Evolutionary Algorithms

Inteligencia Artificial en los Sistemas de Control Autónomo
Máster en Ciencia y Tecnología desde el Espacio

Departamento de Automática

Objectives

- Describe the most relevant EAs

Bibliography

- Eiben, A.E. and Smith, J.E. Introduction to Evolutionary Computing. Springer 2003.

Table of Contents

Genetic Algorithms
Introduction (I)

Introduced by Holland in the 70's

- John H. Holland "Adaptation in Natural and Artificial Systems", MIT Press
- GA is the most popular EA
- Usually EAs confused with GA

Canonical GA (which is not canonical)

- Fixed length strings
- Binary codification
- Holland's Theorem

Representation	Bit strings
Recombination	1-point
Mutation	Bit flip
Parent select	Fitness prop
Survivor select	Generational

Genetic Algorithms
Introduction (II)

GAs are a family of algorithms, with common features

- Representation in strings, named chromosomes
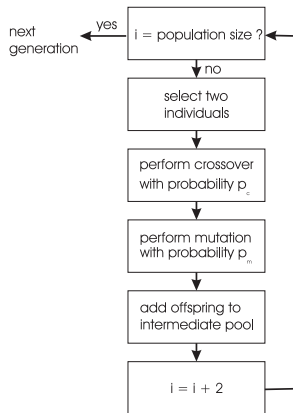- Mutation and recombination
- Usually fixed length

GAs are like a toolbox with customizable components

- Representations, genetic operators, selections mechanism, ...
- These components are interdependent

Rule of thumb: Small genotype changes $\Rightarrow$ Small phenotype changes

Genetic Algorithms
Introduction (III)

Genetic Algorithms

Representation: Binary

I	I	O	I	O	O	O	I

One of the oldest and widely used codifications

- Consequence of Holland's Theorem
- Strong historical influence

Often used to codify non-binary information (not recommended)

- Pure binary codification
- Gray coding
- Custom codification

Genotype   Phenotype

1100 0001 1010 1101 0011 0111   c   a   [t   r   ]

Hint: Use binary codification to represent binary information

Genetic Algorithms

Representation: Integer

4	3	2	1	0	4	2	3	3

Chomosome as a sequence of integers

- More natural codification for many problems
- Optimization of integer values
- Integer representation ($\{1, 2, 3, 4\} = \{$North, East, South, West$\}$)

Genetic Algorithms

Representation: Floating-point

1.1	0.2	3.0	33.2	0.0	-3.2	130.1	88.3	-7.1

Chomosome as a sequence of floating-point values

- Common in optimization problems
- Solutions with continous nature

Genetic Algorithms

Representation: Permutation

4	3	2	5	6	1

Some problems involve order

- Sequence of integers
- No repeated numbers
- Range of valid numbers
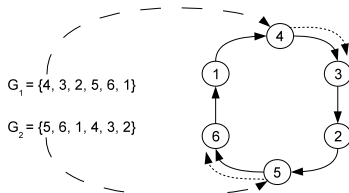- Special genetic operators

Information can be contained in

- The locus (position)
  $$[3, 1, 2, 4] \Rightarrow [C, A, B, D]$$
- The allele (value)
  $$[3, 1, 2, 4] \Rightarrow [B, C, A, D]$$

$G_1 = \{4, 3, 2, 5, 6, 1\}$

$G_2 = \{5, 6, 1, 4, 3, 2\}$

Integer codification to solve TSP

Genetic Algorithms

Mutation

Mutation: Genetic operator that uses one parent

- Introduces randomness into the genotype
- Depends on representation

Main objectives

- Avoid local minima (premature convergence)
- Enhances exploration

Often dependent on the mutation rate

- Significant influence in the algorithm behaviour
- Higher mutation rate, higher exploration

Genetic Algorithms

Mutation for binary representations

Flip bit with probability p_m

```
1 0 1 0 0 0 0 1 0    ⟶    1 0 0 1 0 0 0 0 0
```

Optimal p_m depends on the problem and goals

- Need of high fitness population
- Need of high fitness individual
- Need of genetic diversity
- Modality of the problem
- Algorithm dynamics

