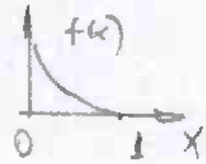


(1) X = proporção do orçamento $\qquad f(x) = k(1-x)^4$

i) $k = ?$ $\qquad \displaystyle\int_0^1 k(1-x)^4\,dx = 1 \Longrightarrow k\int_1^0 -u^4\,du = 1$

$(1-x) = u \quad \begin{cases} x=0 \to u=1 \\ x=1 \to u=0 \end{cases}$

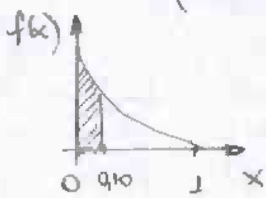
$\frac{du}{dx} = -1 \to -du = dx$

$\Rightarrow -k\,\frac{u^5}{5}\Big|_1^0 = 1$

$\Rightarrow -k\left(\frac{0}{5} - \frac{1}{5}\right) = 1$

$\Rightarrow \frac{k}{5} = 1 \Rightarrow k = 5$

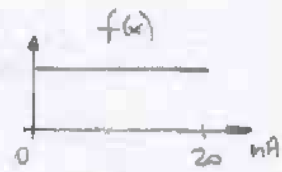
ii) $P(X < 0{,}10) = \displaystyle\int_0^{0{,}10} 5(1-x)^4\,dx = -(1-x)^5\Big|_0^{0{,}10} = -\left[(1-0{,}1)^5 - (1-0)^5\right]$



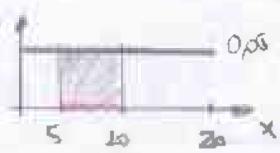
$= -(-0{,}40951)$

$P(X < 0{,}10) \cong 0{,}4096$

(2) X = Corrente que passa no fio de cobre $\qquad f(x) = 0{,}05$



a) $P(5 \leq X \leq 10) = \displaystyle\int_5^{10} 0{,}05\,dx = 0{,}05x\Big|_5^{10} = 0{,}25$



ou fazer (base × altura), uma vez que a área é um retângulo.

b) $\mu = E(x) = \displaystyle\int_0^{20} x \cdot 0{,}05\,dx = \frac{0{,}05\,x^2}{2}\Big|_0^{20} = 0{,}025 \cdot (400 - 0) = 10 \text{ mA}$

$\sigma^2 = Var(x) = \displaystyle\int_0^{20} x^2 \cdot 0{,}05\,dx - \mu^2 = \frac{0{,}05\,x^3}{3}\Big|_0^{20} - 10^2 = \frac{0{,}05}{3} \cdot 8000 - 100 = 33{,}333$

$\sigma = DP(x) = \sqrt{33{,}333} = 5{,}7735 \text{ mA}$

Então $CV = \frac{5{,}7735}{10} \times 100 = 57{,}735\ \%$

Obs: Se tivesse percebido que $f(x) = \frac{1}{b-a}$

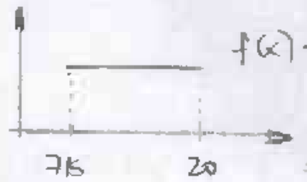
$f(x) = \frac{1}{20} = 0{,}05$, então X segue o modelo Uniforme: $E(x) = \frac{a+b}{2}$

$Var(x) = \frac{(b-a)^2}{12}$

(3) X = profundidade de bioturbação

X é Uniforme de (7,5 a 20)

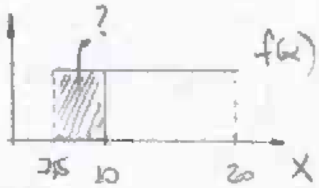
$f(x) = \frac{1}{20-7,5} = \frac{1}{12,5}$



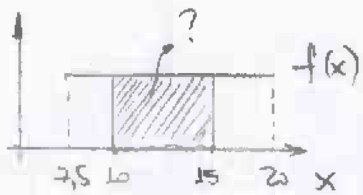
a) $\mu = E(X) = \int_{7,5}^{20} x \cdot f(x)\,dx \xrightarrow{\text{Não precisa}}$ fazer toda essa conta! Se X é Uniforme: $\mu = \frac{a+b}{2} = 13,75$ cm

