ICE503 DSP-Homework#11

1. Figure 1 shows the impulse response for several different LTI systems. Determine the group delay associated with each systems.

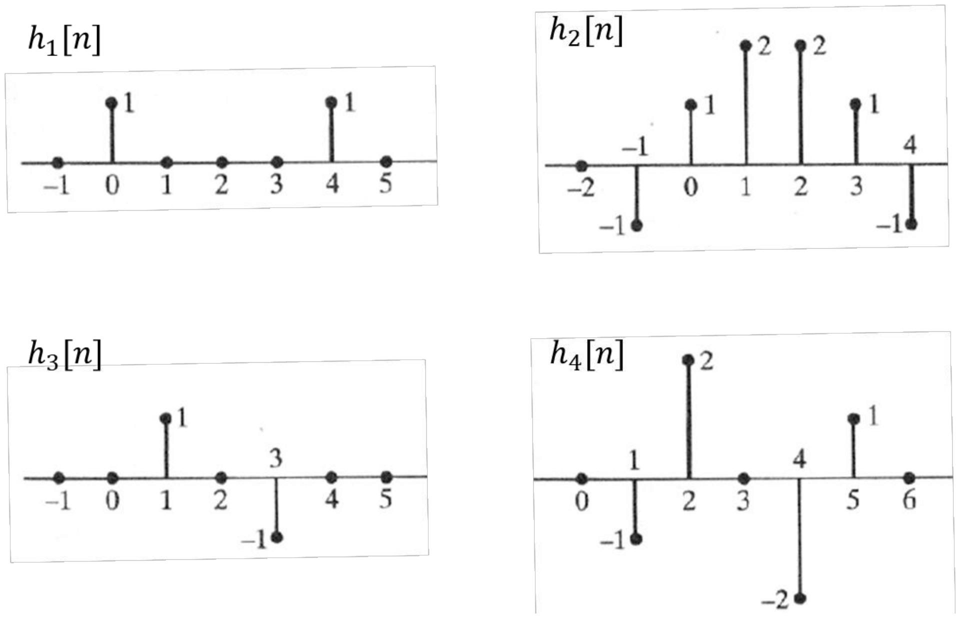

Figure 1: Impulse response for several different LTI systems

2. Figure 2 shows two different interconnections of three systems. The impulse responses $h_1[n]$, $h_2[n]$, and $h_3[n]$ are as shown in Figure 3. Determine whether system A and/or system B is a generalized linear-phase system.

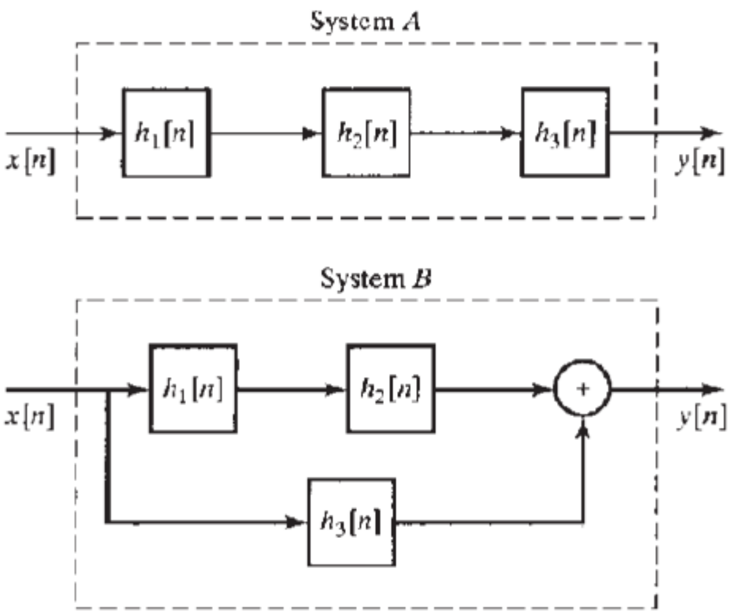

Figure 2: Two different interconnections of three systems

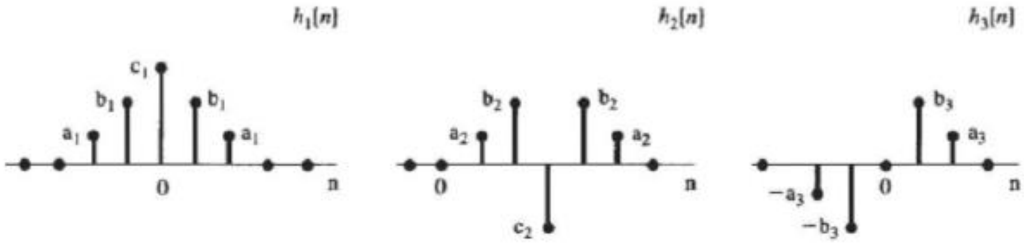

Figure 3 Impulse responses of the three systems

3. MATLAB simulation:

   Using iirnotch function to design a second order IIR notch filter with the notch located at $\omega c = 0.1\pi$ and with the 3 dB bandwidth of 0.001π and use fvtool function sketch the magnitude of the filter in dB and the group delay.

