ICE503 DSP-Homework#4

1. Let $X(e^{j\omega})$ denote the discrete-time Fourier transform (DTFT) of $x[n]$.

$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Proof that

(a) the DTFT of $x[-n]$ is $X(e^{-j\omega})$.

(b) the DTFT of $x^*[n]$ is $X^*(e^{-j\omega})$.

(c) the DTFT of $x^*[-n]$ is $X^*(e^{j\omega})$.

(d) the DTFT of $\text{Re}\{x[n]\}$ is $X_{cs}(e^{j\omega})$.

(e) the DTFT of $x_{cs}[n]$ is $\text{Re}\left\{X\left(e^{j\omega}\right)\right\}$.

2. An LTI system is described as

$$h[n] = \left(\frac{1}{2}\right)^n \mu[n]$$

(a) Determine the DTFT of the system $H\left(e^{j\omega}\right)$.

(b) If the input sequence is $x[n] = \delta[n] - 4\delta[n-2]$, determine the DTFT of the input sequence $X\left(e^{j\omega}\right)$. The output sequence is $y[n] = x[n] * h[n]$, determine the DTFT of the output sequence $Y\left(e^{j\omega}\right)$.

(c) If the input sequence is $x[n] = \cos\left(\frac{\pi n}{5}\right)$, determine the DTFT of the input sequence $X\left(e^{j\omega}\right)$. The output sequence is $y[n] = x[n] * h[n]$, determine the DTFT of the output sequence $Y\left(e^{j\omega}\right)$.

(There is MATLAB simulation in page 2.)

3. MATLAB simulation:

An LTI system is described as

$$h[n] = \left(\frac{1}{3}\right)^n \mu[n]$$

and the input sequence is described as

$$x[n] = 2\delta[n] - \frac{2}{3}\delta[n-1]$$

(a) Use stem function to plot $x[n]$ and $h[n]$ for $0 \leq \mathrm{n} \leq 99$. Plot $x[n]$ in subplot(2,1,1), plot $h[n]$ in subplot(2,1,2), and label each x-axis and y-axis clearly.

(b) Use the definition to calculate the DTFT of $x[n]$ and $h[n]$.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

where $\omega = 0{:}\frac{\pi}{100}{:}6\pi$.

(c) If the output sequence in time domain is $y[n] = x[n] * h[n]$, calculate $Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$ in frequency domain. Use plot function to plot the magnitude and angle part of $X(e^{j\omega})$, $H(e^{j\omega})$ and $Y(e^{j\omega})$ for $\omega = 0{:}\frac{\pi}{100}{:}6\pi$. Plot $|X(e^{j\omega})|$ in subplot(3,2,1), plot $\arg\{X(e^{j\omega})\}$ in subplot(3,2,2), plot $|H(e^{j\omega})|$ in subplot(3,2,3), plot $\arg\{H(e^{j\omega})\}$ in subplot(3,2,4), plot $|Y(e^{j\omega})|$ in subplot(3,2,5), plot $\arg\{Y(e^{j\omega})\}$ in subplot(3,2,6), and label each x-axis and y-axis clearly.

(d) Use the convolution to calculate the output sequence in time domain

$$y[n] = x[n] * h[n],$$

then use the definition to calculate the IDTFT of $Y(e^{j\omega})$.

$$\hat{y}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y\left(e^{j\omega}\right) e^{j\omega n} d\omega$$

Use stem function to plot $y[n]$ and $\hat{y}[n]$ for $0 \leq \mathrm{n} \leq 99$. Plot $y[n]$ in subplot(2,1,1), plot $\hat{y}[n]$ in subplot(2,1,2), and label each x-axis and y-axis clearly.

1.

(a) $\sum_{n=-\infty}^{\infty} x[-n] e^{-j\omega n} = \sum_{n'=\infty}^{-\infty} x[n'] e^{j\omega n'} = \sum_{n'=-\infty}^{\infty} x[n'] e^{-(-j\omega n')} = X(e^{-j\omega})$

$(n' = -n)$

(b) $\sum_{n=-\infty}^{\infty} x^*[n] e^{-j\omega n} = \sum_{n=-\infty}^{\infty} \left(x[n] e^{j\omega n}\right)^* = \sum_{n=-\infty}^{\infty} \left(x[n] e^{-(-j\omega n)}\right)^* = X^*(e^{-j\omega})$

(c) $\sum_{n=-\infty}^{\infty} x^*[-n] e^{-j\omega n} = \sum_{n'=\infty}^{-\infty} x^*[n'] e^{j\omega n'} = \sum_{n'=-\infty}^{\infty} \left(x[n'] e^{-j\omega n'}\right)^* = X^*(e^{j\omega})$

(d) $\sum_{n=-\infty}^{\infty} Re\{x[n]\} e^{-j\omega n} = \sum_{n=-\infty}^{\infty} \frac{1}{2}\{x[n] + x^*[-n]\} e^{-j\omega n} = \frac{1}{2}\left(\sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} + \sum_{n=-\infty}^{\infty} x^*[-n] e^{-j\omega n}\right) = \underbrace{\frac{1}{2}\left[X(e^{j\omega}) + X^*(e^{j\omega})\right]}_{= X_{cs}(e^{j\omega})}$

(e) $\sum_{n=-\infty}^{\infty} X_{cs}[n] e^{-j\omega n} = \sum_{n=-\infty}^{\infty} \frac{1}{2}\{x[n] + x^*[-n]\} e^{-j\omega n} = \frac{1}{2}\{X(e^{j\omega}) + X^*(e^{j\omega})\} = \frac{1}{2} \cdot 2 Re\{X(e^{j\omega})\} = Re\{X(e^{j\omega})\}$

2.

(a)

$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n] e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{2} e^{-j\omega}\right)^n = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$ #

(b)

$x[n] = \delta[n] - 4\delta[n-2]$

$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\delta[n] - 4\delta[n-2]\right) e^{-j\omega n} = e^{-j\omega \cdot 0} - 4e^{-j\omega 2} = 1 - 4e^{-j\omega 2}$ #

$Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega}) = \frac{1-4e^{-j\omega 2}}{1-\frac{1}{2}e^{-j\omega}}$ #

(c) $x[n] = \cos\left(\frac{\pi n}{5}\right)$

$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \cos\left(\frac{\pi n}{5}\right) e^{-j\omega n} = \sum_{n=-\infty}^{\infty} \frac{1}{2}\left(e^{j\frac{\pi}{5}n} + e^{-j\frac{\pi}{5}n}\right) e^{-j\omega n}$

$= \frac{1}{2} \cdot \left\{2\pi\delta\left(n - \frac{\pi}{5}\right) + 2\pi\delta\left(n + \frac{\pi}{5}\right)\right\} = \pi\,\delta\left(n-\frac{\pi}{5}\right) + \delta\left(n+\frac{\pi}{5}\right)$

$Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega}) = \pi \cdot \frac{\delta\left(n-\frac{\pi}{5}\right) + \delta\left(n+\frac{\pi}{5}\right)}{1-\frac{1}{2}e^{-j\omega}}$ #