Rule of thumb: $p_m = \frac{1}{length}$

Genetic Algorithms

Mutation for integer representations

Two main mutations applied to each gene

- **Random resetting**: Choose new random value with p_m
- **Creep mutation**: Add small (positive or genative) random value with p_m

Genetic Algorithms

Mutation for floating-point representations

Set new value with value drawn from a distribution

- **Uniform mutation** Choose new random value from $[L, U]$ with p_m
- **Non-uniform mutation** Usually adding a value drawn from a zero-mean gaussian distribution

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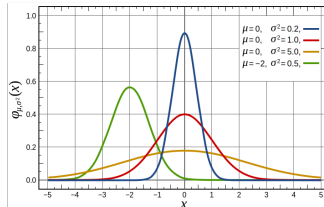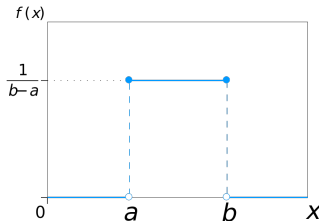
Genetic Algorithms

Mutation for permutation representations

Genes are no longer independent

- No gene mutation, p_m affects the whole chromosome

Swap mutation

Insert mutation

Scramble mutation

Inversion mutation

Genetic Algorithms

Recombination

Recombination creates one individual from two or more parents

- Also known as crossover (specially for two parents)
- Basic feature in GA
- Parents selection mechanism needed

Usually applied to all new individuals

- Not used when elitism is applied
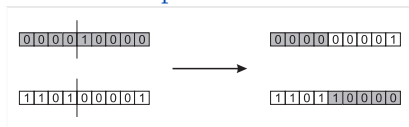- Sometimes applied with $p_c \in [0,5, 1]$

Objectives of recombination

- Combine parents' behavior $\Rightarrow$ No new genetic material
- Constructive role
- Enhances explotation
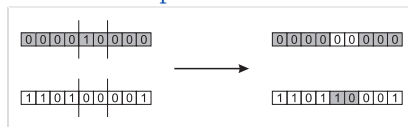
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Recombination: Binary and integer representations

Three crossover mechanisms for binary and integer encodings

One-point crossover

Two-points crossover

Uniform crossover

Genetic Algorithms

Recombination: Floating point representations (I)

Discrete recombination

- Analogous to binary recombination
- No new genetic material

Arithmetic recombination

- Combines the parents' genes
- Weighted sums of genes: $z_i = \alpha x_i + (1 - \alpha)y_i$
- Usually, $\alpha = 0,5$ (average values)
- Different arithmetic recombinations

Genetic Algorithms

Recombination: Floating point representations (II)

Whole arithmetic recombination
(All genes are included)

| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |

| 0.2 | 0.2 | 0.3 | 0.3 | 0.4 | 0.4 | 0.5 | 0.5 | 0.6 |

⟶

| 0.3 | 0.2 | 0.3 | 0.2 | 0.3 | 0.2 | 0.3 | 0.2 | 0.3 |

| 0.2 | 0.2 | 0.3 | 0.3 | 0.4 | 0.4 | 0.5 | 0.5 | 0.6 |

Simple arithmetic recombination
(Similar to one-point crossover)

| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |

| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.5 | 0.5 | 0.6 |

⟶

| 0.3 | 0.2 | 0.3 | 0.2 | 0.3 | 0.2 | 0.3 | 0.2 | 0.3 |

| 0.3 | 0.2 | 0.3 | 0.2 | 0.3 | 0.2 | 0.5 | 0.5 | 0.6 |

Single arithmetic recombination
(Similar to uniform crossover)

| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |

| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.5 | 0.9 |

⟶

| 0.3 | 0.2 | 0.3 | 0.2 | 0.3 | 0.2 | 0.3 | 0.2 | 0.3 |

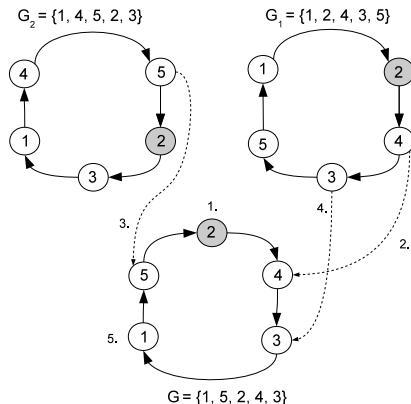| 0.3 | 0.2 | 0.3 | 0.2 | 0.3 | 0.2 | 0.3 | 0.5 | 0.3 |

