Input Generation Method

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The goal of this generation strategy is to have a very sensitive input, which peaks very quickly and has very low objective value at places other than the peaks. To do so, consider a superset S of cardinality n composed of vertices $S = \{v_{i_1}, v_{i_2}, \ldots, v_{i_n}\}$ where each i_k is an index in $1, \ldots, |V|$. We will build rowdy groups out of subsets of S. Let k be the cardinality of the minimum size subset of S that we deem a rowdy group. We can then say that any subset $X \subseteq S$ such that $|X| \geq k$ also forms a rowdy group. The reason for this is that it makes the rowdy groups very hard to separate. Further, to create a very sensitive, non-smooth objective surface we can generate the inputs such that the majority of members in S are very popular. Now because they are both popular and hard to separate it is difficult to seperate the rowdy groups, however the solver is highly incentivized to do so. To see why, consider a superset $S = \{v_{i_1}, v_{i_2}, \ldots, v_{i_n}\}$, and $k = 3$. The number of possible rowdy groups is then

$$\sum_{k=3}^{|S|} \binom{|S|}{k} \approx 2^{|S|}$$

That is the number of possible rowdy groups grows in $O(2^n)$. Suppose we give one, or very few, combinations of rowdy groups where there are either no invalidated edges or very few. In this case, because all members of the rowdy superset are popular, the problem of splitting up the rowdy groups has a solution that is significantly higher in value than approximate solutions that leave some rowdy subsets together. Since we begin with a solution, if we carefully construct the rowdy groups to form supersets of the structure above, and ensure that members of the superset are popular, our solution will likely beat out the majority of solutions. Note that we are not strictly limited to choosing out subsets by size, we can also pick and choose subsets so that not every subset satisfying $k \leq |X| \leq n$ is a rowdy group. The point is generally that the superset be densely filled with subset rowdy groups.