Stat 30280 - Assignment 3

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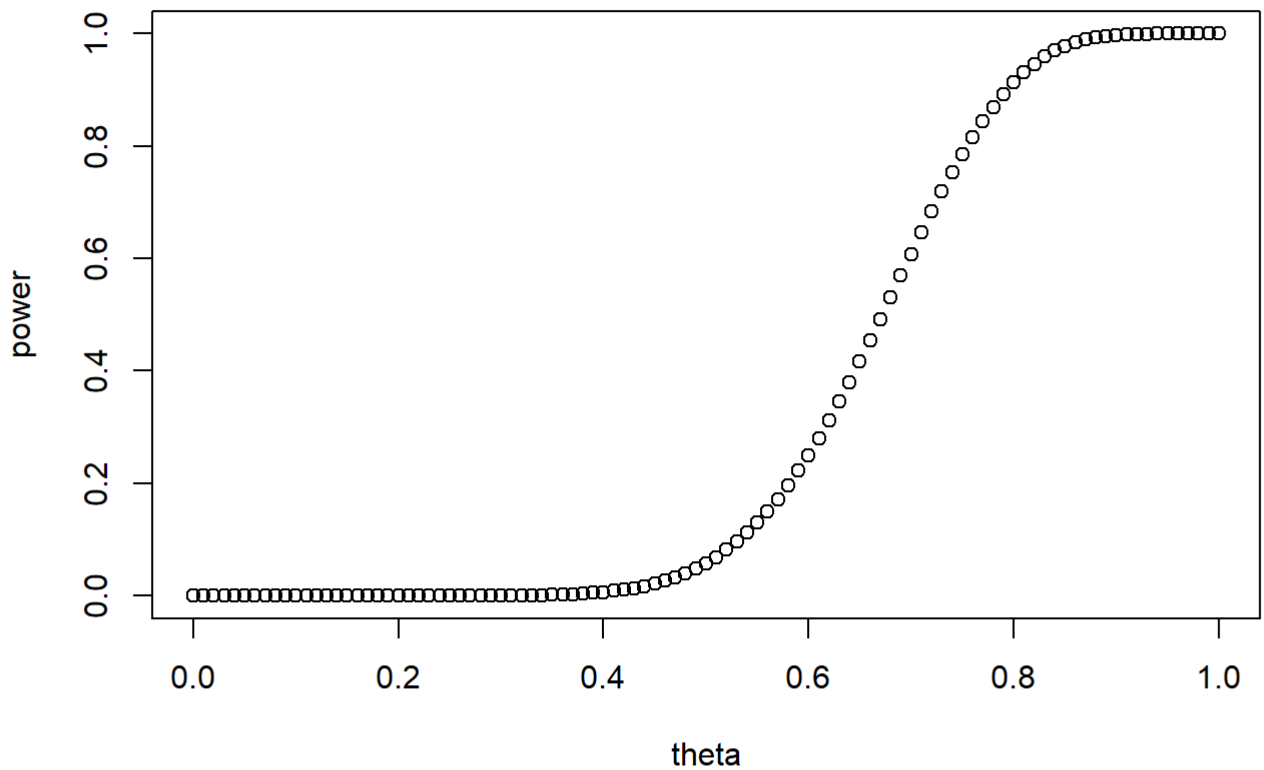
Note unfortunately my answers are not quite in order, as I have joined 2x PDF's, one with written answers, one with answers in R markdown

