ICE503 DSP-Homework#7

1. Consider the system given in Figure 1. You may assume that $R_c(j\Omega)$ is bandlimited, i.e., $R_c(j\Omega) = 0, |\Omega| \geq 2\pi(1000)$, as shown in Figure 2.

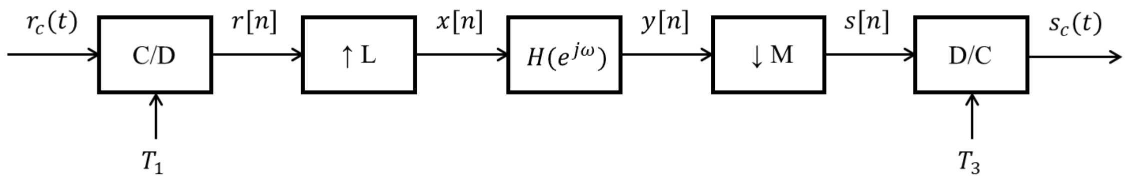


Figure 1. The system.

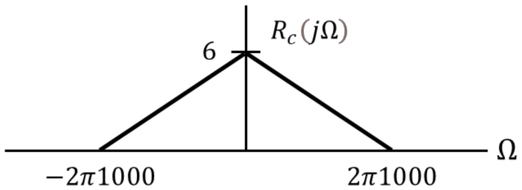


Figure 2. Fourier transform of $r_c(t)$

(a) Consider $L = 3$, $T_1 = \frac{1}{2000}$ second, and the lowpass filter

$$H(e^{j\omega}) = \begin{cases} 1, & 0 \leq \omega \leq \omega_0 \\ 0, & \omega_0 < \omega \leq \pi \end{cases}$$

Sketch $R(e^{j\omega})$ and $X(e^{j\omega})$.

(b) Consider $M = 2$. Choose nonzero values for ω_0 and T_2 such that

$$y[n] = \alpha r_c(nT_2)$$

for some nonzero constant α. (You do not have to determine the value of α.)

(c) Using the value of ω_0 you obtained in (b), determine a choice for T_3 such that

$$s_c(t) = \beta r_c(t)$$

for some nonzero constant β. (You do not have to determine the value of β.)

(d) Consider $M = 3$, if the output signal $s_c(t) = r_c(t - \frac{1}{1000})$, and the value of T_3

is $\frac{1}{2000}$, is it possible to obtain $s[n]$ from $r[n]$? If so, how $H(e^{j\omega})$ should be?

2. Compute the discrete Fourier transform (DFT) of each of the following finite-length sequences considered to be of length N (where N is even):

(e) $x[n] = \begin{cases} 1, & n \text{ even}, \quad 0 \le n \le N-1 \\ 0, & n \text{ odd}, \quad 0 \le n \le N-1 \end{cases}$

(f) $x[n] = \begin{cases} 1, & 0 \le n \le N/2-1 \\ 0, & N/2 \le n \le N-1 \end{cases}$

(g) $x[n] = \begin{cases} \alpha^n, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$

3. MATLAB simulation:

Generate a mixed wave with two sine waves

$$x(t) = \sin(2\pi 20t) + \sin(2\pi 100t)$$

Then, sample the mixed wave $x(t)$ with 1000Hz to obtain $x[n]$.

(a) Downsampling $x[n]$ 5 times with decimate function and downsample function, plot each of the result with $x[n]$.

(b) Downsampling $x[n]$ 8 times with decimate function and downsample function, plot each of the result with $x[n]$.

4. MATLAB simulation:

Download demodata_L4.zip from cyber university (網路大學), you can find EMG_ICA.mat in a file called EMG. After loading EMG_ICA.mat, you will see 8 channels, here we only use the first channel to be the signal.

```
load('EMG_ICA.mat');
org_signal = fdata(:,1);
```

The signal is already sampled with 2000Hz.

(a) First, use rat function and resample function to upsample the signal with 5000Hz. Then, downsample the signal with 2000Hz. Plot the result and the original signal together. What happened to the original signal after upsample then downsample?

(b) First, use rat function and resample function to downsample the signal with 300Hz. Then, upsample the signal with 2000Hz. Plot the result and the original signal together. What happened to the original signal after downsample then upsample?

5. MATLAB simulation:

Generate a cosine wave for 1 second

$x(t) = \cos(2\pi 5t)$.

Then, sample the cosine wave $x(t)$ with 100Hz to obtain $x[n]$.

