The Empirical Frequency of a Pivotal Vote

Author(s): Casey B. Mulligan and Charles G. Hunter

Source: *Public Choice*, Jul., 2003, Vol. 116, No. 1/2 (Jul., 2003), pp. 31-54

Published by: Springer

Stable URL: https://www.jstor.org/stable/30025867

Public Choice **116:** 31–54, 2003.

The empirical frequency of a pivotal vote *

CASEY B. MULLIGAN & CHARLES G. HUNTER
University of Chicago and Lexecon

Accepted 19 June 2002

Abstract. Some economic theories of voting suggest that competition leads to close elections, and that election closeness is a factor for bringing voters to the polls. How often in fact are civic elections decided by one vote? One of every 89,000 votes cast in U.S. Congressional elections, and one of 15,000 in state legislator elections, "mattered" in the sense that they were cast for a candidate that tied or won by one vote. We find an inverse relationship between election size and the frequency of one vote margins. Recounts, and other margin-specific election procedures, are determinants of the pivotal vote frequency.

How often are civic elections decided by one vote? This question has been asked numerous times in the economics literature and results derived from economic and statistical models of voting have been offered as answers (e.g., Beck, 1975; Goode and Mayer, 1975; Margolis, 1977; Chamberlain and Rothschild, 1981; Fischer, 1999). Many of these authors (and others, such as Gelman et al., 1998) have pointed out how the result of the calculation very much depends on how the model is built – for example, should each vote be modeled as an independent draw from a binomial distribution? – and how the model is calibrated. The purpose of this paper is to nonparametrically compute *empirical* frequencies of close elections, and their variation with election turnout, and to compare the empirical frequencies with those predicted by some economic and statistical models.[1]

Perhaps it is common knowledge that civic elections are not often decided by one vote.[2] But the costs of voting are often small, so a precise calculation of the frequency of a pivotal vote can contribute to our understanding of how many, if any, votes might be rationally and instrumentally cast. Furthermore, the distribution of electoral outcomes offers some information about the validity of various models of voting. For example, two economic theories of voting suggest that close elections should be more common than predicted by statistical models. One suggests that competing candidates try to position themselves near or at the preferences of the median voter, which makes close

* We appreciate the comments of Andrew Gelman, Jose Liberti, Howard Margolis, Jeff Russell, Steve Stigler, Economics 396 students, participants at the University of Chicago's Econometrics and Statistics Colloquium, and the financial support of the Smith-Richardson Foundation. Jakob Bluestone provided able research on an earlier version, Mulligan and Hunter (2000).

elections especially likely. The swing voter theory suggests that voters themselves attempt to make elections close, so that the "right" voters are pivotal. Telser's (1982) theory predicts the opposite: that winning majorities will be large, and therefore relatively few elections will be close, because winning majorities are in part liable for costs they create for the losing minority.

Our paper begins by introducing the set of elections studied, and providing an overview of the returns in those elections. Section 2 then calculates frequencies of pivotal votes, for the sample as a whole and as a function of election size. In calculating and interpreting the empirical frequency of close elections, we avoid many of the modeling questions raised in previous studies but we encounter two issues that may be unique to the empirical approach. The first issue, addressed in Section 2, is the calculation of standard errors on the empirical frequencies, since we expect them to be nontrivial when calculating frequencies that are so small. Section 3 then compares the empirical frequencies to probabilities calculated in the theoretical literature. Section 4 examines some of the very close elections in some detail, and raises a second question: how should election return statistics be interpreted when those returns are almost always disputed when an election turns out to be close? Section 5 concludes.

1. Data

Two data sets are used. The first is vote counts for elections to the United States House of Representatives for the election years 1898–1992 (hereafter "US returns"). The second is state legislative election returns in the United States for the years 1968–89 (hereafter "STATE returns").

US returns are reported electronically in ICPSR study #6311, and are divided into two samples. One sample is the "exceptions file," containing those 839 U.S. House elections where a minority candidate won, or a Democrat was not opposed by a Republican, or a Republican was not opposed by a Democrat. These 839 elections are omitted from our tabulations. Another 3228 elections were omitted for which vote counts for Democrat or Republican were omitted (7), zero (3081), or otherwise uncontested (140), leaving 16577 US returns for analysis.

STATE returns are reported electronically in ICPSR study #8907. We analyze only contested general elections in both single- and multi-member districts, except for those in multi-member free-for-all districts of which there are 5,785. Of the remaining elections, 2,478 are reported as having missing votes (at either the candidate or election level) and are excluded. Of the remaining 51,262 elections, 11,047 are coded as being uncontested, and another 179 are de facto uncontested once vote totals for candidates with similar names have been combined. This leaves 40,036 elections included in the analysis.[3]

As reported by the ICPSR, New York STATE candidates running under multiple party labels are listed multiple times, with their votes under each label listed separately. Since the election outcome depends on each candidate's votes under all labels, we aggregated vote totals, in each district and for each post, by name. In 173 cases (out of 93,142 candidate records for the national sample of 40,036 elections satisfying the above selection criteria), we assume that names spelled very similarly were the same candidate. We compute the election margin as the number of votes cast for the winner with the least votes (usually, but not always, there was one winner) minus the number of votes cast for the loser with the most votes.

Summary statistics for both samples are displayed in Table 1. The US returns are for many more years, as compared to STATE returns, but many fewer districts each year. The median election in both samples is not close by any definition; the margin is 22% or 25% of total votes cast. STATE elections are substantially smaller, both in terms of total votes cast and absolute margins. The STATE distribution of total election votes seems to be more skewed than the US distribution, with the STATE average total votes 1.7 times median total votes (compare with 1.1 times for US elections).

Since the samples include several years, two rows in our table are provided to give the reader some information about the cross-section dimension of our data. Those two rows are the number of elections and districts sample in 1988, the most recent election year included in both datasets. The STATE sample has about ten times more districts in its typical cross section than does the US sample. We also see that each district has exactly one election in 1988.

2. The empirical distribution of election returns

Before comparing the returns data with particular voting theories, we first present the overall relationship between election margin and frequency. We then offer some calculations of the empirical frequency of a pivotal vote, and how that frequency varies with election size.

2.1. *The overall relationship between margin and frequency*

Figure 1a shows how the modal percentage US election margin (i.e., the absolute election margin divided by the sum of votes cast for Democratic and Republican candidates) is four or five percent.[4] Beyond four or five percent, the density of percentage election margin appears to decline monotonically.

Table 1. Sample characteristics

	US	STATE
Elections	16,577	40,036
Years	1898–1992 (even)	1968–1989
Elections in 1988	354	3,485
Districts in 1988	354[a]	3,485
Median vote margin	18,021	3,256.5
Median percentage margin	22	25
Total votes:		
min	2,723	25
median	105,987	13,665.5
average	111,370	23,658
max	1,663,687	388,143
Addendum: uncontested elections (dropped from sample)	3,221	11,226

[a] 19 districts were omitted in 1988 because votes were not reported for the Democratic or Republican candidate. 62 more districts were omitted because either Democratic or Republican received zero votes.

Looking at elections decided by less than 100 votes (Figure 1b), margins of less than 25 seem to be particularly rare.

Figure 2a shows how the STATE distribution of percentage election margins is bimodal, with one mode near zero and another near 100%.[5] The distribution appears broadly uniform for elections decided by less than 10%. Looking closer still at elections decided by less than 100 votes (Figure 2b), perhaps margins of less than 25 are more rare, although this is not as noticeable as with the US elections. Margins of less than 5 are rarest.

