$\boxed{\text{Transformada de Laplace}}$ Vitor Martin Simoni - 18.00050-9

$\underline{3.1}$ a) $x(t) = e^{-2t}(1 + sen\,5t) = e^{-2t} + e^{-2t}\,sen\,5t =$

$e^{-2t} + \frac{1}{j2}e^{(-2+j5)t} - \frac{1}{j2}e^{(-2-j5)t}$

$\boxed{s_1 = -2}$ $\boxed{s_2 = -2+j5}$ $\boxed{s_3 = -2-j5}$

b) $x(t) = A\cos(w_0 t) + B\cos(2w_0 t) = \frac{A}{2}\left(e^{w_0 j} + e^{-w_0 j}\right) + \frac{B}{2}\left(e^{2w_0 j} + e^{-2w_0 j}\right)$

$\boxed{\begin{matrix} s_1 = jw_0 \\ s_2 = -jw_0 \end{matrix}}$ $\boxed{\begin{matrix} s_3 = 2jw_0 \\ s_4 = -2jw_0 \end{matrix}}$

c) $x(t) = sen(w_0 t + \varphi) = \frac{1}{2j}\left(e^{(w_0 jt + \varphi)} - e^{(-w_0 jt + \varphi)}\right)$

$\boxed{s_1 = jw_0}$ $\boxed{s_2 = jw_0}$

$\underline{3.2}$ a) $sen(wt) \longrightarrow 5\cos(wt + 20°)$ $s_1 = s_2 \longrightarrow \boxed{\text{Sim}}$

$s_1 = \pm jw$ $s_2 = \pm jw$

b) $\frac{1}{4}sen(10t) \longrightarrow \cos(5t)$ $s_1 \neq s_2 \rightarrow \boxed{\text{Não}}$

$s_1 = \pm 10j$ $s_2 = \pm 5j$

c) $e^{-t} \longrightarrow \frac{1}{2}e^{-2t}$ $s_1 \neq s_2 \rightarrow \boxed{\text{Não}}$

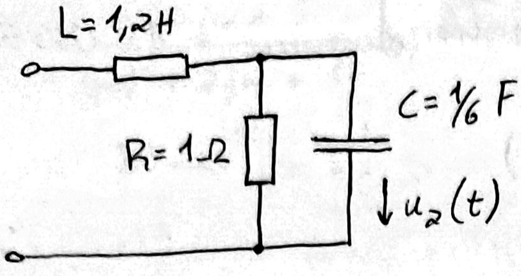
$s_1 = -1$ $s_2 = -2$

d) $e^{-2jt} \longrightarrow -5\cos(2t)$ Se $s_2 = +2j$, $s_1 \neq s_2 \rightarrow \boxed{\text{Não}}$

$s_1 = -2j$ $s_2 = \pm 2j$

e) $-5\cos(2t) \longrightarrow e^{-j2t}$ $s_1 = 2j$ $s_2 = -2j$ $s_1 = s_2 \rightarrow \boxed{\text{Sim}}$

Ex 3.3



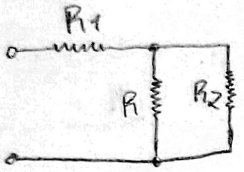
$$u_1(t) = e^{\frac{-3t}{s}} \cdot \cos\left(\frac{-4t}{s}\right) \cdot 1\,V$$

a) $R_0 = \frac{V}{I} = 1\Omega \Rightarrow \tilde{R} = \frac{R}{R_0} = 1\Omega$

$L_0 = \frac{Vt}{I} = 1H \Rightarrow \tilde{L} = \frac{L}{L_0} = 1{,}2H$

$C_0 = \frac{Is}{V} = 1F \Rightarrow \tilde{C} = \frac{C}{C_0} = \frac{1}{6}F$

b)



$R = R$

$R_1 = sL$

$R_2 = \frac{1}{sC}$

$$R // R_2 : \frac{R \cdot R_2}{R + R_2} = \left(\frac{R}{sC}\right) \cdot \left(\frac{1}{R + \frac{1}{sC}}\right) =$$

$$= \left(\frac{R}{\cancel{sC}}\right) \cdot \left(\frac{1}{\frac{sRC+1}{\cancel{sC}}}\right) = \frac{R}{sRC+1}$$

$$H(s) = \frac{U_2(s)}{U_1(s)} = \frac{\frac{R}{sRC+1}}{\frac{R}{sRC+1} + sL} = \boxed{\frac{1}{0{,}2s^2 + 1{,}2s + 1}}$$

c) $u_1(t) = e^{\frac{-3t}{s}} \cdot \cos\left(\frac{-4t}{s}\right) \cdot 1V \Rightarrow u(s) = e^{\frac{-3t}{s}} \cdot \frac{1}{2}\left(e^{\frac{-4j}{s}} + e^{\frac{+4j}{s}}\right)$

