Practical Example - Linear Regression

1. Purpose

> ***Predict the price*** of a used car depending on its specs

2. Data Profiling

- Possibly strong ***explanatory variables***: `Brand` , `Mileage` , `EngineV` , `Year`
- ***Target:*** `Price`

3. Process

Events	Table
read_csv	raw_data
drop column `Model`)	data
drop N/A	data_no_mv
remove 1% higest outliers from `Price`	data_1
remove 1% highest outliers from `Mileage`	data_2
remove abnormal value from `EngineV`	data_3
remove 1% oldest cars from `Year`	data_4
reset_index for data_4	data_cleaned
apply log transformation for the target `Price`	data_cleaned (replace `Price` by `log_price`)
check multicollinearity via VIF	data_no_multicolinearity
get dummies for categorical variables	data_with_dummy
rearrange	data_processed

Importing the relevant libraries

In [1]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns
sns.set() # Turn all Matplotlib's graphs to Seaborn's

import statsmodels.api as sm
from sklearn.linear_model import LinearRegression

import warnings
warnings.filterwarnings('ignore')
```

Loading the data

In [2]:
```
raw_data = pd.read_csv("C:/Users/baoph/OneDrive - Seneca/Documents/365 Data Science/Mac
raw_data.head()
```

Out[2]:

	Brand	Price	Body	Mileage	EngineV	Engine Type	Registration	Year	Model
0	BMW	4200.0	sedan	277	2.0	Petrol	yes	1991	320
1	Mercedes-Benz	7900.0	van	427	2.9	Diesel	yes	1999	Sprinter 212
2	Mercedes-Benz	13300.0	sedan	358	5.0	Gas	yes	2003	S 500
3	Audi	23000.0	crossover	240	4.2	Petrol	yes	2007	Q7
4	Toyota	18300.0	crossover	120	2.0	Petrol	yes	2011	Rav 4

In [3]:
```
raw_data.shape
```

Out[3]:
```
(4345, 9)
```

Preprocessing

Exploring the descriptive statistics of the variables

In [4]:
```
raw_data.describe(include= 'all') # include descriptives for category var too
```

Out[4]:

	Brand	Price	Body	Mileage	EngineV	Engine Type	Registration	Year
count	4345	4173.000000	4345	4345.000000	4195.000000	4345	4345	4345.000000
unique	7	NaN	6	NaN	NaN	4	2	NaN
top	Volkswagen	NaN	sedan	NaN	NaN	Diesel	yes	NaN
freq	936	NaN	1649	NaN	NaN	2019	3947	NaN
mean	NaN	19418.746935	NaN	161.237284	2.790734	NaN	NaN	2006.550058
std	NaN	25584.242620	NaN	105.705797	5.066437	NaN	NaN	6.719097
min	NaN	600.000000	NaN	0.000000	0.600000	NaN	NaN	1969.000000
25%	NaN	6999.000000	NaN	86.000000	1.800000	NaN	NaN	2003.000000
50%	NaN	11500.000000	NaN	155.000000	2.200000	NaN	NaN	2008.000000
75%	NaN	21700.000000	NaN	230.000000	3.000000	NaN	NaN	2012.000000
max	NaN	300000.000000	NaN	980.000000	99.990000	NaN	NaN	2016.000000

Some Notes:

- ***Misssing value***: Look at count row. `Price` and `EngineV` seems to be missing some of values
- ***Unique entries*** of each `cat var` : `Model` has ***312 unique entries***, which is **hard to implement*** the regression (It means we have more than ***300 dummies***)
- Number of car has been register `Registration` = ***'yes'*** is ***significantly high*** (***90%*** total of entries - almost all of them) **--> Won't be useful**
- A lot of the information from `Model` could be engineered from `Brand`, `Year`, and `EngineV` --> **Won't be losing too much variability**

Determining the variables of interest - Drop column(s)

In [5]:

```
data = raw_data.drop(['Model'], axis=1) # Drop [Model] column
data.describe(include='all')
```

Out[5]:

	Brand	Price	Body	Mileage	EngineV	Engine Type	Registration	Year
count	4345	4173.000000	4345	4345.000000	4195.000000	4345	4345	4345.000000
unique	7	NaN	6	NaN	NaN	4	2	NaN
top	Volkswagen	NaN	sedan	NaN	NaN	Diesel	yes	NaN
freq	936	NaN	1649	NaN	NaN	2019	3947	NaN
mean	NaN	19418.746935	NaN	161.237284	2.790734	NaN	NaN	2006.550058
std	NaN	25584.242620	NaN	105.705797	5.066437	NaN	NaN	6.719097
min	NaN	600.000000	NaN	0.000000	0.600000	NaN	NaN	1969.000000
25%	NaN	6999.000000	NaN	86.000000	1.800000	NaN	NaN	2003.000000
50%	NaN	11500.000000	NaN	155.000000	2.200000	NaN	NaN	2008.000000
75%	NaN	21700.000000	NaN	230.000000	3.000000	NaN	NaN	2012.000000
max	NaN	300000.000000	NaN	980.000000	99.990000	NaN	NaN	2016.000000

Dealing with missing values

In [6]:

```
data.isnull().sum()/data.shape[0]*100 # % of missing values for each var
```

Out[6]:

```
Brand           0.000000
Price           3.958573
Body            0.000000
Mileage         0.000000
EngineV         3.452244
Engine Type     0.000000
Registration    0.000000
```