2.2. *What is a pivotal vote?*

To begin, we refer to a "pivot election" as one in which the winner's official and final vote total exceeded the loser's official and final vote total by no more than one vote. A "pivotal vote" is a vote cast for the winner in a pivot election, or cast for either candidate in a tied election. In other words, a "pivotal vote" is one for which it appears that, depending on the tie-breaking coin flip, the election outcome could have been different if it had not been cast.[6]

Our defining "pivotal vote" may seem to be unnecessary, and verbose. But one contribution of our paper is to present some facts suggesting that, even

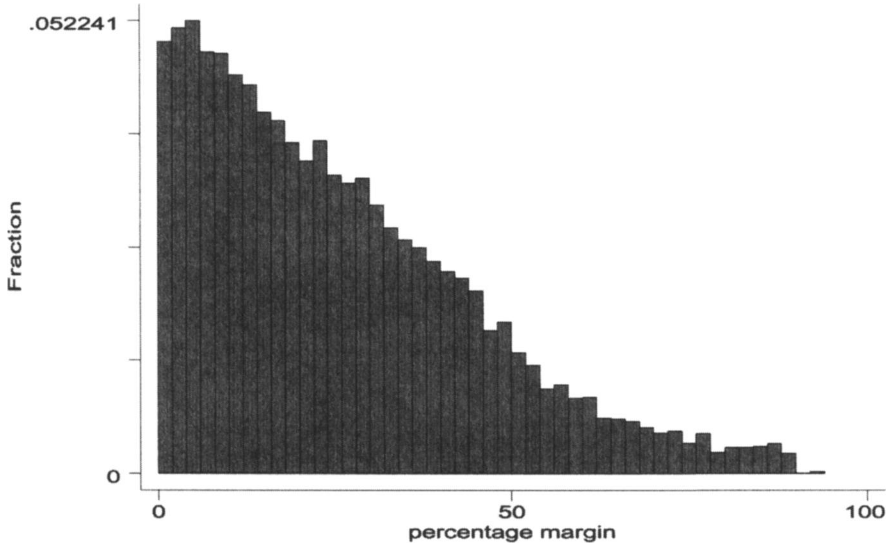

Figure 1a. Percentage margin, all contested U.S. elections

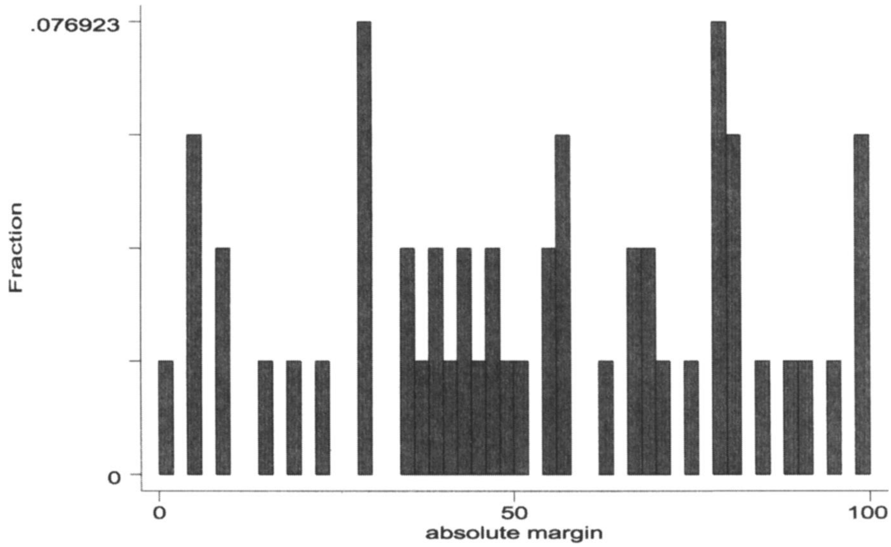

Figure 1b. Absolute margin, U.S. elections within 100 votes

when the official and final returns are known, whether or not the election was a pivot election is unclear because the procedures for determining the official and final returns are both unclear and margin-specific. We discuss this in some detail in Section 4, and for now stick with our definition above. For now, we

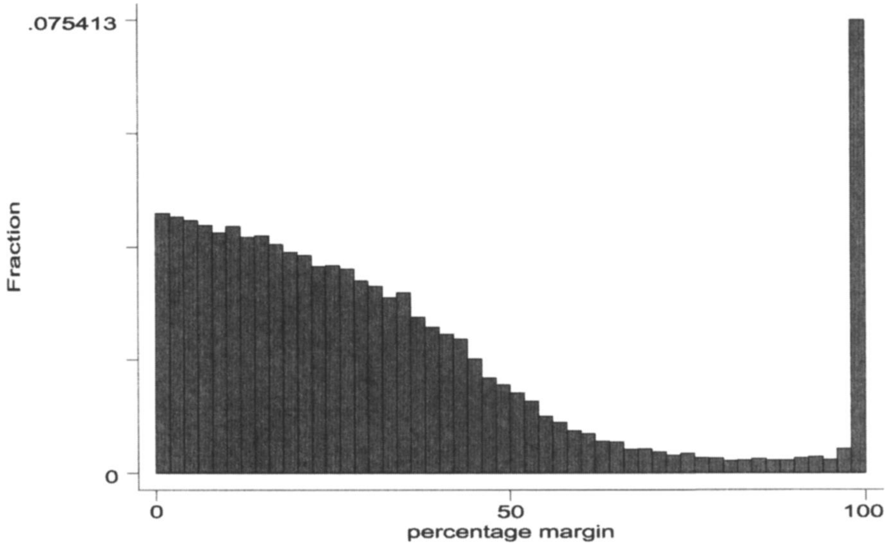

Figure 2a. Percentage margin, all contested STATE elections

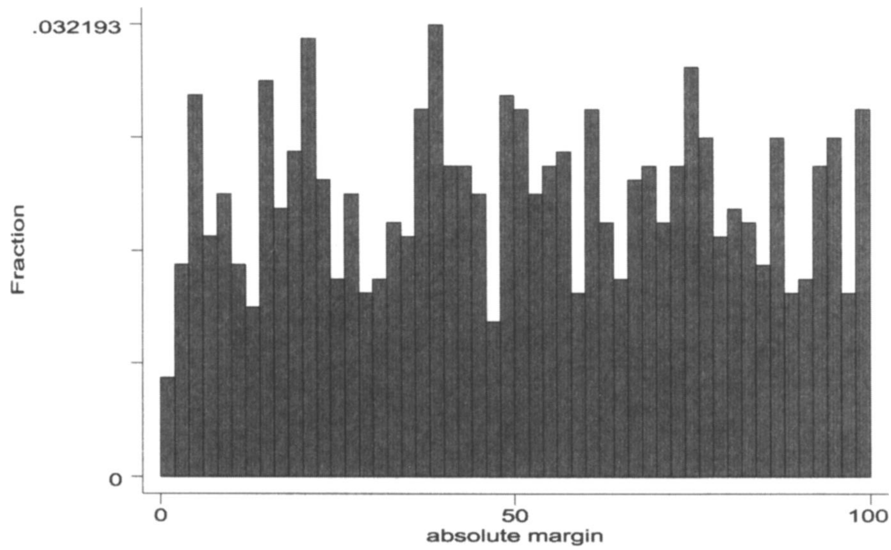

Figure 2b. Absolute margin, STATE elections within 100 Votes

also assume that the vote totals reported in the ICPSR studies are official and final.

Most economic models of voting assume that voters treat their vote as if it had instrumental value, and therefore cast for the candidate whose policies are expected to yield the voter higher income or utility. Pivot elections are interesting because those are the elections in which, at least with perfect foresight, a vote indeed had instrumental value. Of course, a voter typically does not know the precise election returns before he casts his ballot, so a pivot election is only a subjective probability from his point of view, and the expected instrumental value of his vote the product of that probability and the instrumental value of a vote cast in a pivot election. Presumably this probability varies across votes, because those votes are cast in elections of different sizes and different expected closeness and, within elections, cast by different voters with perhaps different assessments of the probability.

