Universidad de Alcalá

Informed Search

ISG
Intelligent Systems Group

Informed search strategies

- Use the problem-specific knowledge beyond the definition of the problem itself to find more efficient solutions that uninformed strategy
  - Best-first search
    - Greedy best-first search
    - A^* search
  - Heuristics
  - Local search algorithms
    - Hill-climbing search
    - Simulated annealing search
    - Local beam search
    - Genetic algorithms

Best-first search

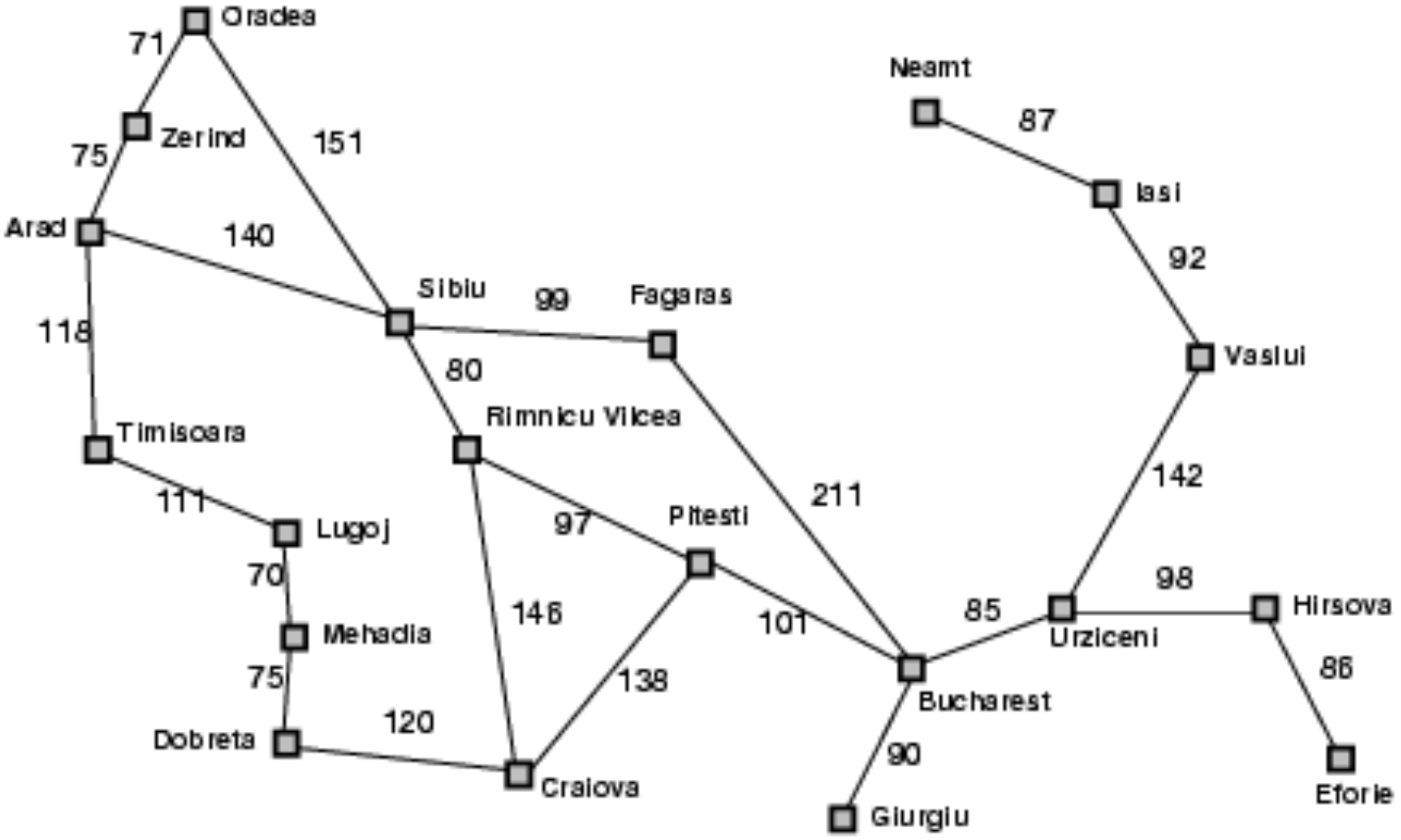
- Idea: use an evaluation function *f(n)* for each node
  - estimate of "desirability"
  - Expand most desirable unexpanded node

- Implementation:

  Order the nodes in fringe in decreasing order of desirability

- Special cases:
  - greedy best-first search
  - A* search

Romania with step costs in km



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
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Informed search strategies

