Numerical Effects in Computer Simulation of Simplified Hodgkin-huxley Model

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ABSTRACT

In this paper, we consider the impact of the discretization effects on the dynamical behaviour of the single-compartment neuron model represented by simplified Hodgkin-Huxley equations. In order to implement numerical simulation, one need to discretize the continuous model of the system or use the discrete operator preserving the main properties of the continuous prototype. However, discrete models can suffer from negative effects caused by the applied method, discretization step and round-off errors. This fact is particularly important for the analysis of nonlinear systems, e.g. the biological neuron models. Within this study, the impact of the application of the most common one-step integration methods is examined through the series of computational experiments, where the dynamical system is put into the resonant and chaotic oscillation modes. The results of the study are visualized as two-parameter dynamical maps and interspike interval histograms.

CCS Concepts

Mathematics of computing → Solvers;
Computing methodologies → Simulation tools

Keywords

Nonlinear dynamics; Numerical integration; Computer simulation; Biological neuron model; Hodjkin-Huxley; Neural chaos

1. INTRODUCTION

Studies of mathematical models of neurons and synapses are intended to establish the nature of real processes in the brain structures. Understanding the operational modes of neural networks is necessary during the development of analysis tools for the brain research and revealing possible pathologies. For instance, the state preceding the epileptic seizure is associated with strong synchronization of neurons in the affected area [1], also it is known that schizophrenia is characterized by abnormal neural oscillations and synchronization [2].

Computer simulation of generation and transmission of nerve impulses is usually carried out for large-scale neural networks. This process implies significant computational costs in case of using complex mathematical models of neurons and synapses. Thus, in order to save resources, numerical methods of low orders are still applied. Sometimes authors, when publishing the results of their research in the field of nonlinear dynamical systems, which also include neural networks and their components, do not indicate the applied numerical techniques. Such cases negatively affect the reproducibility of the reported simulation results.

The purpose of this study is to show that the discrete models obtained with different numerical integration methods can produce qualitatively different simulation results for the same mathematical models of biological neurons with identical parameters. Within the framework of this paper some effects are shown through the simulation of a single neuron represented by the simplified Hodgkin-Huxley model, based on single-step numerical integration methods of first, second and eighth order.

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2. OBSERVED NUMERICAL EFFECTS

The Hodgkin-Huxley system of equations [3] is the classical phenomenological model that established the relations between intrinsic ionic currents and dynamical behavior of the electrically excitable membrane. However, this model is rather complicated for analytical research and time-consuming for computer simulation due to the presence of transcendental functions and the large system dimension. Because of that, simplified models able to describe the same processes were proposed. The most known of these is the FitzHugh–Nagumo model [4], but it ignores such physiological aspects as adherence to Ohm's law and explicit reference to the equilibrium potentials of sodium and potassium ionic currents. An alternative model, devoid of these shortcomings, was proposed by Wilson in [5] and is used in this paper.

2.1 Simplified Hodgkin-huxley Model

The investigated neuron model is described by the following differential equations:

$$
\begin{aligned}
C\frac{dV}{dt} &= -(17.81 + 47.71V + 32.63V^2)(V - 0.55) - \\
&\quad - 26.0R(V + 0.92) + I \qquad (1)\\
\frac{dR}{dt} &= \frac{1}{\tau}(-R + 1.35V + 1.03)
\end{aligned}
$$

where V is the membrane potential or voltage, R is the recovery variable, I is the input current, the capacitance of the membrane $C = 0.8$ μF/cm^2 and the recovery time constant $\tau = 1.9$ ms.

In order to produce the set of discrete operators for the simulation of the dynamical system (1), the following numerical integration methods with the fixed stepsize h were applied: explicit Euler (EE, 1st order), implicit Euler (IE, 1st order), semi-explicit Euler (SEE, 1st order), explicit midpoint (EMP, 2nd order) and explicit Dormand-Prince 78 (DOPRI 78, 8th order).

