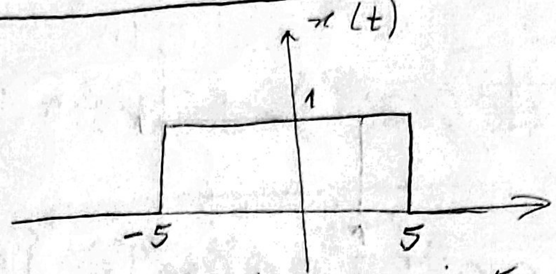
Transformada de Fourier

9.1 b) $x(t) = ret(0{,}1t)$



$$\mathcal{F} = \int_{-\infty}^{+\infty} ret(0{,}1t)\, e^{-j\omega t}\, dt = \int_{-5}^{5} e^{-j\omega t}\, dt = -\left[\frac{e^{-j\omega t}}{j\omega}\right]_{-5}^{5} = -\left[\frac{e^{-j\omega . 5} - e^{+j\omega 5}}{j\omega}\right] =$$

$$\mathcal{F} = \frac{e^{5j\omega} - e^{-5j\omega}}{j\omega} = \frac{2\cancel{j}\, sen(5\omega)}{\cancel{j}\omega} = \boxed{10\, sinc(5\omega)}$$

c) $x(t) = \delta(-4t)$

$$\mathcal{F} = \int_{-\infty}^{+\infty} \delta(-4t)\, e^{-j\omega t}\, dt = \frac{1}{|-4|} \cdot 1 = \boxed{\frac{1}{4}}$$

e) $x(t) = e^{-j\omega t}$

$$\mathcal{F} = \int_{-\infty}^{+\infty} e^{-2j\omega t}\, dt = -\left.\frac{e^{-2j\omega t}}{2j\omega}\right|_{-\infty}^{+\infty} \Rightarrow \lim_{t \to \pm\infty} \frac{-e^{-\infty} + e^{+\infty}}{2j\omega} = \frac{1}{2j\omega} \lim_{t \to \pm\infty} e^{\infty} - \cancelto{0}{\frac{1}{e^{\infty}}} =$$

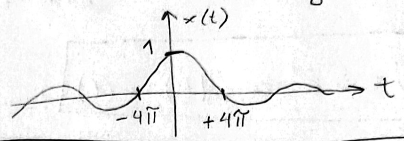
$$\mathcal{F} = \frac{1}{2j\omega} \lim_{t \to \pm\infty} e^{\infty} \rightsquigarrow \boxed{\text{não converge}}$$

Ex 9.5 $x(t) = \text{sinc}(\omega t)$

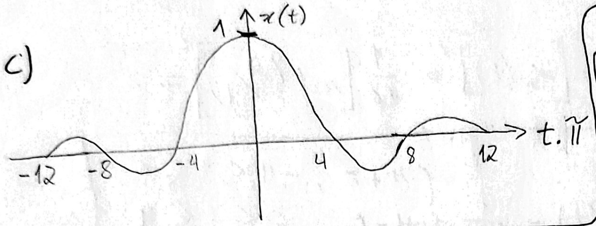
a) $t = n \cdot 4\pi \Rightarrow \text{sinc}(\omega t) = 0$, quando $\omega t = n\pi \;\therefore\; \omega \cdot n 4\pi = n\pi \Rightarrow \boxed{\omega = \frac{1}{4}\,\frac{rad}{s}}$

b) $\int_{-\infty}^{+\infty} x(t)\,dt$ ⇝ função sinc(x): cálculo da área do triângulo

$\therefore \int_{-\infty}^{+\infty} \text{sinc}(t/4)\,dt = \frac{1 \cdot 8\pi}{2} = \boxed{4\pi} \Longrightarrow$



c)



$\mathcal{F} = \int_{-\infty}^{+\infty} \frac{sen(10\pi(t+T))}{10\pi(t+T)} \cdot e^{-j\omega t}\,dt =$

= dualidade entre sinc e rect =

$\text{sinc}(\frac{t}{2}) \longleftrightarrow 2\pi\, \text{rect}(\omega)$

$\therefore \text{sinc}(10\pi t) \longleftrightarrow \frac{\pi}{10\pi}\, \text{rect}\left(\frac{\omega}{20\pi}\right)$

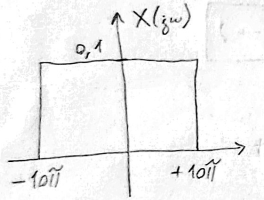
tendo: $\text{sinc}(\pi(t-1)) \longleftrightarrow e^{-j\omega} \cdot \text{rect}\left(\frac{\omega}{2\pi}\right)$

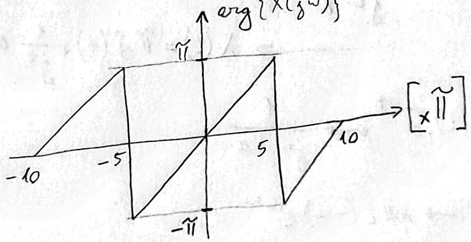
concluimos: $\text{sinc}(10\pi(t-T)) \longleftrightarrow \left[\frac{1}{10}\, \text{rect}\left(\frac{\omega}{20\pi}\right)\right] \cdot e^{j\omega T}$