```
Year            0.000000
dtype: float64
```

Rule of thumb:

> If you are ***removing <5% of the observations***, you are free to ust remove all that have Missing Value

In [7]:
```
data_no_mv = data.dropna(axis=0) # Drop N/A by row
```

In [8]:
```
data_no_mv.describe(include='all')
```

Out[8]:

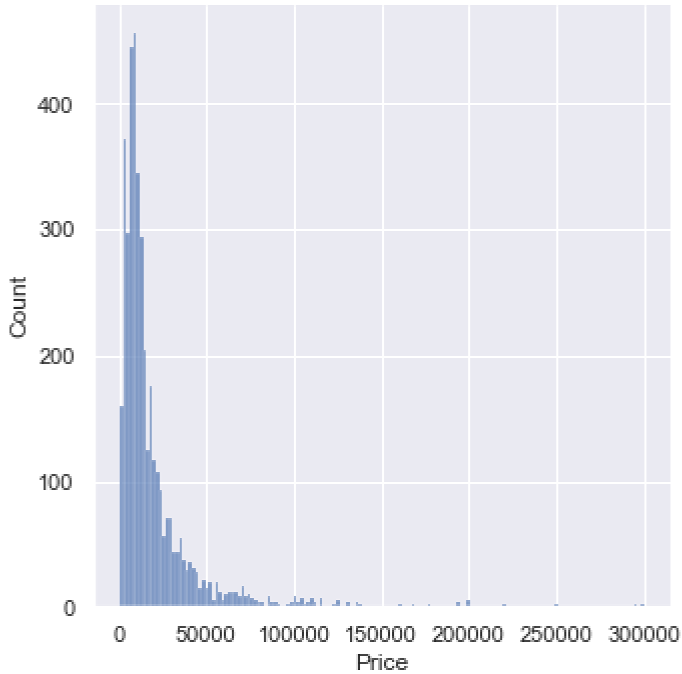
	Brand	Price	Body	Mileage	EngineV	Engine Type	Registration	Year
count	4025	4025.000000	4025	4025.000000	4025.000000	4025	4025	4025.000000
unique	7	NaN	6	NaN	NaN	4	2	NaN
top	Volkswagen	NaN	sedan	NaN	NaN	Diesel	yes	NaN
freq	880	NaN	1534	NaN	NaN	1861	3654	NaN
mean	NaN	19552.308065	NaN	163.572174	2.764586	NaN	NaN	2006.379627
std	NaN	25815.734988	NaN	103.394703	4.935941	NaN	NaN	6.695595
min	NaN	600.000000	NaN	0.000000	0.600000	NaN	NaN	1969.000000
25%	NaN	6999.000000	NaN	90.000000	1.800000	NaN	NaN	2003.000000
50%	NaN	11500.000000	NaN	158.000000	2.200000	NaN	NaN	2007.000000
75%	NaN	21900.000000	NaN	230.000000	3.000000	NaN	NaN	2012.000000
max	NaN	300000.000000	NaN	980.000000	99.990000	NaN	NaN	2016.000000

Exploring the PDFs

1. Distribution of `Price`

In [9]:
```
sns.displot(data_no_mv.Price) # Plot Price Distribution
```

Out[9]:
```
<seaborn.axisgrid.FacetGrid at 0x1867fa064f0>
```



Some Notes:

- `Price` has an ***exponential*** distribution
- For ***optimal results*** we would be looking for a ***normal distribution***
- We have a few ***outliers*** in `Price` --> **Remove the top 1% of observation**

1.1 Dealing with outliers in `Price`

In [10]:
```
q = data_no_mv.Price.quantile(0.99)
data_1 = data_no_mv[data_no_mv.Price < q]
data_1.describe(include='all')
```

Out[10]:

	Brand	Price	Body	Mileage	EngineV	Engine Type	Registration	Year
count	3984	3984.000000	3984	3984.000000	3984.000000	3984	3984	3984.000000
unique	7	NaN	6	NaN	NaN	4	2	NaN
top	Volkswagen	NaN	sedan	NaN	NaN	Diesel	yes	NaN
freq	880	NaN	1528	NaN	NaN	1853	3613	NaN
mean	NaN	17837.117460	NaN	165.116466	2.743770	NaN	NaN	2006.292922
std	NaN	18976.268315	NaN	102.766126	4.956057	NaN	NaN	6.672745
min	NaN	600.000000	NaN	0.000000	0.600000	NaN	NaN	1969.000000
25%	NaN	6980.000000	NaN	93.000000	1.800000	NaN	NaN	2002.750000
50%	NaN	11400.000000	NaN	160.000000	2.200000	NaN	NaN	2007.000000
75%	NaN	21000.000000	NaN	230.000000	3.000000	NaN	NaN	2011.000000

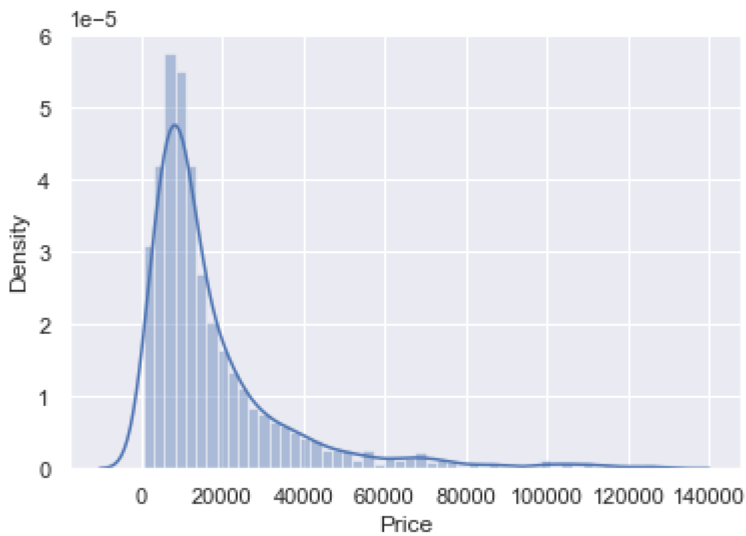
	Brand	Price	Body	Mileage	EngineV	Engine Type	Registration	Year
max	NaN	129222.000000	NaN	980.000000	99.990000	NaN	NaN	2016.000000

Some Notes:

- After removing outliers, the ***MAX(Price)*** is far away higher than the ***MEAN(Price)***, it is still acceptably closer

In [11]:
```
sns.distplot(data_1.Price)
```

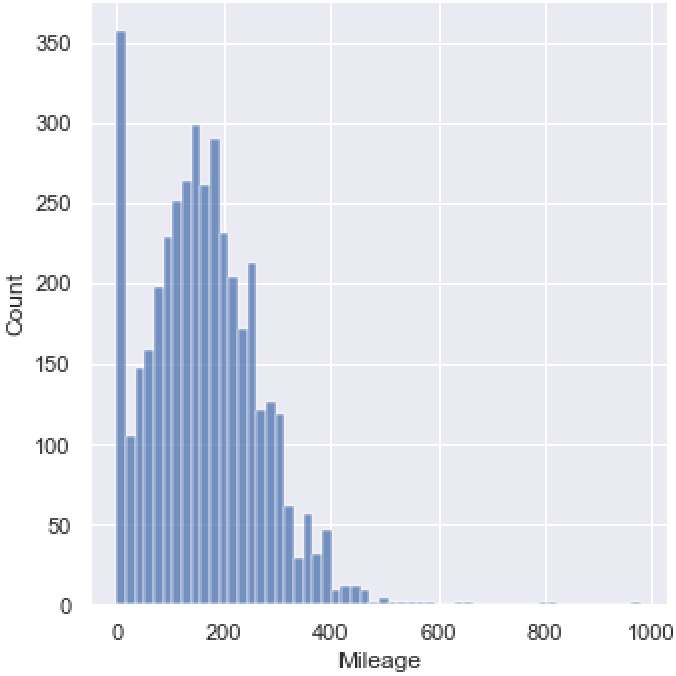
Out[11]:
```
<AxesSubplot:xlabel='Price', ylabel='Density'>
```



2. Distribution of `Mileage`

In [12]:
```
sns.displot(data_no_mv.Mileage)
```

Out[12]:
```
<seaborn.axisgrid.FacetGrid at 0x1860128f700>
```



2.1 Dealing with outliers in `Mileage`

In [13]:

```
q_1 = data_1.Mileage.quantile(0.99)
data_2 = data_1[data_1.Mileage < q_1]
data_2.describe(include='all')
```

Out[13]:

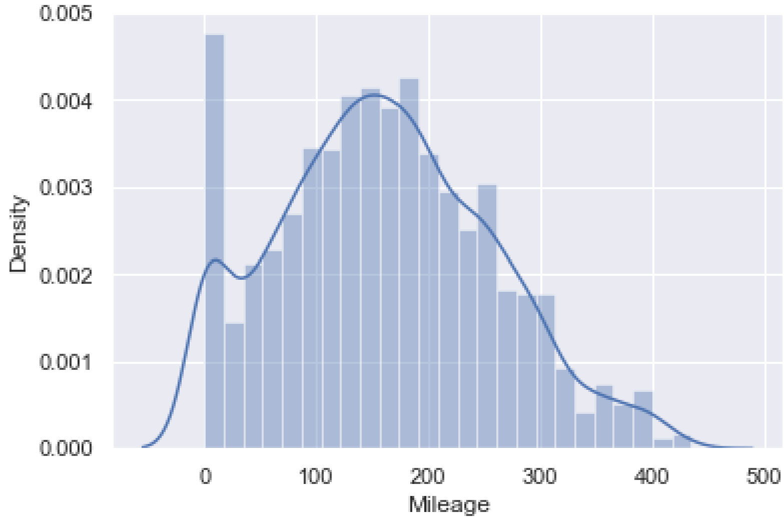
	Brand	Price	Body	Mileage	EngineV	Engine Type	Registration	Year
count	3944	3944.000000	3944	3944.000000	3944.000000	3944	3944	3944.000000
unique	7	NaN	6	NaN	NaN	4	2	NaN
top	Volkswagen	NaN	sedan	NaN	NaN	Diesel	yes	NaN
freq	867	NaN	1511	NaN	NaN	1825	3576	NaN
mean	NaN	17933.880822	NaN	161.484026	2.747612	NaN	NaN	2006.389959
std	NaN	19008.212025	NaN	96.027108	4.980406	NaN	NaN	6.595986
min	NaN	600.000000	NaN	0.000000	0.600000	NaN	NaN	1969.000000
25%	NaN	7000.000000	NaN	92.000000	1.800000	NaN	NaN	2003.000000
50%	NaN	11500.000000	NaN	158.000000	2.200000	NaN	NaN	2007.000000
75%	NaN	21376.250000	NaN	230.000000	3.000000	NaN	NaN	2011.000000
max	NaN	129222.000000	NaN	435.000000	99.990000	NaN	NaN	2016.000000

In [14]:

```
sns.distplot(data_2.Mileage)
```

Out[14]:

```
<AxesSubplot:xlabel='Mileage', ylabel='Density'>
```

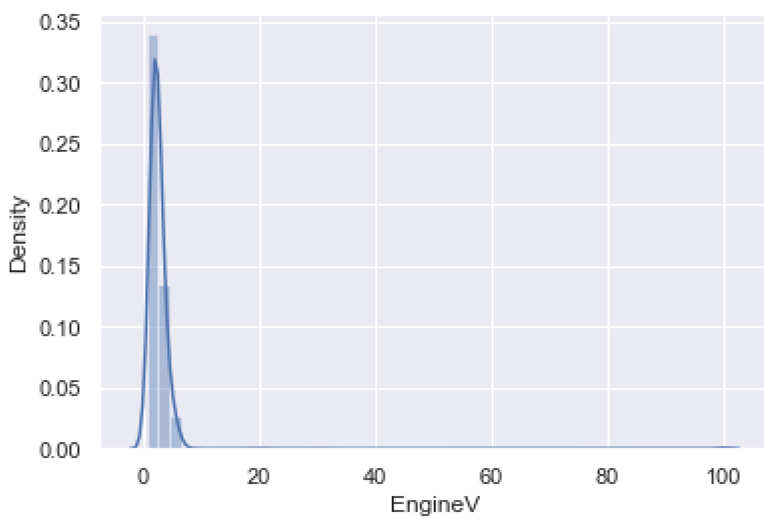


3. Distribution of `EngineV`

In [15]:

```
sns.distplot(data_no_mv.EngineV)
```

Out[15]: `<AxesSubplot:xlabel='EngineV', ylabel='Density'>`



Some Notes:

- Take a look at `EngineV` we see there is many ***strange*** value like ***99.99***. The interval of the `EngineV` normally low ***[0.6; 6.5]*** **--> 99.99 is incorrect entry** (That's a common way to label missing values) **--> Chose the engine volumn below 6.5**

In [16]:

```
EngV = pd.DataFrame(raw_data.EngineV)
EngV = EngV.dropna(axis=0)
EngV.sort_values(by="EngineV")
```

Out[16]:

	EngineV
2512	0.60
188	0.65

	EngineV
3295	1.00
2725	1.00
1923	1.00
...	...
1311	99.99
3114	99.99
1264	99.99
3641	99.99
256	99.99

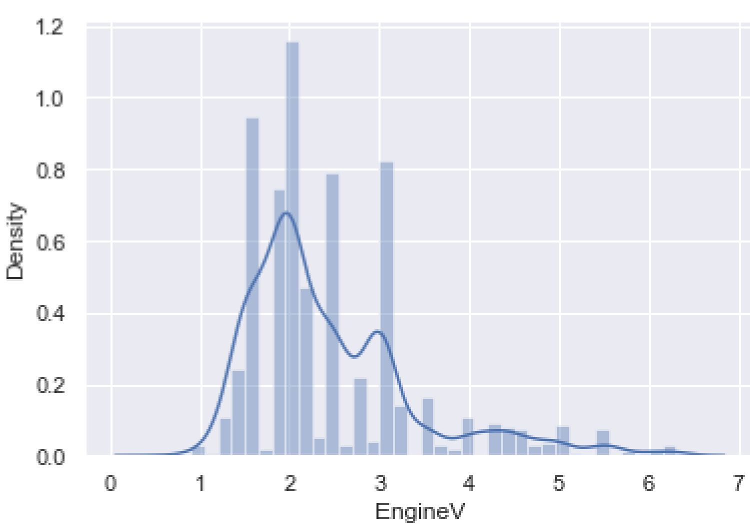
4195 rows × 1 columns

3.1 Remove abnormal value of `EngineV`

In [17]:
```
data_3 = data_2[data_2.EngineV < 6.5]
```

In [18]:
```
sns.distplot(data_3.EngineV)
```

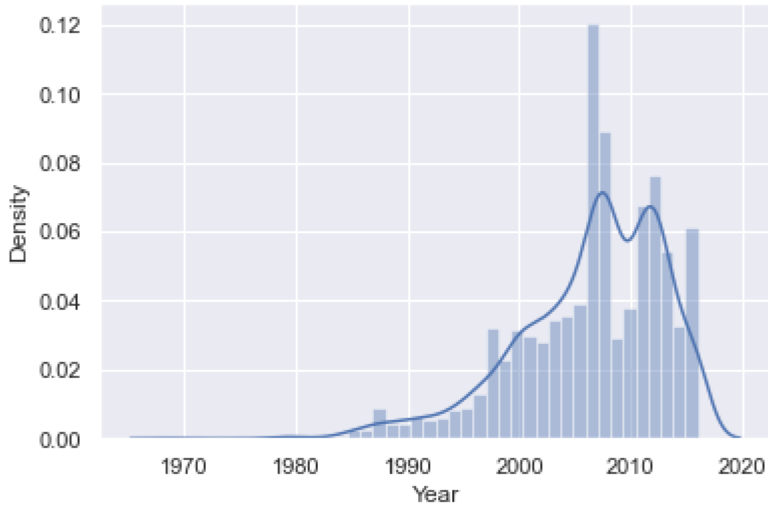
Out[18]:
```
<AxesSubplot:xlabel='EngineV', ylabel='Density'>
```



4. Distribution of `Year`

In [19]:
```
sns.distplot(data_3.Year)
```

Out[19]:
```
<AxesSubplot:xlabel='Year', ylabel='Density'>
```

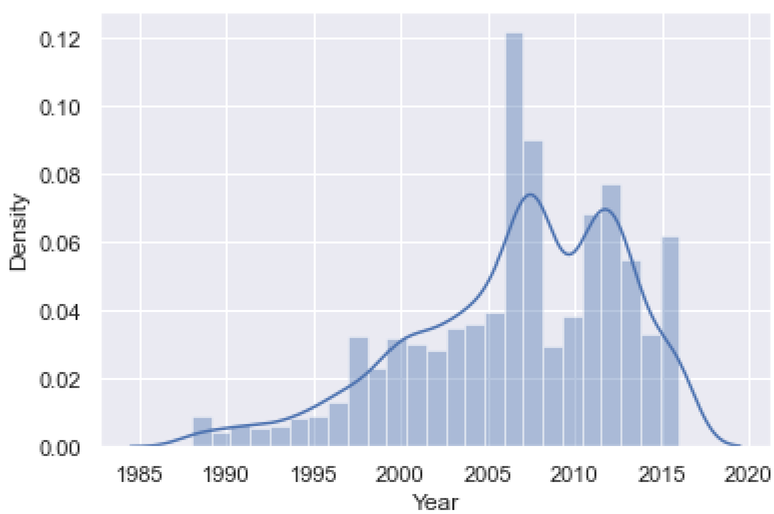


4.1 Remove most vintage car

In [20]:
```
q_3 = data_3.Year.quantile(0.01)
data_4 = data_3[data_3.Year>q_3]
```

In [21]:
```
sns.distplot(data_4.Year)
```

Out[21]: <AxesSubplot:xlabel='Year', ylabel='Density'>



In [22]:
```
data_cleaned = data_4.reset_index(drop=True)
data_cleaned
```

Out[22]:

	Brand	Price	Body	Mileage	EngineV	Engine Type	Registration	Year
0	BMW	4200.0	sedan	277	2.0	Petrol	yes	1991
1	Mercedes-Benz	7900.0	van	427	2.9	Diesel	yes	1999
2	Mercedes-Benz	13300.0	sedan	358	5.0	Gas	yes	2003
3	Audi	23000.0	crossover	240	4.2	Petrol	yes	2007
4	Toyota	18300.0	crossover	120	2.0	Petrol	yes	2011

	Brand	Price	Body	Mileage	EngineV	Engine Type	Registration	Year
...	...	...	...	...	...	...	...	...
3862	Volkswagen	11500.0	van	163	2.5	Diesel	yes	2008
3863	Toyota	17900.0	sedan	35	1.6	Petrol	yes	2014
3864	Mercedes-Benz	125000.0	sedan	9	3.0	Diesel	yes	2014
3865	BMW	6500.0	sedan	1	3.5	Petrol	yes	1999
3866	Volkswagen	13500.0	van	124	2.0	Diesel	yes	2013

3867 rows × 8 columns

Final table for preprocessing step

In [23]:

```
data_cleaned.describe(include='all')
```

Out[23]:

	Brand	Price	Body	Mileage	EngineV	Engine Type	Registration	Year
count	3867	3867.000000	3867	3867.000000	3867.000000	3867	3867	3867.000000
unique	7	NaN	6	NaN	NaN	4	2	NaN
top	Volkswagen	NaN	sedan	NaN	NaN	Diesel	yes	NaN
freq	848	NaN	1467	NaN	NaN	1807	3505	NaN
mean	NaN	18194.455679	NaN	160.542539	2.450440	NaN	NaN	2006.709853
std	NaN	19085.855165	NaN	95.633291	0.949366	NaN	NaN	6.103870
min	NaN	800.000000	NaN	0.000000	0.600000	NaN	NaN	1988.000000
25%	NaN	7200.000000	NaN	91.000000	1.800000	NaN	NaN	2003.000000
50%	NaN	11700.000000	NaN	157.000000	2.200000	NaN	NaN	2008.000000
75%	NaN	21700.000000	NaN	225.000000	3.000000	NaN	NaN	2012.000000
max	NaN	129222.000000	NaN	435.000000	6.300000	NaN	NaN	2016.000000

Checking the OLS assumption

First Assumption: Linearity

In [24]:

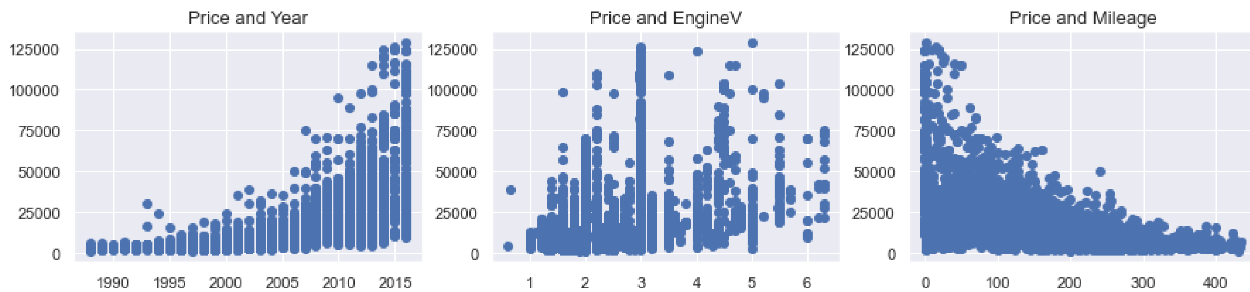
```
fig, axes = plt.subplots(nrows=1, ncols=3, figsize=(15,3))

axes[0].scatter(data_cleaned.Year, data_cleaned.Price)
axes[0].set_title("Price and Year", fontsize=13)

axes[1].scatter(data_cleaned.EngineV, data_cleaned.Price)
axes[1].set_title("Price and EngineV", fontsize=13)
```