(a) Compute the DFT of $x[n]$ with DFT matrix to obtain $X[k]$.

(b) Compute the IDFT of $X[k]$ with DFT matrix to obtain $x[n]$.

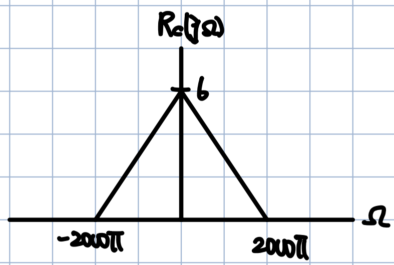
(c) Compute the DFT of $x[n]$ with fft function to obtain $X[k]$.

(d) Compute the IDFT of $X[k]$ with ifft function to obtain $x[n]$.

(e) Use stem function to plot the amplitude of $X[k]$ and $x[n]$ for (a) ~ (d).

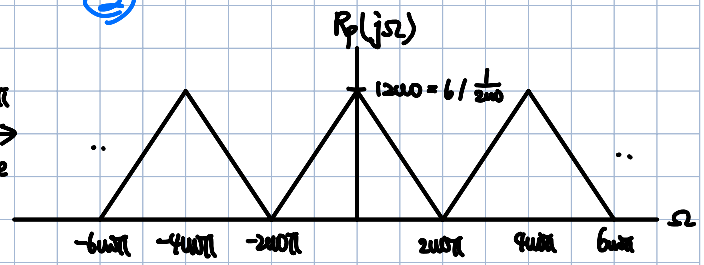
1. (a) ~ (c)

1



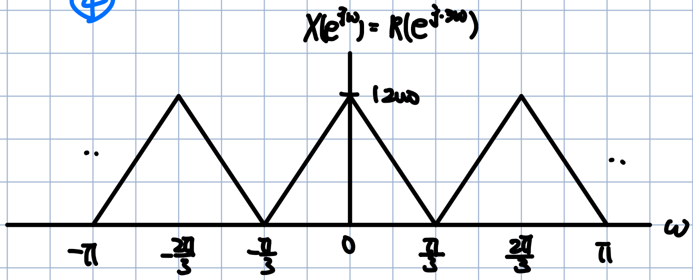
$\frac{2\pi}{T_1} = 4000\pi$ → Sampling rate

2



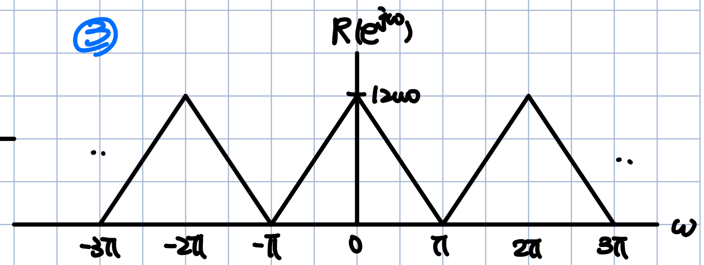
↓ $\Omega T_1 = \omega \Rightarrow \omega = \frac{\Omega}{2000}$

