Gradient Descent with 1 Feature

In [1]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns
sns.set()

import warnings
warnings.filterwarnings("ignore")

pd.set_option('max_columns', 200)
pd.set_option('max_rows', 100)
```

In [2]:

```
import math
```

In [3]:

```
x_train = np.array([1.0, 2.0])
y_train = np.array([300, 500])
```

In [4]:

```
def compute_cost(x,y,w,b):
    m = x.shape[0]
    cost = 0

    for i in range(m):
        f_wb = w * x[i] + b
        cost = cost + (f_wb-y[i])**2

    total_cost = 1/(2*m) * cost

    return total_cost
```

In [5]:

```
def compute_gradient(x,y,w,b):
    m = x.shape[0]

    dj_dw = 0
    dj_db = 0

    for i in range(m):
        f_wb = w * x[i] + b
        dj_dw_i = (f_wb - y[i]) * x[i]
        dj_db_i = f_wb - y[i]
        dj_db += dj_db_i
        dj_dw += dj_dw_i
    dj_dw = dj_dw / m
    dj_db = dj_db / m

    return dj_dw, dj_db
```

In [6]:

```
def gradient_descent(x, y, w_in, b_in, alpha, num_iters, cost_function, gradient_functi
    J_history = [] # Previous values of Cost function
```

```
    p_history = [] # Previous values of w and b
    b = b_in
    w = w_in

    for i in range(num_iters):
        dj_dw, dj_db = gradient_function(x,y,w,b)
        b = b - alpha * dj_db
        w = w - alpha * dj_dw

        if i < 100000:
            J_history.append(cost_function(x,y,w,b))
            p_history.append([w,b])
        if i % math.ceil(num_iters/10) == 0:
            print(f"Iteration {i:4}: Cost {J_history[-1]:0.2e} ",
                  f"dj_dw: {dj_dw: 0.3e}, dj_db: {dj_db: 0.3e} ",
                  f"w: {w: 0.3e}, b:{b:0.5e}")
    return w, b, J_history, p_history
```

In [7]:

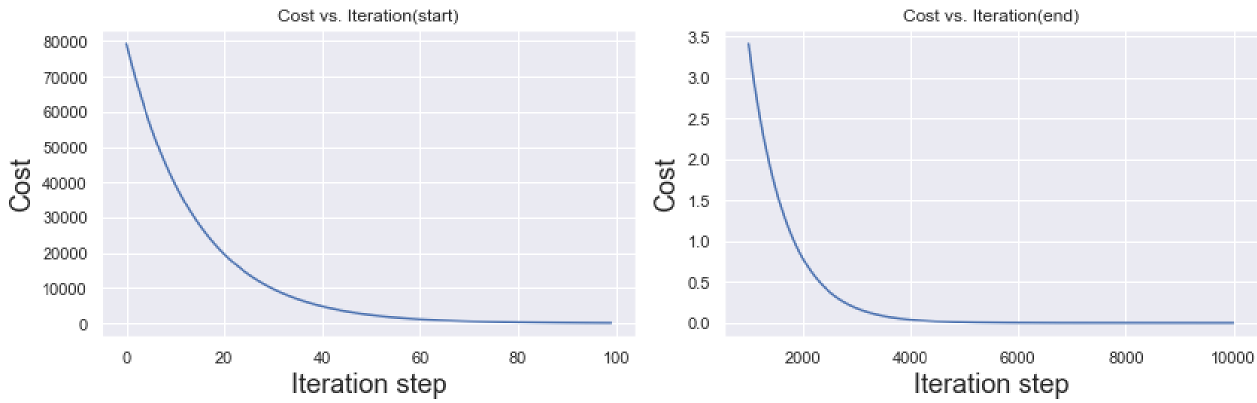
```
w_init = 0
b_init = 0

iterations = 10000
tmp_alpha = 1.0e-2
w_final, b_final, J_hist, p_hist = gradient_descent(x_train, y_train, w_init, b_init, t

print(f"(w,b) found by gradient descent: ({w_final: 8.4f}, {b_final: 8.4f})")
```