It would be nice to know how the subjective probability of a pivotal vote varies according to election and voter characteristics. We attempt to empirically describe some of those variations, but we begin by calculating the empirical frequency of a pivotal vote in our STATE and US elections samples. If subjective probabilities are unbiased, this empirical frequency might be interpreted as an estimate of the average subjective probability in the population of elections with similar election and voter characteristics as those in our samples.

2.3. *Calculations of the frequency of a pivotal vote*

Of the 16577 US returns analyzed, only one (1/16577=0.00006) was decided by a single vote.[7] 41369 votes were cast in that election, or a 0.00002 share of all of the votes in the US elections analyzed.[8] Two others were decided by 4 votes, one by 5, and two by 9 votes. All 16577 (16571/16577 = 0.9996) others were decided by at least 14 votes. Of the 40036 STATE elections (with almost one billion votes) analyzed, two were tied and seven were decided by a single vote (9/40036 = 0.0002). 61,328 votes were cast in those nine elections, or a 0.00006 share of all of the votes in the STATE elections analyzed.

These calculations do not make use of two dimensions of the data that are potentially informative about the subjective probability of a pivotal vote: the frequency of "close" (but not as close as 10 votes) elections and relationships between margins and election size. With a simple model of the probability of a pivotal vote, we can use results from other close – but not pivot – elections to increase the precision of our estimates. For example, suppose that the subjective probability of margin m is independent of m for $m \in [1,M]$, for M very small relative to total votes cast,[9] and that the probability of a tied election is half of that.[10] Then two estimates of the frequency of a pivot election (i.e., the probability of m = 0 or m = 1) are of interest, q and p:

$$q(M) = \frac{3}{2(M+1)} \frac{\sum_i [x_i(M) + x_i(0)]}{\sum_i 1}, \quad p(M) = \frac{3}{2(M+1)} \frac{\sum_i v_i[x_i(M) + x_i(0)]}{\sum_i v_i}$$

$$\text{where } x_i(M) = \begin{cases} \text{i if } m_i \in [0, M] \\ 0 \text{ otherwise} \end{cases}$$

and where m_i and v_i denote the margin and total votes, respectively, in the ith election. p and q are 1.5/(M + 1) times a ratio of sums. q's ratio of sums is the sample frequency of elections with margin less than or equal to M, counting tied elections twice, and dividing this ratio by (M + 1) gives us an estimate of the probability that an election is decided by exactly, say, 1 vote. Multiplying by 3/2 yields an estimate of the probability that an election is either decided by 1 or decided by 0 votes. For example, if M = 9, we have q(M) = (3/20)(90/40036) = 0.0003 in the STATE returns data because there are 86 elections decided by 1, 2, 3, 4, 5, 6, 7, 8, or 9 votes and two tied elections, which means, in expectation, 9 (= 90/10) elections would be decided by exactly 1 vote, 9 by exactly 2 votes, 9 by exactly 9 votes, and 9 by exactly 4 votes – or 13.5 (= 9*3/2) elections decided by exactly 0 or one votes.

p differs from q in only that it weights by votes cast in the election. Hence, while q is an estimate of the probability that a randomly chosen election will be decided by zero or one votes, p is an estimate of the probability that a randomly chosen *ballot* is cast in an election decided by zero or one votes. Hereafter, we refer to q as the "unweighted frequency" and p as the "vote-weighted frequency."

If, in a sample of N elections with the same turnout, the factors determining whether or not one of those elections has $m \leq M$ were independently determined across elections, then the sampling variability of our empirical frequencies is readily calculated from the binomial formulas. For example, we calculate the vote-weighted frequency p's t-ratio $t_q(M)$ as:

$$t_p(M) = \sqrt{\frac{N}{\frac{3}{2p(M)}\frac{M+2}{(M+1)^2} - 1}} \approx \sqrt{\frac{Np}{3}(2M+1)}\frac{M+1}{\sqrt{(M+2)(M+1/2)}}$$

where N is the number of elections sampled. The formula for q's t-ratio is the same, except that we replace p with q. Under some conditions the formula above also applies to samples of elections of different sizes.[11] Because p and q are so much less than one, the squared t-statistic can be very closely approximated by a product of two terms. The first term is the expected number of elections (unweighted for q, and vote weighted for p, and counting tied elections twice) where the margin is less than or equal to M. The second term is a function of M only, and is approximately one. Hence, the confidence intervals for p and q can, relative to those for p(1) and q(1), be shrunk substantially by assuming the probability of a margin m does not vary on [1,M], even for M quite small, and thereby including more "close" elections in the calculation.

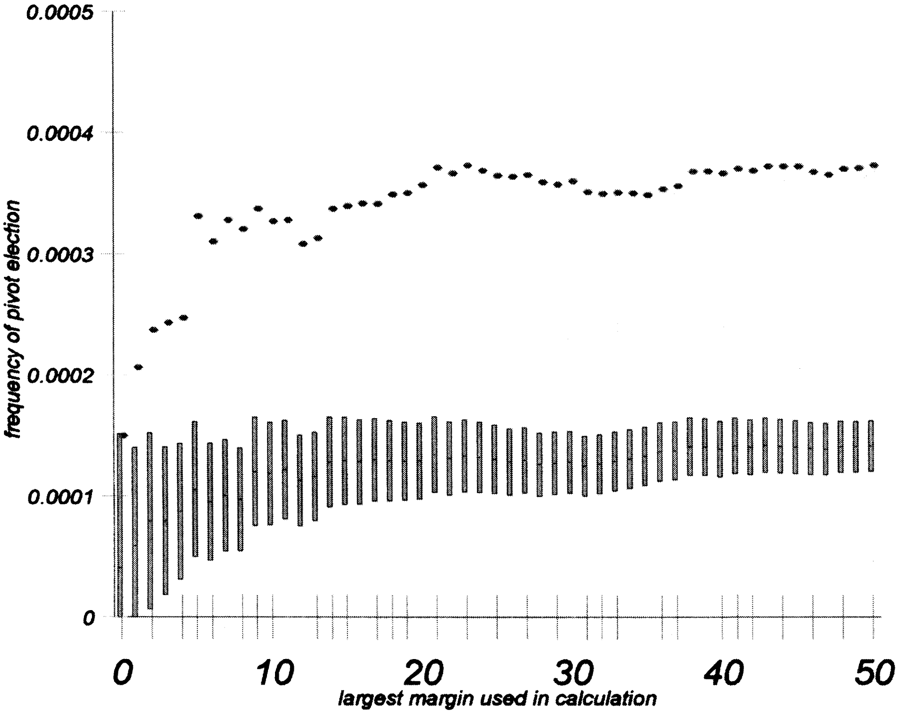

Figure 3. STATE calculations of the frequency of a pivot election (confidence intervals shown for vote-weighted frequency only)

For example, for M = 1 we have only 7 close STATE elections, but we have 92 for M = 10 and 196 for M = 20.

There is some evidence that election closeness is not independently determined across the elections in our data. For example, the election-independence assumption might be violated because we have multiple elections from the same district; conditional on voter turnout, some districts are more likely to have an uncontested election.[12] Hence we suggest that readers interpret our confidence intervals with some caution. Figure 3 graphs p(M) and q(M) vs M for the STATE data, and for p only also display 95% confidence intervals as $p(M)[1 \pm 2/t_p(M)]$.[13] We see that p and q rise with M for M between 0 and 50, which is another version of what we see from the histograms (Figures 1 and 2), namely that margins of 0–4 are substantially less likely than, say, margins of 20–24. We also see the confidence intervals shrink with M but, given the sensitivity of p and q to M for M less than 10, perhaps the increased confidence is not enough to justify using M larger than 4.