```
axes[2].scatter(data_cleaned.Mileage, data_cleaned.Price)
axes[2].set_title("Price and Mileage", fontsize=13)
```

Out[24]: Text(0.5, 1.0, 'Price and Mileage')



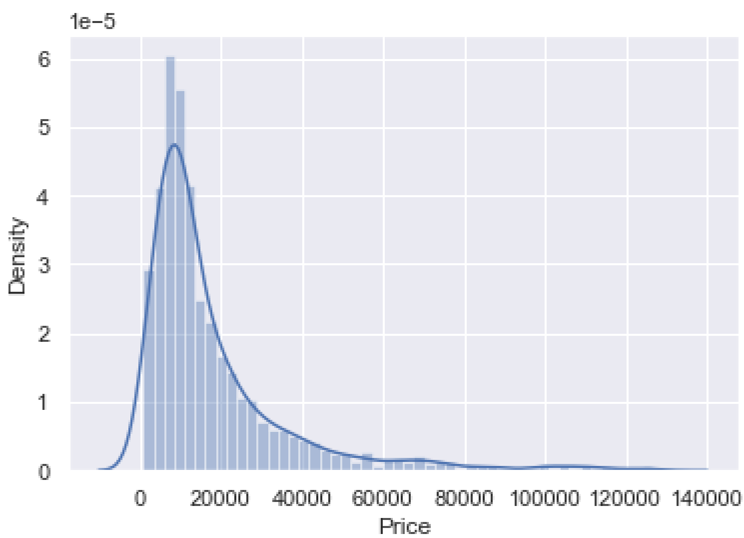
Some Notes:

- We can spot the patterns but definitely not Linear one **--> Should not run Linear Regression this case (Assumption 1)**

--> Should first **transform one or more variables** --> **Log Tranformation** is especially useful when dealing with exponential scatter plots like we do now

In [25]:
```
sns.distplot(data_cleaned.Price)
```

Out[25]: <AxesSubplot:xlabel='Price', ylabel='Density'>



Some Notes:

- `Price` is ***not normally distributed***

--> It's relationship with other normally distributed features is not linear (quite exponential in those cases)

Relaxing the assumption

In [26]:
```
log_price = np.log(data_cleaned.Price)
data_cleaned["log_price"] = log_price
```

```
data_cleaned
```

Out[26]:

	Brand	Price	Body	Mileage	EngineV	Engine Type	Registration	Year	log_price
0	BMW	4200.0	sedan	277	2.0	Petrol	yes	1991	8.342840
1	Mercedes-Benz	7900.0	van	427	2.9	Diesel	yes	1999	8.974618
2	Mercedes-Benz	13300.0	sedan	358	5.0	Gas	yes	2003	9.495519
3	Audi	23000.0	crossover	240	4.2	Petrol	yes	2007	10.043249
4	Toyota	18300.0	crossover	120	2.0	Petrol	yes	2011	9.814656
...	...	...	...	...	...	...	...	...	...
3862	Volkswagen	11500.0	van	163	2.5	Diesel	yes	2008	9.350102
3863	Toyota	17900.0	sedan	35	1.6	Petrol	yes	2014	9.792556
3864	Mercedes-Benz	125000.0	sedan	9	3.0	Diesel	yes	2014	11.736069
3865	BMW	6500.0	sedan	1	3.5	Petrol	yes	1999	8.779557
3866	Volkswagen	13500.0	van	124	2.0	Diesel	yes	2013	9.510445

3867 rows × 9 columns

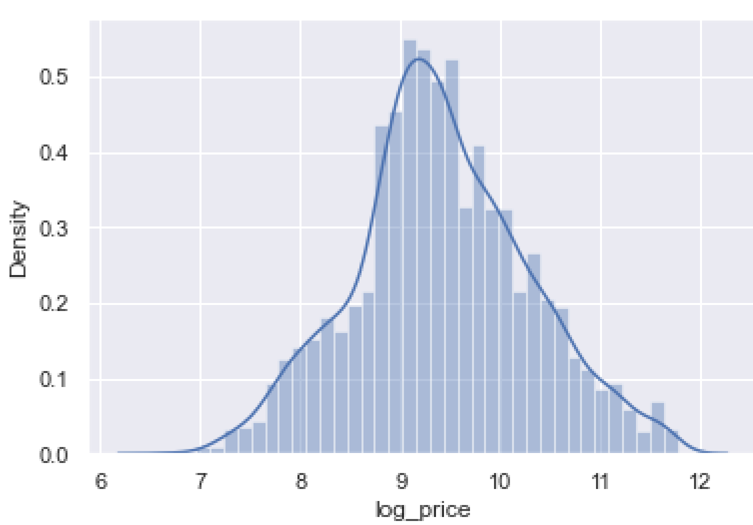
Some Notes:

- Now `log_price` distribution is approiximately ***bell-shaped***
- We can see the ***linear patterns*** in all plots now

In [27]:

```
sns.distplot(data_cleaned.log_price)
```

Out[27]: <AxesSubplot:xlabel='log_price', ylabel='Density'>



In [28]:

```
fig, axes = plt.subplots(nrows=1, ncols=3, figsize=(15,3))
```

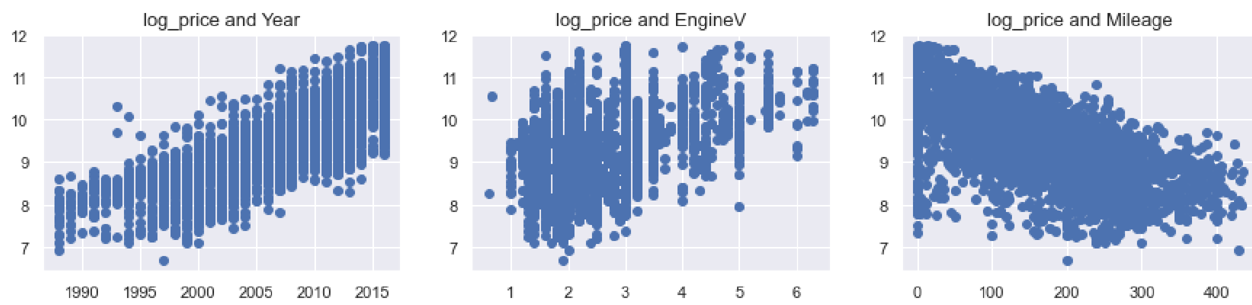
```
axes[0].scatter(data_cleaned.Year, data_cleaned.log_price)
axes[0].set_title("log_price and Year", fontsize=13)

axes[1].scatter(data_cleaned.EngineV, data_cleaned.log_price)
axes[1].set_title("log_price and EngineV", fontsize=13)

axes[2].scatter(data_cleaned.Mileage, data_cleaned.log_price)
axes[2].set_title("log_price and Mileage", fontsize=13)
```

Out[28]:

```
Text(0.5, 1.0, 'log_price and Mileage')
```



In [29]:

```
data_cleaned = data_cleaned.drop("Price", axis=1)
```

Second Assumption: No Endogeneity

Third Assumption: Normality and Homoscedasticity

- ***Normality and zero mean***: $residual-N(0,var^2)$
- ***Homoscedasticity***: as we can see from the graphs, because we already implemented a log transformation.

Fourth Assumption: No Autocorrelation

- The observations we have are ***not coming from time series data or panel data***, so do not need too much effort on that
- There is no reason for the observations to be dependent on each other in this car sales case

Fifth Assumption: Multicollinearity

- It is logical that `Year` and `Mileage` will be correlated. The newer the car. the lower its mileage

---> **Have ground to suspect some degree of multicilinearity in the data**

---> Check multicolinearity through ***Variance Inflation Factor (VIF)***.

> VIF produces a measure that estimates how much larger the square root of the standard error of an estimate is compared to a situation where the variable is completely uncorrelated to the other predictors

In [30]:
```
from statsmodels.stats.outliers_influence import variance_inflation_factor

variables = data_cleaned[["Mileage","Year","EngineV"]]

vif = pd.DataFrame()
vif["VIF"] = [variance_inflation_factor(variables.values, i) for i in range(variables.s
vif["features"] = variables.columns
```

In [31]:
```
vif
```

Out[31]:

	VIF	features
0	3.791584	Mileage
1	10.354854	Year
2	7.662068	EngineV

In [32]:
```
data_no_multicollinearity = data_cleaned.drop("Year", axis=1)
```

$VIF \in [1, inf)$

- $VIF = 1$: **No Multicollinearity**
- $1 < VIF < 5$: **Perfectly okay**
- $5/6/10 < VIF$: **unacceptable** (cut-off line varies from sources to sources)

Some Notes:

- It seems like `Year` is defintely ***too correlated with the other variables*** **---> Only remove `Year`**

Create dummy variables

In [33]:
```
data_with_dummies = pd.get_dummies(data_no_multicollinearity, drop_first=True)
```

Some Notes:

- `Drop_first = True` to make sure that it ***won't create data for the first items of each category***
- If not, we are introducing ***multicolinearity*** to the model

In [34]:
```
data_with_dummies
```

Out[34]:

	Mileage	EngineV	log_price	Brand_BMW	Brand_Mercedes-Benz	Brand_Mitsubishi	Brand_Renault	B
0	277	2.0	8.342840	1	0	0	0	
1	427	2.9	8.974618	0	1	0	0	

	Mileage	EngineV	log_price	Brand_BMW	Brand_Mercedes-Benz	Brand_Mitsubishi	Brand_Renault	B
2	358	5.0	9.495519	0	1	0	0	
3	240	4.2	10.043249	0	0	0	0	
4	120	2.0	9.814656	0	0	0	0	
...	...	...	...	...	...	...	...	
3862	163	2.5	9.350102	0	0	0	0	
3863	35	1.6	9.792556	0	0	0	0	
3864	9	3.0	11.736069	0	1	0	0	
3865	1	3.5	8.779557	1	0	0	0	
3866	124	2.0	9.510445	0	0	0	0	

3867 rows × 18 columns

Check the VIF of the features including the dummies

In [35]:

```
data_with_dummies_branch = data_with_dummies.copy()
data_with_dummies_branch = data_with_dummies_branch.drop("log_price", axis=1)
```

In [36]:

```
VIF = pd.DataFrame()

variables = data_with_dummies_branch

VIF["VIF"] = [variance_inflation_factor(variables.values, i) for i in range(variables.s
VIF["features"] = variables.columns
```

In [37]:

```
VIF
```

Out[37]:

	VIF	features
0	4.459662	Mileage
1	7.841729	EngineV
2	2.294007	Brand_BMW
3	2.868649	Brand_Mercedes-Benz
4	1.641712	Brand_Mitsubishi
5	2.086774	Brand_Renault
6	2.162166	Brand_Toyota
7	2.844515	Brand_Volkswagen
8	1.464260	Body_hatch

	VIF	features
9	1.534059	Body_other
10	3.120431	Body_sedan
11	1.581933	Body_vagon
12	2.470096	Body_van
13	1.689146	Engine Type_Gas
14	1.082037	Engine Type_Other
15	2.498172	Engine Type_Petrol
16	9.641446	Registration_yes

In [38]:

```
VIF[VIF['VIF'] > 5]
```

Out[38]:

	VIF	features
1	7.841729	EngineV
16	9.641446	Registration_yes

Some Notes:

- We can drop the `Registration_yes` columns to optimize the model

Rearrange a bit

In [39]:

```
data_with_dummies.columns
```

Out[39]:

```
Index(['Mileage', 'EngineV', 'log_price', 'Brand_BMW', 'Brand_Mercedes-Benz',
       'Brand_Mitsubishi', 'Brand_Renault', 'Brand_Toyota', 'Brand_Volkswagen',
       'Body_hatch', 'Body_other', 'Body_sedan', 'Body_vagon', 'Body_van',
       'Engine Type_Gas', 'Engine Type_Other', 'Engine Type_Petrol',
       'Registration_yes'],
      dtype='object')
```

In [40]:

```
cols = ['log_price', 'Mileage', 'EngineV', 'Brand_BMW', 'Brand_Mercedes-Benz',
        'Brand_Mitsubishi', 'Brand_Renault', 'Brand_Toyota', 'Brand_Volkswagen',
        'Body_hatch', 'Body_other', 'Body_sedan', 'Body_vagon', 'Body_van',
        'Engine Type_Gas', 'Engine Type_Other', 'Engine Type_Petrol',
        'Registration_yes']
```

In [41]:

```
data_preprocessed = data_with_dummies[cols]
data_preprocessed
```

Out[41]:

	log_price	Mileage	EngineV	Brand_BMW	Brand_Mercedes-Benz	Brand_Mitsubishi	Brand_Renault	B
0	8.342840	277	2.0	1	0	0	0	
1	8.974618	427	2.9	0	1	0	0	

	log_price	Mileage	EngineV	Brand_BMW	Brand_Mercedes-Benz	Brand_Mitsubishi	Brand_Renault	B
2	9.495519	358	5.0	0	1	0	0	
3	10.043249	240	4.2	0	0	0	0	
4	9.814656	120	2.0	0	0	0	0	
...	...	...	...	...	...	...	...	
3862	9.350102	163	2.5	0	0	0	0	
3863	9.792556	35	1.6	0	0	0	0	
3864	11.736069	9	3.0	0	1	0	0	
3865	8.779557	1	3.5	1	0	0	0	
3866	9.510445	124	2.0	0	0	0	0	

3867 rows × 18 columns

Linear Regression Model

Declare the inputs and the targets

In [42]:
```
targets = data_preprocessed['log_price']
inputs = data_preprocessed.drop(['log_price'], axis=1)
```

Scale input variables - Standardization

In [43]:
```
from sklearn.preprocessing import StandardScaler

scaler = StandardScaler()
scaler.fit(inputs)
```

Out[43]:
```
StandardScaler()
```

In [44]:
```
inputs_scaled = scaler.transform(inputs)
```

Some Notes:

- It is not usually recommended to standardize dummy variables

> Scaling has no effect on the predictive power of dummies. Ince scaled, though, they lose all their dummy meaning.

Train Test Split

In [45]:
```
from sklearn.model_selection import train_test_split
```

```
x_train, x_test, y_train, y_test = train_test_split(inputs_scaled, targets, test_size=0
```

Create the regression

In [46]:
```
reg = LinearRegression()
reg.fit(x_train, y_train)
```

Out[46]:
```
LinearRegression()
```

Some Notes:

- In fact, this is a ***log-linear regression*** as the dependent variables is the log of `Price`

In [47]:
```
y_hat = reg.predict(x_train)
```

Check performance visually

In [48]:
```
x = np.linspace(0,13)
y = x
plt.plot(x,y, c="orange", lw=3)

plt.scatter(y_train, y_hat)
plt.xlabel("Targets (y_train)", fontsize=18)
plt.ylabel("Predictions (y_hat)", fontsize=18)
plt.xlim(6,13)
plt.ylim(6,13)
```

Out[48]:
```
(6.0, 13.0)
```

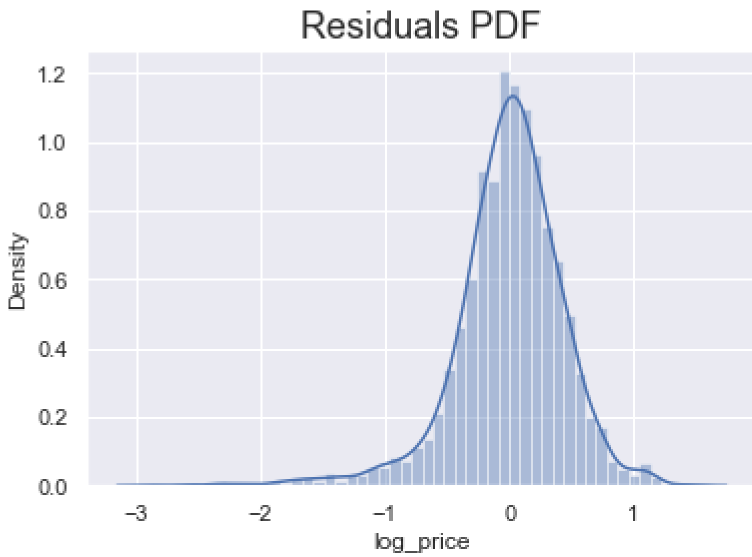
Some Notes:

- The more close the datapoint (`Targets` & `predictions`) to the 45-degree line, the better the model

In [49]:
```
sns.distplot(y_train-y_hat)
```

```
plt.title("Residuals PDF", size=18)
```

Out[49]: Text(0.5, 1.0, 'Residuals PDF')



Some Notes:

- When $Residuals$ ~ $N(0, var^2)$ --> Better the performance **(Normality and Homoscedasticity assumption)**
- There are quite **a much longer tails on the left side*** ---> There are certain observations for which `y_train` - `y_hat` is much lower than the mean (a much higher price is predicted than is observed)

---> `Predictions` tend to **overestimate the** `targets` and **rarely underestimate the** `targets`

R-Squared

In [50]:
```
reg.score(x_train, y_train)
```

Out[50]: 0.744996578792662

Finding the weights and bias

In [51]:
```
reg.intercept_
```

Out[51]: 9.415239458021299

In [52]:
```
reg.coef_
```

Out[52]:
```
array([-0.44871341,  0.20903483,  0.0142496 ,  0.01288174, -0.14055166,
       -0.17990912, -0.06054988, -0.08992433, -0.1454692 , -0.10144383,
       -0.20062984, -0.12988747, -0.16859669, -0.12149035, -0.03336798,
       -0.14690868,  0.32047333])
```

In [53]:
```
reg_summary = pd.DataFrame(columns = ["Features"], data=inputs.columns)
```

```
reg_summary["weights"] = reg.coef_
reg_summary
```

Out[53]:

	Features	weights
0	Mileage	-0.448713
1	EngineV	0.209035
2	Brand_BMW	0.014250
3	Brand_Mercedes-Benz	0.012882
4	Brand_Mitsubishi	-0.140552
5	Brand_Renault	-0.179909
6	Brand_Toyota	-0.060550
7	Brand_Volkswagen	-0.089924
8	Body_hatch	-0.145469
9	Body_other	-0.101444
10	Body_sedan	-0.200630
11	Body_vagon	-0.129887
12	Body_van	-0.168597
13	Engine Type_Gas	-0.121490
14	Engine Type_Other	-0.033368
15	Engine Type_Petrol	-0.146909
16	Registration_yes	0.320473

Testing

In [54]:

```
y_hat_test = reg.predict(x_test)
```

In [55]:

```
plt.plot(x,y,c="orange",lw=3)

plt.scatter(y_test, y_hat_test, alpha=0.2)
plt.xlabel("Targets (y_test)", fontsize=18)
plt.ylabel("Predictions (y_hat_test)", fontsize=18)
plt.xlim(6,13)
plt.ylim(6,13)
```

Out[55]:

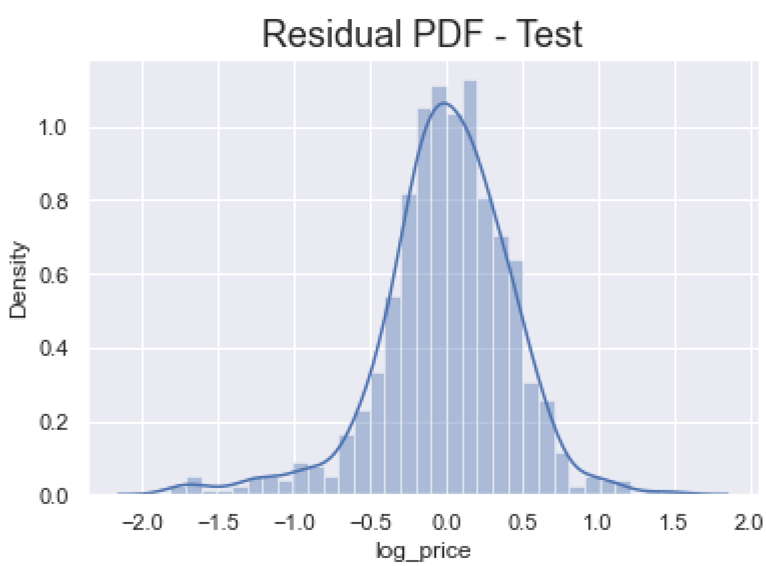
```
(6.0, 13.0)
```

Some Notes:

- For higher prices, we have a high concentration of values around the 45-degree line ---> Our model is very good at predicting higher prices
- For the lower ones, it looks not so amazing, much more scatterd

In [56]:
```
sns.distplot(y_test - y_hat_test)
plt.title("Residual PDF - Test", fontsize=18)
```

Out[56]:
```
Text(0.5, 1.0, 'Residual PDF - Test')
```



R-Squared

In [57]:
```
reg.score(x_test, y_test)
```

Out[57]:
```
0.7726984972665858
```

In [58]:
```
df_pf = pd.DataFrame(np.exp(y_hat_test), columns=["Prediction"])
df_pf.head()
```

Out[58]:

	Prediction
0	10685.501696
1	3499.255242
2	7553.285218
3	7463.963017
4	11353.490075

In [59]:

```
df_pf["Target"] = np.exp(y_test)
df_pf.head()
```

Out[59]:

	Prediction	Target
0	10685.501696	NaN
1	3499.255242	7900.0
2	7553.285218	NaN
3	7463.963017	NaN
4	11353.490075	NaN

In [60]:

```
y_test = y_test.reset_index(drop=True)
y_test
```

Out[60]:

```
0       7.740664
1       7.937375
2       7.824046
3       8.764053
4       9.121509
          ...
769    10.292146
770     9.169518
771     9.814656
772    11.134589
773     9.287301
Name: log_price, Length: 774, dtype: float64
```

In [61]:

```
df_pf["Target"] = np.exp(y_test)
df_pf.head()
```

Out[61]:

	Prediction	Target
0	10685.501696	2300.0
1	3499.255242	2800.0
2	7553.285218	2500.0
3	7463.963017	6400.0
4	11353.490075	9150.0

In [62]:
```
df_pf["Residual"] = df_pf["Target"] - df_pf["Prediction"]
df_pf
```

Out[62]:

	Prediction	Target	Residual
0	10685.501696	2300.0	-8385.501696
1	3499.255242	2800.0	-699.255242
2	7553.285218	2500.0	-5053.285218
3	7463.963017	6400.0	-1063.963017
4	11353.490075	9150.0	-2203.490075
...	...	...	...
769	29651.726363	29500.0	-151.726363
770	10732.071179	9600.0	-1132.071179
771	13922.446953	18300.0	4377.553047
772	27487.751303	68500.0	41012.248697
773	13491.163043	10800.0	-2691.163043

774 rows × 3 columns

In [63]:
```
df_pf["Difference%"] = np.absolute(df_pf["Residual"]/df_pf["Target"]*100)
df_pf
```

Out[63]:

	Prediction	Target	Residual	Difference%
0	10685.501696	2300.0	-8385.501696	364.587030
1	3499.255242	2800.0	-699.255242	24.973402
2	7553.285218	2500.0	-5053.285218	202.131409
3	7463.963017	6400.0	-1063.963017	16.624422
4	11353.490075	9150.0	-2203.490075	24.081859
...	...	...	...	...
769	29651.726363	29500.0	-151.726363	0.514327
770	10732.071179	9600.0	-1132.071179	11.792408
771	13922.446953	18300.0	4377.553047	23.921055
772	27487.751303	68500.0	41012.248697	59.871896
773	13491.163043	10800.0	-2691.163043	24.918176

774 rows × 4 columns

In [64]:
```
df_pf.describe()
```

Out[64]:

	Prediction	Target	Residual	Difference%
count	774.000000	774.000000	774.000000	774.000000
mean	15946.760167	18165.817106	2219.056939	36.256693
std	13133.197604	19967.858908	10871.218143	55.066507
min	1320.562768	1200.000000	-29456.498331	0.062794
25%	7413.644234	6900.000000	-2044.191251	12.108022
50%	11568.168859	11600.000000	142.518577	23.467728
75%	20162.408805	20500.000000	3147.343497	39.563570
max	77403.055224	126000.000000	85106.162329	512.688080

Some Notes:

- The min difference in percentage is 0.06%
- The max difference in percentage is pretty off mark
- For the most of our predictions, we got relatively close (based on percentiles)

---> The lower the difference% we got, the better

In [65]:

```
# Set diplay max_rows
# pd.options.display.max_rows = 999

# set display float_format 2 significant number
pd.set_option("display.float_format", lambda x: "%.2f" % x)

# sort values by Difference%
df_pf.sort_values(by=["Difference%"])
```

Out[65]:

	Prediction	Target	Residual	Difference%
698	30480.85	30500.00	19.15	0.06
742	16960.31	16999.00	38.69	0.23
60	12469.21	12500.00	30.79	0.25
110	25614.14	25500.00	-114.14	0.45
367	42703.68	42500.00	-203.68	0.48
...	...	...	...	...
657	32481.05	6000.00	-26481.05	441.35
162	9954.42	1800.00	-8154.42	453.02
451	35956.50	6500.00	-29456.50	453.18
532	10019.90	1800.00	-8219.90	456.66
639	30628.28	4999.00	-25629.28	512.69

774 rows × 4 columns

Some Notes:

> In those last samples `predictions` are higher than `targets` ---> Maybe we are missing an important factor which drives the price of a used car lower, which maybe the `car_model` that we removed or maybe that car was damaged in some way (the information that we did not initally have)

How to improve our model?

1. Use a different set of variables
2. Remove a bigger part of the ourliers in observations
3. Use different kind of transformations