ICE503 DSP-Homework#2

1. For each of the following systems, determine whether the system is (1) linear, (2) time invariant, and (3) causal.

   (a) $y[n] = a^2 x[n] - b$, a and b are non-zero constant

   (b) $y[n] = x[an - b]$, a and b are non-zero positive constant

   (c) $y[n] = \frac{1}{M}(\mathrm{x[n]} + \sum_{k=1}^{(M-1)/2} x[n-k] + x[n+k])$

   (d) $y[n] = \log_2(|x[n]|)$

2. The system T in Figure 1 is known to be time-invariant. When the inputs to the system are $x_1[n]$, $x_2[n]$, and $x_3[n]$, the responses of the system are $y_1[n]$, $y_2[n]$, and $y_3[n]$ as shown. Determine whether the system T is linear or nonlinear.

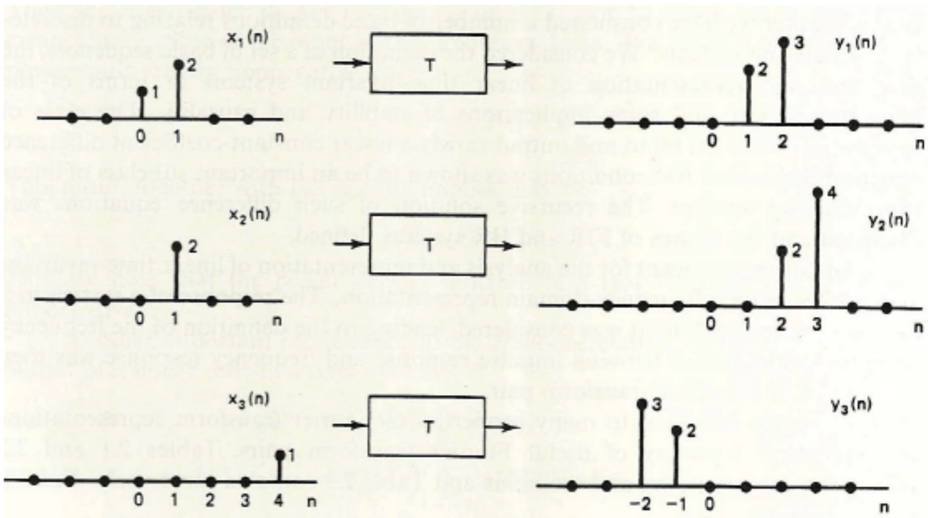

Figure 1: The time-invariant system T

3. MATLAB simulation:

   The input signal is

$$x[n] = \delta[n] + 3\delta[n-1] + 7\delta[n-2] + 2\delta[n-3] + 4\delta[n-4]$$

and the output signal of a 5-point moving average is

$$y[n] = \frac{1}{5}\sum_{k=0}^{4} x[n-k]$$

(a) Use stem function to plot $x[n]$.

(b) Use for loop to calculate $y[n]$.

(c) Use convolution function to calculate $y[n]$.

(The result of $y[n]$ in (b) and (c) should be the same.)

(d) Use stem function to plot y[n].

1. (a) $y[n] = a^2 x[n] - b$

① Let $x[n] = \alpha x_1[n] + \beta x_2[n] \Rightarrow y[n] = a^2(\alpha x_1[n] + \beta x_2[n]) - b = \alpha a^2 x_1[n] + \beta a^2 x_2[n] - b$

$$\neq \alpha(a^2 x_1[n] - b) + \beta(a^2 x_2[n] - b) = \alpha y_1[n] + \beta y_2[n]$$

... not linear ※

② $x_1[n] = x[n-n_0] \Rightarrow y_1[n] = a^2 x_1[n] - b = a^2 x[n-n_0] - b = y[n-n_0]$ ... time invariant ※

③ $y[\overset{\downarrow}{n}] = a^2 x[\overset{\downarrow}{n}] - b$, the output depends on the current input $\Rightarrow$ causal ※

