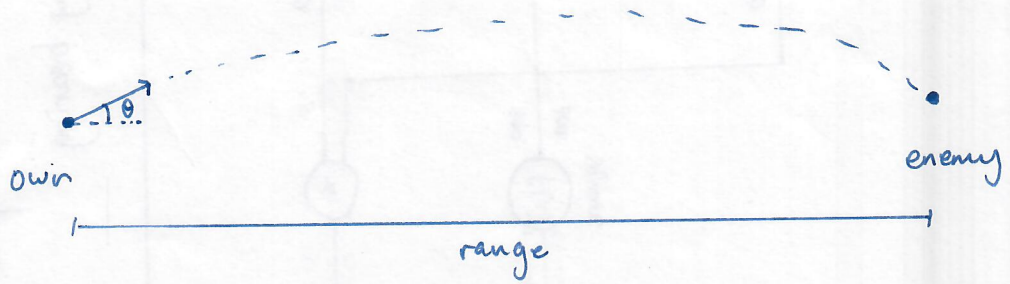
Part 1

Calculate the angle above horizontal that a gun on a ship has to point to hit an enemy ship. (no air / air resistance)
Also calculate bearing of gun given bearing of enemy ship from own ship direction



distance = speed · time

range = horizontal speed · flight time

horizontal speed = $\cos\theta$ · initial speed

flight time = $2 \cdot \frac{\text{vertical speed}}{g}$ (up and down)

flight time = $\frac{2}{g} \cdot \sin\theta \cdot$ initial speed

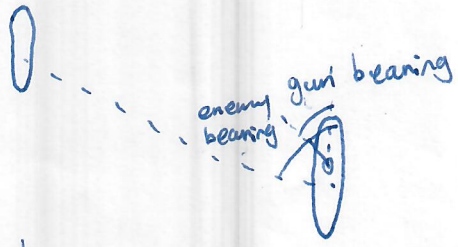
range = $\frac{2}{g} \cdot \sin\theta \cdot \cos\theta \cdot$ initial speed^2

$\sin\theta \cdot \cos\theta = \frac{1}{2}\sin(2\theta)$

range = $\frac{1}{g} \cdot \sin(2\theta) \cdot$ initial speed^2

$$\sin(2\theta) = \frac{g \cdot \text{range}}{\text{initial speed}^2}$$

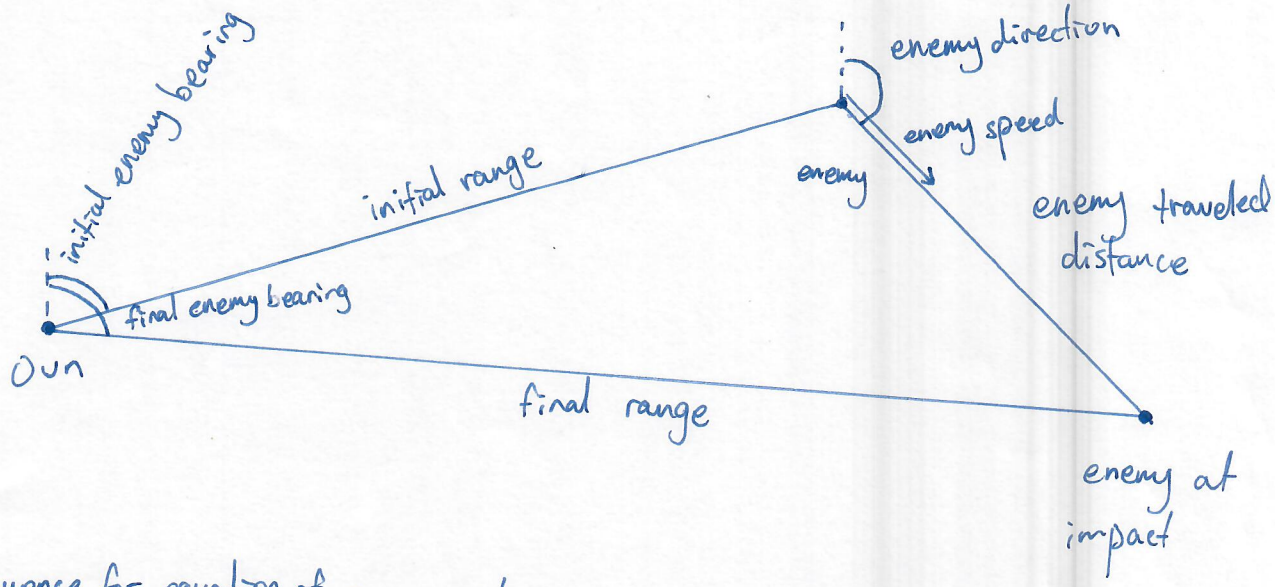
$$\text{gun angle} = \frac{1}{2} \cdot \sin^{-1}\left(\frac{g \cdot \text{range}}{\text{initial speed}^2}\right)$$



gun bearing = enemy bearing

Part 2

Calculate angle above horizon and bearing of gun if own ship is still and enemy ship is moving (no air/air resistance)



Solving sequence for equation of gun angle:

- Convert diagram to geometry problem to find range. deduce expression for it with cosine rule.
- substitute, from part 1, final range as equation for range, flight time as equation for flight time
- rearrange/solve for gun angle (all other variables are known values)

Geometry problem

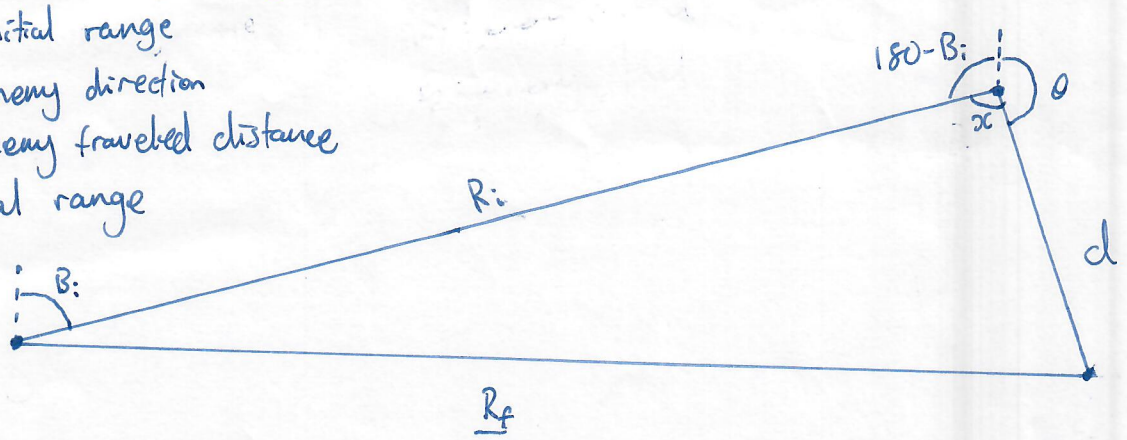
B_i : initial enemy bearing

R_i : initial range

θ : enemy direction

d : enemy traveled distance

R_f : final range



$$x = 360-(180-B_i)-\theta = 180+B_i-\theta$$

$$R_f^2 = R_i^2 + d^2 - 2\cdot R_i \cdot d \cdot \cos(180+B_i-\theta)$$

enemy traveled distance = enemy speed · flight time

$$\underline{R_f}^2 = R_i^2 + \text{enemy speed}^2 \cdot \underline{\text{flight time}}^2 - 2\cdot R_i \cdot \text{enemy speed} \cdot \underline{\text{flight time}} \cdot \cos(180+B_i-\theta)$$

underline = unknowns

From part 1:

$$\underline{\text{flight time}} = \frac{2}{g}\cdot \text{initial speed} \cdot \sin(\text{gun angle})$$

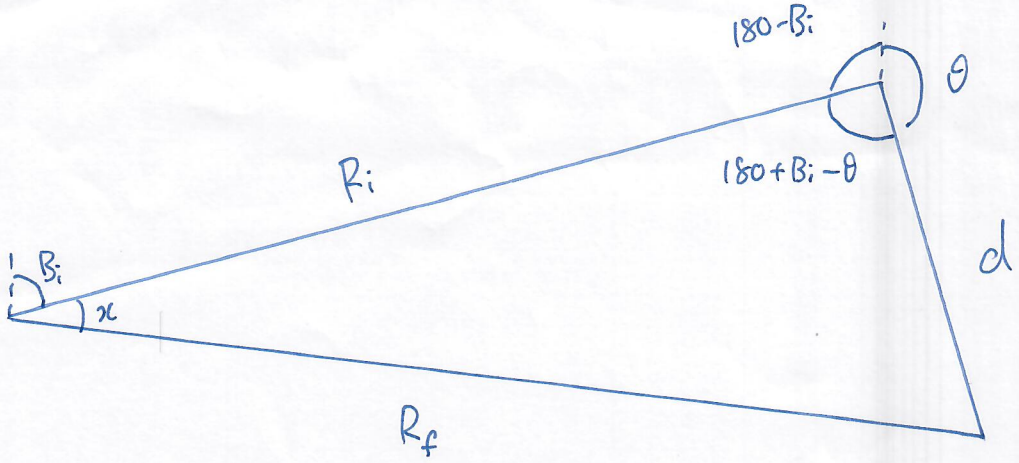
$$\underline{\text{range}} = \frac{1}{g}\cdot \text{initial speed}^2 \cdot \sin(2\cdot \text{gun angle})$$

solve for gun angle:

?

Cheat : numerically solve by using trial and improvement using python, (~~takes computer [illegible]s to generate result to a few dp. in degrees~~)

Calculate gun bearing



$$\text{gun bearing} = B_i + x \overset{\text{mod}}{\%} 360$$

$$\frac{\sin x}{d} = \frac{\sin(180 + B_i - \theta)}{R_f}$$

$$\sin x = \frac{d}{R_f} \sin(180 + B_i - \theta)$$

$$x = \sin^{-1}\left(\frac{d}{R_f} \cdot \sin(180 + B_i - \theta)\right)$$

$$\text{gun bearing} = \text{mod}\left(B_i + \sin^{-1}\left(\frac{d}{R_f} \cdot \sin(180 + B_i - \theta)\right), 360\right)$$