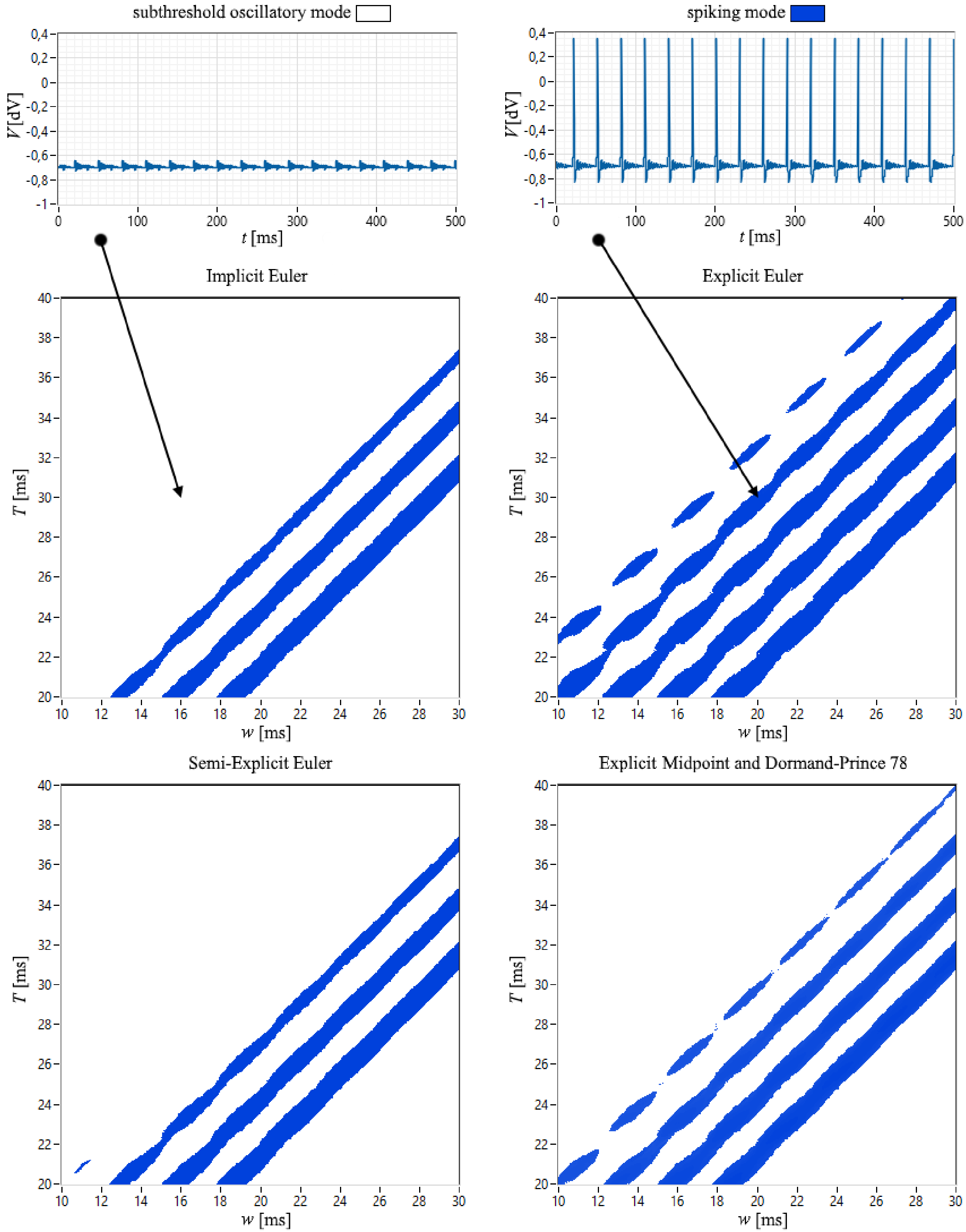

Figure 1. Regions of the spiking mode for the dynamical system (1) regarding the period *T* and the pulse duration *w* of the input current *I* = 4.5 μA in amplitude

2.2 Resonant Spiking

The neuron model, represented by (1), exhibits the coexistence of resting (stationary) and spiking states as well as damped subthreshold oscillations, therefore it can be classified as *bistable resonator*. The first experiment is designed to demonstrate the difference in the behavior of the discrete operators in response to a pulse wave with various values of the period and the pulse duration. We show the result of the simulation as a set of the dynamical maps shown in Figure 1, where the white color regions correspond to the stationary mode and the mode of sub-threshold oscillations, and the blue color shows the regions of spiking mode for the considered discrete operators, implemented with the stepsize h = 0.01 ms: IE, EE, SEE, EMP. The diagram obtained using the DOPRI 78 method is provided as a reference.

One can see from the dynamical maps, that the spiking regions mostly form the diagonal bands, which indicates a linear relationship between the period and the pulse duration. Thus, excitability in this case is determined by the value of the duty cycle. However, note that the number of such bands depends on the applied discrete operator. As it was expected, the discrete operators based on the IE and SEE methods produce the smallest spiking area (only 3 bands). The largest number of spiking regions is obtained by the EE method, while the reference 8th order explicit method demonstrates intermediate result. The diagram of the 2nd order EMP method closely resembles this reference.