Exercise 1g

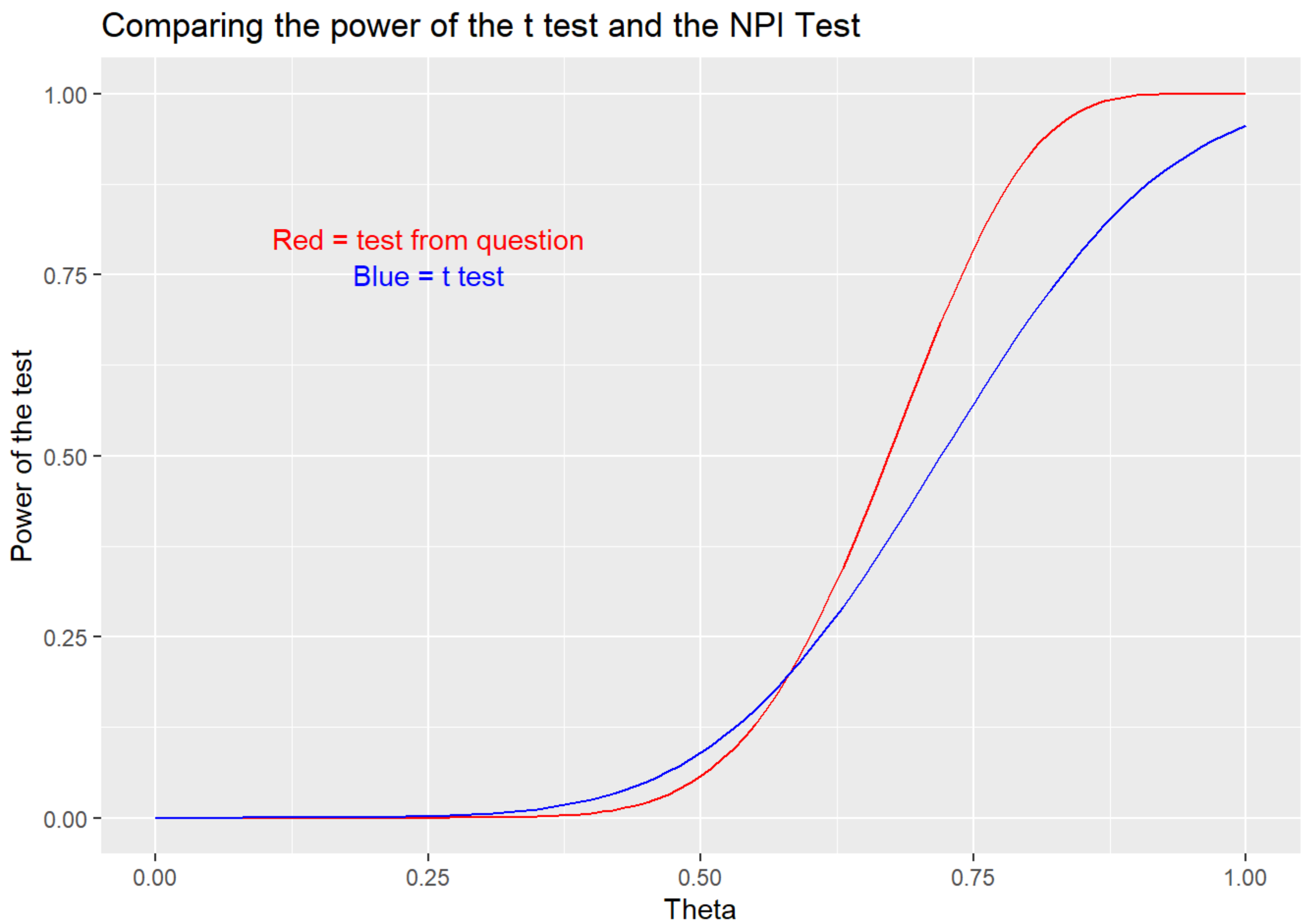
```
theta <- seq(0,1,0.01)
beta <- pbinom(13, 20, theta)
power <- 1- beta
plot(theta,power)
```



We see a low power of the test around theta zero (0.45) and below. As we move to higher theta, the power of the test increases as it is easier to differentiate between H0, and a true theta which is much higher.