Normalizando : $u_1(t) = e^{-3t} \cdot \frac{1}{2} \cdot \left(e^{-4jt} + e^{+4jt}\right) \Rightarrow S_1 = -3 \pm 4j$

$$H(s_1) = \frac{1}{0{,}2(-7-j24) + 1{,}2(-3+j4) + 1} = -0{,}25 \Rightarrow \boxed{H(s_1^*) = -0{,}25}$$

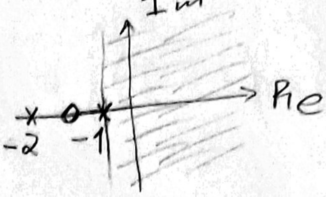
d) $u_2(t) = \frac{H(s_1)e^{s_1 t}}{2} + \frac{H(s_1)^* e^{s_1^* t}}{2} = \boxed{\frac{-e^{-3t}\cos(4t)}{4}}$

Denormalizando : $\boxed{u_2(t) = \frac{-Ve^{\frac{-3t}{s}}\cos\left(\frac{4t}{s}\right)}{4}}$

$\boxed{\text{Ex 4.5}}$ $F(s) = \dfrac{2s+3}{s^2+3s+2}$ $\qquad G(s) = \dfrac{3s+1}{s^2+4s+3}$

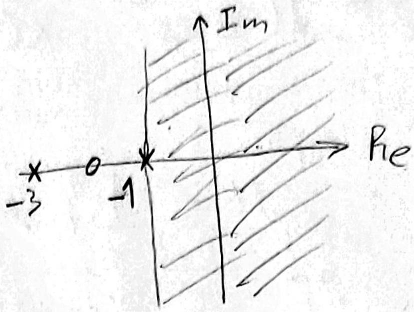
a) Pólos de $F(s)$: $s^2+3s+2=0 \rightarrow p_1 = -1,\; p_2 = -2$

$$\boxed{p_1=-1;\; p_2=-2;\; \text{Re}\{s\} > -1}$$



b) $s^2+4s+3=0 \rightarrow g_1 = -1,\; g_2 = -3$

$$\boxed{\begin{array}{c} g_1=-1;\; g_2=-3 \\ \text{Re}\{s\} > -1 \end{array}}$$



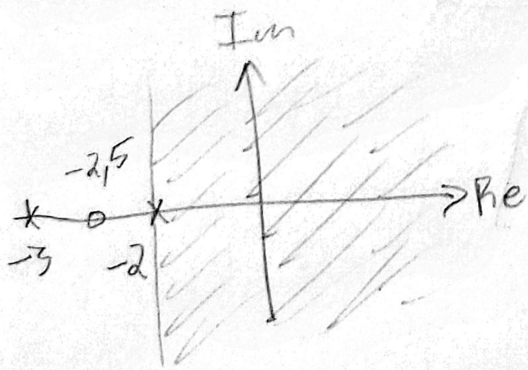
c) $F(s) = \dfrac{2s+3}{(s+1)(s+2)}$; $G(s) = \dfrac{3s+1}{(s+1)(s+3)}$

$(s+2)$

$$X(s) = F(s)+G(s) = \frac{2s+3}{(s+1)(s+2)} + \frac{3s+1}{(s+1)(s+3)} = \frac{(2s+3)(s+3)+(3s+1)(s+2)}{(s+1)(s+2)(s+3)}$$

$$X(s) = \frac{2s^2+9s+9+3s^2+7s+2}{(s+1)(s+2)(s+3)} = \frac{5s^2+16s+11}{(s+1)(s+2)(s+3)} =$$

$$X(s) = \frac{\cancel{(s+1)}(5s+11)}{\cancel{(s+1)}(s+2)(s+3)} = \frac{5s+11}{(s+2)(s+3)} \qquad x_1=-2,\; x_2=-3$$