```
Iteration    0: Cost 7.93e+04  dj_dw: -6.500e+02, dj_db: -4.000e+02  w:  6.500e+00, b:4.
00000e+00
Iteration 1000: Cost 3.41e+00  dj_dw: -3.712e-01, dj_db:  6.007e-01  w:  1.949e+02, b:1.
08228e+02
Iteration 2000: Cost 7.93e-01  dj_dw: -1.789e-01, dj_db:  2.895e-01  w:  1.975e+02, b:1.
03966e+02
Iteration 3000: Cost 1.84e-01  dj_dw: -8.625e-02, dj_db:  1.396e-01  w:  1.988e+02, b:1.
01912e+02
Iteration 4000: Cost 4.28e-02  dj_dw: -4.158e-02, dj_db:  6.727e-02  w:  1.994e+02, b:1.
00922e+02
Iteration 5000: Cost 9.95e-03  dj_dw: -2.004e-02, dj_db:  3.243e-02  w:  1.997e+02, b:1.
00444e+02
Iteration 6000: Cost 2.31e-03  dj_dw: -9.660e-03, dj_db:  1.563e-02  w:  1.999e+02, b:1.
00214e+02
Iteration 7000: Cost 5.37e-04  dj_dw: -4.657e-03, dj_db:  7.535e-03  w:  1.999e+02, b:1.
00103e+02
Iteration 8000: Cost 1.25e-04  dj_dw: -2.245e-03, dj_db:  3.632e-03  w:  2.000e+02, b:1.
00050e+02
Iteration 9000: Cost 2.90e-05  dj_dw: -1.082e-03, dj_db:  1.751e-03  w:  2.000e+02, b:1.
00024e+02
(w,b) found by gradient descent: ( 199.9929,  100.0116)
```

In [8]:

```
fig , (ax1,ax2) = plt.subplots(1,2,figsize=(12,4))
ax1.plot(J_hist[:100])
ax2.plot(1000 + np.arange(len(J_hist[1000:])), J_hist[1000:])
ax1.set_title("Cost vs. Iteration(start)")
ax2.set_title("Cost vs. Iteration(end)")
ax1.set_xlabel("Iteration step", fontsize=18)
ax2.set_xlabel("Iteration step", fontsize=18)
ax1.set_ylabel("Cost", fontsize=18)
```

```
ax2.set_ylabel("Cost", fontsize=18)
plt.tight_layout()
```



In [9]:

```
print(f"1000 sqft house: {w_final*1 + b_final:0.1f} thousand dollars\n2000 sqft house:
```

```
1000 sqft house: 300.0 thousand dollars
2000 sqft house: 500.0 thousand dollars
```

Gradient Descent with Multiple Features

In [10]:

```
import copy, math
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns
sns.set()
```

In [11]:

```
X_train = np.array([[2104,5,1,45],[1416,3,2,40],[852,2,1,35]])
y_train = np.array([460,232,178])
```

In [12]:

```
b_init = 785.1811367994083
w_init = np.array([ 0.39133535, 18.75376741, -53.36032453, -26.42131618])
```

In [13]:

```
def compute_cost(X,y,w,b):
    cost = 0
    row_num = X.shape[0]

    for i in range(row_num):
        cost += 1/(2*row_num) * (np.dot(w,X[i]) + b - y[i])**2
    return cost
```

In [14]:

```
cost = compute_cost(X_train, y_train, w_init, b_init)
cost
```

Out[14]:

```
1.5578904045996674e-12
```

In [15]:

```
def compute_gradient(X,y,w,b):
```

```
    row_num, feature_num = X.shape
    dj_dw = np.zeros((feature_num,))
    dj_db = 0

    for i in range(row_num):
        f_wb = np.dot(X[i],w) + b
        for j in range(feature_num):
            dj_dw[j] += 1/row_num * (f_wb - y[i])*X[i,j]
        dj_db += 1/row_num * f_wb - y[i]


    return dj_dw, dj_db
```

Some Notes:

Compute gradient for each parameter (w,b) for the first time in iteration

- We have n rows and m features in the matrix X_train
- Start with **initial w and b**. ***w is the (mx1) vector*** coressponding to m features
- To compute the ***gradient dj_dw of each feature***:

1. Choose row: 1st row for example
2. Compute f_wb for that row:

   `f_wb = np.dot(w_init,X_train[i]) + b`

3. Calculate the error for that row:

   `error = f_wb - y[i]`

4. Calculate the gradient for each of the weight (coressponding to each feature)

   `dj_dw[j] += 1/m * error*X[i,j]`

- To compute the ***gradient dj_db (for the first time in iteration series)***

  `dj_db += error`

In [16]:

```
tmp_dj_dw, tmp_dj_db = compute_gradient(X_train,y_train,w_init,b_init)
print(f"dj_dw at initial w.b: {tmp_dj_dw}\ndj_db at initial w,b: {tmp_dj_db}")
```

```
dj_dw at initial w.b: [-2.72623574e-03 -6.27197255e-06 -2.21745574e-06 -6.92403377e-05]
dj_db at initial w,b: -580.0000016739251
```

In [17]:

```
def gradient_descent(X,y,w_in,b_in, cost_function, gradient_function, alpha, num_iters)
    J_history = []
    w = w_in
    b = b_in

    for i in range(num_iters):
        dj_dw, dj_db = gradient_function(X,y,w,b)

        w = w - alpha*dj_dw
        b = b - alpha*dj_db

        if i < 100000:
            J_history.append(cost_function(X,y,w,b))

        if i% math.ceil(num_iters/10) == 0:
```

```python
            print(f"Iteration {i:4d}: Cost {J_history[-1]:8.2f} ")
    return w,b,J_history
```

In [18]:

```python
initial_w = np.zeros_like(w_init)
initial_b = 0

iterations = 1000
alpha = 5.0e-7

w_final, b_final, J_hist = gradient_descent(X_train,y_train,initial_w, initial_b,comput
print(f"b_final: {b_final}\nw_final: {w_final}")
```

```
Iteration    0: Cost  2529.48
Iteration  100: Cost   696.13
Iteration  200: Cost   695.20
Iteration  300: Cost   694.27
Iteration  400: Cost   693.36
Iteration  500: Cost   692.45
Iteration  600: Cost   691.54
Iteration  700: Cost   690.65
Iteration  800: Cost   689.76
Iteration  900: Cost   688.88
b_final: 0.2877566511360619
w_final: [ 0.20379445  0.00375025 -0.01125989 -0.06608828]
```

In [19]:

```python
from sklearn.linear_model import LinearRegression
```

In [20]:

```python
reg = LinearRegression()

reg.fit(X_train,y_train)

print(reg.coef_)
print(f"\n{reg.intercept_}")
```

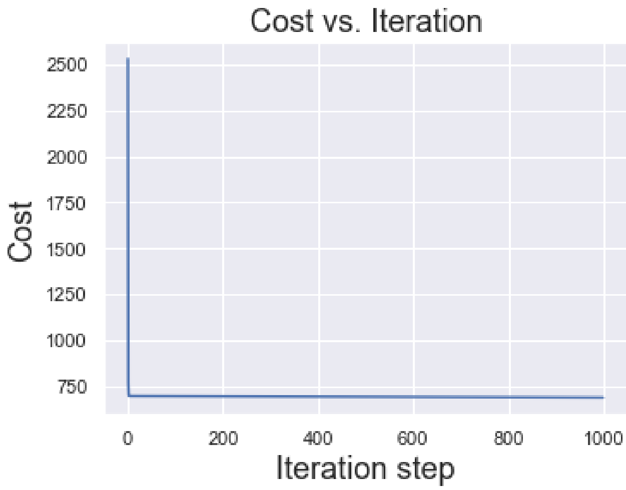
```
[  0.39133535  18.75376741 -53.36032453 -26.42131618]

785.1811367994087
```

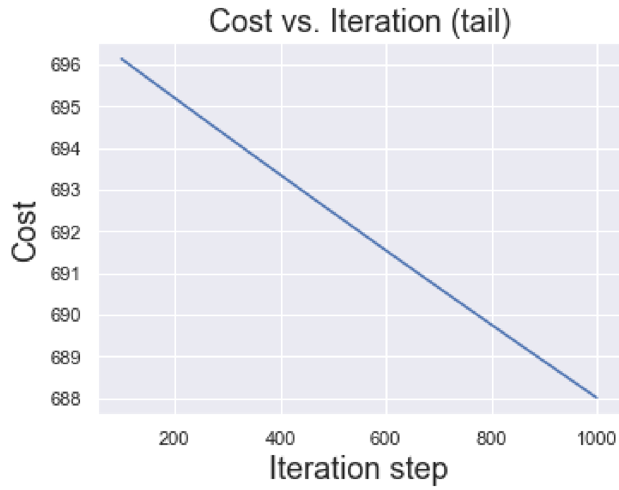
In [21]:

```python
fig, (ax1,ax2) = plt.subplots(1,2,figsize=(12,4))

ax1.plot(np.arange(iterations), J_hist)
ax2.plot(100+np.arange(len(J_hist[100:])), J_hist[100:])
ax1.set_title("Cost vs. Iteration", fontsize=18)
ax2.set_title("Cost vs. Iteration (tail)", fontsize=18)
ax1.set_xlabel("Iteration step", fontsize=18)
ax2.set_xlabel("Iteration step", fontsize=18)
ax1.set_ylabel("Cost", fontsize=18)
ax2.set_ylabel("Cost", fontsize=18)
```

Out[21]:

```
Text(0, 0.5, 'Cost')
```


Cost vs. Iteration
Cost
2500
2250
2000
1750
1500
1250
1000
750
0
200
400
600
800
1000
Iteration step


Cost vs. Iteration (tail)
Cost
696
695
694
693
692
691
690
689
688
200
400
600
800
1000
Iteration step