ICE503 DSP-Homework#8

1. The two 8-point sequence $x_1[n]$ and $x_2[n]$ shown in Figure 1. have DFTs $X_1[k]$ and $X_2[k]$, respectively.

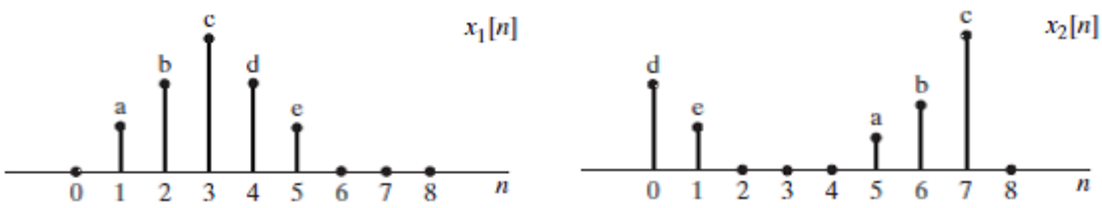


Figure 1. $x_1[n]$ and $x_2[n]$

(a) Determine the relationship between $X_1[k]$ and $X_2[k]$.

(b) Plot the sequence $x_3[n]$ whose DFT is $X_3[k] = W_8^{-3k} X_1[k]$.

2. The even part of a real sequence $x[n]$ is defined by

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

Suppose that x[n] is a real finite-length sequence defined such that $x[n] = 0$ for $n < 0$ and $n \geq N$. Let $X[k]$ denote the N-point DFT of $x[n]$.

(a) Is ~~Re{x[n]}~~ $Re\{X[k]\}$ the DFT of $x_e[n]$?

(b) What is the inverse DFT of ~~Re{x[n]}~~ $Re\{X[k]\}$ in terms of $x[n]$?

3. MATLAB simulation:

Let $x_1[n]$ and $x_2[n]$ be the two 4-points sequences given below

$$x_1[n] = [2,1,2,-3]$$

$$x_2[n] = [-3,2,1,-5]$$

(a) Compute the linear convolution to obtain $x_3[n] = x_1[n] \otimes x_2[n]$.

(b) Compute the DFT of $x_3[n]$ to obtain $X_3[k]$.

(c) Compute the DFT of $x_1[n]$ and $x_2[n]$ to obtain $X_1[k]$ and $X_2[k]$, then multiply them to obtain $X'[k] = X_1[k] \times X_2[k]$.

(d) Use stem function to plot the amplitude and phase of $X_3[k]$ and $X'[k]$. Are $X_3[k]$ and $X'[k]$ the same?

(e) Compute the 4-point circular convolution to obtain $x_4[n] = x_1[n]④x_2[n]$.

(f) Compute the DFT of $x_4[n]$ to obtain $X_4[k]$.

(g) Use stem function to plot the amplitude of $X_4[k]$. Are $X_4[k]$ and $X'[k]$ the same?

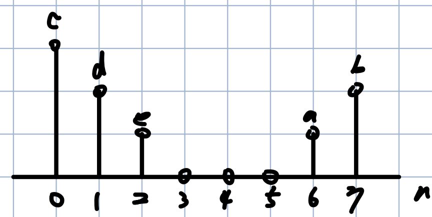
1.

(a)

$$x_2[n] = x_1[\langle n-4\rangle_8] \Rightarrow X_2[k] = W_8^{4k} X_1[k]$$

(b)

$$X_3[k] = W_8^{-3k} X_1[k] \Rightarrow x_3[n] = x_1[\langle n+3\rangle_8]$$



2.

(a) Length of $x[n]$ N from $0 \sim N-1$ $\Rightarrow$ Length of $x_e[n]$ is $2N-1$ from $-N+1 \sim N-1$

$$x_e[n] \begin{cases} \frac{x[n]+x[-n]}{2}, & -N+1 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

$$X_e[k] = \sum_{-N+1}^{N-1} \frac{x[n]+x[-n]}{2} W_{2N-1}^{kn} = \sum_{n=-N+1}^{0} \frac{x[-n]}{2} W_{2N-1}^{kn} + \sum_{n=0}^{N-1} \frac{x[n]}{2} W_{2N-1}^{kn}$$

$$(m=-n) = \sum_{m=-(-N+1)}^{0} \frac{x[m]}{2} W_{2N-1}^{-km} + \sum_{n=0}^{N-1} \frac{x[n]}{2} W_{2N-1}^{kn} = \sum_{n=0}^{N-1} \frac{x[n]}{2} W_{2N-1}^{-kn} + \sum_{n=0}^{N-1} \frac{x[n]}{2} W_{2N-1}^{kn}$$

$$= \sum_{n=0}^{N-1} x[n] \left(\frac{W_{2N-1}^{-kn} + W_{2N-1}^{kn}}{2} \right) = \sum_{n=0}^{N-1} x[n] \left(\frac{e^{j\frac{2\pi k}{2N-1}n} + e^{-j\frac{2\pi k}{2N-1}n}}{2} \right) = \sum_{n=0}^{N-1} x[n] \cos\left(\frac{2\pi k}{2N-1} n\right)$$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{N}n} \Rightarrow Re\{X[k]\} = \sum_{n=0}^{N-1} x[n] \cos\left(\frac{2\pi k}{N} n\right)$$ ... not the same

(b)

$$Re\{X[k]\} = \frac{X[k]+X^*[k]}{2} = \frac{1}{2}\sum_{n=0}^{N-1} x[n] W_N^{kn} + \frac{1}{2}\sum_{n=0}^{N-1} x[n] W_N^{-kn} = \frac{1}{2}\sum_{n=0}^{N-1} (x[n] + x[N-n]) W_N^{kn}$$

?

$$= \frac{1}{2} DFT\{ x[n] + x[N-n] \}$$

$$\frac{1}{N} \sum_{k=0}^{N-1} \left(\sum_{\ell=0}^{N-1} x[\ell] \cos\left(\frac{2\pi k}{N} n\right) \right) W_N^{-nk} = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{\ell=0}^{N-1} x[\ell] \cdot \frac{W_N^{nk} + W_N^{-nk}}{2} W_N^{-nk}$$

$$= \frac{1}{N} \sum_{\ell=0}^{N-1} x[\ell] \sum_{k=0}^{N-1} \frac{1}{2} + \frac{W_N^{-2nk}}{2} = x[n] \cdot \frac{1}{N} \cdot \frac{N-1}{2} \cdot \sum_{k=0}^{N-1} \frac{W_N^{-2nk}}{2} = \frac{N-1}{2N} x[n]$$