$\text{Re}\{s\} > -2$

Ex 4.6 a) $\mathcal{L}\{x(t)\} = X(s) = \int_{-\infty}^{+\infty} x(t).e^{-st}\, dt$

$$\mathcal{L}\{x(t-\tau)\} = e^{-st}.X(s) = \int_{\tau-\infty}^{\tau+\infty} x(t'-\tau).e^{-s(t'-\tau)}\, dt = X(s)$$

$$\int_{-\infty}^{+\infty} x(t'-\tau).e^{-st'}.dt = e^{-st}.X(s)$$

$$\int_{-\infty}^{+\infty} x(t'-\tau).e^{-st'}\, dt = \mathcal{L}\{x(t'-\tau)\}$$

b) $X(s'-\alpha) = \int_{-\infty}^{+\infty} x(t).e^{-(s-\alpha)t}\, dt = \int_{-\infty}^{+\infty} e^{\alpha t} x(t) e^{-s't}\, dt = \mathcal{L}\{e^{\alpha t} x(t)\}$

Ex 5.3 $f(t) = \mathcal{L}\{F(s)\}$; $F(s) = \dfrac{2s-1}{(s+1)^3(s+4)}$; $Re\{s\} > -1$

a) $F(s) = \dfrac{A_1}{(s+1)^1} + \dfrac{A_2}{(s+1)^2} + \dfrac{A_3}{(s+1)^3} + \dfrac{B}{s+4}$

$$B = \lim_{s\to -4} [F(s).(s+4)] = \lim_{s\to -4} \frac{(2s-1)\cancel{(s+4)}}{(s+1)^3 \cancel{(s+4)}} = \frac{-9}{-27} \Rightarrow \boxed{B = 1/3}$$

$$A_1 = \frac{1}{(3-1)!} \lim_{s\to -1} \frac{d^2}{ds^2} \left[\frac{(2s-1)\cancel{(s+1)^3}}{\cancel{(s+1)^3}(s+4)} \right] = \frac{1}{2} \lim_{s\to -1} \left[\frac{-18}{(x+4)^3} \right] = \frac{1}{2} \cdot \frac{(-18)}{27}$$

$$\boxed{A_1 = \frac{-1}{3}}$$

$$A_2 = \lim_{s\to -1} \frac{d}{ds} \left[\frac{(2s-1)}{(s+4)} \right] = \lim_{s\to -1} \frac{9}{(x+4)^2} \Rightarrow \boxed{A_2 = 1}$$

$$A_3 = \lim_{s\to -1} \frac{2s-1}{s+4} = \frac{-3}{3} \Rightarrow \boxed{A_3 = -1}$$

Sendo: $A_1 = \frac{-1}{3}$; $A_2 = 1$; $A_3 = -1$; $B = \frac{1}{3}$

$$F(s) = \frac{-1}{3(s+1)} + \frac{1}{(s+1)^2} - \frac{1}{(s+1)^3} + \frac{1}{3(s+4)}$$

tabela

$$f(t) = \left[\frac{-e^{-t}}{3} + te^{-t} - \frac{t^2 e^{-t}}{2} + \frac{e^{-4t}}{3} \right] \mathcal{E}(t)$$

b) Como visto no item a) $\boxed{A_3 = -1}$ e $\boxed{B = 1/3}$

Igualando: $2s - 1 = A_1 (s+1)^2 (s+4) + A_2 (s+1)(s+4) \ldots$

$\ldots + A_3 (s+4) + B (s+1)^3$

s^3

$A_1 + B = 0 \Rightarrow A_1 = -B \Rightarrow \boxed{A_1 = -1/3}$

s^0

$4A_1 + 4A_2 + 4A_3 + B = -1 \Rightarrow \boxed{A_2 = \frac{1}{4}(-1 - 4A_1 - 4A_3 - B) = 1}$

c) De acordo com o item b):

$$\begin{cases} A_1 + B = 0 \\ 6A_1 + A_2 + 3B = 0 \\ 9A_1 + 5A_2 + A_3 + 3B = 2 \\ 4A_1 + 4A_2 + 4A_3 + B = -1 \end{cases} \xrightarrow{\text{resolvendo na calculadora}} \begin{cases} A_1 = \frac{-1}{3} \\ A_2 = 1 \\ A_3 = -1 \\ B = \frac{1}{3} \end{cases}$$

