ICE503 DSP-Homework#14

1. Draw the direct form I and linear-phase form for the FIR filters
   (a) $H(z) = 2 + z^{-1} + 2z^{-2}$
   (b) $H(z) = 3 - z^{-1} + 2z^{-2} + 2z^{-3} - z^{-4} + 3z^{-5}$

2. To design a FIR filter $h_t[n]$, we can use a rectangular window $w_R[n]$ to window an ideal filter $h_d[n]$. Truncation of $h_d[n]$ to $2M+1$ points is multiplication with a rectangular window.

$$h_t[n] = h_d[n] \cdot w_R[n]$$

$$w_R[n] = \begin{cases} 1 \ , -M \le n \le M \\ 0 \ , \quad \text{otherwise} \end{cases}$$

   Given $M = 7$.

   (a) Calculate $W_R(e^{j\omega})$.

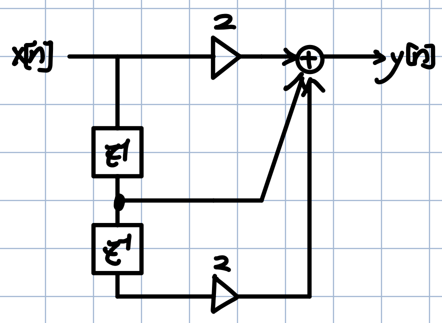
   (b) Plot $|W_R(e^{j\omega})|$, and indicate the magnitude of the mainlobe.

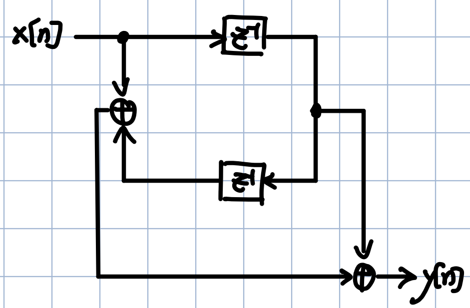
   (c) Indicate the frequency of the mainlobe and sidelobe.

3. Plot the flow graph of a 16 point decimation in time FFT structure, and determine the number of complex multiplications.

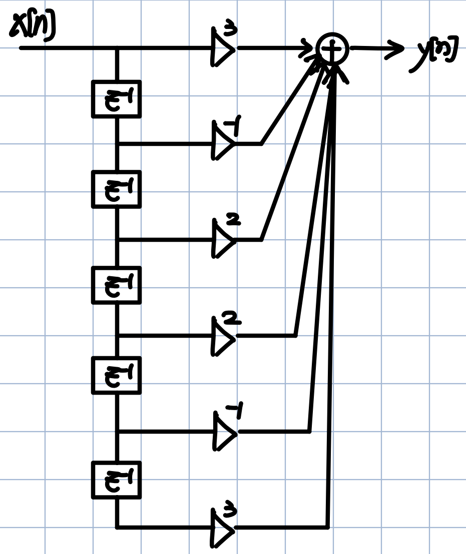
1.

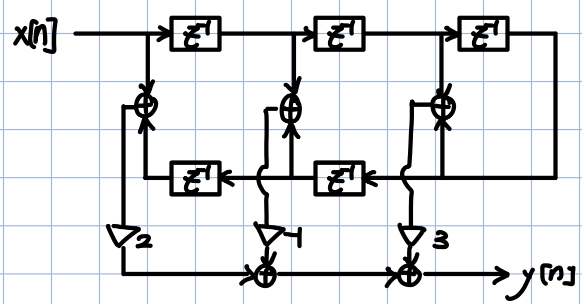
(a) $H(z) = 2 + z^{-1} + 2z^{-2} = 2(1+z^{-2}) + z^{-1}$





(b) $H(z) = 3 - z^{-1} + 2z^{-2} + 2z^{-3} - z^{-4} + 3z^{-5} = 3(1+z^{-5}) - (z^{-1}+z^{-4}) + 2(z^{-2}+z^{-3})$





2.

(a)

$$w_R[n] = \begin{cases} 1, & -M \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

$$W_R(e^{j\omega}) = \sum_{n=-\infty}^{\infty} w_R[n] e^{-j\omega n} = \sum_{n=-M}^{M} 1 \cdot e^{-j\omega n} = \frac{e^{j\omega M}(1-e^{-j\omega(2M+1)})}{1-e^{-j\omega}}$$