- Best-first search
  - **Greedy best-first search**
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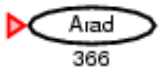
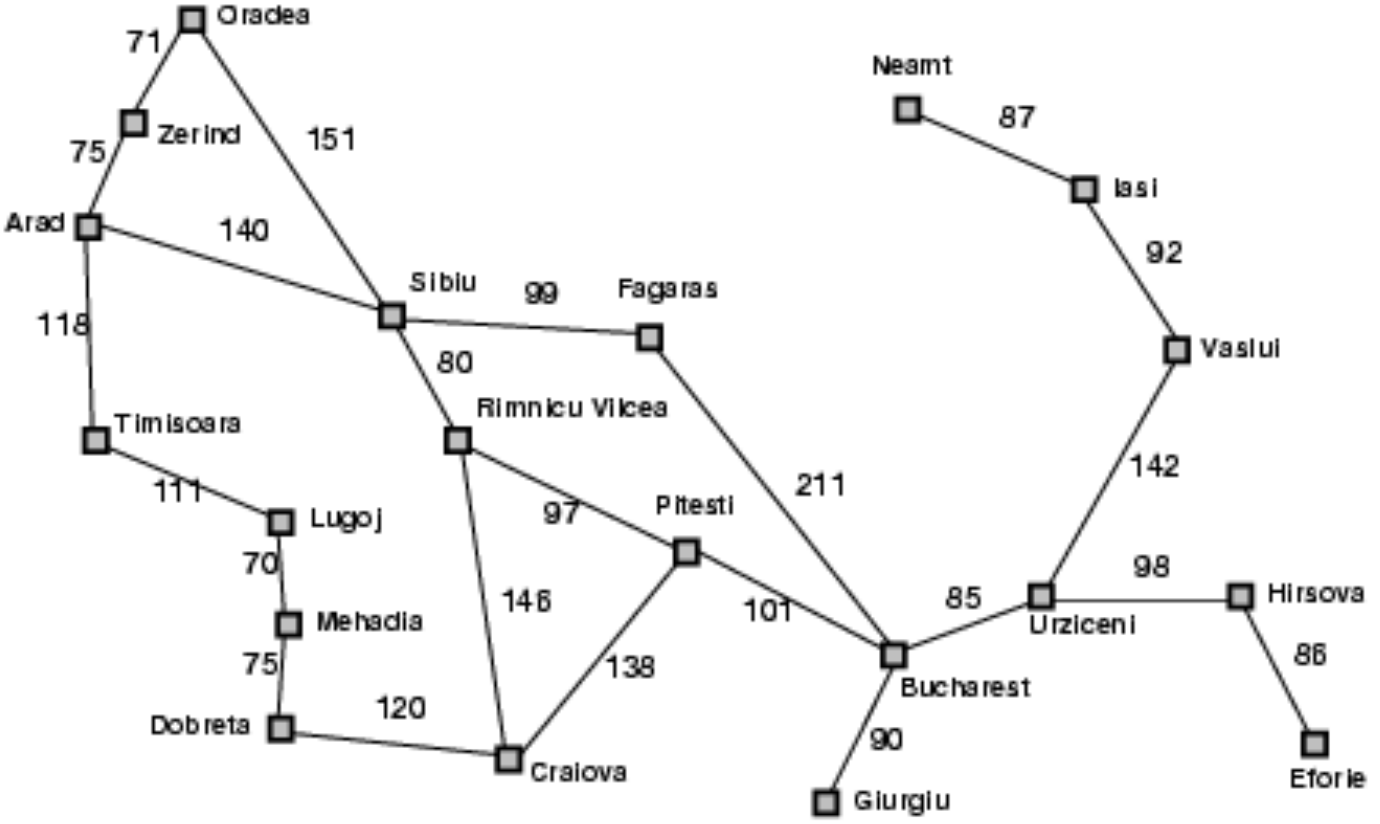
Greedy best-first search

- Evaluation function $f(n) = h(n)$ (heuristic) = estimate of cost from n to *goal*
- Greedy best-first search expands the node that appears to be closest to the goal
- Implementation: as a priority queue to keep the fringe in ascending order of f-values
- e.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest

Greedy best-first search example



Greedy best-first search example



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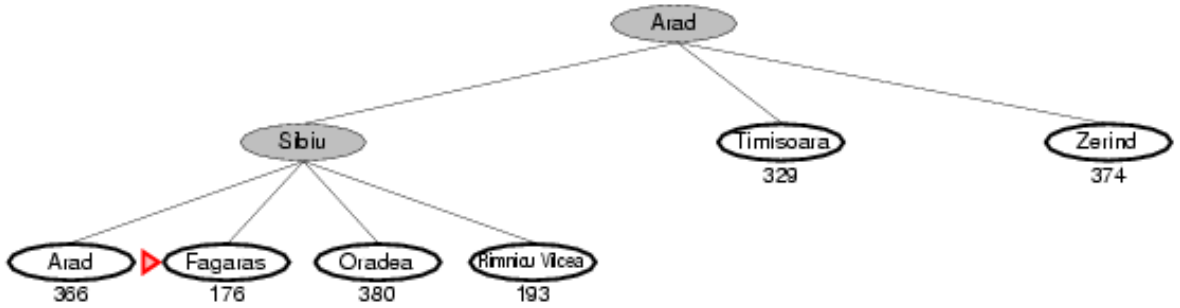
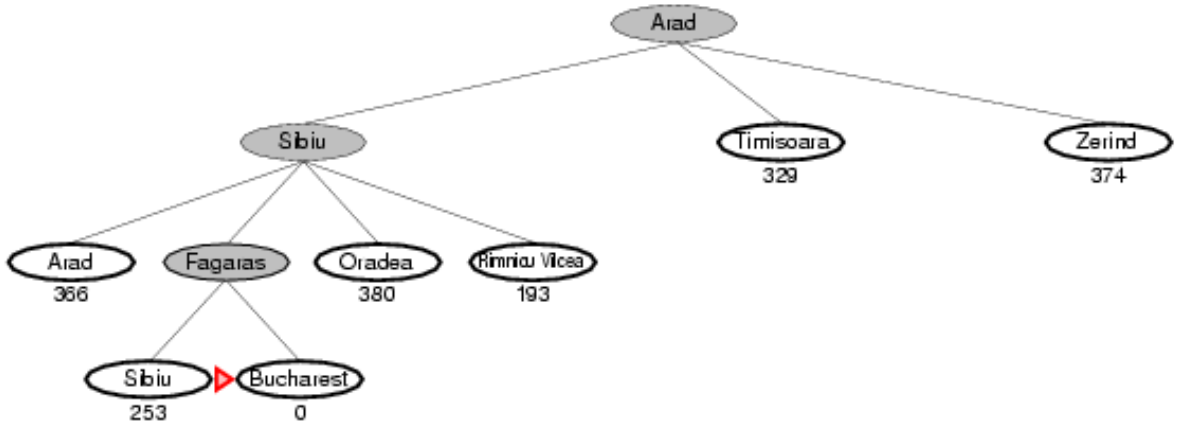
Greedy best-first search example



Greedy best-first search example



Properties of greedy best-first search

- Complete? No – can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →
- Time? $O(b^m)$, but a good heuristic can give dramatic improvement
- Space? $O(b^m)$ -- keeps all nodes in memory
- Optimal? No