Ex 5.4 $F(s) = \frac{s+3}{s^2+2s+5}$

a) $s^2+2s+5=0 \begin{cases} p_1 = -1+2j \\ p_2 = -1-2j \end{cases}$ $F(s) = \frac{A}{(s+1-2j)} + \frac{A^*}{(s+1+2j)}$

$$A = \lim_{s \to -1+2j} \left[\frac{s+3}{s+1+2j}\right] = 0{,}5-0{,}5j$$

$$f(t) = \left[\tfrac{1}{2}(1+j)e^{-(1+2j)} + \tfrac{1}{2}(1-j)e^{-(1-2j)}\right]\mathcal{E}(t)$$

$$f(t) = \left[e^{-t}\cos(2t) + e^{-t}\operatorname{sen}(2t)\right]\mathcal{E}(t)$$

$$\boxed{f(t) = \left[\cos(2t) + \operatorname{sen}(2t)\right]e^{-t}\,\mathcal{E}(t)}$$

b) $F(s) = \frac{s+3}{s^2+2s+5} = \frac{s+3}{(s^2+2s+1)+4} = \frac{(s+1)+2}{(s+1)^2+2^2} = \frac{(s+1)}{(s+1)^2+2^2} + \frac{2}{(s+1)^2+2^2}$

$$f(t) = \left[e^{-t}\cos(2t) + e^{-t}\operatorname{sen}(2t)\right]\mathcal{E}(t)$$

Ex 5.5 $F(s) = \frac{s}{(s+2)(s^2+\omega_0^2)}$; $Re > 0$

$$F(s) = \frac{A}{(s+2)} + \frac{Bs+C}{(s^2+\omega_0^2)}$$

$$A = \lim_{s \to -2}\left[F(s)(s+2)\right] = \frac{-2}{4+\omega_0^2}$$

$$(Bs+C)(s+2) + A(s^2+\omega_0^2) = s$$

s^2) $B + A = 0 \Rightarrow B = \frac{2}{4+\omega_0^2}$

s^0) $2C + A\omega_0^2 = 0 \Rightarrow C = \frac{\omega_0^2}{4+\omega_0^2}$

$$F(s) = \frac{1}{4+\omega_0^2}\left[\frac{-2}{s+2} + \frac{2s}{s^2+\omega_0^2} + \frac{\omega_0^2}{s^2+\omega_0^2}\right]$$

$$\boxed{f(t) = \frac{1}{4+\omega_0^2}\left[-2e^{-2t} + 2\cos(\omega_0 t) + \omega_0 \operatorname{sen}(\omega_0 t)\right]\cdot\mathcal{E}(t)}$$

Ex 6.3 a) Pólos: $(-1+j)$ e $(-1-j)$ Zeros: $(1+j)$ e $(1-j)$

$$H(s) = K\,\frac{(s-1+j)(s-1-j)}{(s+1+j)(s+1-j)} = \frac{K\,(s^2 - s - sj - s + 1 + j + sj - j + 1)}{s^2 + s - sj + s + 1 - j + sj + j + 1}$$

$$\boxed{H(s) = K\,\frac{s^2 - 2s + 2}{s^2 + 2s + 2}}$$

b) Pólos: $(2j)$ e $(-2j)$ Zeros: (1)

$$H(s) = K\,\frac{\cancel{(s-1)}}{(s-2j)(s+2j)\cancel{(s-1)}} = \boxed{K\,\frac{1}{s^2+4}}$$

c) Pólos: (2) e (-2) Zeros: (-1)

$$H(s) = K\,\frac{s+1}{(s-2)(s+2)} = \boxed{K\,\frac{s+1}{s^2-4}}$$

d) Pólos: (0) e (-3) Zeros: (4)

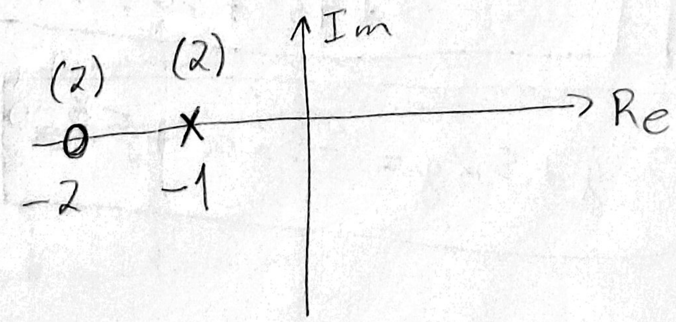
$$\boxed{H(s) = K\,\frac{s-4}{s(s+3)}}$$