Genetic Algorithms

Recombination: Permutation representations

Specialized recombinations

- Partially Mapped Crossover
- Edge Crossover
- Order Crossover
- Cycle Crossover

$G_2 = \{1, 4, 5, 2, 3\}$

$G_1 = \{1, 2, 4, 3, 5\}$

$G = \{1, 5, 2, 4, 3\}$

Genetic Algorithms

Selection

Two purposes for selection

- Parent selection: Individuals to generate offspring
- Survivor selection: Individuals to remplace

Usually same methods applied to both

Genetic Algorithms
Selection: Fitness Proportional Selection

Selection probability proportional to fitness

- Premature convergence
- Lack of selective pressure for close fitness values
- Selective pressure not customizable
- Susceptibility to function transposition

Historically relevant

$$p_i = \frac{f_i}{\sum^{\#}_{}}$$

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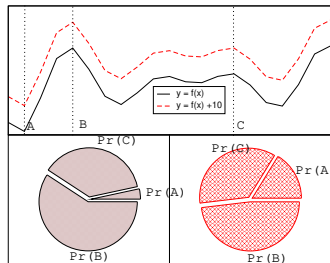
Genetic Algorithms
Selection: Ranking Selection

Selection probability proportional to rank

- Individuals are sorted by fitness
- Arbitrary rank to probability mapping
- Avoid problems with super individuals
- Selective pressure independent of fitness
- Selective pressure not customizable

Linear mapping

$$P_{lin_rank}(i) = \frac{(2 - s)}{\mu} + \frac{2i(s - 1)}{\mu(\mu - 1)}$$

$1{,}0 < s < 2{,}0$

Exponential mapping

$P_{exp_rank}(i) = \frac{1 - e^{-i}}{c}$

c = normalization factor

Genetic Algorithms

Selection: Tournament Selection

Algorithm of tournament size k

1. Select randomly k chromosomes
2. Compute their fitness
3. Select the fittest one
4. Go to 1

Customizable selective pressure

- Depends on k and μ

De facto standard

- Good for parallel computation
- Efficient implementation

Usually $k = 2$ in GA, in GP $k = 7$

Genetic Algorithms

Selection: Survival selection

Two strategies

- Generational (all the population is remplaced)
- Steady-stade (partial remplacement)

Survival selection algorithms

- Fitness-Based Replacement (inverse of the previous ones)
- Age-Based Replacement
- Elitism

Genetic Programming
Introduction (I)

GP is a family of algorithms

- Evolve programs
- Self-programming computers
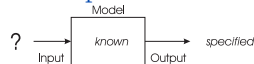- GP, Linear GP, Cartesian GP, EDA, ...

GP introduced by Koza in the 90's

Koza, J.R. *"Genetic Programming: On the Programming of Computers by Means of Natural Selection"*, MIT Press. 1992
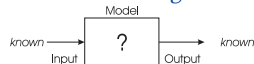
GA and ES focused on optimization

- GP focused on Machine Learning

Optimization

Modelling

Simulation

Genetic Programming
Introduction (II)

Example: Credit scoring problem within a bank. Develop a model describing good customers

Id	Children	Salary	Status	Credit
Id-1	2	45.000	Married	0
Id-2	0	30.000	Single	1
Id-3	1	40.000	Married	1
Id-4	2	60.000	Divorced	1
...				
Id-X	2	50.000	Married	1

Possible model:
```
IF (children=2) AND (Salary>80.000) THEN good ELSE bad
```

Genetic Programming
Introduction (III)

General form

        IF (Formula)
         THEN good
         ELSE bad

In EC terms
Phenotype: Formula
Fitness: Classification accuracy
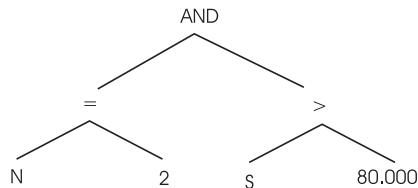
(children=2) AND (Salary>80.000)

