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Propoblity and Bayes rule:

1.a:
denote A as 'identical twins',C as 'fraternal twins' and B as 'twin brother'

P(A ∧ B) = P(A) · P(B) = $\frac{1}{2} \cdot \frac{1}{300}$

P(C ∧ B) = P(C) · P(B) = $\frac{1}{4} \cdot \frac{1}{125}$

That means we can use the conditional probability's formula:

$$P(A|B) \ = \ \frac{\frac{1}{2} \cdot \frac{1}{300}}{\frac{1}{500} + \frac{1}{600}} = \frac{5}{11}$$

1.b
1st cookie jar has 10 almond cookies and 30 chocolate cookies
2nd cookie jar has 20 almond and chocolate cookies

$$A \ - \ chose\ bowl\ 1 \left(\frac{1}{2}\right)$$
$$B - \ chose\ chocolate\ cake\ ()$$

$$P(A|B) \ = \ \frac{P(B|A) \cdot P(A)}{P(B|A) \ + \ P(B|\neg A)} = \frac{\frac{3}{4} \cdot \frac{1}{2}}{\frac{3}{4} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2}} = 0.6$$

2.
denote A as 'given yellow m&m from 1994 bag', B as 'given yelolow and grenn m&m'
so:

$$P(A|B) \ = \ \frac{0.2}{0.2 \ + \ 0.14} \ = \ \frac{10}{17}$$

3.
denote A as 'person is sick' and denote B as 'tested positive' then:

$$P(A|B) \ = \ \frac{P(B|A) \cdot P(A)}{P(B)}$$

we know that:

$$P(B|A) \ = \ 1$$

$$P(B|\neg A) \ = \ 0.1$$

$$P(A) \ = \frac{1}{10000}$$

hence:

$$P(A|B) \ = \ \frac{\frac{1}{10000}}{\frac{1}{10000} \ + \ \left(1 - \frac{1}{10000}\right)0.1} = \frac{100}{10099} \ = 0.99\%$$

2.b

now $P(H) \ = \frac{1}{200}$,hence:

$$P(H|E) \ = \ \frac{\frac{1}{200}}{\frac{1}{200} + \left(1 - \frac{1}{200}\right)0.1} \ = \frac{100}{299} \approx 33\%$$

4.

denote A as 'identical twins',C as 'fraternal twins' and B as 'twin brother'

P(A $\wedge$ B) = P(A) $\cdot$ P(B) = $\frac{1}{2} \cdot \frac{1}{300}$

P(C $\wedge$ B) = P(C) $\cdot$ P(B) =$\frac{1}{4} \cdot \frac{1}{125}$

That means we can use the conditional probability's formula:

$$P(A|B) \ = \frac{\frac{1}{2} \cdot \frac{1}{300}}{\frac{1}{500} + \frac{1}{600}} = \frac{5}{11}$$

Random Variables:

1.

denote X_1, X_2 as the resualt of the first and second throw, and D_i as number divisable by i, We wan D_3, there are 4 sums that achive that, 3,6,9,12

so

$$P(D_i) = P(X_1 + X_2 = 3) \ + \ P(X_1 + X_2 = 6) + P(X_1 + X_2 = 9) + P(X_1 + X_2 = 12)$$

$$= \frac{2}{36} + \frac{5}{36} + \frac{4}{36} + \frac{1}{36} \ = \ \frac{12}{36}$$

and

$$E(D_3) = 6 \cdot D_3 + (-3) \cdot (1 - D_3) = 6 \cdot \frac{12}{36} - 3 \cdot \frac{24}{36} = 0$$

2.
The total number os possible outcomes is 25, hence the probability of geting more then 12 is $\frac{6}{25}$
$\{(3, 10)\ , (4, 10), (4, 9), (5, 8), (5, 9), (5, 10)\}$

the probability of getting 12 us $\frac{4}{25}$ $\{(2\ ,\ 10)\ , (3, 9), (\ 4\ ,\ 8), (\ 5\ ,\ 7)\}$

So :

$$E\ =\ 5 \cdot \frac{6}{25}\ - 6 \cdot \frac{15}{25}\ =\ -2.4$$

3.
There are 8 seats each month.
Denote X_i as the indicator of the i-th seat was taken by a man, So $P(X_i = 1)\ =\ 0.4$
then the mean is $8 \cdot P(X_i = 1)\ =\ 0.4 \cdot 8 = 3.2$

the std is:

$$\sigma\ =\ \sqrt{var(X)}\ =\ \sqrt{8 \cdot 0.4 \cdot 0.6} = 1.38$$

where the second equallity is based on the formula of variance of binomal disterbution

4.
We will use the Z-table to solve this problme, where $x = 30, \mu\ =\ 26,\ \sigma\ =\ 2$
so the Z score is

$$Z\ =\ \frac{x - \mu}{\sigma} = \frac{30 - 26}{2} = 2$$

from the Z table:

$$P(26 < X < 30)\ =\ P(X < 30)\ - 0.5\ = P(0 < Z < 2)\ -\ 0.5\ =\ 0.97725 -\ 0.5 = 0.47725$$

5.
it is the area under the triangle, so

$$P(X \geqslant 3) \ = \frac{0.4 \cdot 2}{2} = 0.4$$

and P(X>3) is

6.
This is the binomial disterbution where the probobilty of an employee having a child is 0.6 so

$$\binom{4}{3}(0.6)^3 \cdot 0.4 = \frac{216}{625}$$

7.
looking at the graph we can see it is symetrical and the mean is 0, and the expecter value is the mean value so the expected value is 0