Ex 6.4 a) $\ddot{y} + 2\dot{y} + y = \ddot{x} + 4\dot{x} + 4x \Rightarrow s^2y(s) + 2sy(s) + y(s) = s^2x(s) + 4sx(s) + 4x(s)$

$$y(s)(s^2+2s+1) = x(s)(s^2+4s+4) \Rightarrow H(s) = \frac{y(s)}{x(s)} = \frac{s^2+4s+4}{s^2+2s+1} = \frac{(s+2)^2}{(s+1)^2}$$

x Pólos: (-1)

o zeros: (-2)



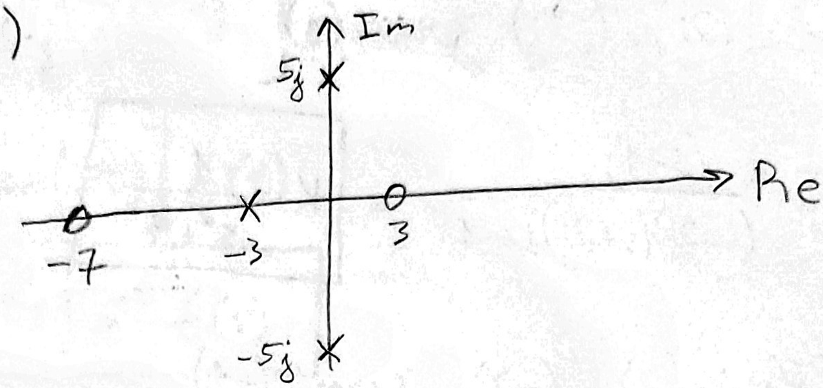
b) $\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 25\frac{dy}{dt} + 75y = \frac{d^2x}{dt^2} + 4\frac{dx}{dt} - 21x$

$y(s)(s^3 + 3s^2 + 25s + 75) = x(s)(s^2 + 4s - 21)$

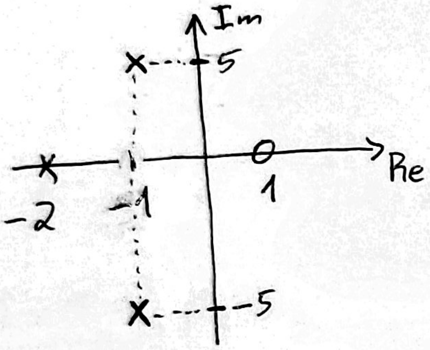
$H(s) = \frac{y(s)}{x(s)} = \frac{s^2 + 4s - 21}{s^3 + 3s^2 + 25s + 75} = \frac{(s-3)(s+7)}{(s+3)(s-5j)(s+5j)}$

x Pólos: (-3); $(+5j)$; $(-5j)$

o zeros: $(+3)$; (-7)



Ex 6.5 | $H(0) = 1$



Pólos: (-2); $(-1+5j)$; $(-1-5j)$

zeros: (1)

$H(s) = \frac{K.(s-1)}{(s+2)(s+1+5j)(s+1-5j)}$

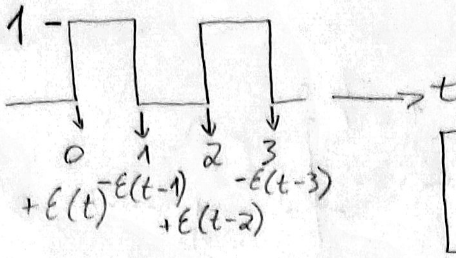
$H(0) = 1 = K.\frac{-1}{52} \Rightarrow K = -52$

Expandindo: $\boxed{H(s) = \frac{-52s + 52}{s^3 + 4s^2 + 30s + 52}} = \frac{y(s)}{x(s)}$

$$\boxed{\dddot{y} + 4\ddot{y} + 30\dot{y} + 52y = -52\dot{x} + 52x}$$

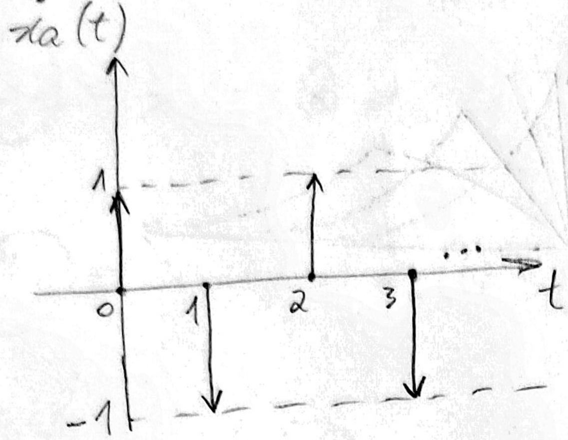
Ex 8.3

a)