Similar to depth-first search

Each state has b successors (branching factor)

d is the depth of the shallowest solution

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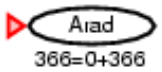
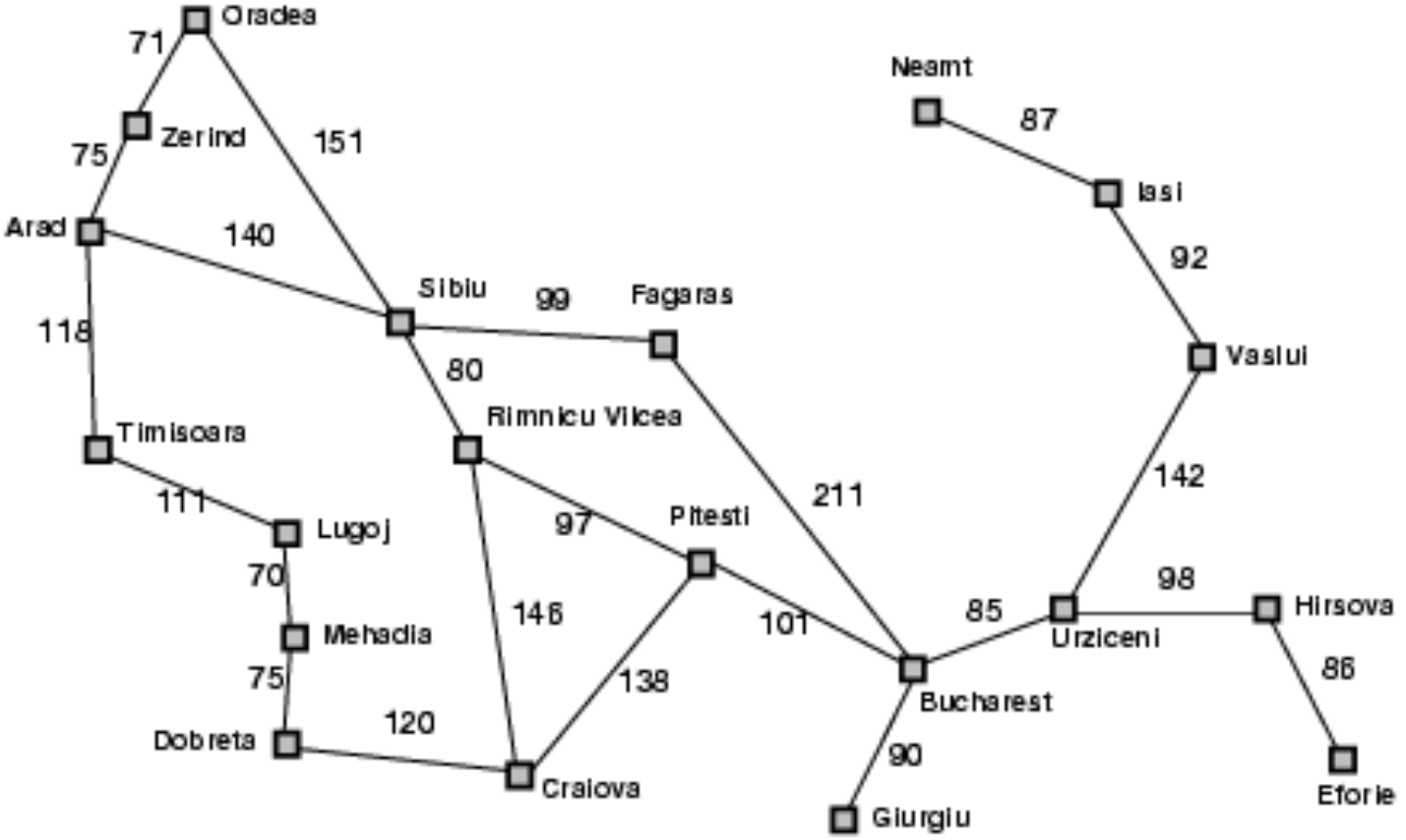
A* search

- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
  - $g(n)$ = cost so far to reach n
  - $h(n)$ = estimated cost from n to goal
  - $f(n)$ = estimated total cost of path through n to goal
- A * is optimal if h(n) is an admissible heuristic such that h(n) never overestimates the cost to reach the goal

A* search example



A* search example



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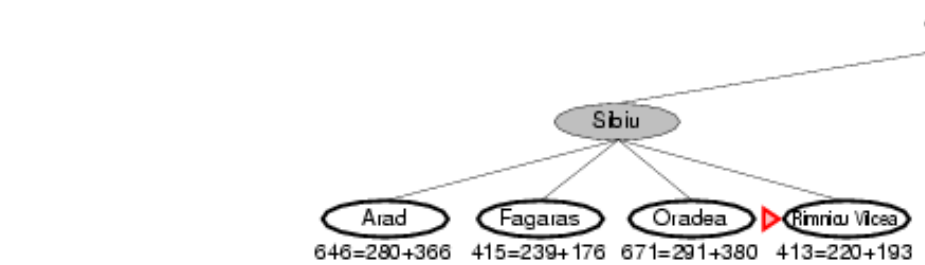

