ICE503 DSP-Homework#5

1. The block diagram of a causal LTE system is shown in Figure 1.

   (a) Write the rational polynomials of the system $H(z)$.

   (b) Use partial fraction to find the LCCDE $h[n]$ of this system.

   (c) Write the difference equation that characterizes the system with $x[n]$ and $y[n]$.

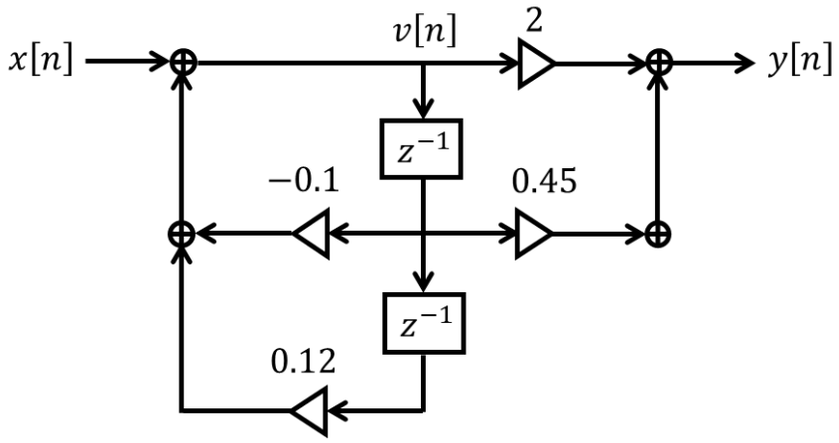


Figure 1: The block diagram of an LTE system

2. When the input to an LTE system is

$$x[n] = \left(\frac{1}{3}\right)^n \mu[n] + 3^n \mu[-n-1]$$

   the output is

$$y[n] = 5\left(\frac{1}{3}\right)^n \mu[n] - 5\left(\frac{3}{4}\right)^n \mu[n]$$

   (a) Find the ZT of $x[n]$ and $y[n]$, and indicate their ROC.

   (b) Find the rational polynomials of the system $H(z)$, plot the pole(s) and zeros(s) and indicate the ROC.

   (c) Determine whether the system is causal and stable.

   (d) Find the impulse response $h[n]$ of the system.

   (e) Write the difference equation that characterizes the system.

3. MATLAB simulation:

   In the question 2(b), you might have sketched the pole(s) and zeros(s) of the system $H(z)$, and you might have found the difference equation in the question 2(e)

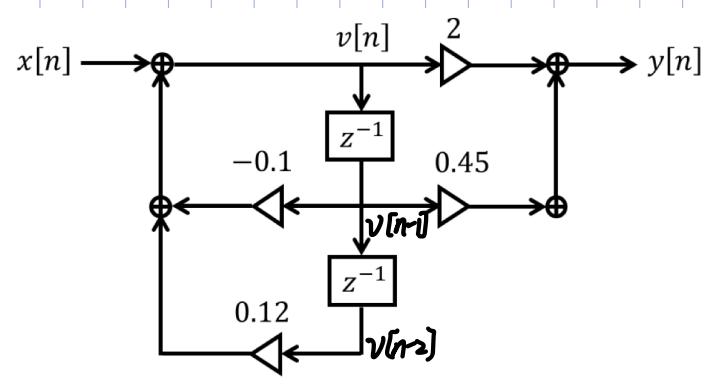
   (a) Use zplane function to plot the pole(s) and zeros(s) of the system $H(z)$

   (The result should be the same as your answer in question 2(b).)

   (b) Use freqz function to calculate a 100-point frequency response vector and the corresponding angular frequency vector, then plot the magnitude and phase response of the frequency response vector.

1.

(a)



$v[n] = x[n] - 0.1\, v[n-1] + 0.12\, v[n-2]$ — ①

$y[n] = 2v[n] + 0.45\, v[n-1]$ — ②

From ①: $x[n] = v[n] + 0.1\, v[n-1] - 0.12\, v[n-2]$

$X(z) = V(z) + 0.1 z^{-1} V(z) - 0.12 z^{-2} V(z)$

②: $y[n] = 2v[n] + 0.45\, v[n-1]$

$Y(z) = 2V(z) + 0.45 z^{-1} V(z)$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{Y(z)/V(z)}{X(z)/V(z)} = \frac{2+0.45z^{-1}}{1+0.1z^{-1}-0.12z^{-2}} \quad \text{※}$$

(b) $$\frac{2+0.45z^{-1}}{1+0.1z^{-1}-0.12z^{-2}} = \frac{2+0.45z^{-1}}{(1+0.4z^{-1})(1-0.3z^{-1})} = \frac{A}{1+0.4z^{-1}} + \frac{B}{1-0.3z^{-1}}$$

$\Rightarrow 2+0.45z^{-1} = A(1-0.3z^{-1}) + B(1+0.4z^{-1})$

① Let $z = 0.3$ $\Rightarrow$ $2+0.45\cdot 0.3^{-1} = B(1+0.4\cdot 0.3^{-1}) \Rightarrow \frac{7}{2} = B\cdot\frac{7}{3} \Rightarrow B = \frac{3}{2} = 1.5$

