ICE503 DSP-Homework#13

1. We know that any rational system can be expressed as

$$H(z) = H_{min}(z)H_{ap}(z)$$

where $H_{min}(z)$ is a minimum-phase system and $H_{ap}(z)$ is an all-pass system. Now, for each of the following system, please specify the $H_{min}(z)$, $H_{ap}(z)$ and make sure $|H(z)| = |H_{min}(z)|$.

(a) $H(z) = \frac{(1-3z^{-1})(1-\frac{1}{4}z^{-1})}{(1-\frac{3}{4}z^{-1})(1-\frac{4}{3}z^{-1})}$

(b) $H(z) = \frac{(1-2z^{-1})}{(1+\frac{1}{3}z^{-1})}$

2. Consider the causal LTI system with the system function

$$H(z) = \frac{D - Mz^{-1}}{(C - Hz^{-1} + Iz^{-2})(A + Nz^{-1})}$$

where $C = 1, H = \frac{1}{2}, I = \frac{1}{3}, A = 1, N = \frac{1}{4}, D = 1, M = \frac{1}{5}$

(a) Draw the signal flow graphs for implementations of the system in each of the following.

I. Direct form I

II. Direct form II

III. Cascade form with first- and second-order sections of direct form II

IV. Parallel form with first- and second-order sections of direct form II

V. Transposed direct form I

VI. Transposed direct form II

(b) Write the difference equations for the flow graph of (a)-VI., and show that this system has the correct system function.

MATLAB simulation (Design notch filter):

A spectrogram is a visual representation of the spectrum of frequencies in a signal as they vary with time. Spectrograms can be used to identify spoken words phonetically and to analyse the various calls of animals. They are often used in the fields of music, sonar, radar, and speech processing. The idea of a spectrogram is plot a sequence of short DFTs of the input signal using overlapping windows. Let N be the DFT length, w[n] be a window function for $n = 0, 1, \cdots, N-1$ (e.g., the Hann window $w[n] = \frac{1}{2}(1 - \cos(2\pi n/N))$, and M be the number of samples to shift after each DFT. For the causal input sequence x[n], the energy in the k-th frequency bin of the i-th window is given by

$$X_i[k] = \left| \sum_{n=0}^{N-1} x[iM + n] w[n] e^{-2\pi jk/N} \right|^2$$

for $k = 0, 1, \cdots, N-1$ If x is a column vector, then $X(:, i) = abs(fft(x((1:N) + i * M).* w)).\hat{}2$ computes this expression for the i-th block and all k using one line of MATLAB. If the signal is real, then one typically plots only the positive frequencies $k = 0, 1, \cdots, \lfloor N/2 \rfloor$. For a real signal, one can plot this in MATLAB using colormaps with

```
image(X(1:floor(N/2),:));
colormap(hot(256));
colorbar;
xlabel('Time');
ylabel('Frequency Bin Number');
```

(a) Following Homework#9, we plot the spectrogram of the sound file guitar4.wav and guitar4_plusInterfer.wav with and without the interference, this week we need to design a second-order IIR notch filter to remove the interference and sketch the magnitude of the filter in dB and the group delay. (Using Matlab function **iirnotch, fvtool** and **filter**)

(b) Plot the spectrogram of the sound file guitar4_plusInterfer.wav after notch filter.

5. MATLAB simulation:

Use fdatool/filterDesigner to design filters. Show the parameter that you choose and the magnitude response the filters. Specify the structure and the order of the filters.

1. Design a highpass Butterworth filter. The normalized passband edge is at 0.8 and stopband edge is exactly at 0.7. The stopband attenuation is 40 dB
2. Design a lowpass Kaiser window filter. The normalized passband edge is exactly at 0.22 and stopband edge is at 0.29. The stopband attenuation is 40 dB

1.