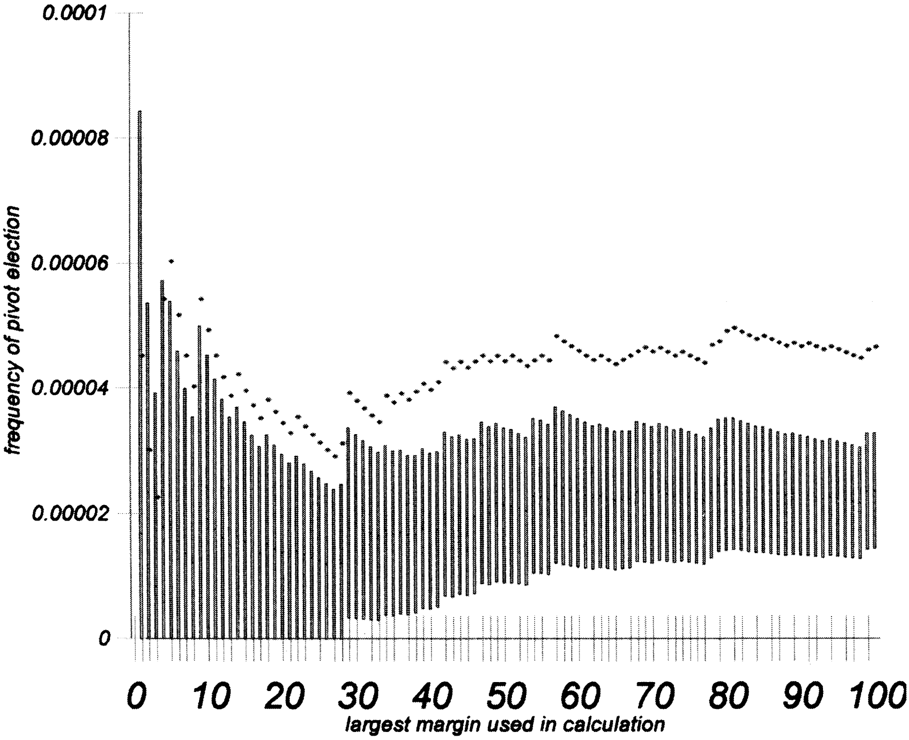

Figure 4. U.S. calculations of the frequency of a pivot election (confidence intervals shown for vote-weighted frequency only)

There are at least 2 STATE elections with margin m, for any integer m $\in$ [1,50], but there are much fewer very close US elections. For example, there were no US elections that tied, or were decided by 2, 3, 6, 7, or 8 votes. Hence a much larger M is needed to estimate the frequency of a pivotal vote from the US sample with some confidence. Figures 4a and 4b graph p(M) and q(M) vs M for the US data on M $\in$ [0,100], and also display 95% p confidence intervals. M does not affect the point estimates as noticeably as in the STATE sample.

2.4. *The frequency of a pivotal vote as a function of election size*

Our data can be used to relate the frequency of a pivot election to election size. We begin by dividing the STATE sample into 3 subsamples, with equal numbers of elections, by total votes in the election and compute p(M) separately for each subsample. Since we expect this probability to vary inversely with election size, we graph in Figure 5 each subsample's product p(M)v where v is the average number of votes cast in the corresponding subsample.[14]

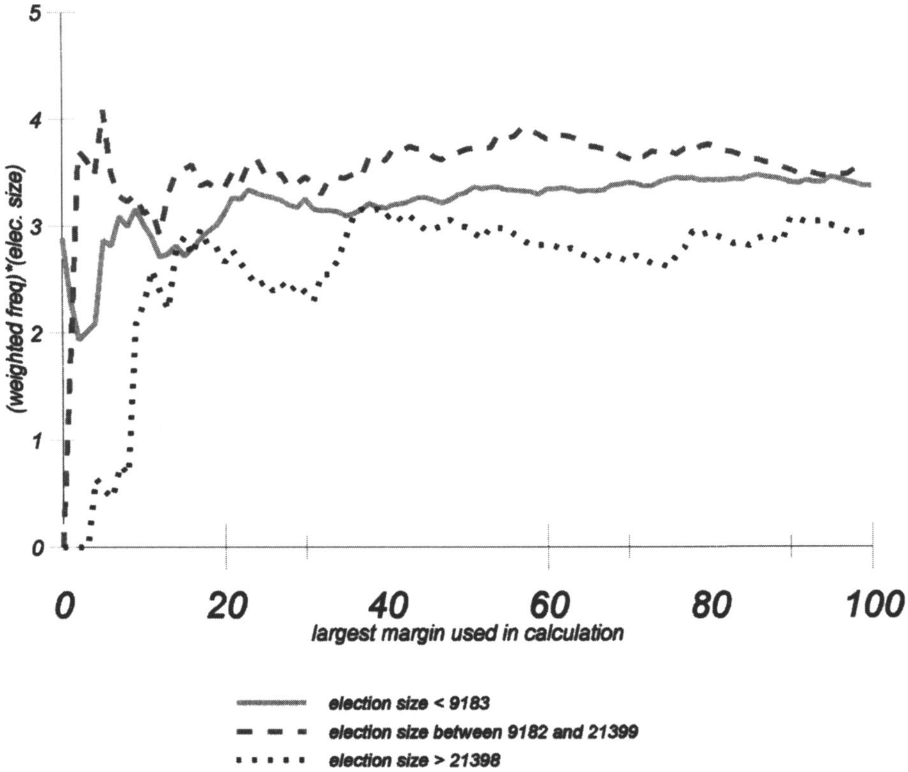

Figure 5. STATE frequency of a pivot election, by election size and M, expressed as a proportion of (1/Election Size)

As we follow any one of the series to the right, election size is held constant and we see how the calculation of p for one of the subsamples varies with M, the largest election margin used in the calculation. In this sense, Figure 5 just disaggregates Figure 3, which is why we see the same pattern: p estimates tend to rise with M for small M. As we compare one series with another for a given M, we see how election size is related to the p estimate. Because the estimated frequency express as a proportion of (1/election size) – i.e., is multiplied by average election size for the corresponding subsample – two series are exactly equal when the two estimated frequencies are exactly proportional to the inverse of average election size. We see that the three series do not coincide for small M – especially M < 10 – so that the frequency of elections decided by less than 10 votes does not seem to vary with the inverse of election size. For example, at M = 4, estimated p is 0.00001 and 0.00038 for the small and large election subsample, respectively, which in comparison with the medium subsample (estimated p of 0.00024) is five and 1.7 times

