$

30.

$$\int_{0^+}^{1} f(\ln\left(\frac{1}{t^t}\right)^w)dt = \sum_{m\geq 0}\frac{f^{(m)}(0)}{(m+1)^{m+1}}w^m = -\int_{0^+}^{1}\ln t\; f(\ln\left(\frac{1}{t^t}\right)^w)dt$$

31.

$$\sum_{n\geq 0}\frac{f^{(n)}(0)}{n^n}z^n = f(0) + z\int_0^1 f'(-zt\ln t)dt$$

32.

$$\int_{0^+}^{1}\frac{t^m(\ln t)^n}{(t^t)^w}dt = (-1)^n n!\sum_{k\geq 0}\binom{n+k}{n}\frac{w^k}{(m+k+1)^{n+k+1}}$$

33.

$$f(e^x) = 1+\sum_{k\geq 1} f^{(k-1)}(1)\sum_{m\geq k}\left\{ {m \atop k} \right\}\frac{x^m}{m!}$$

34.

$$\int t^m(\ln t)^n dt = \frac{(-1)^n n! t^{m+1}}{(m+1)^{n+1}}\sum_{0\leq j\leq n}\frac{[-(m+1)\ln t]^j}{j!}$$

35.

$$\int_{0^+}^{1}\frac{dt}{1+wt\ln t} = \sum_{m\geq 0}\frac{m!}{(m+1)^{m+1}}w^m, \quad |w|<e$$

36.

$$\rho_n^x := \sum_{m\geq 2}\frac{m^x}{m!}(\ln m)^n$$

$$\rho_0^n = e\sum_{k=1}^{n}\left\{ {n \atop k} \right\} - 1$$

$$\rho_0^{n+1} - \rho_0^n = \sum_{k=1}^{n} k\left\{ {n \atop k} \right\}$$

$$\left(\frac{d}{dx}\right)^{(m)}\rho_n^x = \rho_{n+m}^x$$

$$\rho_n^x = \sum_{k\geq 0}\rho_{n+k}^w\frac{(x-w)^k}{k!}$$

$$\rho_0^n - \rho_0^m = \sum_{k\geq 1}\rho_k^m\frac{(n-m)^k}{k!}$$

$$\rho_0^{n-1} + \rho_1^n - \rho_0^n = \sum_{k\geq 0}\rho_{k+1}^{n-1}\frac{k}{(k+1)!}$$

37.

$$\cos\frac{\pi}{5} = \frac{1+\sqrt{5}}{4}$$

38.

$$\cos\frac{\pi}{7} = \frac{1}{6}\left[1+2\sqrt{7}\cos\frac{\theta}{3}\right], \quad \cos\theta = \frac{-1}{2\sqrt{7}}, \quad \sin\theta = \frac{3\sqrt{3}}{2\sqrt{7}}, \quad \theta \sim 1.76092193$$

39.

$$\cos\frac{\pi}{7} = x \quad \Longrightarrow \quad 0 = 8x^3 - 4x^2 - 4x + 1$$

40. Given the well known cubic equation (not mine):

$$ax^3 + bx^2 + cx + d = 0$$

$$\Longrightarrow \quad x \;=\; \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) + \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}}$$

$$+ \; \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) - \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} - \frac{b}{3a}$$

we then have:

$$\Longrightarrow \quad \cos\frac{\pi}{7} \;=\; \frac{1}{6} + \frac{1}{6}\sqrt[3]{\frac{21}{2}}\left(\sqrt[3]{-\frac{1}{3}+\sqrt{3}i} + \sqrt[3]{-\frac{1}{3}-\sqrt{3}i}\right)$$