(a) $H(z) = \dfrac{(1-3z^{-1})(1-\frac{1}{4}z^{-1})}{(1-\frac{3}{4}z^{-1})(1-\frac{4}{3}z^{-1})}$, zeros : $3, \frac{1}{4}$

poles : $\frac{3}{4}, \frac{4}{3}$

$$= \frac{1-\frac{1}{4}z^{-1}}{1-\frac{3}{4}z^{-1}} \; \frac{1-3z^{-1}}{1-\frac{4}{3}z^{-1}} = \underbrace{\frac{(1-\frac{1}{4}z^{-1})(3-z^{-1})}{(1-\frac{3}{4}z^{-1})(\frac{4}{3}-z^{-1})}}_{H_{min}(z)} \cdot \underbrace{\frac{(1-3z^{-1})(\frac{4}{3}-z^{-1})}{(1-\frac{4}{3}z^{-1})(3-z^{-1})}}_{H_{ap}(z)}$$

all zeros inside u.c.

zeros : $\frac{3}{4}, 3$

poles : $\frac{4}{3}, \frac{1}{3}$

reciprocal

$H_{ap}(z)$ ... poles and zeros should be conjugate reciprocal pairs

$|H_{min}(z)| = |H(z)|$ $\quad |H_{ap}(z)| = 1$

(b) $H(z) = \dfrac{1-2z^{-1}}{1+\frac{1}{3}z^{-1}} = \underbrace{\dfrac{2-z^{-1}}{1+\frac{1}{3}z^{-1}}}_{H_{min}(z)} \; \underbrace{\dfrac{1-2z^{-1}}{2-z^{-1}}}_{H_{ap}(z)}$

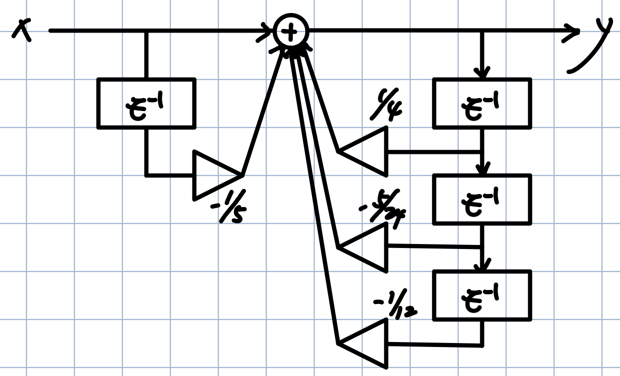
zero : 2

pole : $-\frac{1}{3}$

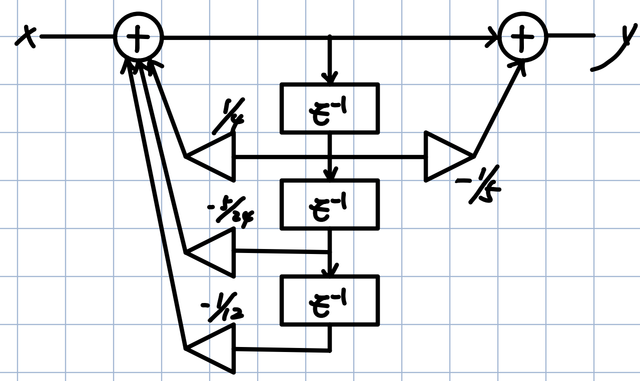
2. (a)

$$H(z) = \frac{1-\frac{1}{5}z^{-1}}{\left(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}\right)\left(1+\frac{1}{4}z^{-1}\right)} = \frac{1-\frac{1}{5}z^{-1}}{1-\frac{1}{4}z^{-1}+\frac{5}{24}z^{-2}+\frac{1}{12}z^{-3}}$$