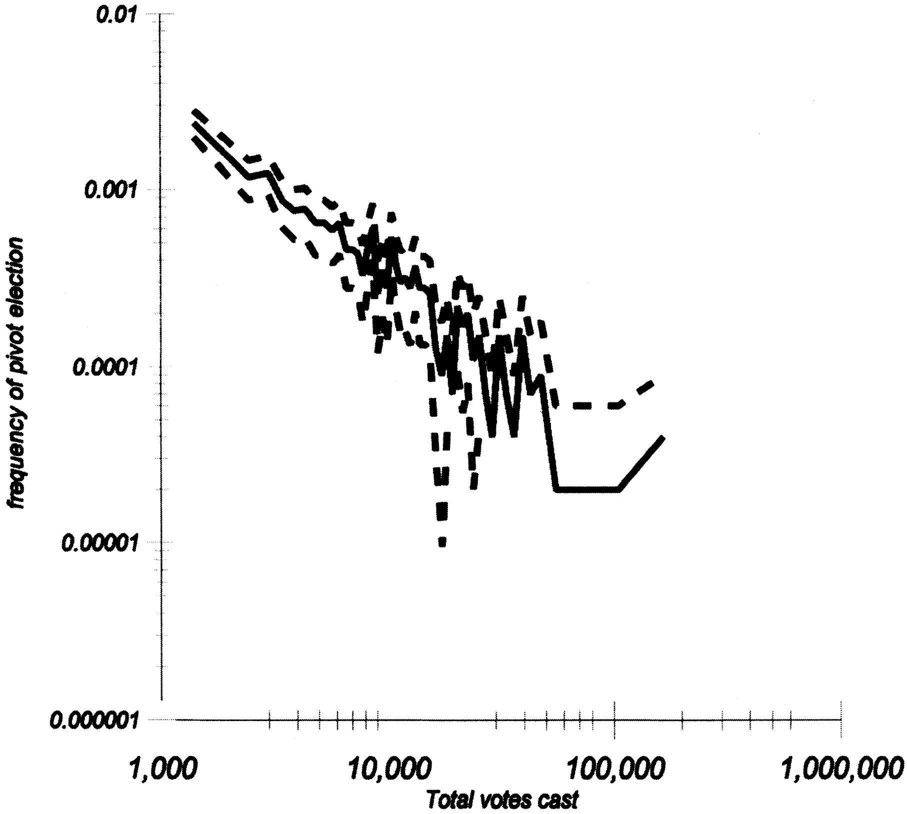

Figure 6. Frequency of a pivot STATE election, as a function of votes cast

less likely then we would expect if these frequencies varied inversely with election size. The election-size inverse rule agrees much more closely with our estimates for M closer to 100.

To analyze more subtle relationships between election size and the frequency of a pivotal vote, we need to have M fairly large or else estimates would be dominated by sampling error. We set M = 100 for the STATE sample, so that there are 996 elections with m < M, and make 50 equally sized subsamples by total votes. Estimates of q,[15] and a confidence interval for q are displayed in Figure 6, as functions of election size, for each of the 50 subsamples. Both axes are on a logarithmic scale, and the relationship is apparently linear with slope equal to –1, so the probability of a pivotal vote appears to vary with 1/v.

We set M = 500 for the US sample, so that there are 304 elections with m < M, and make 34 subsamples by total votes.[16] Estimates of q, and a confidence interval for q are displayed in Figure 7, as functions of election size, for each

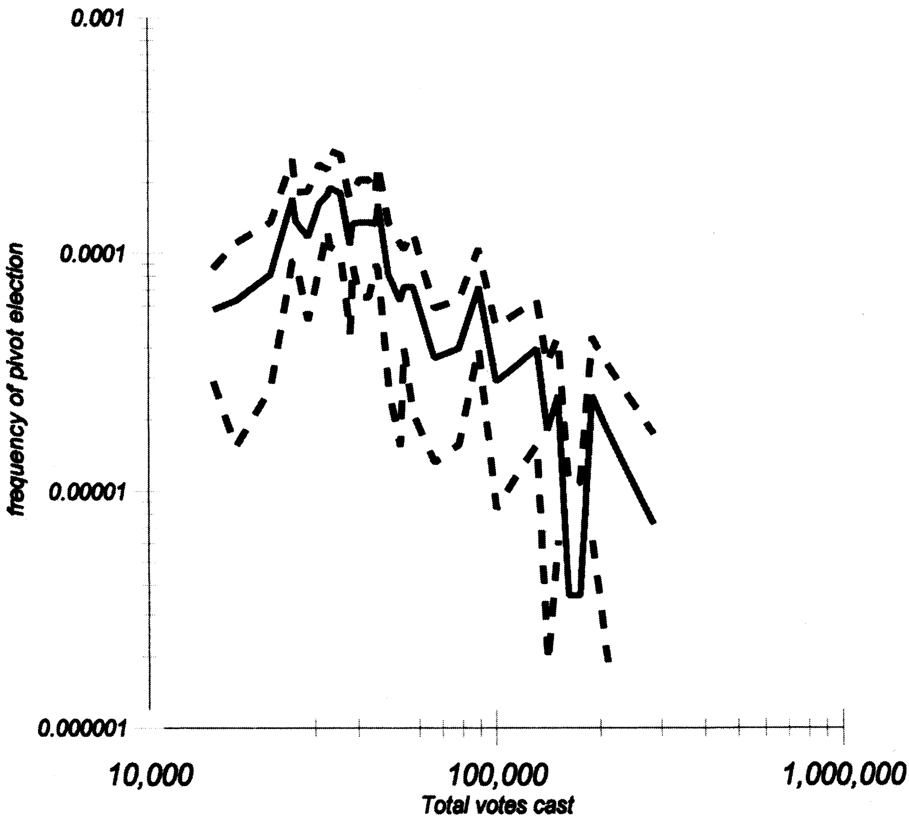

Figure 7. Frequency of a pivot U.S. election, as a function of votes cast

of the subsamples. Both axes are on a logarithmic scale. Notice that the US sample has larger elections than the STATE sample, with little overlap in terms of total votes cast between Figures 6 and 7. For elections with more than 30,000 or 40,000 votes, the U.S. relationship between pivot election frequency and total votes cast is apparently linear with slope equal to –1, as with the STATE elections. However, the frequency of a pivot US election *increases* with total votes cast for elections smaller than 30,000 or 40,000 votes. We conjecture that the increase is due to a response of voter turnout to closeness and that this response is small enough that we see deviations from 1/v only in a small subsample of our elections, but leave verifying our conjecture to future research.[17]

3. Modeling votes as independent binomial variables

3.1. *Theoretical calculations of the probability of a pivot election*

The most common voting model in the literature used to study the probability of a pivotal vote might be called the "independent binomial" model. The model election is between two candidates, say, Democrat and Republican. There is a fixed number of voters v. A vote for the Democrat is denoted z = 1 and z = 0 for the Republican. Denoting as z_j the vote of the jth voter ($j \in \{1, \ldots, v\}$), the election margin is (as a share of total votes cast):

$$\left| 2\frac{\sum_j z_j}{v} - 1 \right|$$

Modeling each vote as independently drawn from a binomial distribution with "success probability" Π, the sum (i.e., aggregate Democrat votes) is the sum of i.i.d. binomial variables with expectation $v\Pi$ and variance $\Pi(1\text{-}\Pi)v$. The expected Democratic percentage margin is $\Pi - (1 - \Pi) = 2\Pi - 1$.

Computing the probability that Democrat votes total exactly v/2 is straightforward, although extremely tedious and difficult to relate precisely to one of the key parameters, v:

$$\Pr\left\{\sum_j z_j = v/2 \,|\, \Pi, \text{v even}\right\} = \binom{v}{v/2} \Pi^{v/2}(1 - \Pi)^{v/2} \tag{1}$$

This is the probability of a pivotal vote conditional on the success probability Π. Whether or not voters know the success probability has been discussed in the literature (e.g., Good and Mayer, 1975; Chamberlain and Rothschild, 1981; Gelman et al., 1998; Fischer, 1999), and in the later case the probability (from the perspective of a voter) is computed by integrating (1) over a prior distribution F for Π:

$$\Pr\left\{\sum_j z_j = v/2 \,|\, \text{v even}\right\} = \int_0^1 \binom{v}{v/2} \Pi^{v/2}(1 - \Pi)^{v/2} dF(\Pi) \tag{2}$$

Of course, we do not know Π for any one of our 56,613 elections, let alone know what voters in each election knew about Π. But, in order to generate predictions for the occurrence of pivot elections in our data, the relevant calculation is functionally equivalent to a voter's: integrate over a prior distribution for Π. The only difference is in the interpretation, with the voters having expectations about Π for his election, and our having priors for the distribution of Π across elections.