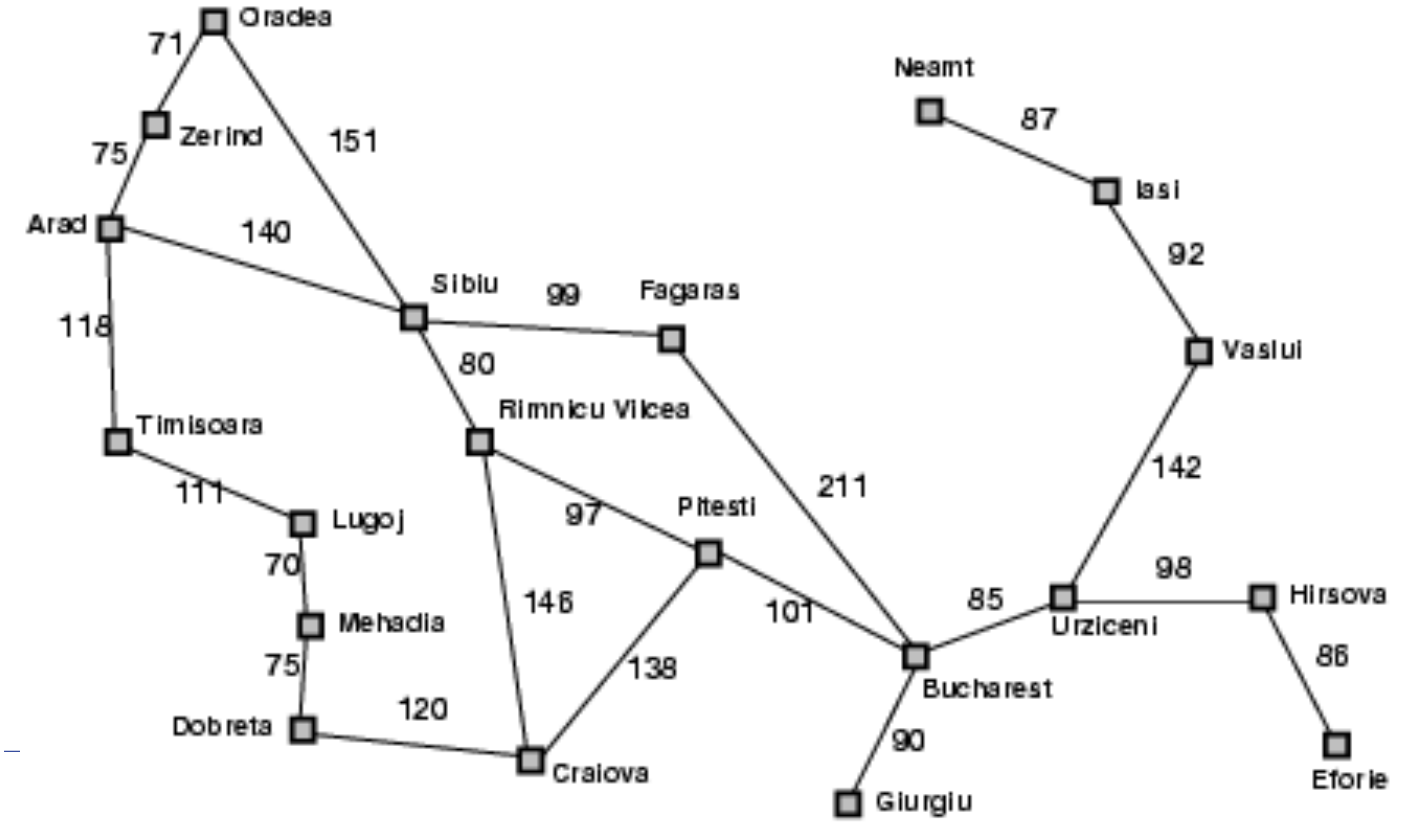

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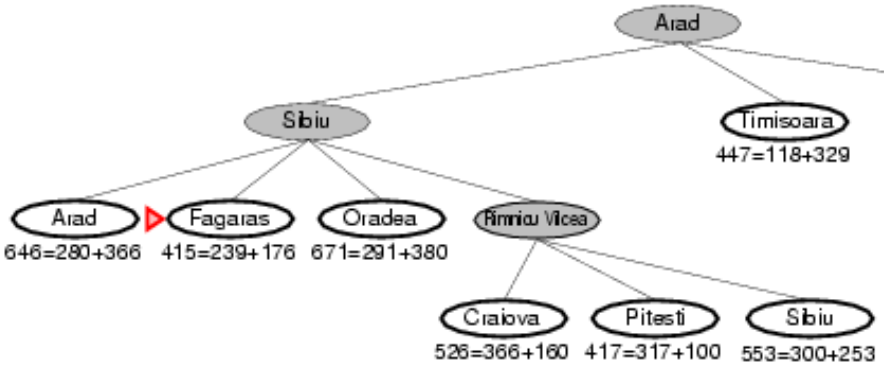

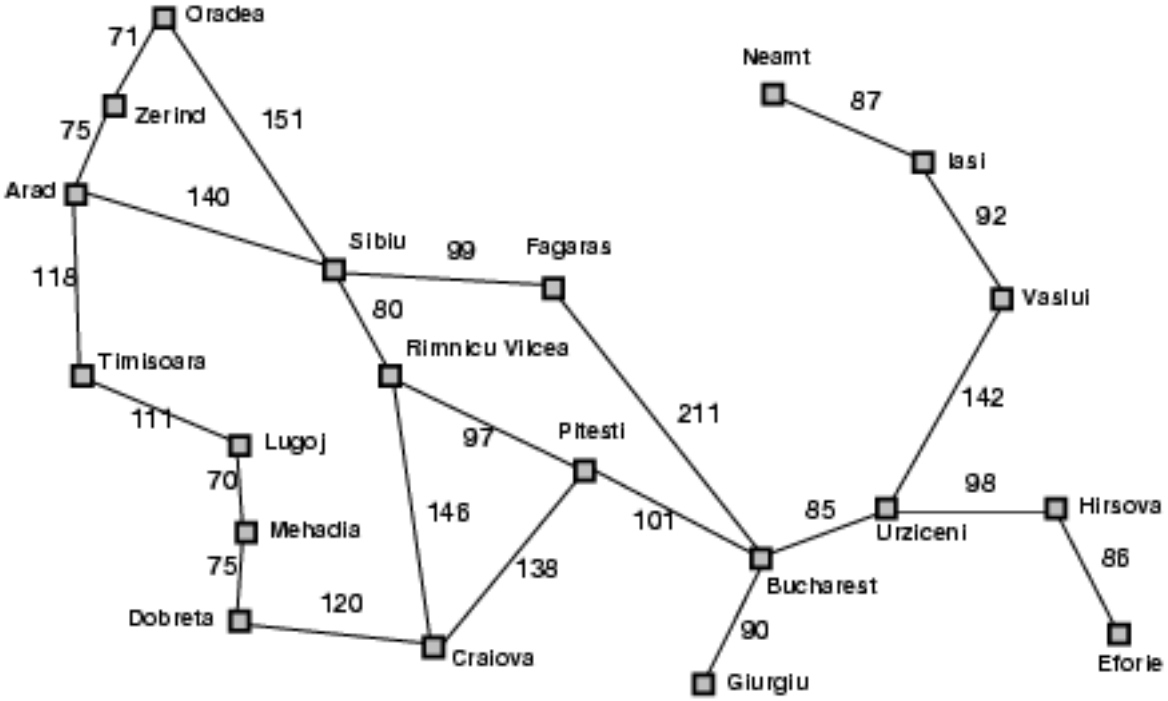

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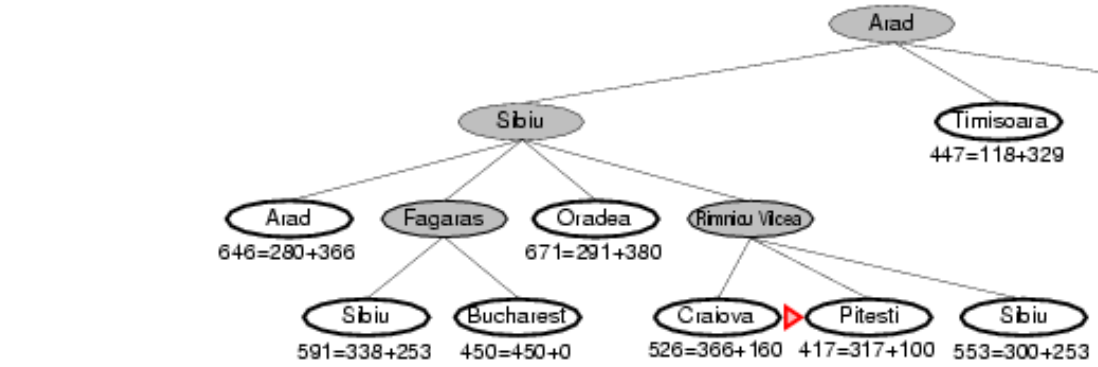