$x_a(t) = \varepsilon(t) - \varepsilon(t-1) + \varepsilon(t-2) - \dots$

$$\boxed{x_a(t) = \sum_{k=0}^{\infty} \left[(-1)^k \cdot \varepsilon(t-k) \right]}$$

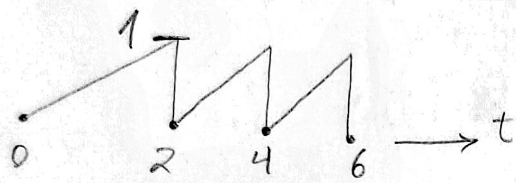


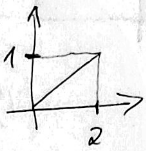
$$\dot{x}_a(t) = \sum_{k=0}^{\infty} \left[(-1)^k \cdot \dot{\varepsilon}(t-k) \right]$$

δ

$$\boxed{\dot{x}_a(t) = \sum_{k=0}^{\infty} \left[(-1)^k \cdot \delta(t-k) \right]}$$

b)





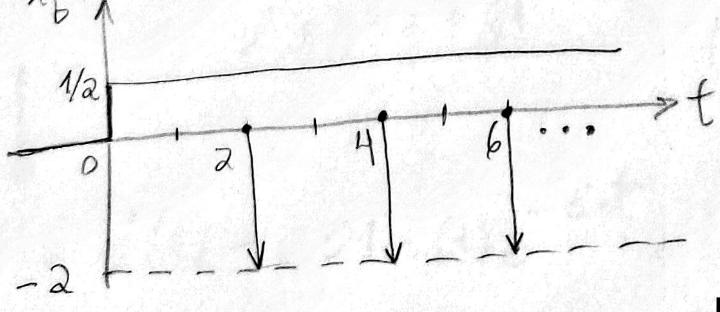
$m = \frac{\Delta y}{\Delta x} = \frac{1}{2} \cdot t$

$$x_b(t) = \varepsilon(t) \cdot \frac{t}{2} - \varepsilon(t-2) - \varepsilon(t-4) - \dots = \boxed{\varepsilon(t) \cdot \frac{t}{2} - \sum_{k=1}^{\infty} \left[\varepsilon(t-2k) \right]}$$

$$\dot{x}_b(t) = \frac{d}{dt} \left[\varepsilon(t) \cdot \frac{t}{2} \right] - \sum_{k=1}^{\infty} \left[\frac{d}{dt} \left(\varepsilon(t-2k) \right) \right]$$

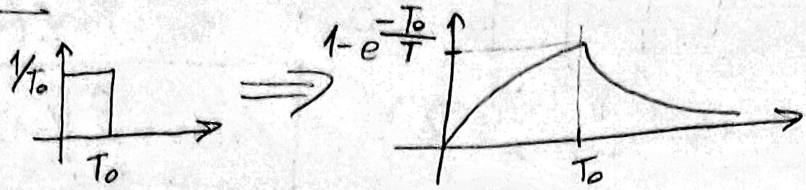
$$\dot{x}_b(t) = \cancelto{0}{\delta(t) \cdot \frac{t}{2}} + \varepsilon(t) \cdot \frac{1}{2} - \sum_{k=1}^{\infty} \left[\delta(t-2k) \right]$$

$$\boxed{\dot{x}_b(t) = \frac{\varepsilon(t)}{2} - \sum_{k=1}^{\infty} \delta(t-2k)}$$

