# ICE503 DSP-Homework#3

1. For each of the following systems, determine whether the system is stable.

   (a) $y[n] = \cos(\ x[n]\ )$

   (b) $y[n] = r^n x[n],\ r > 1$
   (c) $y[n] = nx[n]$

   (d) $y[n] = \frac{1}{n} x[n]$

2. An LTI system is described as

$$y[n] + 2y[n-1] - 8y[n-2] = 80x[n]$$

   and the initial conditions are

$$y[-1] = -8, \qquad y[-2] = -9$$

   If the input sequence is $x[n] = u[n]$, determine the LCCDE of y[n] for n ≥ 0.

3. MATLAB simulation:

   (a) Implement the LCCDE for question 2, and determine the output y[n] for $0 \leq n \leq 4$.

   (b) Use for loop to implement the system

$$y[n] + 2y[n-1] - 8y[n-2] = 80x[n]$$

   and determine the output y[n] for $0 \leq n \leq 4$.

   (c) Use filtic and filter function to determine the output y[n] for $0 \leq n \leq 4$.

   (The result of (a), (b) and (c) should be the same.)

4. MATLAB simulation:

   (a) Generate a sinusoidal signal.

$$x[n] = \begin{cases} \cos(\frac{1}{5}\pi n), & \text{for } -5 \leq n \leq 5 \\ 0, & \text{for } -10 \leq n < -5 \text{ and } 5 < n \leq 10 \end{cases}$$

   (b) Use stem function to plot the autocorrelation of $x[n]$.

1.

(a) $y[n] = \cos(x[n]) \Rightarrow -1 \le \cos(x[n]) \le 1 \quad \forall x[n] \Rightarrow$ stable ※

(b) $y[n] = r^n x[n],\ r > 1 \Rightarrow \lim_{n\to\infty} r^n = \infty \Rightarrow$ not stable ※

(c) $y[n] = n\,x[n] \Rightarrow \lim_{n\to\infty} n \cdot x[n] = \infty \Rightarrow$ not stable ※

(d) $y[n] = \frac{1}{n} x[n] \Rightarrow \lim_{n\to 0} \frac{1}{n} x[n] = \infty \Rightarrow$ not stable ※

2.

$y[n] + 2y[n-1] - 8y[n-2] = 80\,x[n], \quad y[-1] = -8,\ y[-2] = -9$

① complementary solution : $(y_c[n] = \lambda^n,\ y_p[n] = 0)$

$\lambda^n + 2\lambda^{n-1} - 8\lambda^{n-2} = 0 \Rightarrow \lambda^{n-2}(\lambda^2 + 2\lambda - 8) = 0 \Rightarrow \lambda = 2, -4$

$y_c[n] = \alpha_1 2^n + \alpha_2 (-4)^n$

② particular solution: $(y_p[n] = \beta)$

For large "n", $\beta + 2\beta - 8\beta = 80 \Rightarrow \beta = -16$

③ $y[n] = y_c[n] + y_p[n] = \alpha_1 2^n + \alpha_2 (-4)^n - 16$

$n = 0 \Rightarrow y[0] + 2y[-1] - 8y[-2] = 80\,x[0]$

$(\alpha_1 + \alpha_2 - 16) + 2\cdot(-8) - 8\cdot(-9) = 80$

$\alpha_1 + \alpha_2 = 40 \ \dots$ ①

$n = 1 \Rightarrow y[1] + 2y[0] - 8y[-1] = 80\,x[1]$

$(2\alpha_1 - 4\alpha_2 - 16) + 2(\alpha_1 + \alpha_2 - 16) - 8\cdot(-8) = 80$

$4\alpha_1 - 2\alpha_2 = 64 \Rightarrow 2\alpha_1 - \alpha_2 = 32 \ \dots$ ② $\qquad$ ① & ② $\Rightarrow \alpha_1 = 24,\ \alpha_2 = 16$

$y[n] = 24 \cdot 2^n + 16(-4)^n - 16$ ※