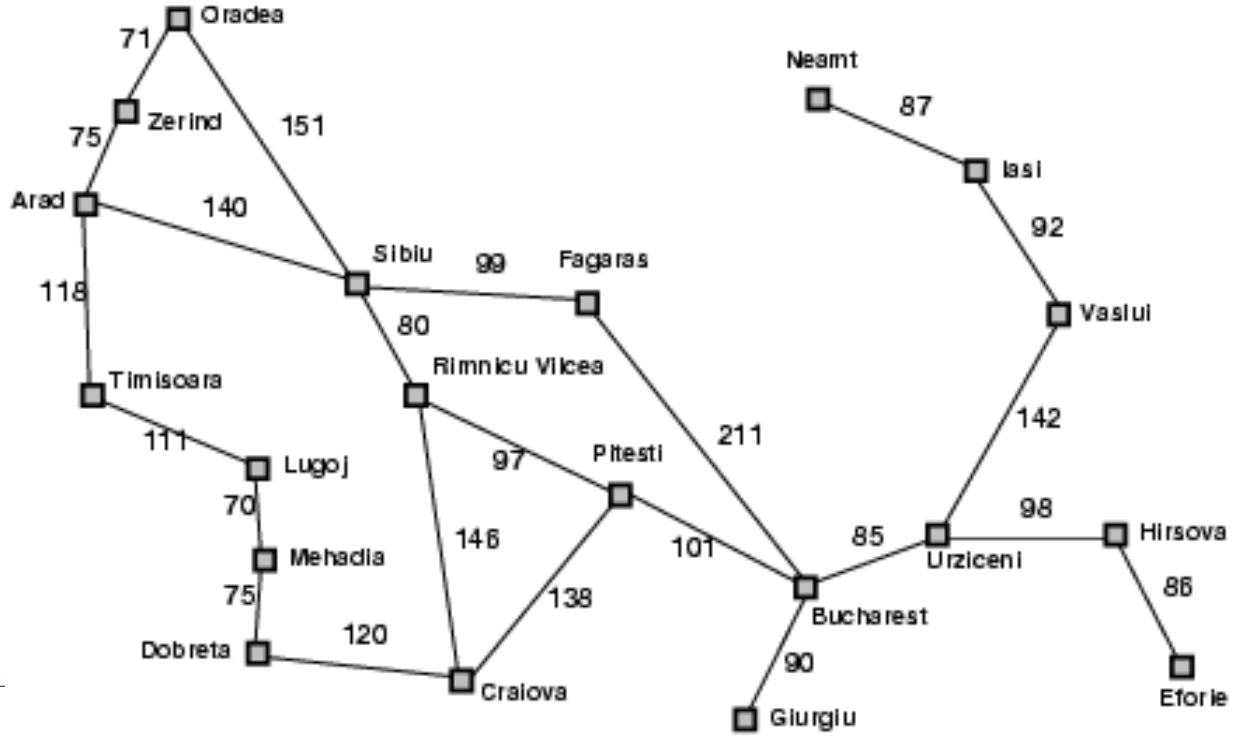

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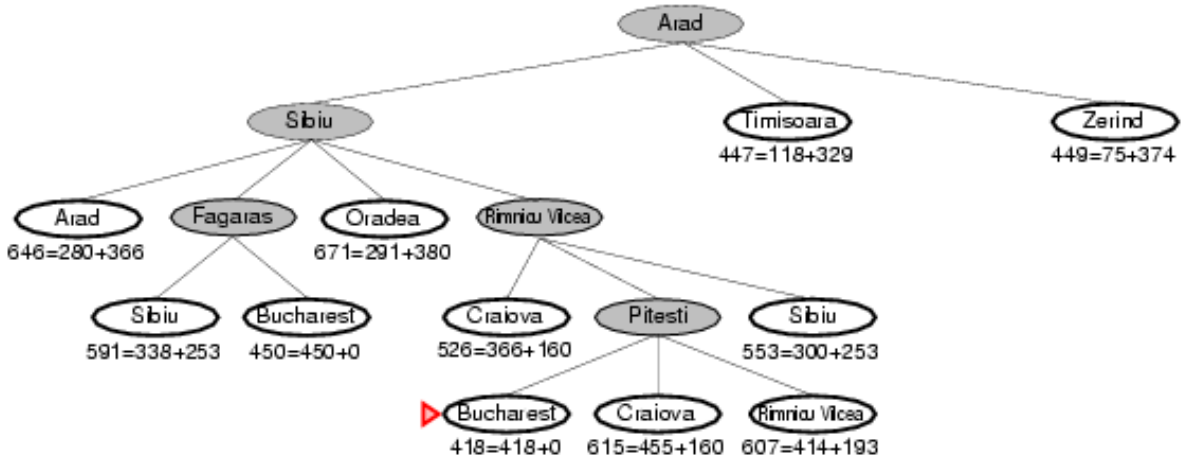

Informed search strategies

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Admissible heuristics

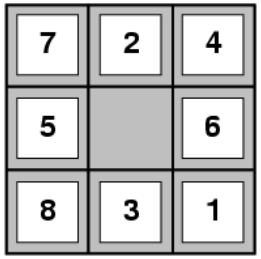
- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If $h(n)$ is admissible, A* using `TREE-SEARCH` is optimal

Properties of A*

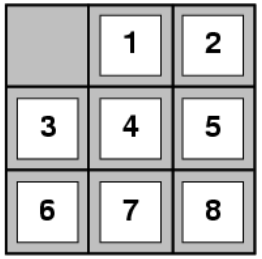
- Complete? Yes (unless there are infinitely many nodes with f ≤ *f(G)*)
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