② Let $z = -0.4$ $\Rightarrow$ $2+0.45\cdot(-0.4)^{-1} = A(1-0.3\cdot(-0.4)^{-1}) \Rightarrow \frac{7}{8} = A\cdot\frac{7}{4} \Rightarrow A = \frac{4}{8} = 0.5$

$$\Rightarrow H(z) = \frac{0.5}{1+0.4z^{-1}} + \frac{1.5}{1-0.3z^{-1}} \Rightarrow h[n] = 0.5\cdot(-0.4)^n u[n] + 1.5\cdot 0.3^n u[n] \quad \text{※}$$

(c) $$\frac{Y(z)}{X(z)} = \frac{2+0.45z^{-1}}{1+0.1z^{-1}-0.12z^{-2}} \Rightarrow Y(z)(1+0.1z^{-1}-0.12z^{-2}) = X(z)(2+0.45z^{-1})$$

$$\Rightarrow Y(z) + 0.1z^{-1}Y(z) - 0.12z^{-2}Y(z) = 2X(z) + 0.45z^{-1}X(z)$$

$$\Rightarrow y[n] + 0.1\, y[n-1] - 0.12\, y[n-2] = 2x[n] + 0.45\, x[n-1] \quad \text{※}$$

2.

(a) $\rightarrow$ ※ $-\lambda^n u[-n-1] \leftrightarrow \frac{1}{1-\lambda z^{-1}}$

$$x[n] = \underbrace{\left(\frac{1}{3}\right)^n u[n]}_{RS} + \underbrace{3^n u[-n-1]}_{LS} \Rightarrow |z| > \frac{1}{3} \wedge |z| < 3 \Rightarrow \frac{1}{3} < |z| < 3$$

$$y[n] = \underbrace{5\cdot\left(\frac{1}{3}\right)^n u[n]}_{RS} - \underbrace{5\left(\frac{3}{4}\right)^n u[n]}_{RS} \Rightarrow |z| > \frac{1}{3} \cdot |z| > \frac{3}{4} \Rightarrow |z| > \frac{3}{4}$$

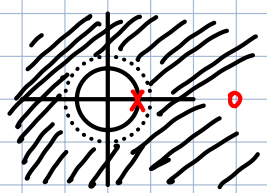
(b)

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-3z^{-1}}, \quad Y(z) = \frac{5}{1-\frac{1}{3}z^{-1}} - \frac{5}{1-\frac{3}{4}z^{-1}}$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{5}{1-\frac{1}{3}z^{-1}} - \frac{5}{1-\frac{3}{4}z^{-1}}}{\frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-3z^{-1}}} = \frac{5(1-\frac{3}{4}z^{-1}-1+\frac{1}{3}z^{-1})(1-\frac{1}{3}z^{-1})(1-3z^{-1})}{(1-3z^{-1}-1+\frac{1}{3}z^{-1})(1-\frac{1}{3}z^{-1})(1-\frac{3}{4}z^{-1})} = \frac{-\frac{25}{12}z^{-1}(1-3z^{-1})}{-\frac{8}{9}z^{-1}(1-\frac{3}{4}z^{-1})}$$

$$= \frac{\frac{25}{32}(1-3z^{-1})}{(1-\frac{3}{4}z^{-1})} \Rightarrow \text{poles: } \frac{3}{4}, \text{ zeros: } 3$$

ROC : $|z| > \frac{3}{4}$



(c)

if $|z| > |\lambda| \Rightarrow$ causal, $|z| < |\lambda| \Rightarrow$ anti-causal

Since $|z| > \frac{3}{4} \Rightarrow$ causal ※

ROC includes the unit circle $\Rightarrow$ stable ※

(d)

$$H(z) = \frac{25}{32}\cdot\frac{1-3z^{-1}}{1-\frac{3}{4}z^{-1}} = \frac{25}{32}\cdot\frac{(1-\frac{3}{4}z^{-1})\cdot 4 - 3}{1-\frac{3}{4}z^{-1}} = \frac{25}{32}\left(4 - \frac{3}{1-\frac{3}{4}z^{-1}}\right)$$

$$\Rightarrow h[n] = \frac{25}{32}\left(4\delta[n] - 3\cdot\left(\frac{3}{4}\right)^n u[n]\right) = \frac{25}{8}\delta[n] - \frac{75}{32}\cdot\left(\frac{3}{4}\right)^n u[n] \text{ ※}$$

(e)

$$H(z) = \frac{25}{32}\cdot\frac{1-3z^{-1}}{1-\frac{3}{4}z^{-1}} = \frac{25-75z^{-1}}{32-24z^{-1}} = \frac{Y(z)}{X(z)}$$

$$\Rightarrow Y(z)(32-24z^{-1}) = X(z)(25-75z^{-1})$$

$$\Rightarrow 32\,y[n] - 24\,y[n-1] = 25\,x[n] - 75\,x[n-1] \text{ ※}$$