Part 1i

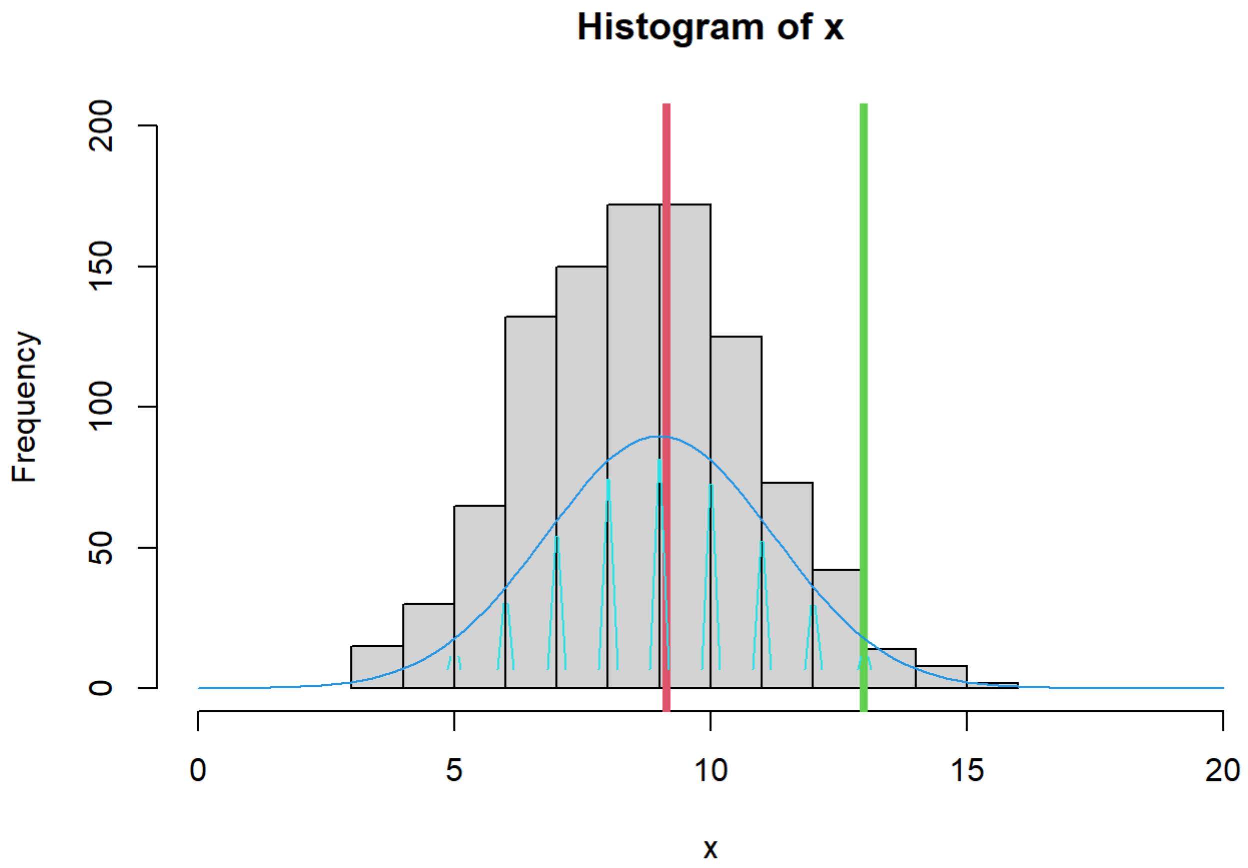
```
library(ggplot2)
n <- 20
x1 <- c(0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,1,1,1)
stdev <- (n * 0.45 * 0.55 ) ^ 0.5
power_t <- power.t.test(n=n,delta=theta-0.45,sig.level=0.05,alternative="one.sided",sd=sd(x
1))$pow


p <- ggplot() + geom_line(aes(x=theta, y= power), color = "red") + geom_line(aes(x=theta, y=p
ower_t), color = "blue") + xlab("Theta") + ylab("Power of the test") + ggtitle("Comparing the
power of the t test and the NPI Test") + annotate(geom = "text", label = "Blue = t test", x=
0.25, y=0.75, color = "blue") + annotate(geom = "text", label = "Red = test from question", x
=0.25, y=0.8, color = "red")
print(p)
```



We see that at low theta, the t test is more powerful
We see that once theta increases beyond ~ 0.6, the test derived for different values of theta in the question becomes more powerful.

Exercise 1j

```
set.seed(501)
x <- rbinom(1000, 20, 0.45)
hist(x, xlim = c(0,20), ylim = c(0,200))
abline(v=13, col =3, lwd = 4)
abline(v=mean(x), col = 2, lwd =4)
curve(500 * dnorm(x,9,2.22), col = 4, add=TRUE)
curve(500 * dbinom(x,20,0.45),col = 5, add=TRUE, type="c")
```




```
Rej_Reg_Percentage <- sum(c(x > 13))/length(x)
Rej_Reg_Percentage
```

```
## [1] 0.024
```