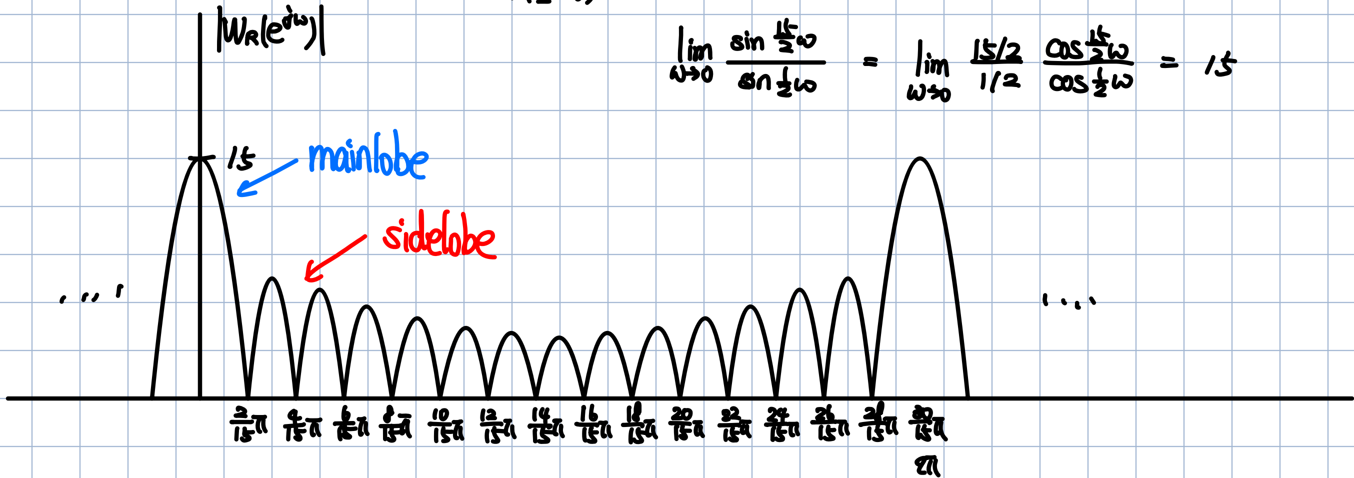
$$= \frac{e^{j\omega M} e^{-j\omega\frac{2M+1}{2}}\left(e^{j\omega\frac{2M+1}{2}} - e^{-j\omega\frac{2M+1}{2}}\right)}{e^{-j\omega\frac{1}{2}}\left(e^{j\omega\frac{1}{2}} - e^{-j\omega\frac{1}{2}}\right)} = e^{j\omega\left(M-\frac{2M+1}{2}+\frac{1}{2}\right)} \frac{2j\sin\left(\frac{2M+1}{2}\omega\right)}{2j\sin\left(\frac{1}{2}\omega\right)}$$

$$= \frac{\sin\left((2M+1)\frac{\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$ ∴ periodic sinc

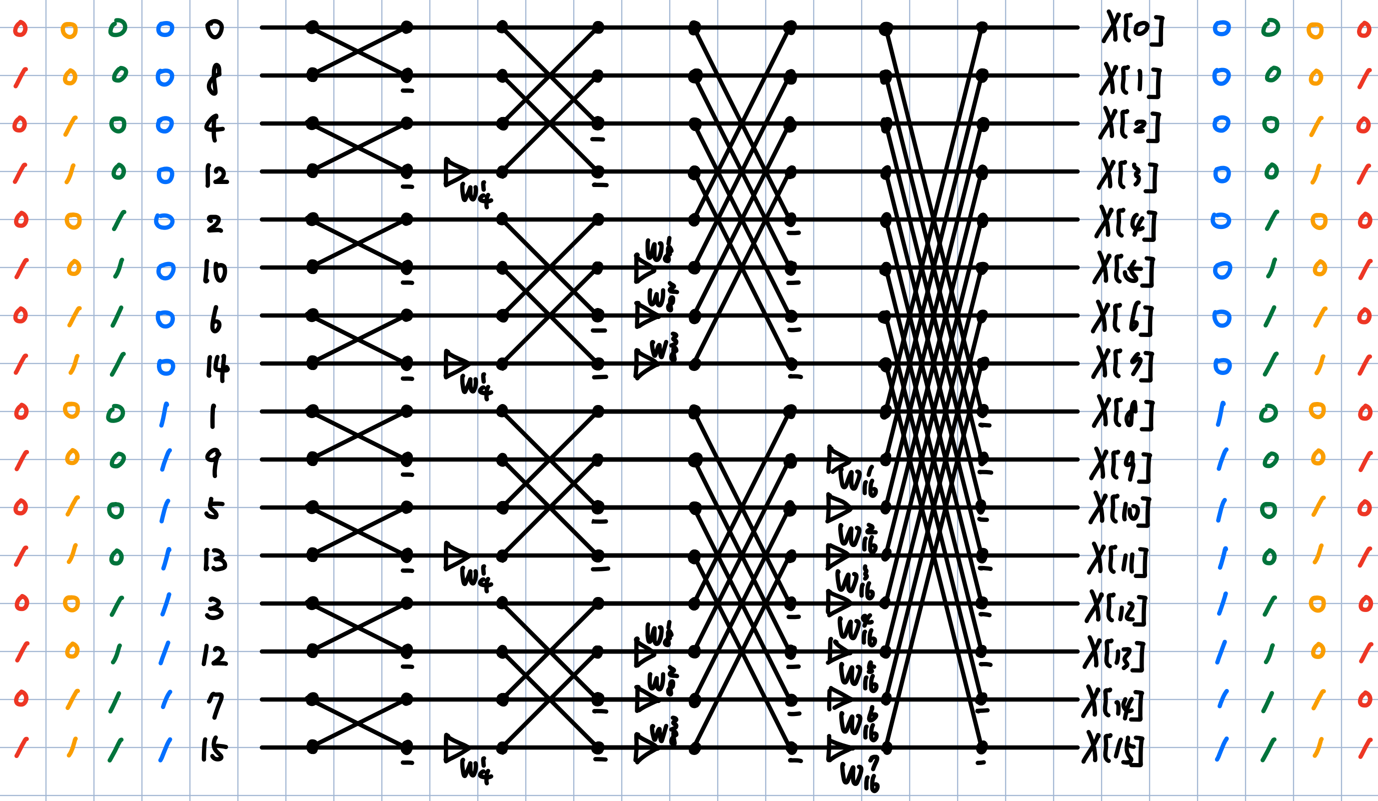
(b)

$M = 7 \Rightarrow \dfrac{\sin\left(\frac{15}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} = 0$ when $\frac{15}{2}\omega = k\pi \Rightarrow \omega = k\frac{2\pi}{15}$

$$\lim_{\omega\to 0} \frac{\sin\frac{15}{2}\omega}{\sin\frac{1}{2}\omega} = \lim_{\omega\to 0} \frac{15/2}{1/2} \frac{\cos\frac{15}{2}\omega}{\cos\frac{1}{2}\omega} = 15$$



3. 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15



# of complex mul. = 17