$\sigma^2 = Var(X) = \int_{7,5}^{20} x^2 \cdot f(x)\,dx - \mu^2 \xrightarrow{\text{Idem}} \sigma^2 = \frac{(b-a)^2}{12} = \frac{(20-7,5)^2}{12} \cong 13,02$ cm^2

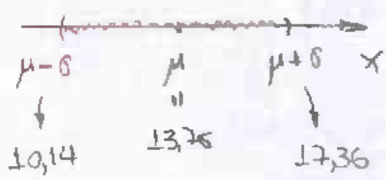
b) • $P(X < 10) = \frac{1}{12,5} \cdot (10 - 7,5) = 0,20$



• $P(10 \leq X \leq 15) = \frac{1}{12,5}(15-10) = 0,40$



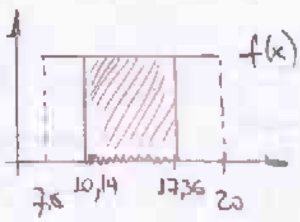
c) •



Então: $P(10,14 \leq X \leq 17,36) = ?$

$= \frac{1}{12,5}(17,36 - 10,14)$

$= 0,5776$



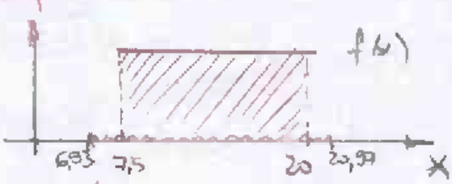
$\mu = 13,75$ cm

$\sigma = 3,61$ cm

•

Então: $P(6,53 \leq X \leq 20,97) = ?$

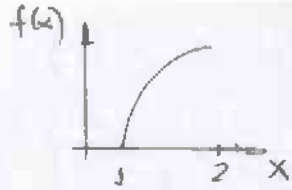
Note que os limites desta probabilidade extrapolam o domínio de X!



Assim: $P(6,53 \leq X \leq 20,97) = 1$

④

X = demanda de gás na semana ($\times 1000$ L)

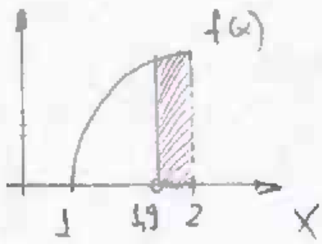


$$f(x) = k\left(1 - \frac{1}{x^2}\right)$$

a) $$\int_1^2 k\left(1 - \frac{1}{x^2}\right)dx = 1 \Rightarrow k\left[x + \frac{1}{x}\right]_1^2 = 1$$

$$\Rightarrow k\left[\left(2 + \frac{1}{2}\right) - (1 + 1)\right] = 1 \Rightarrow \underline{k = 2}$$

b) $$P(X > 1{,}9) = \int_{1{,}9}^2 2\left(1 - \frac{1}{x^2}\right)dx = 2\left[x + \frac{1}{x}\right]_{1{,}9}^2 \cong \underline{0{,}1474}$$



c) $\mu = E(X) = ?$

$$\mu = \int_1^2 x \cdot f(x)dx = \int_1^2 x \cdot 2\left(1 - \frac{1}{x^2}\right)dx = 2\int_1^2\left(x - \frac{1}{x}\right)dx$$

$$= 2\left[\frac{x^2}{2} - \ln(x)\right]_1^2 = 2\left[\left(2 - \frac{1}{2}\right) - \left(\ln(2) - \cancel{\ln(1)}^{\,0}\right)\right]$$

$\mu \cong \underline{1{,}614 \text{ mil litros}}$

$\sigma^2 = Var(X) = ?$

$$\sigma^2 = \int_1^2 x^2 \cdot f(x)dx - \mu^2 = \int_1^2 x^2 \cdot 2\left(1 - \frac{1}{x^2}\right)dx - (1{,}614)^2$$

$$= 2\int_1^2 (x^2 - 1)dx - 2{,}605 = 2\left[\frac{x^3}{3} - x\right]_1^2 - 2{,}605$$

$$= 2\left[\left(\frac{8}{3} - \frac{1}{3}\right) - (2 - 1)\right] - 2{,}605 \cong 0{,}062$$

Então $\sigma = \sqrt{0{,}062} \cong 0{,}248$ mil litros