4



← ↑3

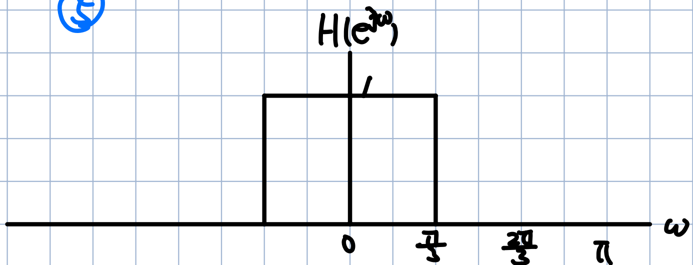
3



(a) ※

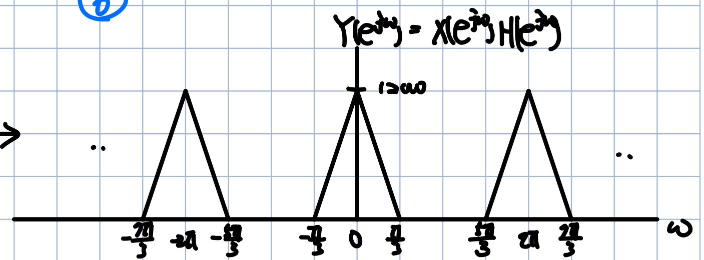
↕

5



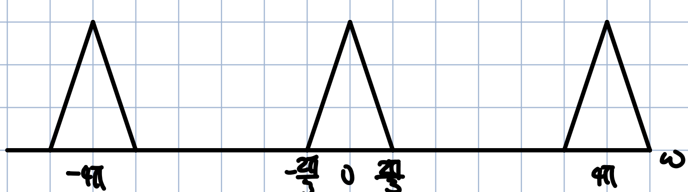
4 × 5 →

6

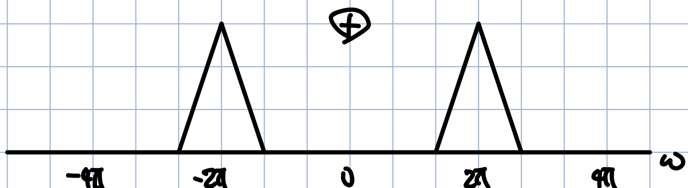


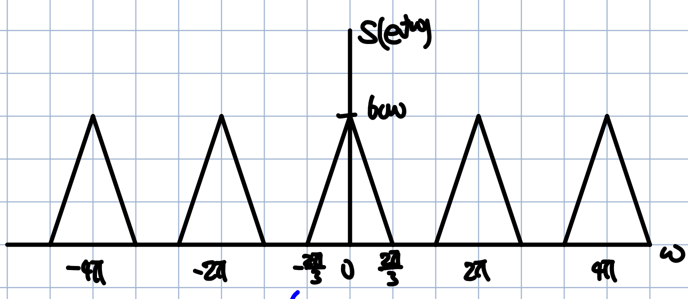
select $\omega_0 = \min\left(\frac{\pi}{L}, \frac{\pi}{M}\right) = \frac{\pi}{3}$ ※, 將 $y[n]$ 還原成 $r_c(t)$, $\frac{\pi}{3}$ 要變回 $2000\pi \Rightarrow \Omega T_2 = \omega \Rightarrow 2000\pi T_2 = \frac{\pi}{3} \Rightarrow T_2 = \frac{1}{6000}$ ※

7 ↓2 $X_d(e^{j\omega}) = \frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j\cdot\frac{\omega-2\pi i}{M}}\right)$



⊕

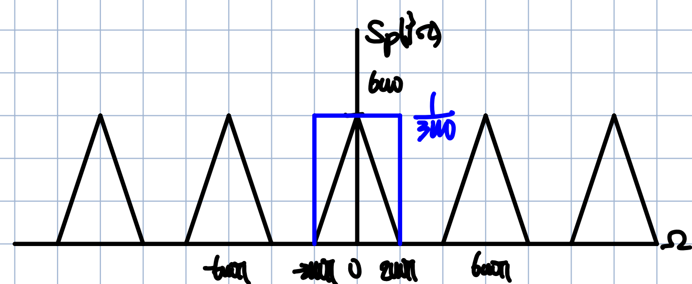


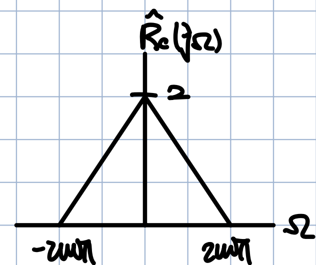


↓ 要將 $\frac{2\pi}{3}$ 還原成 2000π

$\Rightarrow f \times 3000 \Leftrightarrow T_3 = \frac{1}{3000}$ ※

$\Omega = \frac{\omega}{T_3}$





(d)

$$R(e^{j\omega}) = \frac{1}{T_1}\cdot R_c(j\Omega)\Big|_{\Omega=\frac{\omega}{T_1}} = 200\cdot R_c(j\cdot 200\omega)$$

$$S_c(t) = r_c\left(t-\frac{1}{100}\right) \Rightarrow S_c(j\Omega) = R_c(j\Omega)\cdot e^{-j\Omega\cdot\frac{1}{100}}$$

$$S_c(e^{j\omega}) = \frac{1}{T_3} S_c(j\Omega)\Big|_{\Omega T_3=\omega} = \frac{1}{T_3} R_c(j\Omega)\cdot e^{-j\frac{\Omega}{100}}\Big|_{\Omega=\frac{\omega}{T_3}} = 200\cdot R_c(j200\omega)e^{-j2\omega} = R_c(e^{j\omega})\cdot e^{-j2\omega}$$