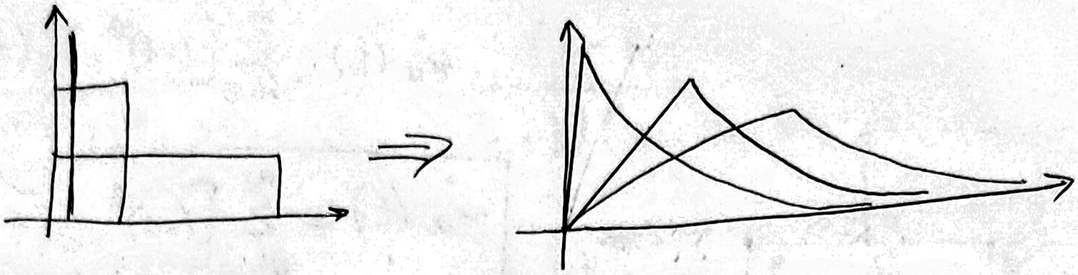


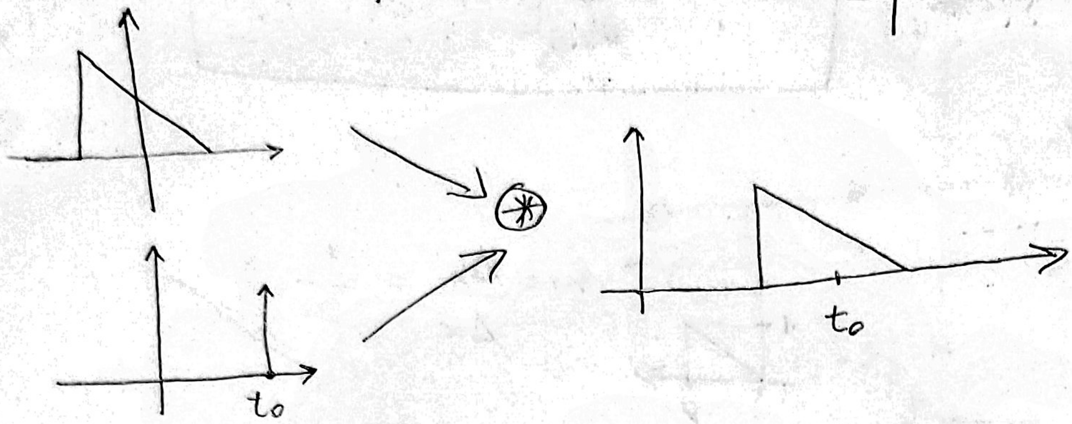
Ex 8.7 Rascunho:

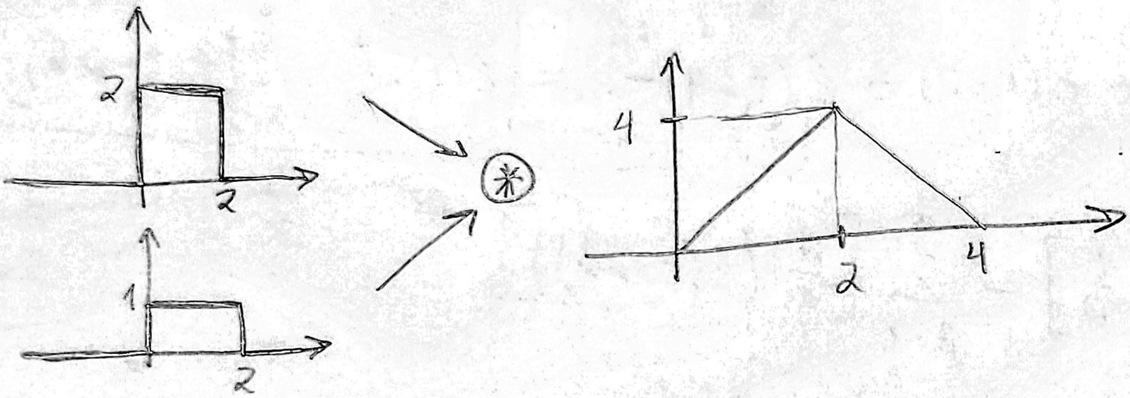
pulso retangular:

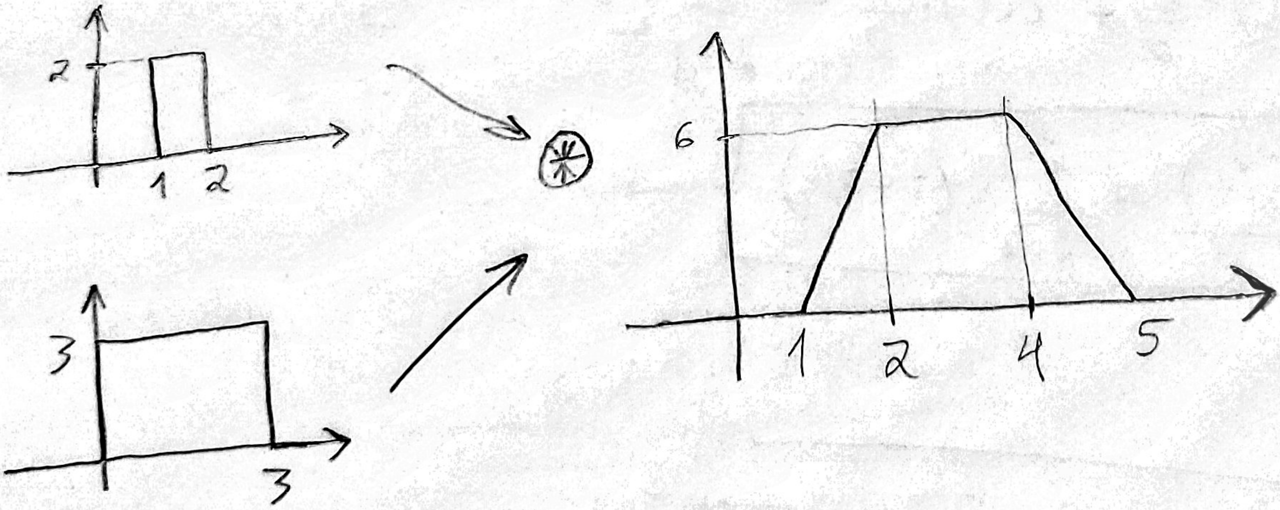


pulso curto:









De acordo com o rascunho / resumo :

$y_1 = x_2(t) \circledast x_3(t)$ $\quad$ $y_5 = x_3(t) \circledast x_6(t)$

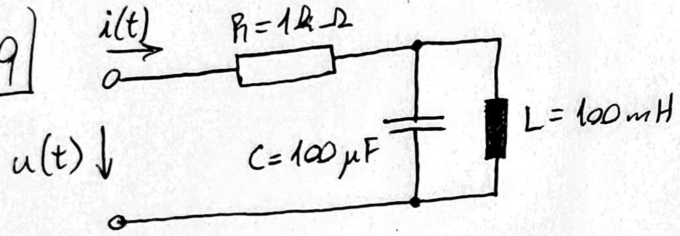
$y_2 = x_1(t) \circledast x_2(t)$ $\quad$ $y_6 = x_5(t) \circledast x_6(t)$

$y_3 = x_4(t) \circledast x_6(t)$ $\quad$ $y_7 = x_1(t) \circledast x_5(t)$

$y_4 = x_1(t) \circledast x_3(t)$ $\quad$ $y_8 = x_3(t) \circledast x_5(t)$

$y_9 = x_1(t) \circledast x_6(t)$

Ex 8.9



$$\left.\begin{array}{l} R = R \\ \left.\begin{array}{l} C = sC \\ L = \frac{1}{sL} \end{array}\right\} \text{paralelo} \end{array}\right\} \text{série}$$

a)

$$H(s) = \frac{I(s)}{U(s)} = \frac{1}{Z(s)} = \frac{1}{R + \frac{1}{sC + \frac{1}{sL}}} = \frac{(s^2+1).LC}{s^2 RLC + sL + R}$$

$R = 1\Omega$ $\quad$ $L = 0{,}1H$ $\quad$ $C = 100F$

$$H(s) = \frac{10s^2+1}{10s^2+0{,}1s+1} = \boxed{\frac{s^2+0{,}1}{s^2+0{,}01s+0{,}1}}$$

b) $H(s) = 1 + \dfrac{-0{,}01s}{s^2+0{,}01s+0{,}1} = 1 - \dfrac{0{,}01s}{(s+\sigma_1)^2+\omega_1^2}$

$$h(t) = \delta(t) - 0{,}01\,\varepsilon(t)\,e^{-\sigma_1 t}\cos(\omega_1 t)$$

$\sigma_1 = 0{,}005$

$\omega_1 = \sqrt{0{,}1 - 0{,}005^2} \cong 1$

$A = -0{,}01$

$B = \frac{A\sigma_1}{\omega_1} \cong 1{,}58 \cdot 10^{-4}$

$h(t) = \delta(t) + e^{-\sigma_1} \cdot e(t) A_1 \cos(\omega_1 t + \varphi_1)$

$\rightarrow A_1 \cong 0{,}01$

$\varphi_1 \cong -89°$

c) $Re\{s\} > -\sigma_1$

d) $ROC\{I(s)\} = ROC\{H(s)\} \cap ROC\{U(s)\}$

$ROC\{I(s)\} \in Im$ → se região de conv. de $I(s)$ contiver o eixo Im.

$\boxed{\therefore \sigma_0 > 0, \ \forall \omega_0 \,/\, \omega_0 \in \mathbb{C}}$ → todo valor de ω_0 que pertença aos números complexos