Interestingly, not much has to be known about F to compute the unconditional probability of a pivotal vote, because (2)'s integrand can be neglected everywhere except in the neighborhood of $\Pi = 1/2$. So (2) simplifies to (3):

$$\Pr\left\{\sum_j z_j = v/2 \,|\, v \text{ even}\right\} \approx f(1/2)\int_0^1 \binom{v}{v/2} \Pi^{v/2}(1-\Pi)^{v/2} d\Pi = \frac{f(1/2)}{v+1} \tag{3}$$

where f is the density function corresponding to F.

(3) calculates the probability of a tied election conditional on an even number of votes. Of course, this probability is literally zero when the number of votes is odd. But, with v odd, (3) does calculate half of the probability that the election margin is exactly one (either in favor of Democrat or of Republican) so, heuristically, we can compute the probability of a pivot election as 1.5 times (3):[18]

$$\Pr\left\{\sum_j z_j = v/2 \text{ or } \sum_j z_j = (v \pm 1)/2\right\} \approx 1.5\frac{f(1/2)}{v+1} \tag{4}$$

The probability of a pivotal vote is just (3) because only half of the votes in a pivot election with odd total votes are pivotal.

If $f(1/2)$ were a free parameter, the independent binomial model can be made consistent with any probability of a pivot election by picking the right value for $f(1/2)$. But suppose we tried to calibrate $f(1/2)$ without our empirical calculations of p and q, but only with the kind of information that could be obtained from small samples of election returns. For example, even a rather small subsample of our STATE elections sample would show us that something between 10 and 11 percent of STATE elections are decided by a margin of less than five percentage points, and thereby suggest that $f(1/2)$ is about 2.1.[19] Then, from the formula (4), the independent binomial model predicts that the probability of a pivot election among elections with turnout v is about $3.15/v$.[20]

This prediction from the calibrated independent binomial model is very close to the estimates we made by exploiting our large data set for the purpose of focusing on the very close elections. For example, Figure 6's estimate of the frequency of a pivot election, for elections of size v = 20,000, is about 1/6000, so $f(1/2) = (1/6000)(2/3)(20001) \approx 2.2$. In other words, Figure 6 confirms almost exactly the predictions of the independent binomial model.

We suspect, but leave it to future research to confirm, that Figure 6 does not present a tough test of the independent binomial model. One reason we have this suspicion is that $f(\Pi)$ may be fairly constant over a broad range including $\Pi = 1/2$. For example, we have experimented with some binomial voting models in which different votes in the same election are correlated, and have found that such models imply a probability of a tie election that is significantly smaller than that implied by the independent binomial model. However, dependence also reduces the fraction of elections decided by, say 5 percentage points, and hence the value with which we would calibrate $f(1/2)$. In other words, as long as $f(\Pi)$ is fairly constant over a broad range, the calibrated independent binomial model may deliver the correct calculation of the probability of a pivot election, even when votes are not independent, because it combines an error in the formula (4) with an offsetting error in the calibration of $f(1/2)$.

Another reason why Figure 6 may provide only a weak test is that it uses $M = 100$. Figure 3's estimates of the frequency of a pivot election are smaller for $M < 10$. Figure 5 displays a similar pattern, and suggests that the 1/v property is not so obvious with the smaller M. If these findings are not the artifact of sampling error, then they are inconsistent with our version of the independent binomial model and any other model in which the probability of margin m does not vary significantly with m in the range $m \in [1, 100]$. Below we suggest that there may be one or two model modifications producing such variation.

3.2. *Recounts and other margin-specific election procedures*

Election procedures are simple, and independent of margin, in the binomial model above. In particular, votes are counted for both candidates, and the winner is the one with the highest count. If election procedures are not independent of margin, this can affect our calculations of pivot frequencies, and interpretations of those calculations. To see this, consider a illustrative model of such procedures.

In any particular election, votes $z \in \{0, 1\}$ are indexed by $j = 1, \ldots, v$ – as in the binomial model. However, these votes are initially measured by election officials with error, so that the combined quantity $(z_j+\varepsilon_j)$ is observed, where $\varepsilon_j \in \{-1, 0, 1\}$ is a measurement error.[21] Denoting the true and initially observed absolute margins in the ith election as m_i and $\hat{m}_i$, respectively, we have:

$$m_i = \left|2\sum_j z_j - v\right|$$

$$\hat{m}_i = \left| 2\sum_j (z_j + \varepsilon_j) - v \right|$$

If the initially observed margin exceeds g_i in the ith election, then the official election margin is $\hat{m}_i$. Otherwise, a "recount" is held to eliminate[22] measurement errors, and the official margin is the true margin, m_i. g_i varies by election, because these procedures may depend on jurisdiction, or on the willingness of candidates to demand (and pay for) a recount.[23]

Although this is a stylized model of margin-specific election procedures, it clearly reveals how such procedures could reduce the frequency of close elections as measured by the official margin – some close elections are misclassified because $\hat{m}_i > g_i$, but no "far" elections are misclassified as close. Furthermore, a vote can have (instrumental) value even when the election in which it was not cast in a pivot election (as we have defined it) because a single vote can affect whether there is a recount, which in turn has some probability of changing the outcome of the election.

3.3. *Other extensions of the binomial model*

There are three types of extensions of the binomial model that may be relevant for interpreting our findings. First, dependence across voters could be modeled. Such dependence might be created by correlation in the latent factors affecting a person's vote (e.g., his response to campaign advertisements), although we suggest above that this may not produce a distribution of election margins that is far from uniform in the neighborhood of 0. But dependence created by strategic interactions as in the "swing voter models" (e.g., Feddersen and Pesendorfer, 1996) might make especially close elections more likely, because the purpose of those strategies is to create an election that is close. A second and related extension might allow for competition among candidates, competition which tends to make an election close, and competition which may be (endogenously) more intense when the election is close.

Both of these extensions could cause frequencies to be higher for closer elections. But does that mean that elections decided by one percent are more common than elections decided by five percent? Or that elections decided by one vote are more common than elections decided by eleven votes? The latter seems unlikely if voters and candidates are, prior to the final election count, are unable to distinguish a close election from a very close election. We leave answering these questions to future research, but point out that our data do not show the very close elections to be particularly common – if anything, margins of 0, 1, 2, 3, and 4 are relatively uncommon.

A third extension is to allow turnout to respond to closeness of the election. This would raise the vote-weighted frequency of a pivot election. It would also cause the relationship between that frequency and election size to deviate from the 1/v formula. But perhaps our finding that deviations from 1/v are uncommon implies that turnout is not very responsive to closeness.

4. Can a vote ever be pivotal?: some evidence on margin-specific voting procedures

4.1. *What really happened to the ballots in the pivot elections?*

The calculations above assume that the vote totals reported in the ICPSR studies are official and final. Systematically verifying this assumption for each of our 56,613 elections is beyond the scope of our project, but we have done some research that reflects on this assumption. Namely, for those 10 elections coded as tied or decided by one vote, we searched for newspaper articles written after the election to verify that the election was in fact close, and to verify that the winner and vote margin were coded "correctly" in our file. Somewhat to our surprise, we learned from these ten cases that the procedure for determining the winner may be flexible and margin-specific, and report some of our findings in Table 2.