(b) $y[n] = x[an-b]$

① Let $x'[an-b] = \alpha x_1[an-b] + \beta x_2[an-b] \Rightarrow y'[n] = x'[an-b] = \alpha x_1[an-b] + \beta x_2[an-b] = \alpha y_1[n] + \beta y_2[n]$

... linear ※

② Let $x'[an-b] = x[an-b-n_0] \Rightarrow y'[n] = x'[an-b] = x[an-b-n_0] \neq x[a(n-n_0)-b] = y[n-n_0]$ ... not time invariant ※

③ Let $a=5$ and $b=1$, say that $n=2 \Rightarrow y[2] = x[5\cdot 2 - 1] = x[9]$ ... not causal ※

(c) $y[n] = \frac{1}{M}\left(x[n] + \sum_{k=1}^{\frac{M-1}{2}} x[n-k] + x[n+k]\right)$

① Let $x'[n] = \alpha x_1[n] + \beta x_2[n] \Rightarrow y'[n] = \frac{1}{M}\left(x'[n] + \sum_{k=1}^{\frac{M-1}{2}} x'[n-k] + x'[n+k]\right)$

$$= \frac{1}{M}\left(\alpha x_1[n] + \beta x_2[n] + \sum_{k=1}^{\frac{M-1}{2}} \alpha x_1[n-k] + \beta x_2[n-k] + \alpha x_1[n+k] + \beta x_2[n+k]\right)$$

$$= \frac{1}{M}\left(\alpha x_1[n] + \beta x_2[n] + \sum_{k=1}^{\frac{M-1}{2}} \alpha(x_1[n-k] + x_1[n+k]) + \sum_{k=1}^{\frac{M-1}{2}} \beta(x_2[n-k] + x_2[n+k])\right)$$

$$= \frac{1}{M}\cdot\alpha\left(x_1[n] + \sum_{k=1}^{\frac{M-1}{2}} x_1[n-k] + x_1[n+k]\right) + \frac{1}{M}\cdot\beta\left(x_2[n] + \sum_{k=1}^{\frac{M-1}{2}} x_2[n-k] + x_2[n+k]\right)$$

$= \alpha y_1[n] + \beta y_2[n]$ ... linear ※

② Let $x'[n] = x[n-n_0] \Rightarrow y'[n] = \frac{1}{M}\left(x'[n] + \sum_{k=1}^{\frac{M-1}{2}} x'[n-k] + x'[n+k]\right) = x[n-n_0] + \sum_{k=1}^{\frac{M-1}{2}} x[n-k-n_0] + x[n+k-n_0] = y[n-n_0]$

...time invariant

③ From term $x[n+k]$, say that $n=0 \Rightarrow y[0] = \frac{1}{M} x[0] + \sum_{k=1}^{\frac{M-1}{2}} x[0-k] + \underset{\substack{\uparrow \\ \text{future signal}}}{x[0+k]} \Rightarrow$ not causal ※

(d) $y[n] = \log_2 |x[n]|$

... not linear

① Let $x[n] = \alpha x_1[n] + \beta x_2[n] \Rightarrow y[n] = \log_2 |x[n]| = \log_2 |\alpha x_1[n] + \beta x_2[n]| \neq \alpha \log_2 |x_1[n]| + \beta \log_2 |x_2[n]|$

② Let $x'[n] = x[n-n_0] \Rightarrow y'[n] = \log_2 |x'[n]| = \log_2 |x[n-n_0]| = y[n-n_0]$ ... time invariant ※

③ Only depends on current signal ⇒ causal ※

2.

$x_3[n] = x_1[n-4] - x_2[n-4]$, if T is linear, then $y_3[n] = y_1[n] - y_2[n]$

$y_3[n] = 2\delta[n+1] + 3\delta[n+2]$, $y_1[n] - y_2[n] = 2\delta[n-1] + \delta[n-2] - 4\delta[n-3]$,

$y_3[n] \neq y_1[n] - y_2[n] \Rightarrow$ not linear ※