Ex 9.6 $x(t) = \text{sinc}(10\pi(t+T))$, $T = 0{,}2$

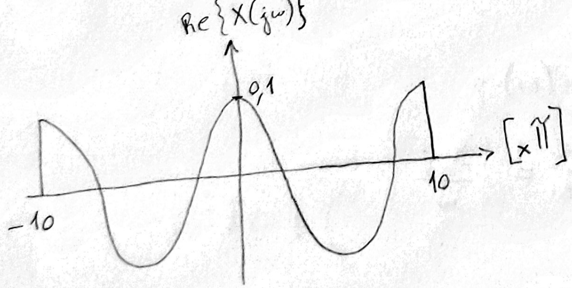
a) $|X(j\omega)|$ e $\arg\{X(j\omega)\}$

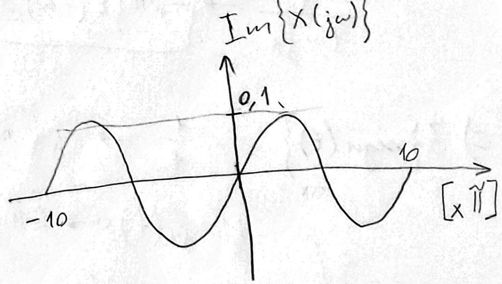
$\text{sinc}(10\pi(t+T)) \longleftrightarrow \frac{1}{10}\, \text{ret}\left(\frac{\omega}{20\pi}\right) \cdot e^{j\omega T} = X(j\omega)$





b) $\text{Re}\{X(j\omega)\}$ e $\text{Im}\{X(j\omega)\}$





Ex 9.7 $X_1 = \dfrac{5j\omega + 5}{(j\omega)^2 + 2j\omega + 17}$; $X_2 = \text{sinc}(2\omega)$; $X_3 = \left(\text{sinc}(2\omega)\right)^2$

$X_1(s) = \dfrac{5s+5}{s^2+2s+17} = 5 \cdot \dfrac{(s+1)}{(s+1)^2 + 4^2}$, sabendo $\mathcal{L}^{-1}\left\{\dfrac{s-a}{(s-a)^2+b^2}\right\} = e^{at}\cos(bt)$

$\boxed{x(t) = 5 e^{-t} \cos(4t) \cdot \varepsilon(t)}$

$\mathcal{F}^{-1}\{X_2\} \Rightarrow$ dualidade

$\text{rect}(t) \circ\!\!-\!\!\bullet\ \text{sinc}\left(\frac{\omega}{2}\right)$

$\text{sinc}\left(\frac{t}{2}\right) \circ\!\!-\!\!\bullet\ 2\pi\,\text{rect}(-\omega) = 2\pi\,\text{rect}(\omega)$

$\boxed{\therefore \mathcal{F}^{-1}\{X_2\} = \frac{1}{4}\text{rect}(t/4)}$

$x_3(t) = [x_2(t)]^2 = \dfrac{1}{16}\left[\text{rect}(t/4)\right]^2 \Rightarrow$

$\Rightarrow$ $\boxed{x_3(t) = \dfrac{1}{16}\begin{cases} 4+t, & -4 < t \leq 0 \\ 4-t, & 0 < t < 4 \\ 0 \end{cases}}$

Ex 9.12 a) $\mathcal{F}\left\{\pi\delta(t) + \frac{1}{jt}\right\} \rightarrow x(t) = \varepsilon(t) \circ\!\!-\!\!\bullet\ X(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$

$\rightarrow X(t) = \pi\delta(t) + \frac{1}{jt} \circ\!\!-\!\!\bullet\ \boxed{2\pi\,\varepsilon(-\omega)}$

b) $\mathcal{F}\left\{\dfrac{1}{t-ja}\right\} \Rightarrow \varepsilon(t)\cdot e^{-at} \circ\!\!-\!\!\bullet\ \dfrac{1}{j\omega + a}$

dualidade: $\dfrac{1}{jt+a} \circ\!\!-\!\!\bullet\ 2\pi\,\varepsilon(-\omega)\,e^{a\omega}$

$\dfrac{1}{t-ja} \circ\!\!-\!\!\bullet\ \boxed{-2\pi j\,\varepsilon(-\omega)\,e^{a\omega}}$

c) $\mathcal{F}\{\text{sign}(t)\} \Rightarrow \dfrac{1}{t} \circ\!\!-\!\!\bullet\ -j\pi\,\text{sign}(\omega)$

$-j\pi\,\text{sign}(t) \circ\!\!-\!\!\bullet\ 2\pi \cdot \dfrac{1}{-\omega} = \dfrac{-2\pi}{\omega}$

$\div -j\pi$ $\quad$ $\div -j\pi$

$\text{sign}(t) \circ\!\!-\!\!\bullet\ \boxed{\dfrac{2}{j\omega}}$