We report in the fifth column the candidate vote totals as reported in the computer file which, with one exception, we found (according to newspaper reports) to be the official vote totals recorded with the state election commission. But, except in the five cases indicated in bold, it was later and different vote totals, reported in the next Table 2 column, that determined the winner of the election. Generally speaking, it seems that the counting of votes is not an exact science and, much like the model above, exact counts are not attempted unless the official (and error-ridden) margin is close. Reasons for errors include human arithmetic errors (as in the federal election), vote machine errors, misplacement of ballots (especially mail-in),[24] and judgements about the legality of a particular ballot. These later counts typically reveal a larger margin of victory than officially recorded – as in four cases shown in the Table – and the official counts are not adjusted when the winner does not change. In one Rhode Island case, another election was held to break the originally "tied" election, even though according to (our reading of) Rhode Island election law, rerunning the election is not one of the legal options. Exhaustive legal research on the conduct of close elections is beyond the scope of our paper, but perhaps one moral of the ten stories told in Table 2 is that election procedures are flexible when official counts are close, and cannot be accurately modeled as the simple counting of a fixed population of ballots

Table 2. General elections coded with margins of 0 or 1. Were they really decided by one vote?

State or fed?	State	Yr	Dist/ off[a]	Returns official	Returns latest	Newspaper account (source)
ST	RI	78	29/S	4110–4110	2546–2038[b]	After some argument whether a disputed 8221st ballot should be included, the tie was broken with another election run two months later with returns 2546–2038 (*Evening Bulletin*, 11/12,14,16/1978 and 1/4,12/1979).
ST	NM	80	19/H	2327–2327	**2327–2327**	(*Albuquerque Journal*, 5, 6, 11–13, 20, 21, 23 and 25–27 Nov. 1980 and 7 Dec. 1980 and 24 Jan. 1981, Associated Press 6 Nov. 1980)
FED	NY	10	36/H	20685–20684	20690–20684	Another recount broke the tie by one, when an arithmetic error was discovered. The winner wrote to the *New York Times* that five more of his votes were uncovered, there were further machine errors in the loser's favor, and both parties recognize that the election was not that close. (*New York Times* 11/11,12/1910)
ST	ME	82	69/H	1387–1386	**1387–1386**	Unofficial counts said it was tied, but all subsequent counts were 1387–1386 (*Bangor Daily News* 4, 16 and 18 Nov. and 1 Dec. 1982)
ST	MA	82	8/S	5352–5351	5378–5347	A recount was requested, revealing (13 days after the election) a 31 vote margin (*The Union Leader*, Manchester 9, 11 and 16 Nov. 1982).
ST	RI	70	26/H	1760–1759	**1760–1759**	1760–1759 was the count after a recount or two (Evening Bulletin, 18 Nov. 1970).
ST	WI	68	25/H	6522–6521	6523–6521	Margin was 3 votes before a recount, and a case was filed with the circuit court, where the judge said the margin was two votes. An election committee investigated, but could make no proclamation of the true margin (*The Milwaukee Journal* 6, 14 and 21 Nov. 1968; 7, 16 and 23–24 Jan. 1969)
ST	MO	70	116/H	4819–4818	**4819–4818**	A recount showed the same vote totals (*Kansas City Star* 6 and 7 Nov. 1970)
ST	ND	78	27/S	2459–2458	2448–2454	A first recount yielded the one vote margin. Another recount 25 days after the election showed a six vote margin *for the other candidate*, who took the office after subsequent appeals were reversed. (*Fargo-Moorhead Forum*; 10, 15, 16 and 28 Nov. 1978; 1, 5, 7, 8 and 12 Dec. 1978; 7 Jan. 1979)
ST	UT	80	44/H	1931–1930	**1931–1930**	(*The Salt Lake Tribune*, 6 Nov. 1980; *Deseret News* 18–19 Nov. 1980)

[a]"dist" is state or federal district number, and "off" is Senate or House

[b]the "late count" was from another election run two months later to break the tie

without regards for the situations of the candidates, their parties, and other legal and political factors. In other words, even when the election is over, one cannot know for sure whether or not the outcome hinged on one vote.

Up to this point, we have used the fifth "official" column to compute the frequency of a pivot election. One alternative calculation is to substitute the sixth column for the fifth in those ten cases where we have the data. In other words, rather than two ties and eight one-vote victories in the combined data set of 56,613 elections (an unweighted pivot election frequency $q(1) = (10+2)/56613 = 0.0002$), we have one tie and four one-vote victories (a frequency $q(1) = (5+1)/56613 = 0.0001$). This is a 50% reduction in the unweighted frequency and, since 76% of the votes analyzed in Table 2 were not cast in a pivot election according to the "latest" numbers, a 76% reduction in the vote-weighted frequency. On the other hand, we might have increased these frequencies if we had verified official margins in some of the other 56,603 elections decided by a margin of two or more votes.

4.2. *Dips in the margin density near zero*

Our Figure 3 also suggests that voting procedures might be margin-specific, at least in the STATE elections. Remember that Figure 3's point estimates would be flat if the sample frequency of vote margins were independent of the margin m in the neighborhood of a tie. Instead, we see that the frequency is nearly 50% higher for $m > 4$ vs. $m \leq 4$, and noted above how the frequency for $m \leq 4$ is still too high because the official vote totals are not those determining the winners.

Is the change in frequency with m due to the behavior of voters, or candidates? A complete answer is beyond the scope of this paper, but it may be that the procedures similar to those revealed by our newspaper investigations (which transformed the "official" counts to counts determining the winner) also played out *before* the official count was determined.[25] In other words, think of elections as being counted in three stages: a first count, which is followed by an independent "official" count only if the first count is close, which is in turn followed by an independent "latest" count only if the official count is close. If we had data on all three counts, we would expect, in the first count, to find the frequency of any margin m to be independent of m (for m small), but the find a dip near $m = 0$ in the official count (as we did in Figure 3), and an even bigger dip in the "latest" count (as our Table 2 suggests).

5. Conclusions

If we take the official counts as perfectly identifying pivotal votes, our empirical findings are perhaps best summarized in reference to the binomial voting model that has been used in the literature to calculate the probability of a pivotal vote. First, the frequency of a pivotal ballot (i.e., one cast for a winner in an election decided by one vote, or for either candidate in a tied election), and its relationship with total votes v cast in the election, is according to one of our estimates closely approximated by 2/v, which is quite close to the formula implied by a simply calibrated independent binomial model. Second, smaller U.S. Congressional elections are an interesting exception to the 2/v rule, because the frequency of a pivotal ballot appears to *increase* with total votes cast. Third, another of our estimates utilizing only elections decided by fewer than 5 votes deviates significantly from 2/v, because the frequency of (official) STATE election margin m is smaller than for $m \geq 5$, and not varying inversely with v for $m < 5$. For example, only two STATE elections tied and only 29 were decided by 1–4 votes while 39 were decided by 11–14 votes, 46 by 21–24 votes, 29 by 31–34 votes, 46 by 41–44 votes, 49 by 51–54 votes, etc.

It might also be argued that the official counts do not accurately identify pivotal votes, because officially close elections would be recounted. So, conditional on a recount, a pivotal vote is one cast for a candidate winning by no more than one *according to the recount.* Some preliminary investigation of recounts suggest that the frequency of a pivotal ballot would only be 1/v, rather than 2/v, according to this definition. 1/v is perhaps a departure from the calibrated binomial model, although not from one that is modified to include margin-specific recounts. Such a model would not only emphasize the value of a ballot because it might be pivotal (i.e., the election might be decided by one vote in the final recount), but also the value of a ballot in affecting the probability of recounts that would change the winner of the election.

We believe that the theory of voting can be enhanced by fitting it to the empirical distribution of election returns documented here, but we leave the theoretical analysis to future research (Becker and Mulligan, 1999 is one attempt). Perhaps some relevant questions are "Is the Swing Voter Model consistent with the empirical frequency of swing votes?", or "Can models of candidate competition for votes explain the empirical distribution of election returns?" Future empirical research could also investigate the prevalence of votes in uncontested elections – these were omitted from our calculations but are conspicuous in our data and probably relevant to the theory of voting.