$$r[n] \to \boxed{\uparrow L\ (3)} \to x[n] \to \boxed{H(e^{j\omega})} \to y[n] \to \boxed{\downarrow M\ (3)} \to s[n]$$

$$X(e^{j\omega}) = R(e^{j\omega L}) = R(e^{j3\omega})$$

$$Y(e^{j\omega}) = \begin{cases} 3S(e^{j3\omega}), & |\omega|<\frac{\pi}{3} \\ 0, & \text{otherwise} \end{cases} = \begin{cases} 3R(e^{j3\omega})e^{-j6\omega}, & |\omega|<\frac{\pi}{3} \\ 0, & \text{otherwise} \end{cases}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{3\cancel{R(e^{j3\omega})}e^{-j6\omega}}{\cancel{R(e^{j3\omega})}} = 3e^{-j6\omega},\ |\omega|<\frac{\pi}{3}$$ ※

2.

(a)

$$x[n] = \begin{cases} 1, & n \text{ even},\ 0\le n\le N-1 \\ 0, & n \text{ odd},\ 0\le n\le N-1 \end{cases}$$

$$X[k] = X(e^{j\omega})\Big|_{\omega=\frac{2\pi k}{N}} = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi k}{N}n} = e^{-j\frac{2\pi k}{N}\cdot 0} + e^{-j\frac{2\pi k}{N}\cdot 2} + \cdots + e^{-j\frac{2\pi k}{N}\cdot(N-2)}$$

$$= \frac{1\cdot\left(1-e^{-j\frac{2\pi k}{N}\cdot 2\cdot\frac{N}{2}}\right)}{1-e^{-j\frac{2\pi k}{N}\cdot 2}} = \frac{1-e^{-j2\pi k}}{1-e^{-j\frac{4\pi}{N}\cdot k}} \Rightarrow \text{when}$$

① $k=0$, $\frac{1-1}{1-1} = \frac{0}{0}$

② $k=\frac{N}{2}$, $\frac{1-e^{-j\pi N}}{1-e^{-j2\pi N}}$ (N is even) $= \frac{0}{0}$

$$\lim_{k\to 0,\frac{N}{2}} \frac{1-e^{-j2\pi k}}{1-e^{-j\frac{4\pi}{N}\cdot k}} = \lim_{k\to 0,\frac{N}{2}} \frac{j2\pi e^{-j2\pi k}}{j\frac{4\pi}{N}e^{-j\frac{4\pi}{N}\cdot k}} = \frac{N}{2} \Rightarrow X[k] = \begin{cases} \frac{N}{2}, & k=0,\frac{N}{2} \\ 0, & \text{otherwise} \end{cases}$$ ※

(b)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{N}\cdot n} = e^{-j\frac{2\pi k}{N}\cdot 0} + \cdots + e^{-j\frac{2\pi k}{N}\cdot(\frac{N}{2}-1)} = \frac{1-e^{-j\frac{2\pi k}{N}\cdot\frac{N}{2}}}{1-e^{-j\frac{2\pi k}{N}}} = \frac{1-e^{-j\pi k}}{1-e^{-j\frac{2\pi}{N}k}}$$

when ① : k is odd $\rightarrow \dfrac{2}{1-e^{-j\frac{2\pi}{N}k}}$

② : even $\rightarrow 0$

③ : $0 \rightarrow \dfrac{0}{0} \rightarrow \displaystyle\lim_{k\to 0} \frac{1-e^{-j\pi k}}{1-e^{-j\frac{2\pi}{N}k}} = \lim_{k\to 0} \frac{j\pi e^{-j\pi k}}{j\frac{2\pi}{N} e^{-j\frac{2\pi}{N}k}} = \frac{N}{2} \frac{e^{-j\pi 0}}{e^{-j\frac{2\pi}{N}0}} = \frac{N}{2}$

$$X[k] = \begin{cases} \dfrac{N}{2} & , k=0 \\ \dfrac{2}{1-e^{-j\frac{2\pi}{N}k}} & , k \text{ is odd} \\ 0 & , k \text{ is even} \end{cases}$$

(c)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{N}\cdot n} = \sum_{n=0}^{N-1} \alpha^n e^{-j\frac{2\pi k}{N}\cdot n} = \frac{1-\alpha^n e^{-j\frac{2\pi k}{N}N}}{1-\alpha e^{-j\frac{2\pi k}{N}}} = \frac{1-\alpha^n}{1-\alpha e^{-j\frac{2\pi}{N}k}}$$