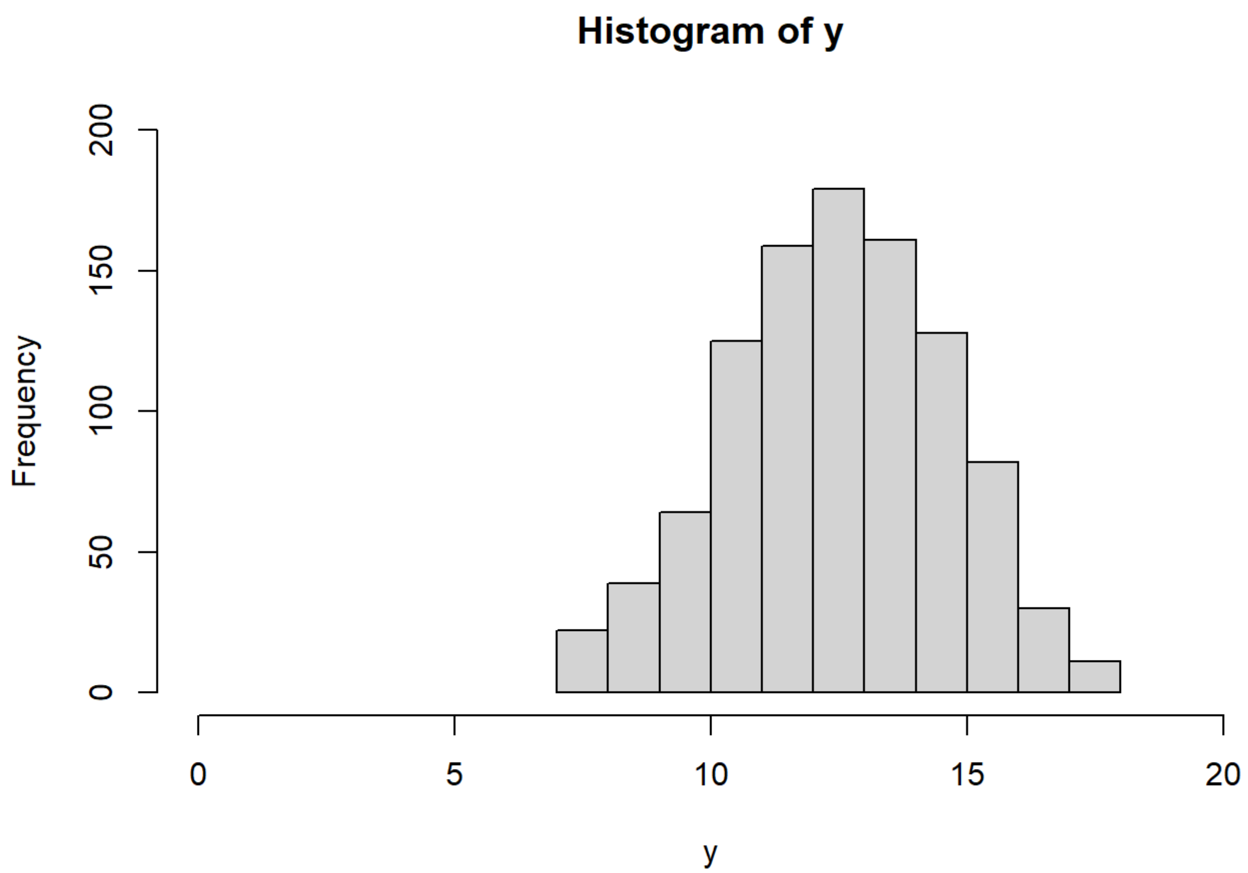
Our kappa was chosen to represent the beginning of a rejection region at 95% confidence level that the true theta was not 0.45. We can see at high n, binomial and normal approximation work well using variance as nqp, mean as n xbar

kappa shown with green vertical line successfully represents a region where more extreme x, would lead to a lower than 5% confidence that the true distribution is binomial with theta = 0.45

We see that the percentage of results from the simulation falling within the rejection region is 2.4% this is close to alpha / 2. Originally as it seems to be a one tailed test, I was expecting alpha to be the result. Perhaps this is infact a two tailed test, and we have just calculated one kappa, for the top end.

Exercise 1k

```
y <- rbinom(1000, 20, 0.65)
y_bar <- mean(y)
x_bar <- mean(x)
sigma_y <- sd(y)
hist(y, xlim = c(0,20), ylim = c(0,200))
```




```
stdev <- (n * 0.45 * 0.55 ) ^ 0.5
power_t_simul <- power.t.test(n=20, delta = y_bar /20 - x_bar /20, sd = sigma_y, sig.level =
0.05, alternative="one.sided")
power_t_theory <- power.t.test(n=n,delta=0.65-0.45,sig.level=0.05,alternative="one.sided",sd=
stdev)
power_t_simul
```

```
##
##      Two-sample t test power calculation
##
##               n = 20
##           delta = 0.1915
##              sd = 2.160161
##       sig.level = 0.05
##           power = 0.0854258
##     alternative = one.sided
##
## NOTE: n is number in *each* group
```

```
power_t_theory
```

```
##
##      Two-sample t test power calculation
##
##               n = 20
##           delta = 0.2
##              sd = 2.22486
##       sig.level = 0.05
##           power = 0.08603024
##     alternative = one.sided
##
## NOTE: n is number in *each* group
```

We see that the theoretical power curve result gained in (i), and the experimental result from (k) adhere well to each other.