ICE503 DSP-Homework#9

1. Suppose we have two four-point sequences $x[n]$ and $h[n]$ as follow:

$$x[n] = \sin\left(\frac{\pi n}{2}\right), n = 0,1,2,3$$

$$h[n] = 2^n, \qquad n = 0,1,2,3$$

   (a) Calculate the four-point DFT $X[k]$.

   (b) Calculate the four-point DFT $H[k]$.

   (c) Calculate $y[n] = x[n] \; ④ \; h[n]$ by doing the circular convolution directly.

   (d) Calculate $y[n]$ of Part (c) by multiplying the DFTs of $x[n]$ and $h[n]$ and performing an inverse DFT.

2. MATLAB simulation:

   The idea of a spectrogram is plotting a sequence of short DFTs of the input signal using overlapping windows. If the signal is real, then one typically plots only the positive frequencies $k = 0,1,\ldots,\frac{N}{2} - 1$.

   (a) Download guitar4.wav from cyber university（網路大學）and use audioread function to obtain the sampled data $x[n]$ and the sample rate F_s.

   (b) Create a Hann window as the overlapping window

$$w[n] = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{N}\right)\right), n = 0,1,\ldots,N-1$$

   where N is the DFT length. Here you need to choose a suitable $N = 2^m$ so that the bandwidth of the DFT frequency bins is around 20Hz. Plot $w[n]$.

   (c) Let $M = \frac{N}{4}$ be the number of samples to shift after each DFT. The energy in the k-th frequency bin of the i-th window is given by

$$X_i[k] = \left| \sum_{n=0}^{N-1} x[iM+n]w[n]e^{-\frac{j2\pi k}{N}} \right|^2, k = 0,1,\dots,N-1$$

Write a MATLAB function "myspectrogram.m" that computes the short DFTs of the input signal for each window.

```
X = myspectrogram(x,N,w,M)
% x is the sampled data x[n]
% N is the DFT point
% w is the overlapping window
% M the number of samples to shift after each DFT
```

(d) Use the following code to plot the spectrogram

```
image(t,f,X(1:floor(N/2),:));  % t is time and f is frequency
colormap(hot(256));
colorbar;
```

1.

(a)

$$W_4^k - W_4^{3k}$$

$$X[k] = \sum_{n=0}^{3} \sin\left(\frac{\pi n}{2}\right) e^{-j\frac{2\pi k}{4}n} = e^{-j\frac{2\pi k}{4}\cdot 1} - e^{-j\frac{2\pi k}{4}\cdot 3} = e^{-j\frac{\pi}{2}k} - e^{-j\frac{3\pi}{2}k} \quad k=0,1,2,3$$

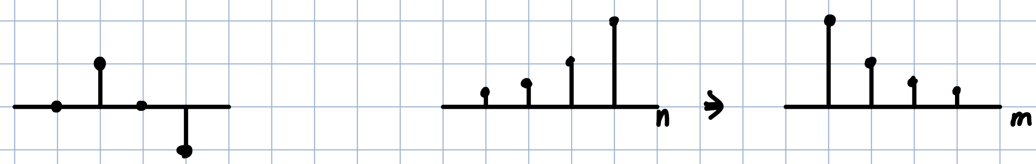
(b)

$$H[k] = \sum_{n=0}^{3} 2^n e^{-j\frac{2\pi k}{4}n} = \frac{1-(2e^{-j\frac{\pi}{2}k})^4}{1-2e^{-j\frac{\pi}{2}k}} = \frac{1-16e^{-j2\pi k}}{1-2e^{-j\frac{\pi}{2}k}} = \frac{-15}{1-2e^{-j\frac{\pi}{2}k}} = \begin{cases} 15 \\ \frac{-15}{1+2j} \\ -5 \\ \frac{-15}{1-2j} \end{cases}$$

$$\hookrightarrow 1 + 2W_4^k + 4W_4^{2k} + 8W_4^{3k}$$

(c)

$x[n]$: 0 1 0 -1 , $h[n]$ = 1 2 4 8



$$\begin{bmatrix} 1 & 8 & 4 & 2 \\ 2 & 1 & 8 & 4 \\ 4 & 2 & 1 & 8 \\ 8 & 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ -6 \\ 3 \end{bmatrix} \Rightarrow y[n] = 6\delta[n] - 3\delta[n-1] - 6\delta[n-2] + 3\delta[n-3]$$

(d)

$$Y[k] = X[k]H[k] = (W_4^k - W_4^{3k})(1 + 2W_4^k + 4W_4^{2k} + 8W_4^{3k})$$

$$= W_4^k + 2W_4^{2k} + 4W_4^{3k} + 8W_4^{4k} - W_4^{3k} - 2W_4^{4k} - 4W_4^{5k} - 8W_4^{6k}$$

$$= W_4^k + 2W_4^{2k} + 3W_4^{3k} + 6W_4^{4k} - 4W_4^{5k} - 8W_4^{6k}$$

$$= W_4^k + 2W_4^{2k} + 3W_4^{3k} + 6 - 4W_4^k - 8W_4^{2k}$$

$$= 6 - 3W_4^k - 6W_4^{2k} + 3W_4^{3k}$$

$\Rightarrow$ IDFT : $6\delta[n] - 3\delta[n-1] - 6\delta[n-2] + 3\delta[n-3]$