1.

(a) $h_1[n] = \delta[n] + \delta[n-4] \Rightarrow H_1(e^{j\omega}) = 1 + e^{-j\omega 4} = e^{-j\omega 2}(e^{j\omega 2} + e^{-j\omega 2}) = e^{-j\omega 2}(2\cos 2\omega)$

$\theta(\omega) = -2\omega$

$\tau_g(\omega) = -\frac{d\theta}{d\omega} = 2$

(b) $h_2[n] = -\delta[n+1] + \delta[n] + 2\delta[n-1] + 2\delta[n-2] + \delta[n-3] - \delta[n-4]$

$H_2(e^{j\omega}) = -e^{j\omega} + 1 + 2e^{-j\omega} + 2e^{-j\omega 2} + e^{-j\omega 3} - e^{-j\omega 4}$

$= e^{-j\omega 1.5}(-e^{j2.5} + e^{j\omega 1.5} + 2e^{j\omega 0.5} + 2e^{-j\omega 0.5} + e^{-j\omega 1.5} - e^{-j\omega 2.5})$

$= e^{-j\omega 1.5}(-2\cos 2.5\omega + 2\cos 1.5\omega + 4\cos 0.5\omega)$

$\theta(\omega) = -1.5\omega$

$\tau_g(\omega) = 1.5$ ※

(c) $h_3[n] = \delta[n-1] - \delta[n-3] \Rightarrow H_3(e^{j\omega}) = e^{-j\omega} - e^{-j\omega 3} = e^{-j\omega 2}(e^{j\omega} - e^{-j\omega}) = 2je^{-j\omega 2}\sin 2\omega$

$= e^{-j(2\omega - \frac{\pi}{2})} 2\sin 2\omega$

$\theta(\omega) = -(2\omega - \frac{\pi}{2})$

$\tau_g(\omega) = 2$ ※

(d) $h_4[n] = -\delta[n-1] + 2\delta[n-2] - 2\delta[n-4] + \delta[n-5]$

$H_4(e^{j\omega}) = -e^{-j\omega} + 2e^{-j\omega 2} - 2e^{-j\omega 4} + e^{-j\omega 5} = e^{-j\omega 3}(-e^{j\omega 2} + 2e^{j\omega} - 2e^{-j\omega} + e^{-j\omega 2})$

$= e^{-j\omega 3}(-2j\sin 2\omega + 4j\sin\omega)$

$= e^{-j(3\omega - \frac{\pi}{2})}(-2\sin 2\omega + 4\sin\omega)$

$\theta(\omega) = -(3\omega - \frac{\pi}{2})$

$\tau_g(\omega) = 3$ ※

2.

$$H_1(e^{j\omega}) = a_1 e^{j\omega 2} + b_1 e^{j\omega} + c_1 + b_1 e^{-j\omega} + a_1 e^{-j\omega 2} = c_1 + 2b_1 \cos\omega + 2a_1 \cos 2\omega$$

$$H_2(e^{j\omega}) = a_2 e^{-j\omega} + b_2 e^{-j\omega 2} - c_2 e^{-j\omega 3} + b_2 e^{-j\omega 4} + a_2 e^{-j\omega 5} = e^{-j\omega 3}\left(-c_2 + 2b_2 \cos\omega + 2a_2 \cos 2\omega\right)$$

$$H_3(e^{j\omega}) = -a_3 e^{j\omega 2} - b_3 e^{j\omega} + b_3 e^{j\omega} + a_3 e^{-j\omega 2} = e^{-j\frac{\pi}{2}}\left(2b_3 \sin\omega + 2a_3 \sin 2\omega\right)$$

System A:

$$H(e^{j\omega}) = H_1(e^{j\omega})\, H_2(e^{j\omega})\, H_3(e^{j\omega})$$

$$= \left(c_1 + 2b_1 \cos\omega + 2a_1 \cos 2\omega\right)\left[e^{-j\omega 3}\left(-c_2 + 2b_2 \cos\omega + 2a_2 \cos 2\omega\right)\right]\left[e^{-j\frac{\pi}{2}}\left(2b_3 \sin\omega + 2a_3 \sin 2\omega\right)\right]$$

$$= e^{-j\left(3\omega + \frac{\pi}{2}\right)} \cdots\cdots$$

generalized linear phase system #

System B:

$$H_B(e^{j\omega}) = H_1(e^{j\omega})\, H_2(e^{j\omega}) + H_3(e^{j\omega})$$

$$= e^{-j3\omega} \cdots\cdots + e^{-j\frac{\pi}{2}} \cdots$$

different terms

Not generalized linear phase system #