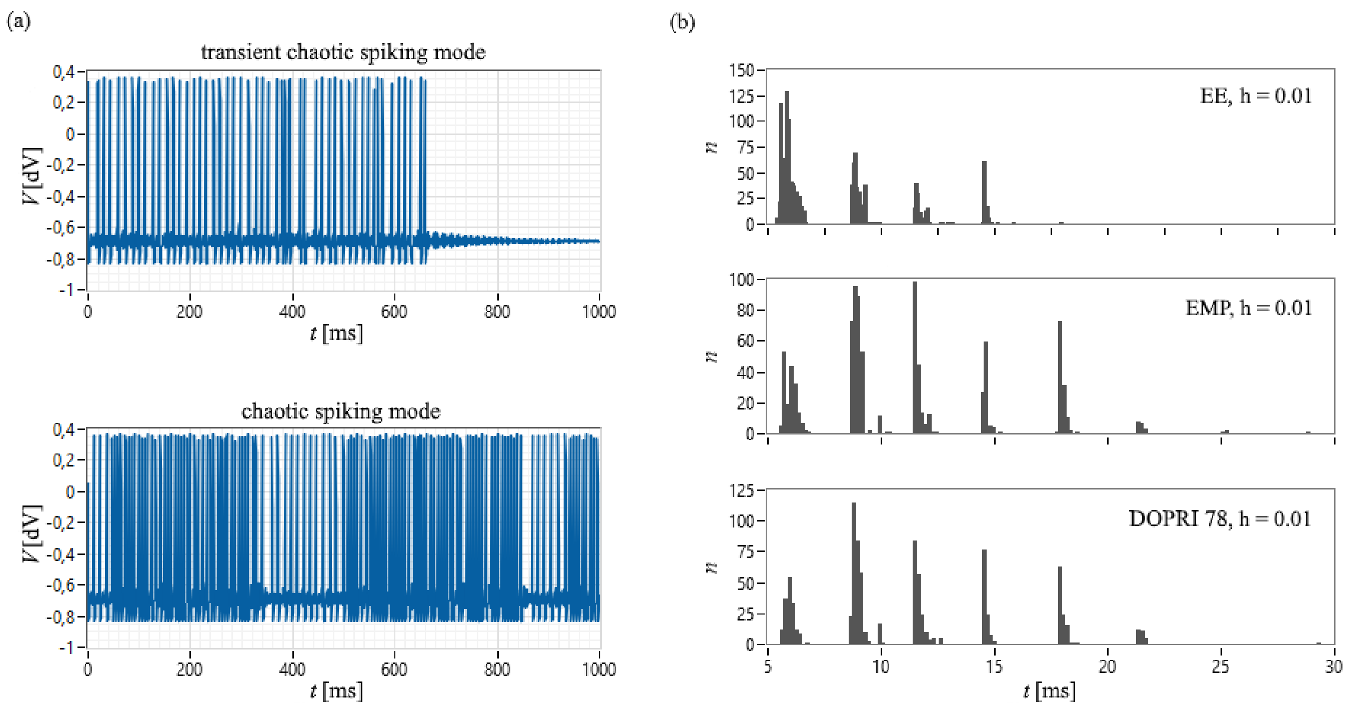

Figure 2. (a) Chaotic spiking modes in response to the input current I_0 = 7.5 µA with addition of the 264.6 Hz sinusoidal component with the amplitude A = 0.7 µA (b) Interspike interval histograms for the various explicit methods of different order

2.3 Chaotic Spiking

The second experiment was conducted for the case when dynamical system (1) is in a chaotic mode, and interspike interval histograms were chosen as an analysis tool. This type of diagram displays the number of intervals n with respect to their duration time t, and is usually applied to examine the statistical features of neurons in response to the noisy input.

Figure 2 (b) shows the diagrams (150 bins) for the EE, EMP and DOPRI 78 methods with the same stepsize h = 0.01 ms for the long-term (104 ms) simulation. The main observation for the explicit methods is that the discrete operators based on them do not share quantitative properties of the interval distribution. The use of the discrete operator on the basis of the 1st order EE method even leads to a loss of the long-interval peaks in the histogram.

Interesting results were obtained for the IE and SEE methods. In case of the IE application, the system simply entered the stationary mode in response to the input current, which was supposed to plunge it into chaos. The operator based on the SEE method has put the system into the transient chaotic mode. The lifetime of the transient spiking interval in this mode sensitively depends on the initial conditions. In view of these facts it seems inappropriate to apply interspike interval histograms on a par with the investigated explicit methods.

Thus, low-order numerical methods are unable to preserve even the statistical properties of the original mathematical model in a chaotic mode of oscillations.

3. CONCLUSIONS

The results of this study can be useful for those, who are looking for a digital realization of dynamical systems representing spiking neurons, e.g. based on the DSP technology. In this paper we briefly described some undesirable numerical effects that are inherent in digital computer models of biological neurons on the basis of numerical methods of low accuracy orders. In addition to the mathematical model under consideration, similar effects were observed for the classical Hodgkin-Huxley model. The coupled neuron models were left beyond the scope of the paper, but previously obtained results for the small network with memristive synapses [6] suggest that sensitivity to a discrete operator significantly increases in the presence of chaotic transients.

4. ACKNOWLEDGMENTS

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