Admissible heuristics for 8-puzzle

- Heuristic: produce a solution in a reasonable time frame, good enough to solve the problem
- The average cost for the 8-puzzle are approx. 22 steps. Here are 26 steps.
- Branching factor is approx. 3
  - Empty in the middle, 4 mov
  - Empty in the corner, 2 mov
  - Rest cases, 3 mov



Start State



Goal State

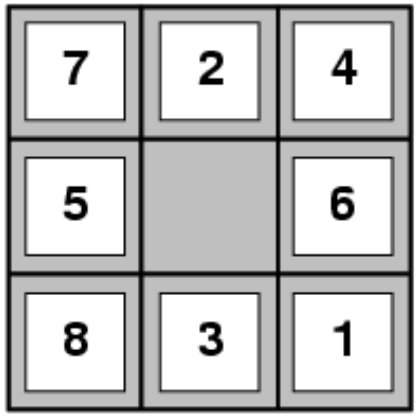
- Depth-first search will look 3^{22} states
- If we keep track of repeated states, we could reduce to 170.000
- In the15-puzle = 10^{13}

Admissible heuristics for 8-puzzle

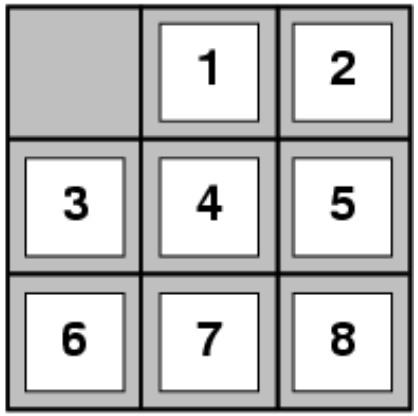
E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., + horizontal and vertical distance from desired location of each tile)



Start State



Goal State

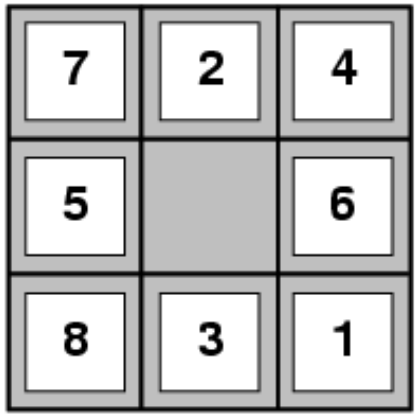
- $h_1(S) = ?$
- $h_2(S) = ?$

Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



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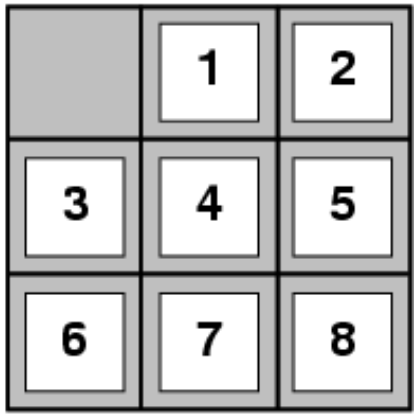

Goal State

- $h_1(S) = ?$ 8
- $h_2(S) = ?$ 3+1+2+2+2+3+3+2 = 18

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible) then h_2 dominates h_1
- h_2 is better for search

- Typical search costs (average number of nodes expanded):

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

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Local search algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution
- State space = set of "complete" configurations. Find configuration satisfying constraints, e.g., n-queens
- In such cases, we can use local search algorithms
- Keep a single "current" state, try to improve it
- Work with one current state and generally moves to the neighboring state
- The paths followed by the search are not retained
  - They use little memory
  - You can find reasonable solutions in large state spaces or infinite

Example: *n*-queens

- Put *n* queens on an $n \times n$ board with no two queens on the same row, column, or diagonal

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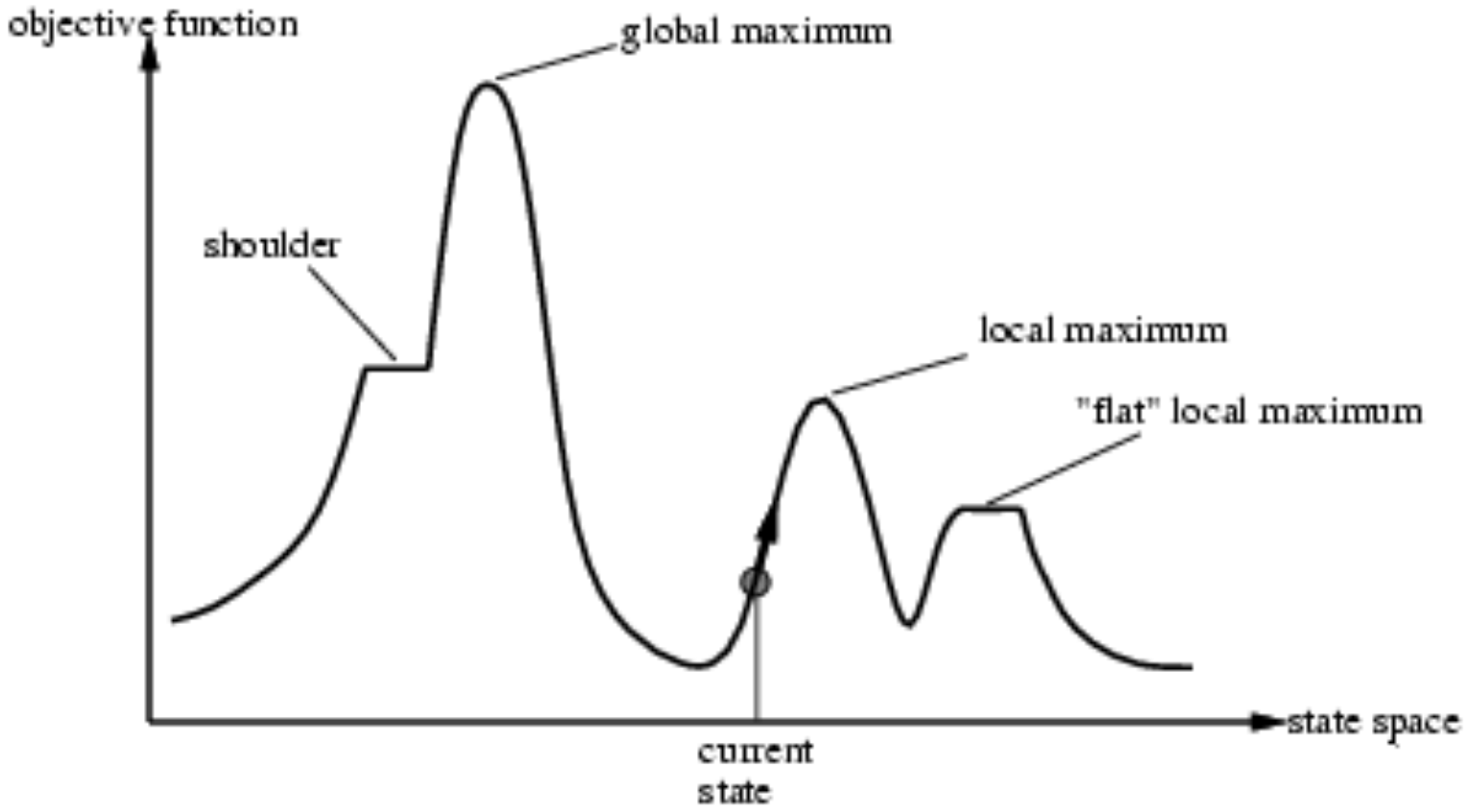
Hill-climbing search