Notes

1. A related exercise was carried out by Arbuthnot (1710), who looked at the empirical frequency (namely zero) of years when male births in London were exactly the same in number as female births and compared the frequency to that from a calibrated binomial model. We owe this point to Steve Stigler (1986, and personal communication).
2. Although the Federal Election Commission appears to claim otherwise: "'Just' one vote can and often does make a difference in the outcome of an election." (http://www.fec.gov/pages/faqs.htm)
3. As a further check for missing elections, we examined the chronological intervals between elections for each district. If the intervals between elections were spaced irregularly then these districts were flagged for further inspection. Of the 59,525 general elections in the database, we found that there were 1495 chronological "gaps". Upon inspection of these gaps we found that 586 were cases in which a senate election was held at a two year interval rather than the usual four year interval; 280 were due to specially ordered state-wide elections; 129 were due to redistricting (e.g., changing from a single-member district to a multi-member district with positions); leaving 24 for which we found no explanation in either the raw data or in the codebook. Among those 24 gaps, we have so far checked three in the newspaper and all three were verified to be special cases when elections were not held.
4. If uncontested elections had been included, 100% would have been the modal percentage margin.
5. Uncontested elections are omitted from the sample, but the histogram shows how there are quite a few barely contested elections.
6. When the winner in a tied election is determined by a fair coin flip, there is a 50% chance that leaving one of the pivotal votes (by our definition) uncounted would leave the election outcome unchanged. In other words, prior to the coin flip, any of the votes cast in a tied election has a 50% chance of being cast for the loser. In an election decided by one vote, those cast for the winner would have a 50% chance of being cast for the loser if the winner's vote total were reduced by one.
7. That election was in 1910 for the Representative for New York's 36th congressional district, with the Democratic candidate winning 20685–20684. See Section 4 for more details on this and other apparently pivot elections.
8. 20684 of those votes "did not matter" in the sense that they were cast for the losing candidate, so we are left with a 0.00001 share of all of the votes in U.S. elections that mattered.
9. In elections with thousands of votes cast, it seems that voters cannot, before the votes are tallied, have a much different subjective probability for the outcome of $m = 1$ or $m = 2$, or $m = 3$, etc. Can a voter in an election with 1000 votes really claim to know that the outcome 504–496 any more (or less) likely than 502–408? Hence, as long as M is small relative to total votes cast, we assume the subjective probability of margin m is independent of m for $m \in [1, M]$. See also Gelman et al. (1995: 26) and Gelman et al. (1998), who make a suggestion like this for calculating probabilities using ICPSR study #6311.
10. With an even number v of voters, there only one way Democrat and Republican can tie (namely, both get v/2), but two ways they can differ by two votes (namely, D gets (v/2)–1, or (v/2)+1), two ways they can differ by four votes (namely, D gets (v/2)–2, or (v/2)+2), etc. When v is odd, there are two ways to differ by one vote, two ways to differ by three votes, etc.

11. See Mulligan and Hunter (2000) for more details.
12. About half of our 40,036 STATE elections have an election margin of less than 25% of the total votes cast. Among these, the 5040 district-office fixed effects predict 30% of the variance of the percentage margin, with an adjusted R-squared of 0.06. The F-statistic is 1.27 for the null hypothesis that district-office fixed effects have no joint predictive power, with critical value 1.00. This calculation ignores the bias involved with selecting elections based on their percentage margin, but suggests that district-office fixed effects are of some limited help in predicting closeness.
13. q's confidence interval is not displayed to avoid cluttering the Figure, but the reader may guess from the formula that it is somewhat wide because q is larger than p.
14. Those averages are 5570, 14236, and 51169, respectively.
15. Since elections are grouped by total votes, any subsample's weighted (p) and unweighted (q) frequencies are very similar.
16. The subsamples are not equally sized, but rather are larger with more total votes.
17. A number of other studies focus on the effect of closeness on turnout. One of them, Grofman et al. (1998) partitions Congressional elections data according to whether it was a presidential election year. Interestingly, our Figure 7 would better match the 1/v model if our sample were limited to presidential election years.

    We also point out that 79% of the 5075 U.S. elections with less than 40,000 total votes cast (i.e., those on the upward sloping and flat parts of Figure 7's curve) occurred prior to 1920. If we were to draw Figure 7 separately for the years 1898–1918 and 1920–92, both Figures would suggest that the 1/v model does not fit the smaller U.S. elections.
18. With an even number of total votes, the probability of a pivot election is approximately twice (3). If even and odd total votes are equally likely, the probability of a pivot election is approximately the average of (3) and twice (3).
19. 2.1 = 10 or 11 percent divided by 0.05. An estimate of 2.1 would also be obtained if we counted only elections with margin less than one percent, or only those with margin less than 0.1 percent.
20. As emphasized by Fisher (1999), the formula (3) yields a *very* different estimate then, say, (1) evaluated at some average percentage election margin. For example, setting v = 100,000 and $\Pi = .61$ (roughly the averages for the US sample), (1) yields an estimated probability of a pivot election of 10^{-1108} (here I follow Margolis (1977) and others in the liteture using the normal density to compute (1)). Compare 10^{-1108} to my US sample frequency of 0.00005.
21. Reasons for errors include vote machine errors, misplacement of ballots, and judgements about the legality of particular ballot.
22. This is for simplicity only. A more realistic model would have separate (nonzero) measurement errors before and after recount, perhaps with a smaller variance after the recount.
23. Perhaps the demand for recounts tends to be greater in larger elections because more is at stake, and this explains the pattern we see in Figure 5 where the p(M) rises most with M for the larger elections.
24. Mail in ballots are a problem because they are usually not counted and are sometimes not filed according to the correct office, district, etc. Even when one of the races is close enough to count, most others in the jurisdiction are not, and it usually is not known whether one of the mail ins was misplaced with another district's mail ins, which are not being counted (*Evening Bulletin*, 18 November 1970).

25. Indeed, our newspaper investigations revealed some cases of this, but we did not sytematically report them in the Table since our objective was to calculate the sixth column.

References

Arbuthnot, J. (1710). An argument for devine providence, taken from the constant regularity observed in the births of both sexes. *Philisophical Transactions of the Royal Society of London* 27: 186–190.

Beck, N. (1975). A note on the probability of a tied election. *Public Choice* 23: 75–80.

Becker, G.S. and Mulligan, C.B. (1999). Is voting rational or instrumental? Working paper, University of Chicago.

Chamberlain, G. and Rothschild, M. (1981). A note on the probability of casting a decisive vote. *Journal of Economic Theory* 25: 152–162.

Feddersen, T. and Pesendorfer, W. (1996). The swing voter's curse. *American Economic Review* 86: 408–424.

Fischer, A.J. (1999). The probability of being decisive. *Public Choice* 101: 267–283.

Gelman, A. et al. (1995). *Bayesian data analysis*. London: Chapman & Hall.

Gelman, A., King, G. and Boscardin, J. (1998). Estimating the probability of events that have never occurred: When is your vote decisive? *Journal of the American Statistical Association* 93: 1–9.

Good, I.J. and Mayer, L.S. (1975). Estimating the efficacy of a vote. *Behavioral Science* 20: 25–33.

Grofman, B., Collet, C. and Griffin, R. (1998). Analyzing the turnout-competition link with aggregate cross-sectional data. *Public Choice* 95: 233–246.

Margolis, H. (1977). Probability of a tie election. *Public Choice* 31: 135–138.

Mulligan, C.B. and Hunter, C.G. (2000). The empirical frequency of a pivotal vote. University of Chicago Harris School Working Paper 00.25.

Stigler, S.M. (1986). *The history of statistics*. Cambridge, MA: Harvard University Press.

Telser, L.G. (1982). Voting and paying for public goods: An application of the theory of the core. *Journal of Economic Theory* 27: 376–409.