Genetic Programming

Representation (I)

GP representation differs in two aspects
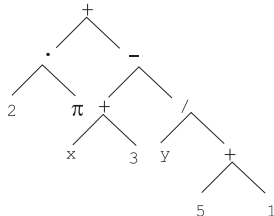
- Nonlinear structure
- Variable size

New representation and genetic operators
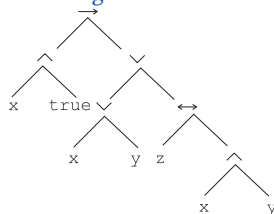
- Same selection (done in phenotipic space)

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Genetic Programming

Representation (II)

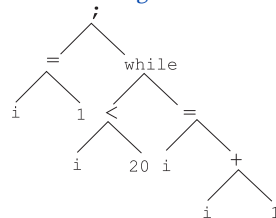Arithmetic formula

$$\left(2\pi + \left((x+3) - \frac{y}{5+1}\right)\right)$$

Logical formula

$$(x \wedge \text{true}) \rightarrow$$
$$((x \vee y) \vee (z \leftrightarrow (x \vee y)))$$

Program

```
^^I^^I^^Ii=1;
^^I^^I^^Iwhile (i<20)
^^I^^I^^I^^Ii = i+1;
^^I^^I^^I}
^^I^^I^^I
```

Genetic Programming

Representation (III)

Two types of nodes

- **Function set** Internal nodes. It has an ssociated number of attributes
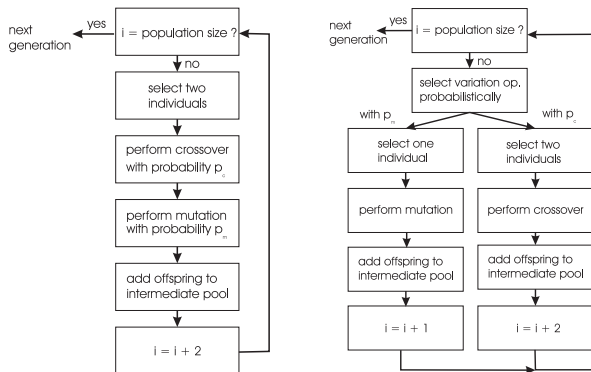- **Terminal set** Leaves of the tree

Danger: Inviable trees

- Grammar-aware GP variants
- Strongly Typed Genetic Programming (STGP), Grammatical Evolution (GE), ...

(Complex representation example)

Genetic Programming
Mutation (I)

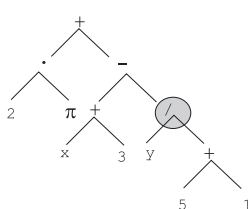### Application of genetic operators in GP contrast to GA
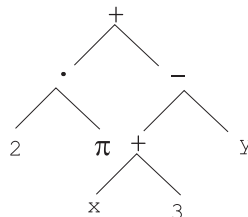
Genetic Programming
Mutation (II)

Subtree mutation

1. Select a random node
2. Delete subtree
3. Add new random subtree

Parameters

- Probability of choosing a terminal node

Highly correlated with code bloat

Genetic Programming

Mutation (III)

Alternative mutation operators

- Size-fair subtree mutation
- Node replacement mutation (point mutation)
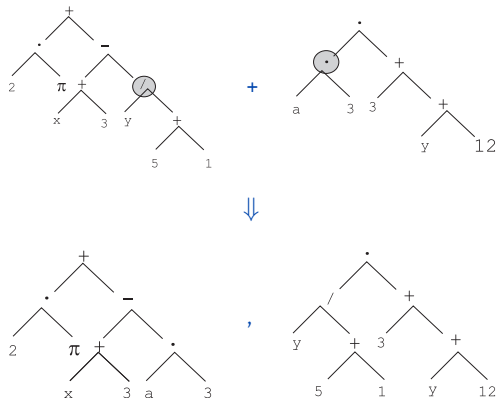- Hoist mutation
- Shrink mutation

