1 Probability and Counting

Counting

1. How many ways are there to permute the letters in the word MISSISSIPPI?

   - The answer is obtained by finding the total number of permutation and dividing by the number of permutation of the individual letters. The total number of permutations is 11! because there are 11 characters in that word. For example the number of permutations of the four characters S is 4!, as such the number of permutations is

   $$\frac{11!}{1!4!4!2!} = 34650.$$

2. (a) How many 7-digit phone numbers are possible, assuming that the first digit can't be a 0 or a 1?

   - Assuming all other digits are possible there are $10-2 = 8$ possible digits for the first position and 10 for the remaining $7 - 1 = 6$ positions as such the number of phone numbers is

   $$8 * 10^6 = 8000000.$$

   (b) Re-solve (a), except now assume also that the phone number is not allowed to start with 911 (since this is reserved for emergency use, and it would not be desirable for the system to wait to see whether more digits were going to be dialed after someone has dialed 911).

   - The total count of numbers starting with 911 is 10^4 as such removing those yields

   $$8000000 - 10^4 = 7990000.$$

3. Fred is planning to go out to dinner each night of a certain week, Monday through Friday, with each dinner being at one of his ten favorite restaurants.

   - How many possibilities are there for Fred's schedule of dinners for that Monday through Friday, if Fred is not willing to eat at the same restaurant more than once?
     - There are 5 days and 10 restaurants which are picked without repetation. As such he as 10 choices on the first day, 9 choices on the second day and so on, as such in total

     $$\frac{10!}{5!} = 30240.$$

- How many possibilities are there for Fred's schedule of dinners for that Monday through Friday, if Fred is willing to eat at the same restaurant more than once, but is not willing to eat at the same place twice in a row (or more)?
  - There are 10 possibilities on the first day. On the second day he can choose any restaurant which is different from the first one. Same situation on the third day and so on. As such in total

$$10 * 9^4 = 65610.$$

4. A round-robin tournament is being held with n tennis players; this means that every player will play against every other player exactly once.

   (a) How many possible outcomes are there for the tournament (the outcome lists out who won and who lost for each game)?

   - Every player will have exactly $n - 1$ opponents, summing up over all players gives $n(n-1)$, but of course we doublecounted everything because every opponent is also a player. As such the total number of matches is $\frac{n(n-1)}{2}$. Because every match is either a win or a loss the total number of outcomes is

$$2^{\frac{n(n-1)}{2}}.$$

   (b) How many games are played in total?

   - As mentioned in the first part, it's $\frac{n(n-1)}{2}$.

5. A knock-out tournament is being held with 2n tennis players. This means that for each round, the winners move on to the next round and the losers are eliminated, until only one person remains. For example, if initially there are $24 = 16$ players, then there are 8 games in the first round, then the 8 winners move on to round 2, then the 4 winners move on to round 3, then the 2 winners move on to round 4, the winner of which is declared the winner of the tournament. (There are various systems for determining who plays whom within a round, but these do not matter for this problem.)

   (a) How many rounds are there?

   (b) Count how many games in total are played, by adding up the numbers of games played in each round.

   (c) Count how many games in total are played, this time by directly thinking about it without doing almost any calculation.