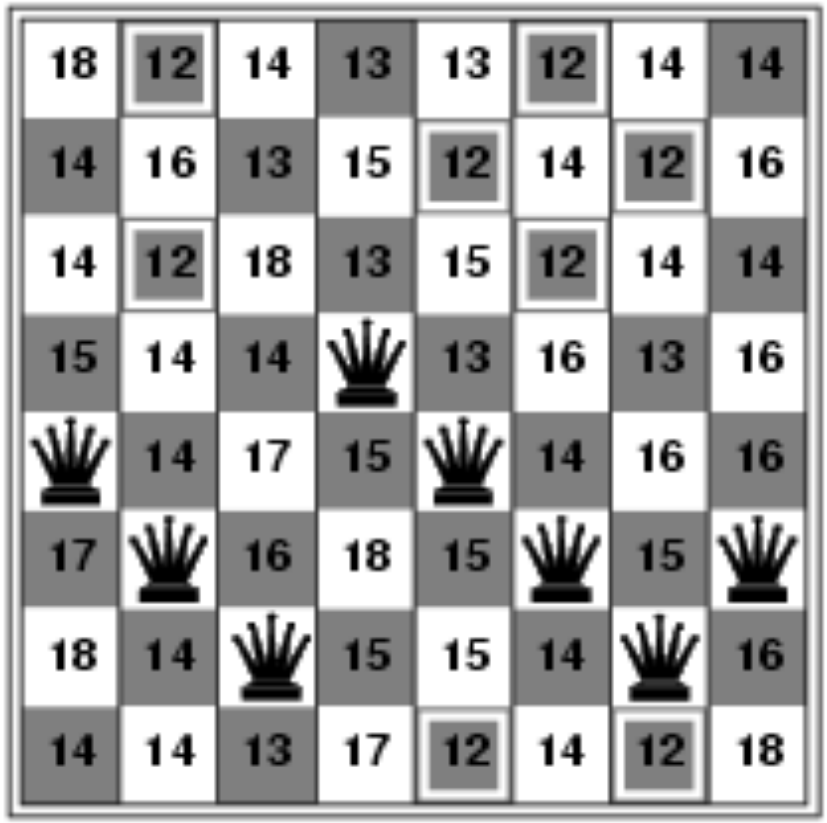
- It's just a loop that moves in the direction of increasing value
  - Ends when it reaches a peak where no neighbor has a higher value
  - The search tree is not kept, just a data structure of the current node to check the goal condition and its objective function value
- "Like climbing Everest in thick fog with amnesia"

Hill-climbing search

- Problem: depending on initial state, can get stuck in local maxima

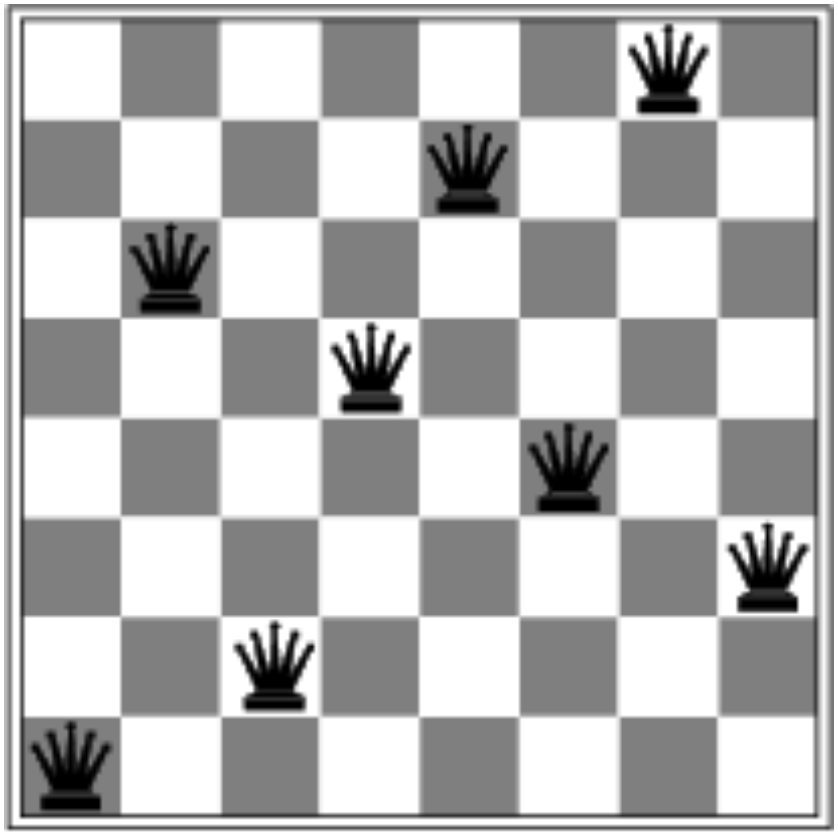


Hill-climbing search: 8-queens problem



- *h* = number of pairs of queens that are attacking each other, either directly or indirectly
- *h = 17* for the above state
- The figure also shows the values of all successors, top successors have h = 12

Hill-climbing search: 8-queens problem



- A local minimum with h = 1 (obtained in 5 steps)