Genetic Programming
Recombination (I)

Subtree crossover

1. Take a random node from both parents

2. Swap subtrees

Parameters

- Probability of choosing a terminal node

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Genetic Programming

Recombination (II)

Alternative recombination operators

- Homologous crossover
- Uniform crossover
- Size-fair crossover
- Node replacement mutation (point mutation)
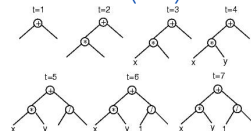- Hoist mutation
- Shrink mutation
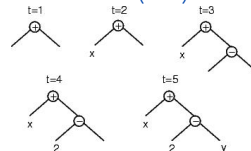
Genetic Programming

Initialization

Three initialization methods

- **Full**. Introduces non-terminals nodes until max depth
- **Grow**. Introduces terminal or non-terminal with equal probability
- **Ramped half-n-half**. Applies full or grow with equal probability

Full (D=2)

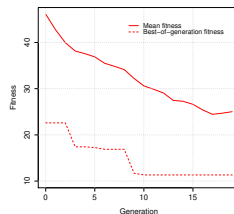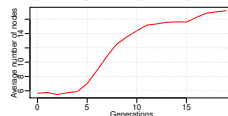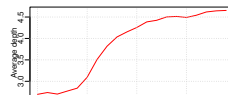Grow (D=2)

Genetic Programming
Bloat in Genetic Programming

Code bloat: Uncontrolled grow of tree sizes
- Intrinsic to variable-length representations
- Undesirable effects
- Perhaps, the worse problem in GP

Countermeasures
- Depth limitation in genetic operators
- Parsimony pressure
- Tree plunning
- Multiobjective techniques

Genetic Programming

Example of reporting

Cuadro 1: Main parameters used to obtain the approximations for secrets ID in the Genetic Tango attack against David-Prasad authentication protocol.

Parameter	ID
Population	500
Generations	10
Terminal Set	A, B, D, E, F, P_{ID1}, P_{ID2}
Function set	And, or, xor
Fitness	Hamming distance to secret
Fitness tags	5
Fitness sessions	100
Min. depth	1
Max. depth	3
Selection	Lexicographic tournament
Tournament size	4
Crossover	0.9
Reproduction	0.1
Elitism size	1
Terminals	0.1
Non terminals	0.9
Initialization	Rampled H-H

Genetic Algorithms
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Genetic Programming
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Evolution Strategies
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Working with an Evolutionary Algorithm
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Evolution Strategies

Introduction (I)

Introduced by Rechenberg and Schwefel in the 60's

- Motivated by wing shape optimization
- Real-function optimization

ES properties

- Emphasis on mutation
- Mutation is gaussian noise
- Self-adaptation

Representation	Real-valued vectors
Recombination	Discrete
Mutation	Gaussian perturbation
Parent selection	Uniform
Survivor selection	(μ, λ) or $(\mu + \lambda)$
Speciality	Self-adaptation

Evolution Strategies
Introduction (II)

Example of basic ES

- Representation: Vector of real values
- Recombination: Not used
- Mutation: Gaussian noise with step-size σ

Adaptative σ (1/5 rule)

- Theorerical foundations
- Based on the ratio of success mutations (p_s)
- After k iterations a new σ is computed

$$\sigma = \begin{cases} \sigma/c & \text{if } p_s > 1/5, \\ \sigma \cdot c & \text{if } p_s < 1/5, \\ \sigma & \text{if } p_s = 1/5 \end{cases}$$

where $0,817 \leq c \leq 1$ is a parameter

Evolution Strategies

Representation

Nowdays ES is usually self-adapted

- Step size (σ) is included in the genotype
- Evolution includes variables and parameters

One or more σ values

- One σ: $\left\langle \underbrace{x_1, x_2, ..., x_n}_{\bar{x}}, \sigma \right\rangle$

- Several: $\sigma : \left\langle \underbrace{x_1, x_2, ..., x_n}_{\bar{x}}, \underbrace{\sigma_1, \sigma_2, ..., \sigma_{n_\sigma}}_{\bar{\sigma}} \right\rangle$

