ICE503 DSP-Homework#10

1. A causal linear time-invariant system has the system function

$$H(z) = \frac{(1 - 1.5z^{-1} - z^{-2})(1 + 0.9z^{-1})}{(1 - z^{-1})(1 + 0.7jz^{-1})(1 - 0.7jz^{-1})}$$

   (a) Write the difference equation that characterizes the system with $x[n]$ and $y[n]$.

   (b) Plot the pole-zero diagram and indicate the region of convergence for the system function.

   (c) Sketch $|H(e^{j\omega})|$.

2. Consider a causal linear time-invariant system function $H(z)$ and real impulse response. $H(z)$ evaluated for $z = e^{j\omega}$ is shown in Figure 1.

   (a) Carefully sketch a pole-zero plot for $H(z)$ showing all information about the pole and zero locations that can inferred from the figure.

   (b) Specify whether the system is stable.

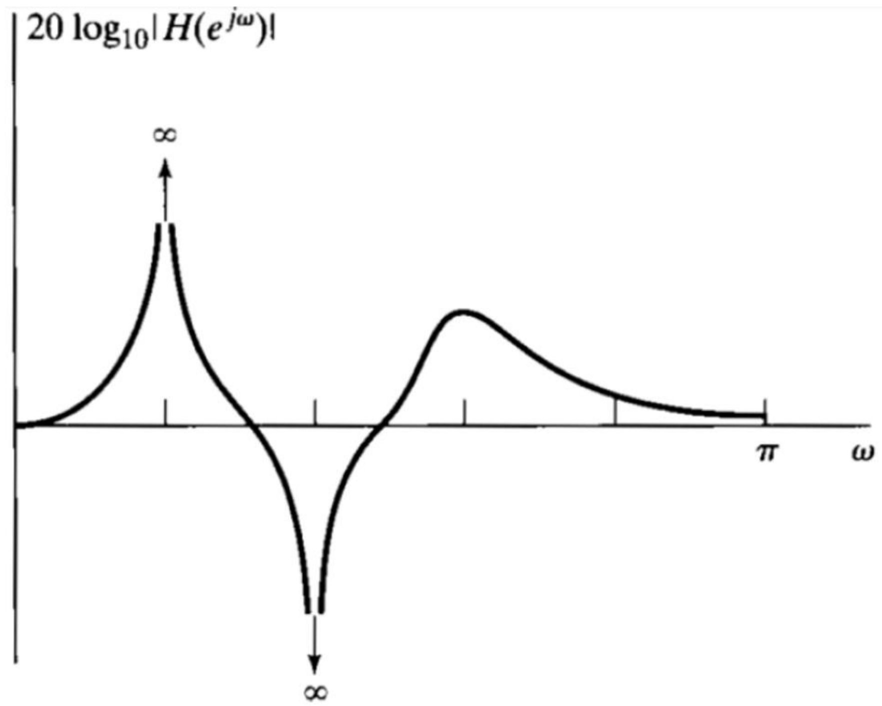


Figure 1. A causal linear time-invariant system function $H(z)$

3. Matlab Simulation

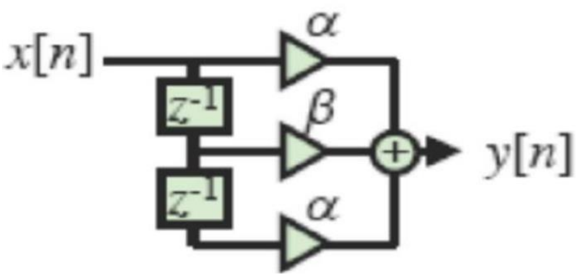
Separate the following information in frequency.

$$x[n] = A\cos(\omega 1 n) + B\cos(\omega 2 n)$$

with construct $H(e^{j\omega})$

$$H(e^{j\omega}) = \begin{cases} |H(e^{j\omega_1})| & \sim \quad 1, \\ |H(e^{j\omega_2})| & \sim \quad 0, \end{cases}$$

Where $\omega_1 = 0.1$ and $\omega_2 = 0.4$. Consideer a 3 pt FIR filters with $h[n] = \{\alpha\ \beta\ \alpha\}$. Sketch the frequency response and compare the output signal with input signals.



1.

(a)

$$H(z) = \frac{(1-1.5z^{-1}-z^{-2})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7jz^{-1})(1-0.7jz^{-1})} = \frac{1-0.6z^{-1}-2.35z^{-2}-0.9z^{-3}}{1-z^{-1}+0.49z^{-2}-0.49z^{-3}} = \frac{Y(z)}{X(z)}$$

$$\Rightarrow (1-z^{-1}+0.49z^{-2}-0.49z^{-3})\,Y(z) = (1-0.6z^{-1}-2.35z^{-2}-0.9z^{-3})\,X(z)$$

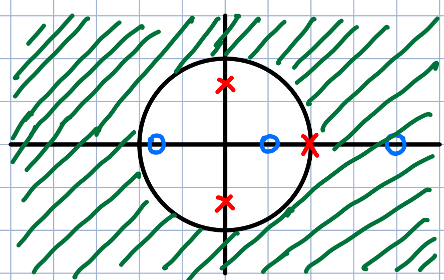
$$\Rightarrow y[n]-y[n-1]+0.49\,y[n-2]-0.49\,y[n-3] = x[n]-0.6\,x[n-1]-2.35\,x[n-2]-0.9\,x[n-3]$$

(b)

$$H(z) = \frac{(1-1.5z^{-1}-z^{-2})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7jz^{-1})(1-0.7jz^{-1})} = \frac{(1-2z^{-1})(1-0.5z^{-1})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7jz^{-1})(1-0.7jz^{-1})}$$, causal $\Rightarrow$ ROC : $|z|>1$

poles : 1 , $\pm 0.7j$

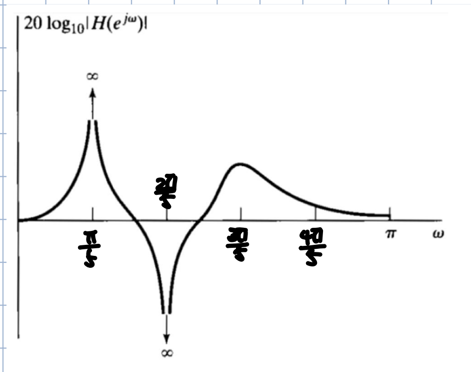
zeros : 2 , 0.5, -0.9

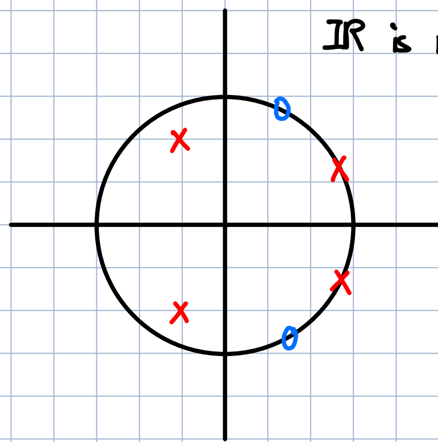


(c)

2.

(a)





IR is real ⇒ complex conjugate

$20 \log |H(e^{j\frac{\pi}{5}})| = \infty$ ⇒ pole

$20 \log |H(e^{j\frac{2\pi}{5}})| = -\infty$ ⇒ zero

$20 \log |H(e^{j\frac{3\pi}{5}})|$ has a peak ⇒ pole inside unit circle