I. Direct form I

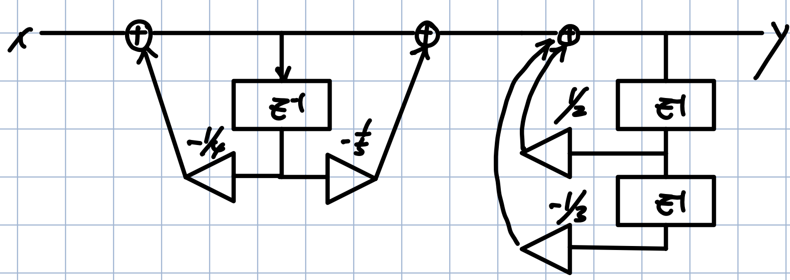


II. Direct form II

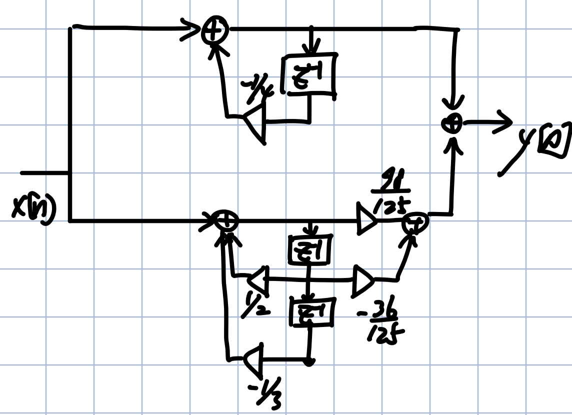


III :

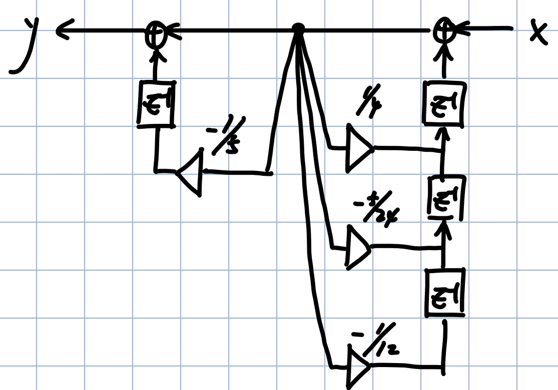
$$H(z) = \underbrace{\frac{1-\frac{1}{5}z^{-1}}{1+\frac{1}{4}z^{-1}}}_{\text{order}=1} \cdot \underbrace{\frac{1}{1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}}}_{\text{order}=2} = \frac{\frac{27}{125}}{1+\frac{1}{4}z^{-1}} + \frac{\frac{98}{125}-\frac{36}{125}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}}$$



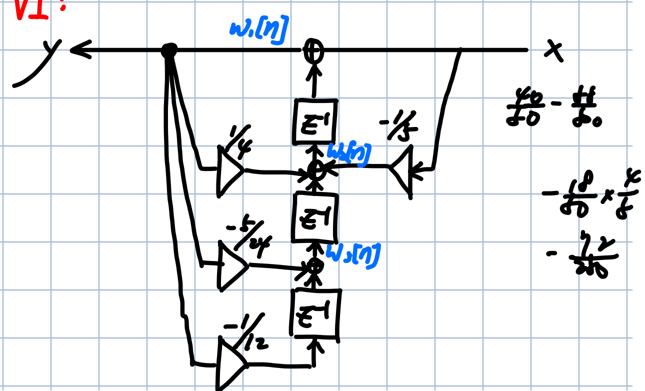
IV :



V :



VI :



(b)

$$\begin{cases} y[n] = w_1[n] \\ w_1[n] = w_2[n-1] + x[n] \\ w_2[n] = \frac{1}{4}y[n] + w_3[n-1] - \frac{1}{5}x[n] \\ w_3[n] = -\frac{4}{25}y[n] - \frac{1}{12}y[n-1] \end{cases}$$

$= w_2[n-1] + x[n] = \frac{1}{4}y[n-1] + w_3[n-2] - \frac{1}{5}x[n-1] + x[n]$

$= \frac{1}{4}y[n-1] - \frac{4}{25}y[n-2] - \frac{1}{12}y[n-3] + x[n] - \frac{1}{5}x[n-1]$

$\Rightarrow y[n] - \frac{1}{4}y[n-1] + \frac{4}{25}y[n-2] + \frac{1}{12}y[n-3] = x[n] - \frac{1}{5}x[n-1]$

$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1-\frac{1}{5}z^{-1}}{1-\frac{1}{4}z^{-1}+\frac{4}{25}z^{-2}+\frac{1}{12}z^{-3}}$ ∴ same