Evolution Strategies
Mutation

Genetic operators to modify σ

- Mutation with one step size:

$$x_i' = x_i + N_i(0, \sigma')$$
$$\sigma' = \sigma \cdot e^{\cdot N(0,\tau)}, \tau \propto 1/\sqrt{n}$$

  τ is analogous to learning rate in ANN

- Mutation with n step sizes:

$$x_i' = x_i + N_i(0, \sigma_i)$$
$$\sigma' = \sigma \cdot e^{\cdot N(0,\tau') + N_i(0,\tau}$$

with $\tau' \propto 1/\sqrt{2n}$ and $\tau \propto 1/\sqrt{2\sqrt{n}}$

Evolution Strategies

Recombination

Secondary operator in ES

- **Discrete recombination**. Like uniform crossover in GA
- **Intermediate recombination**. Like arithmetic crossover in GA

ES tends to use **global recombination**

- More than two parents

Genetic Algorithms
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Genetic Programming
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Evolution Strategies
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Working with an Evolutionary Algorithm
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Evolution Strategies

Parent and survivor selection

The whole population is seen as parent

- Select individual with uniform probability
- No selective pressure in parent selection

After creating the offspring, the λ fittests individuals are selected

- Deterministic procedure

Two selection mechamisms depending on who can be selected

- (μ, λ) **selection**. Only the offpring.
- $(\mu + \lambda)$ **selection**. Parents and offpring

(μ, λ) selection is more popular
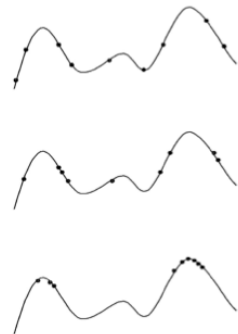
Working with an Evolutionary Algorithm
Search phases

Initial phase: Random distribution, high genetic diversity
Advanced phase: Begins to converge
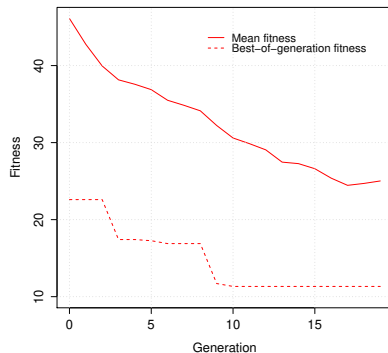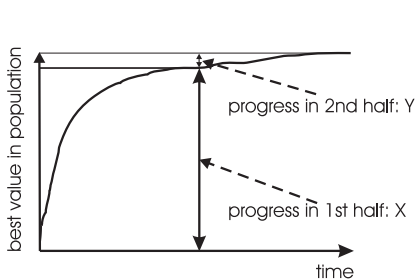**Convergence**: Around one or few points, low genetic diversity

Premature convergence if population not located in global maxima

(Animation)

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Working with an Evolutionary Algorithm

Fitness dynamics

Few long runs or many short runs?

Genetic Algorithms
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Genetic Programming
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Evolution Strategies
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Working with an Evolutionary Algorithm
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Working with an Evolutionary Algorithm
When EAs are useful

Genetic Algorithms
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Genetic Programming
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Evolution Strategies
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Working with an Evolutionary Algorithm
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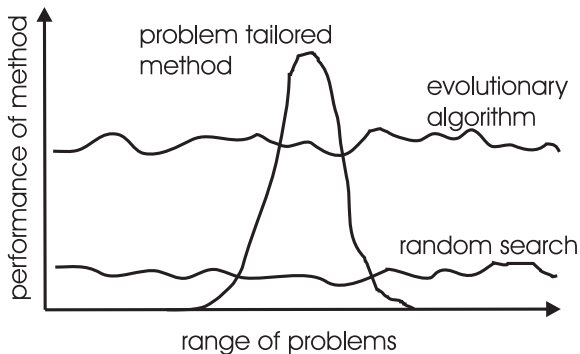
Working with an Evolutionary Algorithm
Advanced EAs

- Multiobjective Evolutionary Algorithms (MOEAs)
- Optimization with constrains
- Coevolution
- Dinamic optimization
- Islands models
- Memetic algorithms
- Hyperheuristics