Hill-climbing search

- The algorithm gets stuck for several reasons::
  - Local Maximum: it is a peak that is higher than each of its neighbours, but lower than the maximum overall
  - Ridges: cause a sequence of local maxima that make navigation difficult
  - Plateau (flat): can lead to a local maximum where there is no ascendant exit or a terrace to advance
- In the 8-queens, it gets stack in 86% and solve 14% cases
- If we allow lateral movements with the hope that we find a terrace (limiting them if reach a local maximum, e.g.100):→ 94% success
- Variants:
  - Stochastically (randomly chooses upward movements)
  - Random restart (the initial states are generated randomly)

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Simulated annealing search

- Simulated annealing is the process of tempering or hardening metals by heating and then cooling them gradually
- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency
- It combines hill-climbing with random generation successor
- One can prove: If *T* decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1
- Widely used in VLSI layout, airline scheduling, etc

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Local beam search

- Idea: Keep track of *k* states rather than just one
  - Start with *k* randomly generated states
  - At each iteration, all the successors of all *k* states are generated
  - If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat
  - Alternatively stochastic LBS randomly choose k successors, with the probability of choosing a successor as an increasing function of its value

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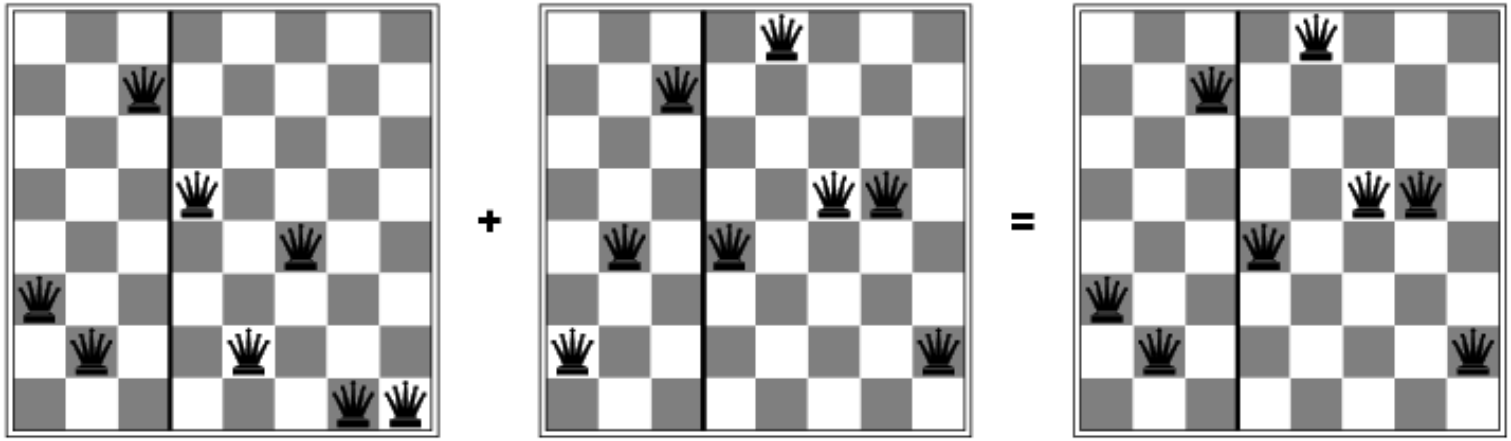
Genetic algorithms

- A successor state is generated by combining two parent states
- Start with *k* randomly generated states (population)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (fitness function). Higher values for better states
- Produce the next generation of states by selection, crossover, and mutation

Genetic algorithms

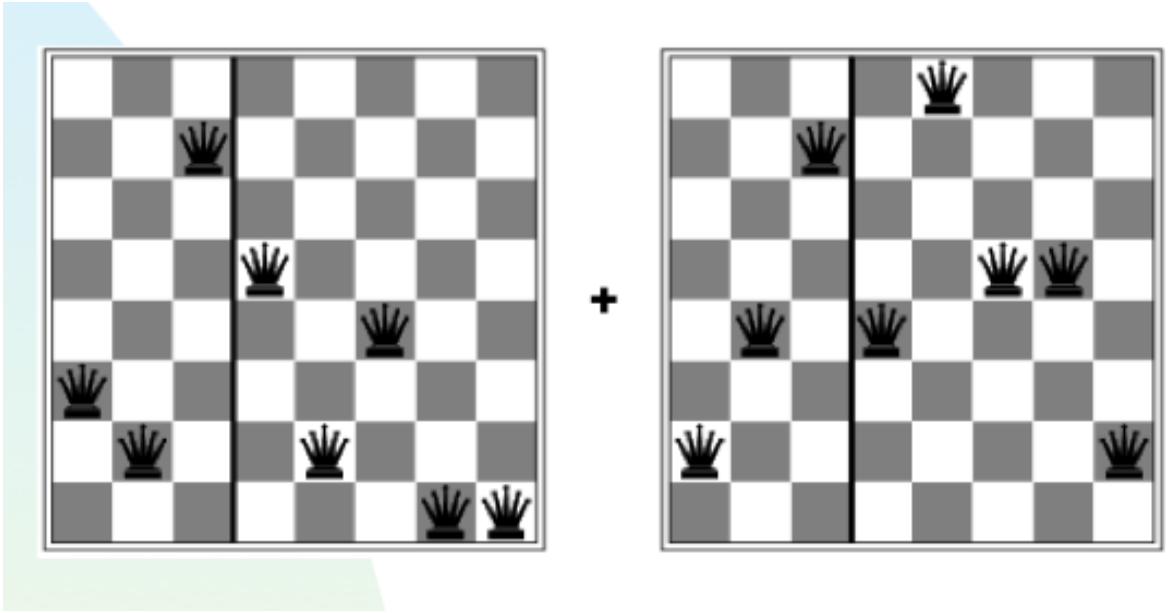
- The initial population is represented by chains of 8 digits representing states of 8 queens

Genetic algorithms



- It corresponds to the 2 parents of figure (c)
  327 52411 (5 attacked queens)
  247 48552 (4 attacked queens)

Genetic algorithms



- Fitness function: number of NON-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)
- Right: 247 48552 (4 attacked queens) = 28 – 4 = 24

  24/(24+23+20+11) = 31%
- Left: 327 52411 (5 attacked queens) = 28 – 5 = 23

  23/(24+23+20+11) = 29%

Genetic algorithms

probabilities

- Crossing points are chosen and d) the offspring are created crossing the chain parental crosspoint
- In e) each position is subject to random mutation with a small probability